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Positive Mathematical Programming with Generalized Risk

by

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Abstract
Price risk in a mathematical programming framework has been confined for a long time to a constant risk aversion specification originally introduced by Freund in 1956. This paper extends the treatment of risk in a mathematical programming framework along the lines suggested by Meyer (1987) who demonstrated the equivalence of expected utility and a wide class of probability distributions that differ only by location and scale. This paper shows how to formulate a PMP specification that allows the estimation of the preference parameters and calibrates the model to the base data within an admissible small deviation. The PMP approach under generalized risk allows also the estimation of output supply elasticities. The approach is applied to a sample of large farms.

Keywords: positive mathematical programming, generalized risk, output supply elasticities

JEL: C6

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Introduction

The treatment of risk in a mathematical programming framework has been confined to an exponential utility function with a constant absolute risk aversion coefficient. This is the strategy originally proposed by Freund (1956) who appealed to the expected utility (EU) approach and assumed that random prices were normally distributed. These assumptions lead to a linear mean-variance specification of expected net revenue defined as total expected revenue minus a risk premium that corresponds to half the variance of revenue multiplied by the constant absolute risk aversion coefficient. This mathematical programming approach has serious limitations as only a rare entrepreneur may possess risk preferences that exhibit constant absolute risk aversion regardless of the firm size and the market environment.

Many other approaches have been proposed in the literature to deal with risk and uncertainty. Among them, the mean-standard deviation (MS) approach has had a long history [Fisher (1906), Hicks (1933), Tintner (1941), Markowitz (1952), Tobin (1958)] but it has not been applied in a mathematical programming context. Meyer (1987) presented a remarkable reconciliation between the EU and the MS approaches that may be fruitful in a PMP analysis of economic behavior under risk. The major objective of Meyer is to find consistency conditions between the EU and the MS approaches in such a way that an agent who ranks the alternatives according to the value of some function defined over the first two moments of the random payoff would rank in the same way those alternatives by means of the expected value of some utility function defined over the same payoffs. It turns out that the location and scale (LS) condition is the crucial link to establish the consistency between the EU and the MS approaches. We reproduce here Meyer's argument (1987, p. 423):

"Assume a choice set in which all random variables Y_i (with finite means and variances) differ from one another only by location and scale parameters. Let X be the random variable obtained from one of the Y_i using the normalizing transformation $X = (Y_i - \mu_i)/\sigma_i$ where μ_i and σ_i are the mean and standard deviation of Y_i. All Y_i, no matter which was selected to define X, are equal *in distribution* to $\mu_i + \sigma_i X$. Hence, the expected utility from Y_i for any agent with utility function $u(\ )$ can be written as

$$
\begin{aligned}
EU(Y_i) &= \int_a^b u(\mu_i + \sigma_i x)\,dF(x) \\
&\equiv V(\mu_i, \sigma_i),
\end{aligned}
\tag{1}
$$

where a and b define the interval containing the support of the normalized random variable X."

"… under the LS condition, various popular and interesting hypotheses concerning absolute and relative risk-aversion measures in the EU setting can be translated into equivalent properties concerning $V(\mu_i, \sigma_i)$."

The structure of absolute risk is measured by the slope of the indifference curves in the (μ, σ) space that is represented as

$$
AR(\mu, \sigma) = \frac{-V_\sigma(\mu, \sigma)}{V_\mu(\mu, \sigma)}
\tag{2}
$$

where $V_\mu(\mu, \sigma)$ and $V_\sigma(\mu, \sigma)$ are first partial derivatives of the $V(\mu, \sigma)$ function. Some properties of this risk measure are as follows:

1. Risk aversion is associated with $AR(\mu,\sigma) > 0$, risk neutrality with $AR(\mu,\sigma) = 0$ and risk propensity with $AR(\mu,\sigma) < 0$.

2. If $u(\mu + \sigma x)$ displays decreasing (constant, increasing) absolute risk aversion for all $\mu + \sigma x$, then $\frac{\partial AR(\mu,\sigma)}{\partial \mu} < (=,>)\ 0$ for all μ and $\sigma > 0$.

3. If $u(\mu + \sigma x)$ displays increasing (constant, decreasing) relative risk aversion for all $\mu + \sigma x$, then $\frac{\partial AR(t\mu,t\sigma)}{\partial t} > (=,<)\ 0$ for $t > 0$.

Saha (1997) formulated an MS utility function that conforms to Meyer's specification

(3) $$V(\mu,\sigma) = \mu^{\theta} - \sigma^{\gamma}$$

and assumed that $\theta > 0$. According to this MS utility function, the absolute risk measure (*AR*) is specified as

(4) $$AR(\mu,\sigma) = \frac{-V_{\sigma}(\mu,\sigma)}{V_{\mu}(\mu,\sigma)} = \frac{\gamma}{\theta}\mu^{(1-\theta)}\sigma^{(\gamma-1)}.$$

Hence, risk aversion, risk neutrality and risk propensity are associated with $\gamma > (=,<)\ 0$, respectively.

Decreasing, constant and increasing absolute risk aversion (with $\gamma > 0$) is defined by

(5) $$\frac{\partial AR(\mu,\sigma)}{\partial \mu} = \frac{(1-\theta)\gamma}{\theta}\mu^{-\theta}\sigma^{(1-\gamma)} < (=,>)\ 0$$

and, therefore, by $\theta > 1, \theta = 1, \theta < 1$, respectively. Decreasing, constant and increasing absolute risk (with $\gamma < 0$) is defined by $\theta < 1, \theta = 1, \theta > 1$.

Decreasing, constant and increasing relative risk aversion is defined (with $\gamma > 0$) by

(6) $$\frac{\partial AR(t\mu,t\sigma)}{\partial t}|_{t=1} = (\gamma-\theta)AR < (=,>)\ 0$$

and, therefore, by $\theta>\gamma, \theta=\gamma, \theta<\gamma$, respectively.

The risk analysis of Meyer (1987) describes and admits all possible combinations of risk behavior. Saha's (1997) implementation of it, for example, admits absolute risk aversion behavior that may be decreasing, when $\theta>1$ and $\gamma>0$, in association with either increasing relative risk aversion when $\gamma>\theta>0$ or decreasing relative risk aversion when $0<\gamma<\theta$. In any given sample of economic agents' performance, therefore, the prevailing combination of risk behavior is an empirical question.

Application to PMP

Suppose N farmers produce J crops using I limiting inputs and a linear technology. Let us assume that the $(J\times 1)$ vector of crops' market prices is a random variable $\tilde{\mathbf{p}}$ with mean $E(\tilde{\mathbf{p}})$ and variance-covariance matrix Σ_p. A $(J\times 1)$ vector $\mathbf{c}$ of accounting unit variable costs is also known. Farmer's availability of limiting resources is given by the $(I\times 1)$ vector $\mathbf{b}$. The linear technology is specified by the $(I\times J)$ matrix A. The unknown output levels are given by the $(J\times 1)$ vector $\mathbf{x}$. Furthermore, farmer has knowledge of previously realized levels of outputs that are listed as $\mathbf{x}_{obs}$. Random wealth is defined by previously accumulated wealth, $\bar{w}$, augmented by the current net revenue. Assuming a MS utility function under this scenario, mean wealth is defined as $\mu=\bar{w}+(E(\tilde{\mathbf{p}})-\mathbf{c})'\mathbf{x}$ with standard deviation equal to $\sigma=(\mathbf{x}'\Sigma_p\mathbf{x})^{1/2}$.

Then, a primal PMP-MS model is specified as follows:

$$\text{(7)} \qquad \max_{\mathbf{x},\theta,\gamma,h} V(\mu,\sigma) = \mu^{\theta} - \sigma^{\gamma} = [\bar{w} + (E(\tilde{\mathbf{p}}) - \mathbf{c})'\mathbf{x})]^{\theta} - (\mathbf{x}'\Sigma_p \mathbf{x})^{\gamma/2}$$

$$\text{subject to} \qquad \begin{array}{ll} A\mathbf{x} \le \mathbf{b} & \text{dual variable } \mathbf{y} \\ \mathbf{x} = \mathbf{x}_{obs} + \mathbf{h} & \text{dual variable } \boldsymbol{\lambda} \end{array}$$

where $\mathbf{h}$ is a vector of deviations from the realized and known output levels. The first set of constraints forms the structural (technological) relations while the second set constitutes the calibration constraints. The corresponding dual constraints turn out to be

$$\text{(8)} \qquad \gamma(\mathbf{x}'\Sigma_p \mathbf{x})^{(\gamma/2-1)} \Sigma_p \mathbf{x} + A'\mathbf{y} + \boldsymbol{\lambda} \ge \theta[\bar{w} + (E(\tilde{\mathbf{p}}) - \mathbf{c})'\mathbf{x}]^{(\theta-1)}[E(\tilde{\mathbf{p}}) - \mathbf{c}] \, .$$

The complexity of the estimation problem becomes clear by considering the nonlinearity of relation (8). Parameters θ and γ are usually unknown as are the optimal output levels, $\mathbf{x}$, the deviations, $\mathbf{h}$, from the observed output levels, $\mathbf{x}_{obs}$, the optimal dual variables, $\mathbf{y}$, and the Lagrange multipliers, $\boldsymbol{\lambda}$. Furthermore, it is often the case that also the market price of some input – say land – is known for a homogeneous area or even for a single farm. The PMP methodology, therefore, should use this information, $\mathbf{y}_{obs}$, that will be treated in the form of the observed output levels as

$$\text{(9)} \qquad \mathbf{y} = \mathbf{y}_{obs} + \mathbf{u}$$

where $\mathbf{u}$ is an $(I \times 1)$ vector of deviations from the observed input prices. Using a least-squares approach for the estimation of deviations $\mathbf{h}$ and $\mathbf{u}$, it turns out that, by the symmetric duality of least squares (LS), $\mathbf{h} = \boldsymbol{\lambda}$ and $\mathbf{u} = \boldsymbol{\psi}$, where $\boldsymbol{\psi}$ is the vector of Lagrange multipliers associated with constraint (9). To show this result, consider the following LS problem

$$\text{(10)} \qquad \begin{array}{lll} & \min LS = \mathbf{h}'\mathbf{h}/2 + \mathbf{u}'\mathbf{u}/2 & \\ \text{subject to} & \mathbf{x} = \mathbf{x}_{obs} + \mathbf{h} & \text{dual variables } \boldsymbol{\lambda} \\ & \mathbf{y} = \mathbf{y}_{obs} + \mathbf{u} & \text{dual variables } \boldsymbol{\psi} \end{array}$$

The corresponding Lagrangean function is

$$L = \mathbf{h}'\mathbf{h}/2 + \mathbf{u}'\mathbf{u}/2 + \boldsymbol{\lambda}'(\mathbf{x} - \mathbf{x}_{obs} - \mathbf{h}) + \boldsymbol{\psi}'(\mathbf{y} - \mathbf{y}_{obs} - \mathbf{u}) \tag{11}$$

and first order necessary conditions of L with respect to $\mathbf{h}$ and $\mathbf{u}$ are

$$\begin{aligned} \frac{\partial L}{\partial \mathbf{h}} &= \mathbf{h} - \boldsymbol{\lambda} = \mathbf{0} \\ \frac{\partial L}{\partial \mathbf{u}} &= \mathbf{u} - \boldsymbol{\psi} = \mathbf{0} \end{aligned} \qquad \text{Q.E.D.} \tag{12}$$

A crucial issue regards parameters θ and γ. On the one hand, an economic entrepreneur wishes to maximize her utility of wealth while minimizing the disutility of its risk. On the other hand, it is a fact that high levels of wealth are associated with high risk. Another fact is that this entrepreneur has already made her choices of a production plan, $\mathbf{x}_{obs}$, in the face of output price risk. It is also likely that she does not know (or that she is not even aware of) parameters θ and γ. The challenge, therefore, is to infer – from her decisions – the values of parameters θ and γ that could explain the behavior of this entrepreneur in a rational and reasonable fashion.

We will assume that this entrepreneur is risk averse, implying that $\theta > 0$ and $\gamma > 0$. Furthermore, for any given level of expected wealth, a high level of utility will be achieved with the highest admissible level of parameter θ, where admissibility depends on the technology, the limiting input constraints, the observed production plan and the observed input prices.

An alternative viewpoint, one that mimics the relationship – referred to above – between high levels of wealth and high levels of its standard deviation, would postulate that high levels of utility (of wealth) are associated with high levels of its risk disutility.

Therefore, for any given level of the standard deviation of wealth, the parameter γ should acquire the highest admissible value, given the observed production plan and input prices.

Phase I PMP Model

Thus, for estimation purposes, we assume that parameters θ and γ will be maximized together with the minimization of deviations $\mathbf{h}$ and $\mathbf{u}$ in a least-squares objective function subject to relevant primal and dual constraints and their associated complementary slackness conditions. This choice leads to the following phase I model

$$\min LS = \mathbf{h}'\mathbf{h}/2 + \mathbf{u}'\mathbf{u}/2 - \theta^2 - \gamma^2 \tag{13}$$

subject to

$$A\mathbf{x} \leq \mathbf{b} + \mathbf{u} \tag{14}$$

$$\theta[\bar{w} + (E(\tilde{\mathbf{p}}) - \mathbf{c})'\mathbf{x}]^{(\theta-1)}[E(\tilde{\mathbf{p}}) - \mathbf{c}] \leq A'\mathbf{y} + \mathbf{h} + \gamma(\mathbf{x}'\boldsymbol{\Sigma}_p\mathbf{x})^{(\gamma/2-1)}\boldsymbol{\Sigma}_p\mathbf{x} \tag{15}$$

$$\mathbf{x} = \mathbf{x}_{obs} + \mathbf{h} \tag{16}$$

$$\mathbf{y} = \mathbf{y}_{obs} + \mathbf{u} \tag{17}$$

$$\mathbf{y}'(\mathbf{b} + \mathbf{u} - A\mathbf{x}) = 0 \tag{18}$$

$$\mathbf{x}'\{A'\mathbf{y} + \mathbf{h} + \gamma(\mathbf{x}'\boldsymbol{\Sigma}_p\mathbf{x})^{(\gamma/2-1)}\boldsymbol{\Sigma}_p\mathbf{x} - \theta[\bar{w} + (E(\tilde{\mathbf{p}}) - \mathbf{c})'\mathbf{x}]^{(\theta-1)}[E(\tilde{\mathbf{p}}) - \mathbf{c}]\} = 0 \tag{19}$$

$$\mathbf{h}'(\mathbf{x}_{obs} + \mathbf{h} - \mathbf{x}) = 0 \tag{20}$$

$$\mathbf{u}'(\mathbf{y}_{obs} + \mathbf{u} - \mathbf{y}) = 0 \tag{21}$$

with $\mathbf{x} \geq \mathbf{0}, \mathbf{y} \geq \mathbf{0}, \theta \geq 0, \gamma \geq 0$, $\mathbf{h}$ and $\mathbf{u}$ free. Constraints (20) and (21) are redundant and can be omitted without loss of information.

Constraints (14) represent the structural (technological) relations of input demand being less-than-or-equal to the effective input supply. Constraints (15) represent the dual relations with marginal utility of the production plan being less-than-or-equal to its marginal cost. Here marginal cost has two parts: the marginal cost due to limiting and variable inputs,

$A'\mathbf{y}+\mathbf{h}$, and the marginal cost of risk due to the random variability of output prices, $\gamma(\mathbf{x}'\boldsymbol{\Sigma}_p\mathbf{x})^{(\gamma/2-1)}\boldsymbol{\Sigma}_p\mathbf{x}$. Constraints (16) and (17) are the calibration relations. Constraints (18) – (21) are the corresponding complementary slackness conditions.

Judging from the dual constraint (15), the input shadow prices are measured in utility units. To achieve a dollar measure of these input prices, $\mathbf{y}$, it is sufficient to divide them by the quantity $\theta[\bar{w}+(E(\tilde{\mathbf{p}})-\mathbf{c})'\mathbf{x}]^{(\theta-1)}$.

Phase II PMP Model

Phase II of the PMP methodology deals with the estimation of a cost function that embodies all the technological and behavioral information revealed in phase I. Typically, a marginal cost function expresses a portion of the dual constraints in a phase I PMP model. In the absence of risk, PMP marginal cost is defined as $A'\mathbf{y}+(\mathbf{c}+\mathbf{h})$, where $A'\mathbf{y}$ stands for the marginal cost due to limiting inputs and $(\mathbf{c}+\mathbf{h})$ for the effective marginal cost due to variable inputs. In this risky case, marginal cost is given by right-hand-side of relation (15) where all the elements are measured in utility units. We desire to obtain a dollar expression of marginal cost, as in the familiar relation $MC \geq E(\tilde{\mathbf{p}})$. To achieve this result, the elements of relation (15) will be divided by the term $\theta[\bar{w}+(E(\tilde{\mathbf{p}})-\mathbf{c})'\mathbf{x}]^{(\theta-1)}$ to write

(22) $$MC \geq E(\tilde{\mathbf{p}})$$

$$\mathbf{c}+\frac{1}{\theta}[\bar{w}+(E(\tilde{\mathbf{p}})-\mathbf{c})'\mathbf{x}]^{(1-\theta)}[A'\mathbf{y}+\mathbf{h}]+\frac{\gamma}{\theta}[\bar{w}+(E(\tilde{\mathbf{p}})-\mathbf{c})'\mathbf{x}]^{(1-\theta)}(\mathbf{x}'\boldsymbol{\Sigma}_p\mathbf{x})^{(\gamma/2-1)}\boldsymbol{\Sigma}_p\mathbf{x} \geq E(\tilde{\mathbf{p}}).$$

In relation (22), all the monetary terms are measured in dollars. The marginal cost of limiting and variable inputs is represented by $\left\{\mathbf{c}+\frac{1}{\theta}[\bar{w}+(E(\tilde{\mathbf{p}})-\mathbf{c})'\mathbf{x}]^{(1-\theta)}[A'\mathbf{y}+\mathbf{h}]\right\}$. The marginal cost of risky output prices is given by $\left\{\frac{\gamma}{\theta}[\bar{w}+(E(\tilde{\mathbf{p}})-\mathbf{c})'\mathbf{x}]^{(1-\theta)}(\mathbf{x}'\boldsymbol{\Sigma}_p\mathbf{x})^{(\gamma/2-1)}\boldsymbol{\Sigma}_p\mathbf{x}\right\}$.

The cost function selected to synthesize the technological and behavioral relations of phase I is expressed as a modified Leontief cost function such as

(23) $$C(\mathbf{x},\mathbf{y}) = (\mathbf{f}'\mathbf{x})(\mathbf{g}'\mathbf{y}) + (\mathbf{g}'\mathbf{y})(\mathbf{x}'Q\mathbf{x})/2 + (\mathbf{f}'\mathbf{x})[(\mathbf{y}^{1/2})'G\mathbf{y}^{1/2}] .$$

A cost function is linear homogeneous and concave in input prices, $\mathbf{y}$. Therefore, matrix G is negative definite. Furthermore, a cost function is increasing in output levels. Thus, matrix Q is positive definite. Parameters $\mathbf{f}$ and $\mathbf{g}$ are introduced to give flexibility to the cost function.

The marginal cost function associated with cost function (23) is given by

(24) $$MC_{\mathbf{x}} = \frac{\partial C}{\partial \mathbf{x}} = (\mathbf{g}'\mathbf{y})\mathbf{f} + (\mathbf{g}'\mathbf{y})Q\mathbf{x} + \mathbf{f}[(\mathbf{y}^{1/2})'G\mathbf{y}^{1/2}] .$$

The derivative of the cost function with respect to input prices corresponds to Shephard lemma that produces the demand function for inputs:

(25) $$\frac{\partial C}{\partial \mathbf{y}} = (\mathbf{f}'\mathbf{x})\mathbf{g} + \mathbf{g}(\mathbf{x}'Q\mathbf{x})/\mathbf{2} + (\mathbf{f}'\mathbf{x})[\Delta(\mathbf{y}^{-1/2})'G\mathbf{y}^{1/2}] = A\mathbf{x}$$

where the term $\Delta(\mathbf{y}^{-1/2})$ represents a diagonal matrix with elements $y_i^{-1/2}$ on the main diagonal.

With knowledge of the solution components resulting from the phase I model (13)-(21), $\hat{\mathbf{x}},\hat{\mathbf{y}},\hat{\mathbf{h}},\hat{\mathbf{u}},\hat{\theta},\hat{\gamma}$, a phase II model's objective is to estimate the parameters of the cost function, $\mathbf{f},\mathbf{g},Q,G$. This task is accomplished by means of the following specification

(26) $$\min Aux = \mathbf{f}'\mathbf{f}/2 + \mathbf{g}'\mathbf{g}/2$$

subject to

(27) $$(\mathbf{g}'\hat{\mathbf{y}})\mathbf{f} + (\mathbf{g}'\hat{\mathbf{y}})Q\hat{\mathbf{x}} + \mathbf{f}[(\hat{\mathbf{y}}^{1/2})'G\hat{\mathbf{y}}^{1/2}] =$$
$$\mathbf{c} + \frac{1}{\hat{\theta}}[\bar{w} + (E(\tilde{\mathbf{p}}) - \mathbf{c})'\hat{\mathbf{x}}]^{(1-\hat{\theta})}[A'\hat{\mathbf{y}} + \hat{\mathbf{h}}] + \frac{\hat{\gamma}}{\hat{\theta}}[\bar{w} + (E(\tilde{\mathbf{p}}) - \mathbf{c})'\hat{\mathbf{x}}]^{(1-\hat{\theta})}(\hat{\mathbf{x}}'\boldsymbol{\Sigma}_p\hat{\mathbf{x}})^{(\hat{\gamma}/2-1)}\boldsymbol{\Sigma}_p\hat{\mathbf{x}} \geq E(\tilde{\mathbf{p}})$$

(28) $$(\mathbf{f}'\hat{\mathbf{x}})\mathbf{g} + \mathbf{g}(\hat{\mathbf{x}}'Q\hat{\mathbf{x}})/\mathbf{2} + (\mathbf{f}'\hat{\mathbf{x}})[\Delta(\hat{\mathbf{y}}^{-1/2})'G\hat{\mathbf{y}}^{1/2}] = A\hat{\mathbf{x}}$$

(29) $$Q = LDL'$$

(30) $$I = QQ^{-1}$$

(31) $$\mathbf{f}'\hat{\mathbf{x}} \geq 0$$

(32) $$\mathbf{g}'\hat{\mathbf{y}} \geq 0$$

with $D \geq 0, \mathbf{f}$ and $\mathbf{g}$ free . The minimization of the $\mathbf{f}$ and $\mathbf{g}$ parameters is a reasonable objective given that $\mathbf{f}$ and $\mathbf{g}$ are introduced merely to give flexibility to the cost function and act as intercepts of the marginal cost and the input demand functions, respectively.

Relation (27) represents $MC \geq E(\tilde{\mathbf{p}})$. Relation (28) is Shephard lemma. Relation (29) is the Cholesky factorization of the Q matrix with D as a diagonal matrix with positive elements on the main diagonal and L is a unit lower triangular matrix. Relation (30) defines the inverse of the Q matrix. This operation is of interest for computing the supply elasticities of the various outputs. Relations (31) and (32) guarantee that the cost function is increasing in output. Finally, the objective function (26) defines a least-squares approach for the estimation of parameters $\mathbf{f}$ and $\mathbf{g}$.

Calibrating Equilibrium Model

With the parameter estimates of the cost function derived from phase II model (26)-(32), $\hat{\mathbf{f}}, \hat{\mathbf{g}}, \hat{Q}, \hat{G}$, it is possible to set up a calibrating equilibrium model to be used for policy analysis. Such a model takes on the following structure

(33) $$\min CSC = \mathbf{y}'\mathbf{z}_p + \mathbf{x}'\mathbf{z}_d = 0$$

subject to

(34) $$(\hat{\mathbf{f}}'\mathbf{x})\hat{\mathbf{g}} + \hat{\mathbf{g}}(\mathbf{x}'\hat{Q}\mathbf{x})/\mathbf{2} + (\hat{\mathbf{f}}'\mathbf{x})[\Delta(\mathbf{y}^{-1/2})'\hat{G}\mathbf{y}^{1/2}] + \mathbf{z}_p = \mathbf{b} + \hat{\mathbf{u}}$$

(35) $$(\hat{\mathbf{g}}'\mathbf{y})\hat{\mathbf{f}} + (\hat{\mathbf{g}}'\mathbf{y})\hat{Q}\mathbf{x} + \hat{\mathbf{f}}[(\mathbf{y}^{1/2})'\hat{G}\mathbf{y}^{1/2}] = E(\tilde{\mathbf{p}}) + \mathbf{z}_d$$

with $\mathbf{x} \geq \mathbf{0}, \mathbf{y} \geq \mathbf{0}, \mathbf{z}_p \geq \mathbf{0}, \mathbf{z}_d \geq \mathbf{0}$. The objective function represents the complementary slackness conditions (*CSC)* of constraints (34) and (35) with an optimal value of zero. The variables $\mathbf{z}_p$ and $\mathbf{z}_d$ are surplus variables of the primal and the dual constraints, respectively. The solution of model (33)-(35) calibrates precisely the solution obtained from the phase I model (13)-(21), that is $\hat{\mathbf{x}}_{LS} = \hat{\mathbf{x}}_{CSC}$ and $\hat{\mathbf{y}}_{LS} = \hat{\mathbf{y}}_{CSC}$. The calibrating model, then, can be used to trace the response to changes in the output expected prices and the supply of limiting inputs.

PMP with Generalized Risk and Price Supply Elasticities

It may be of interest to estimate price supply elasticities for the various commodity outputs involved in a PMP approach. The supply function for outputs is derivable from relation (24) by equating it to the expected market output prices, $E(\tilde{\mathbf{p}})$, and inverting the marginal cost function:

(36) $$\mathbf{x} = -Q^{-1}\mathbf{f} - Q^{-1}\mathbf{f}[(\mathbf{y}^{1/2})'G(\mathbf{y}^{1/2})]/(\mathbf{g}'\mathbf{y}) + [1/(\mathbf{g}'\mathbf{y})]Q^{-1}E(\tilde{\mathbf{p}}) \quad \text{output supply function}$$

that leads to the supply elasticity matrix

(37) $$\Xi = \Delta[E(\tilde{\mathbf{p}})]\left[\frac{\partial \mathbf{x}}{\partial E(\tilde{\mathbf{p}})}\right]\Delta[(\mathbf{x}^{-1})] = \Delta[E(\tilde{\mathbf{p}})]Q^{-1}\Delta[(\mathbf{x}^{-1})]/(\mathbf{g}'\mathbf{y}) \quad \text{output supply elasticities}$$

where matrices $\Delta[E(\tilde{\mathbf{p}})]$ and $\Delta[\mathbf{x}^{-1}]$ are diagonal with elements $E(\tilde{p}_j)$ and x_j^{-1}, respectively, on the main diagonals. Relation (37) includes all the own- and cross-price elasticities for all the output commodities admitted in the model.

Endogenous and Disaggregated Output Supply Elasticities

PMP has been applied frequently to analyze farmers' behavior to changes in agricultural policies. A typical empirical setting is to map out several areas in a region (or state) and to assemble a representative farm for each area (or to treat each area as a large farm). When supply elasticities are exogenously available (say the own-price elasticities of crops) at the regional (or state) level (via econometric estimation or other means), a connection of all area models can be specified by establishing a weighted sum of all the areas endogenous own-price elasticities and the given regional elasticities. The weights are the share of each area's expected revenue over the total expected revenue of the region.

Let us suppose that exogenous own-price elasticities of supply are available at the regional level for the all the J crops, say $\bar{\eta}_j, j = 1,...,J$. Then, the relation among these exogenous own-price elasticities and the corresponding areas' endogenous elasticities can be established as a weighted sum such as

$$(38) \qquad \bar{\eta}_j = \sum_{n=1}^{N} w_{nj}\eta_{nj}$$

where the weights are the areas' expected revenue shares in the region (state)

$$(39) \qquad w_{nj} = \frac{E(p_{nj})x_{nj}}{\sum_{t=1}^{N} E(p_{tj})x_{tj}}$$

and

$$(40) \qquad \eta_{nj} = E(\tilde{p}_{nj})Q_n^{jj}x_{nj}^{-1} / (\mathbf{g}_n'\mathbf{y}_n)$$

where Q_n^{jj} is the jth element of the nth farm (area) on the main diagonal in the inverse of the Q_n matrix. The phase II model that executes the estimation of the disaggregated (endogenous) output supply elasticities for a region (state) that is divided into *N* areas takes on the following specification:

(41) $$\min Aux = \mathbf{f'f}/2 + \mathbf{g'g}/2$$

subject to

(42) $$(\mathbf{g'\hat{y}})\mathbf{f} + (\mathbf{g'\hat{y}})Q\mathbf{\hat{x}} + \mathbf{f}[(\mathbf{\hat{y}}^{1/2})'G\mathbf{\hat{y}}^{1/2}] =$$
$$\mathbf{c} + \frac{1}{\hat{\theta}}[\bar{w} + (E(\mathbf{\tilde{p}}) - \mathbf{c})'\mathbf{\hat{x}}]^{(1-\hat{\theta})}[A'\mathbf{\hat{y}} + \mathbf{\hat{h}}] + \frac{\hat{\gamma}}{\hat{\theta}}[\bar{w} + (E(\mathbf{\tilde{p}}) - \mathbf{c})'\mathbf{\hat{x}}]^{(1-\hat{\theta})}(\mathbf{\hat{x}}'\boldsymbol{\Sigma}_p\mathbf{\hat{x}})^{(\hat{\gamma}/2-1)}\boldsymbol{\Sigma}_p\mathbf{\hat{x}} \geq E(\mathbf{\tilde{p}})$$

(43) $$(\mathbf{f'\hat{x}})\mathbf{g} + \mathbf{g}(\mathbf{\hat{x}}'Q\mathbf{\hat{x}})/\mathbf{2} + (\mathbf{f'\hat{x}})[\Delta(\mathbf{\hat{y}}^{-1/2})'G\mathbf{\hat{y}}^{1/2}] = A\mathbf{\hat{x}}$$

(44) $$Q = LDL'$$

(45) $$I = QQ^{-1}$$

(46) $$\mathbf{f'\hat{x}} \geq 0$$

(47) $$\mathbf{g'\hat{y}} \geq 0$$

(48) $$\Xi = \Delta[E(\mathbf{\tilde{p}})]Q^{-1}\Delta[\mathbf{\hat{x}}^{-1}]/(\mathbf{g'\hat{y}})$$ endogenous own- and cross supply elasticities

(49) $$w_{nj} = \frac{E(\tilde{p}_{nj})\hat{x}_{nj}}{\sum_{t=1}^{N} E(\tilde{p}_{tj})\hat{x}_{tj}}$$ expected revenue weights

(50) $$\eta_{nj} = E(\tilde{p}_{nj})Q_n^{jj}\hat{x}_{nj}^{-1}$$ endogenous own-price elasticities

(51) $$\bar{\eta}_j = \sum_{n=1}^{N} w_{nj}\eta_{nj}$$ disaggregation of exogenous elasticities

with $D_n > 0, g$ and f free and $f'x > 0$ and $g'y > 0$.

Empirical Implementation of PMP_MDS with Supply Elasticities – Large Farms

The PMP-MS approach described in previous sections was applied to three samples of representative farms (small, medium and large) with *N* = 14 observations in each sample. We report the estimates of the large-farm sample. There are four crops: sugar beet (BRB), soft wheat (TEN), corn (MAS) and barley (ORZ). There is only one limiting input: land.

From Phase I Model (13)-(21)

---- 1014 VARIABLE theta.L coefficient of the Mean
PARAMETER $\hat{\theta}$

1 1.0477336, 2 1.0539034, 3 1.0790223, 4 1.0726528, 5 1.0008581
6 1.0386790, 7 1.0782421, 8 1.0819357, 9 1.0842247, 10 1.0667055
11 1.0707482, 12 1.0092212, 13 1.0015747, 14 1.0398168

PARAMETER $\hat{\gamma}$

1 1.2445509, 2 1.2534702, 3 1.2445841, 4 1.2582925, 5 1.1532393
6 1.2118774, 7 1.2747046, 8 1.2746439, 9 1.3160656, 10 1.2601629
11 1.2541866, 12 1.1653607, 13 1.1526221, 14 1.2162482

All fourteen farmers exhibit decreasing absolute risk aversion, $\hat{\theta} > 1$. All farmers exhibit increasing relative risk aversion, $\hat{\gamma} > \hat{\theta}$.

PARAMETER risk aversion

1 0.0123526, 2 0.0066949, 3 0.0052539, 4 0.0038768, 5 0.0072606
6 0.0037938, 7 0.0053129, 8 0.0036410, 9 0.0109963, 10 0.0051071
11 0.0057658, 12 0.0040152, 13 0.0069743, 14 0.0040743

PARAMETER wealth derivative of absolute risk aversion

1 -0.0008419, 2 -0.0005320, 3 -0.0010481, 4 -0.0006543
5 -0.0000427, 6 -0.0007684, 7 -0.0007369, 8 -0.0005705
9 -0.0006823, 10 -0.0006540, 11 -0.0009364, 12 -0.0002642
13 -0.0000770, 14 -0.0007285

PARAMETER $\mathbf{x}_{obs}$ observed output levels

	BRB	TEN	MAS	ORZ
1	1.133424E+3	305.4032351	341.3693403	18.2398722
2	3.103783E+3	861.7445535	478.4465107	59.8025522
3	1.547978E+3	450.7937871	881.9748433	7.6887358
4	3.488354E+3	821.3934271	1.493332E+3	51.1247151
5	959.1102412	468.2848696	478.9261801	28.2406037
6	942.2039951	801.1288268	1.283591E+3	152.5812215
7	1.600731E+3	695.8293118	899.4739570	66.9718421
8	3.507549E+3	1.212855E+3	1.237584E+3	98.0497703
9	1.050537E+3	332.3773404	498.0150725	63.6696198
10	3.473678E+3	952.5199370	774.7402863	84.0070376
11	1.245722E+3	765.1689195	501.9673305	59.5366249
12	3.276145E+3	1.100168E+3	742.9419407	177.9744313
13	877.0970595	380.9171917	564.6091640	76.2122654
14	1.430946E+3	768.6901276	1.309392E+3	67.7906102

PARAMETER $\hat{\mathbf{x}}$ - optimal crop levels in phase I model

	BRB	TEN	MAS	ORZ
1	1.133841E+3	304.9003333	341.8497521	18.1976713
2	3.104476E+3	861.6030580	478.2897000	60.0045325
3	1.548519E+3	450.5967725	881.6756889	8.0748979
4	3.488766E+3	820.9405594	1.493348E+3	51.5339016
5	959.3913739	467.6686456	479.2238883	28.4105088
6	942.7001436	800.6885278	1.283737E+3	152.8370373
7	1.601383E+3	695.5367076	899.7048529	67.0263115
8	3.508096E+3	1.213032E+3	1.237782E+3	97.6121283
9	1.051278E+3	332.5400598	497.9904886	63.4433605
10	3.474201E+3	952.0932429	774.8693405	84.2813824
11	1.246301E+3	764.7621817	502.2088444	59.6874786
12	3.276536E+3	1.099640E+3	743.0977719	178.2457393
13	877.3783649	380.3006737	564.9044723	76.3831745
14	1.431446E+3	768.2501195	1.309532E+3	68.0502120

PARAMETER $\hat{\mathbf{h}}$ - deviations from $\mathbf{x}_{obs}$

	BRB	TEN	MAS	ORZ
1	0.4174666	-0.5029018	0.4804118	-0.0422010
2	0.6929796	-0.1414955	-0.1568107	0.2019803
3	0.5410446	-0.1970146	-0.2991544	0.3861621
4	0.4118370	-0.4528677	0.0164491	0.4091865
5	0.2811327	-0.6162240	0.2977082	0.1699051
6	0.4961485	-0.4402990	0.1464425	0.2558158
7	0.6521659	-0.2926042	0.2308959	0.0544695
8	0.5473309	0.1770229	0.1981659	-0.4376420
9	0.7415965	0.1627194	-0.0245839	-0.2262593
10	0.5230429	-0.4266941	0.1290542	0.2743447
11	0.5790732	-0.4067378	0.2415139	0.1508537
12	0.3910239	-0.5283951	0.1558312	0.2713080
13	0.2813054	-0.6165180	0.2953083	0.1709091
14	0.4999870	-0.4400081	0.1403054	0.2596018

PARAMETER $\mathbf{y}_{obs}$ - observed land price

	land
1	0.7488488
2	0.6918612
3	0.6875297
4	0.6751747
5	0.5976591
6	0.5482897
7	0.7807244

8	0.9028318
9	0.7826706
10	0.8180012
11	0.6382583
12	0.6690701
13	0.6482950
14	0.4651773

PARAMETER $\hat{\mathbf{y}}$ - optimal land shadow price

	land
1	0.7011229
2	0.6918300
3	0.7039447
4	0.6752100
5	0.5592771
6	0.5418763
7	0.7850682
8	0.8798274
9	0.7883638
10	0.8300010
11	0.6320721
12	0.6334454
13	0.6095794
14	0.4586504

PARAMETER $\hat{\mathbf{u}}$ - deviations from $\mathbf{y}_{obs}$

	land
1	-0.0477259
2	-0.0000312
3	0.0164150
4	0.0000353
5	-0.0383820
6	-0.0064134
7	0.0043438
8	-0.0230044
9	0.0056932
10	0.0119998
11	-0.0061862
12	-0.0356247
13	-0.0387156
14	-0.0065269

From Phase II Model (41)-(51)

PARAMETER $\hat{\mathbf{f}}$ - cost function

	BRB	TEN	MAS	ORZ
1	0.0000653	0.0001451	-0.0000117	0.0000331
2	0.0000034	0.0001862	-0.0000184	0.0000266
3	0.0000520	-0.0000421	0.0001262	-0.0000431
4	0.0000984	0.0000557	0.0000454	-0.0000180
5	-0.0000004	0.0001434	-0.0000043	0.0000279
6	-0.0000017	0.0000817	0.0001031	0.0000653
7	0.0000149	0.0001685	0.0001578	0.0000861
8	0.0000302	0.0000661	0.0000133	-0.0000755
9	0.0000027	0.0000918	0.0000552	0.0000650
10	0.0000255	-0.0000454	0.0001127	-0.0000021
11	0.0000091	0.0000534	0.0000004	0.0001596
12	0.0001058	0.0000472	-0.0000977	0.0001108
13	0.0000470	0.0001276	0.0000847	0.0000720
14	-0.0000201	0.0000823	0.0000628	0.0000800

PARAMETER $\hat{\mathbf{g}}$ - cost function

	land
1	0.0003585
2	0.0002878
3	0.0002938
4	0.0003300
5	0.0002626
6	0.0004394
7	0.0001911
8	0.0002395
9	0.0002208
10	0.0001927
11	0.0002067
12	0.0002257
13	0.0004519
14	0.0004226

PARAMETER $\hat{Q}$ - output Q cost matrix

	BRB	TEN	MAS	ORZ
1 .BRB	3.5766596	2.0304287	2.0500501	0.6164998
1 .TEN	2.0304287	47.8437788	0.3102451	-2.5415393
1 .MAS	2.0500501	0.3102451	14.9656720	-0.9009641
1 .ORZ	0.6164998	-2.5415393	-0.9009641	564.1352158
2 .BRB	0.8435185	0.1292506	0.1331886	1.2845607
2 .TEN	0.1292506	43.8267742	-0.7140416	-0.4320476
2 .MAS	0.1331886	-0.7140416	24.7303847	-31.4843316

2 .ORZ	1.2845607	-0.4320476	-31.4843316	445.7819448
3 .BRB	3.8648558	-3.0660507	1.7800281	0.3514769
3 .TEN	-3.0660507	26.9534000	-0.1935468	0.0885825
3 .MAS	1.7800281	-0.1935468	19.2685997	-0.1701037
3 .ORZ	0.3514769	0.0885825	-0.1701037	698.7678037
4 .BRB	6.6813868	-30.8158856	7.1415274	-27.2223751
4 .TEN	-30.8158856	265.7267571	-64.3143559	-6.5562355
4 .MAS	7.1415274	-64.3143559	27.2730941	-4.4670674
4 .ORZ	-27.2223751	-6.5562355	-4.4670674	2.258088E+3
5 .BRB	3.7016579	-1.3363396	-1.2132346	5.7027102
5 .TEN	-1.3363396	98.6132833	-0.8253365	-8.5565144
5 .MAS	-1.2132346	-0.8253365	28.7377785	-2.2994488
5 .ORZ	5.7027102	-8.5565144	-2.2994488	676.4951271
6 .BRB	4.6186034	2.5868735	-3.2956426	-3.4178200
6 .TEN	2.5868735	33.3589173	-8.5447821	-0.8719286
6 .MAS	-3.2956426	-8.5447821	27.5221995	-38.7808014
6 .ORZ	-3.4178200	-0.8719286	-38.7808014	458.5303488
7 .BRB	2.0576087	2.0717072	-1.6564472	5.2831453
7 .TEN	2.0717072	44.7863761	-5.4215109	-7.0962166
7 .MAS	-1.6564472	-5.4215109	46.1100101	-1.20101E+2
7 .ORZ	5.2831453	-7.0962166	-1.20101E+2	1.867644E+3
8 .BRB	7.5378978	-13.5542979	-1.8241472	-12.1807925
8 .TEN	-13.5542979	57.7520158	-0.8422275	-0.2635546
8 .MAS	-1.8241472	-0.8422275	15.2211084	-0.4430574
8 .ORZ	-12.1807925	-0.2635546	-0.4430574	430.6839545
9 .BRB	0.9484116	1.7191084	2.0590982	3.0688872
9 .TEN	1.7191084	86.0603983	-1.2719675	-0.1352004
9 .MAS	2.0590982	-1.2719675	40.5260275	-1.4666695
9 .ORZ	3.0688872	-0.1352004	-1.4666695	330.0746279
10.BRB	9.3084306	-28.3240064	2.3871528	2.0422720
10.TEN	-28.3240064	115.8447614	0.3047887	-94.0965509
10.MAS	2.3871528	0.3047887	61.7031154	-2.41502E+2
10.ORZ	2.0422720	-94.0965509	-2.41502E+2	3.354817E+3
11.BRB	5.8989199	-1.7047572	-3.5498756	32.4758510
11.TEN	-1.7047572	49.5014397	1.2219108	-6.2735772
11.MAS	-3.5498756	1.2219108	38.0966888	-5.4214388
11.ORZ	32.4758510	-6.2735772	-5.4214388	677.4220909

12.BRB	20.6277152	-41.6116112	-6.2709677	-7.2622065
12.TEN	-41.6116112	143.2772995	1.3017304	-7.6196831
12.MAS	-6.2709677	1.3017304	31.2555880	0.2639772
12.ORZ	-7.2622065	-7.6196831	0.2639772	341.9679113
13.BRB	8.6677069	-0.7066885	-4.9818734	-8.4910604
13.TEN	-0.7066885	38.0117053	0.2173645	0.0006766
13.MAS	-4.9818734	0.2173645	29.5682186	-12.4291226
13.ORZ	-8.4910604	0.0006766	-12.4291226	334.5387765
14.BRB	3.2058314	-4.4309557	-2.5932924	2.9317909
14.TEN	-4.4309557	49.5956557	-1.5069791	-3.8834446
14.MAS	-2.5932924	-1.5069791	22.7353560	-6.7900495
14.ORZ	2.9317909	-3.8834446	-6.7900495	539.4756237

PARAMETER $\hat{Q}^{-1}$ - Q inverse matrix

	BRB	TEN	MAS	ORZ
1 .BRB	0.3113363	-0.0129625	-0.0424073	-0.0004664
1 .TEN	-0.0129625	0.0214488	0.0013378	0.0001129
1 .MAS	-0.0424073	0.0013378	0.0726111	0.0001683
1 .ORZ	-0.0004664	0.0001129	0.0001683	0.0017739
2 .BRB	1.1945217	-0.0037608	-0.0120084	-0.0042939
2 .TEN	-0.0037608	0.0228420	0.0007931	0.0000890
2 .MAS	-0.0120084	0.0007931	0.0445768	0.0031837
2 .ORZ	-0.0042939	0.0000890	0.0031837	0.0024806
3 .BRB	0.2980208	0.0337062	-0.0271939	-0.0001608
3 .TEN	0.0337062	0.0409159	-0.0027030	-0.0000228
3 .MAS	-0.0271939	-0.0027030	0.0543832	0.0000273
3 .ORZ	-0.0001608	-0.0000228	0.0000273	0.0014312
4 .BRB	0.3639290	0.0452692	0.0122006	0.0045429
4 .TEN	0.0452692	0.0144081	0.0222262	0.0006315
4 .MAS	0.0122006	0.0222262	0.0859468	0.0003816
4 .ORZ	0.0045429	0.0006315	0.0003816	0.0005002
5 .BRB	0.2787983	0.0036795	0.0116947	-0.0022639
5 .TEN	0.0036795	0.0102029	0.0004563	0.0000996
5 .MAS	0.0116947	0.0004563	0.0353064	0.0000272
5 .ORZ	-0.0022639	0.0000996	0.0000272	0.0014986
6 .BRB	0.2495785	-0.0107594	0.0330801	0.0046377

6 .TEN	-0.0107594	0.0334745	0.0103098	0.0008554
6 .MAS	0.0330801	0.0103098	0.0498071	0.0044787
6 .ORZ	0.0046377	0.0008554	0.0044787	0.0025959
7 .BRB	0.5219096	-0.0224922	0.0144580	-0.0006321
7 .TEN	-0.0224922	0.0237695	0.0028681	0.0003384
7 .MAS	0.0144580	0.0028681	0.0269857	0.0017053
7 .ORZ	-0.0006321	0.0003384	0.0017053	0.0006482
8 .BRB	0.2676269	0.0633686	0.0358022	0.0076447
8 .TEN	0.0633686	0.0323338	0.0094364	0.0018217
8 .MAS	0.0358022	0.0094364	0.0705428	0.0010909
8 .ORZ	0.0076447	0.0018217	0.0010909	0.0025403
9 .BRB	1.2873488	-0.0267206	-0.0666922	-0.0122765
9 .TEN	-0.0267206	0.0121798	0.0017494	0.0002612
9 .MAS	-0.0666922	0.0017494	0.0281460	0.0007459
9 .ORZ	-0.0122765	0.0002612	0.0007459	0.0031472
10.BRB	0.4484414	0.1114373	-0.0093765	0.0021776
10.TEN	0.1114373	0.0365998	-0.0010298	0.0008846
10.MAS	-0.0093765	-0.0010298	0.0229499	0.0016289
10.ORZ	0.0021776	0.0008846	0.0016289	0.0004388
11.BRB	0.2485566	0.0065536	0.0212876	-0.0116848
11.TEN	0.0065536	0.0204127	-0.0000619	-0.0001256
11.MAS	0.0212876	-0.0000619	0.0281213	-0.0007961
11.ORZ	-0.0116848	-0.0001256	-0.0007961	0.0020288
12.BRB	0.1394363	0.0404619	0.0262582	0.0038424
12.TEN	0.0404619	0.0187317	0.0073272	0.0012710
12.MAS	0.0262582	0.0073272	0.0369516	0.0006924
12.ORZ	0.0038424	0.0012710	0.0006924	0.0030336
13.BRB	0.1337547	0.0023475	0.0243257	0.0042986
13.TEN	0.0023475	0.0263500	0.0002304	0.0000681
13.MAS	0.0243257	0.0002304	0.0387822	0.0020583
13.ORZ	0.0042986	0.0000681	0.0020583	0.0031748
14.BRB	0.4038351	0.0374391	0.0481508	-0.0013191
14.TEN	0.0374391	0.0236890	0.0058528	0.0000407
14.MAS	0.0481508	0.0058528	0.0499869	0.0004096
14.ORZ	-0.0013191	0.0000407	0.0004096	0.0018663

PARAMETER $\hat{G}$ - input G cost matrix

	land
1 .land	-2.08452E+4
2 .land	-3.94773E+4
3 .land	-2.44938E+4
4 .land	-1.95820E+4
5 .land	-5.62237E+4
6 .land	-4.46220E+4
7 .land	-1.62368E+4
8 .land	-3.65667E+4
9 .land	-4.04197E+4
10.land	-4.02674E+4
11.land	-7.51964E+4
12.land	-2.59564E+4
13.land	-2.39889E+4
14.land	-9.01456E+4

PARAMETER $\hat{\mathbf{f}}'\hat{\mathbf{x}}$

1 0.1148425, 2 0.1636634, 3 0.1725346, 4 0.4557730, 5 0.0654597
6 0.2061026, 7 0.2888122, 8 0.1951070, 9 0.0649497, 10 0.1325372
11 0.0618992, 12 0.3457535, 13 0.1430823, 14 0.1221670

PARAMETER $\hat{\mathbf{g}}'\hat{\mathbf{y}}$

1 0.0002513, 2 0.0001991, 3 0.0002068, 4 0.0002228, 5 0.0001469
6 0.0002381, 7 0.0001500, 8 0.0002107, 9 0.0001740, 10 0.0001600
11 0.0001306, 12 0.0001430, 13 0.0002755, 14 0.0001938

PARAMETER $\hat{\Xi}$ - supply elasticity matrix

	BRB	TEN	MAS	ORZ
1 .BRB	0.4370057	-0.0676613	-0.1974308	-0.0407865
1 .TEN	-0.0973419	0.5989745	0.0333212	0.0528408
1 .MAS	-0.3065527	0.0359626	1.7409426	0.0758189
1 .ORZ	-0.0032730	0.0029474	0.0039185	0.7757036
2 .BRB	0.9275877	-0.0105225	-0.0605262	-0.1725105
2 .TEN	-0.0148452	0.3248811	0.0203195	0.0181736
2 .MAS	-0.0474018	0.0112797	1.1421315	0.6502004
2 .ORZ	-0.0161160	0.0012034	0.0775601	0.4816843
3 .BRB	0.3535859	0.1374318	-0.0566667	-0.0365850
3 .TEN	0.2336297	0.9746281	-0.0329056	-0.0303049
3 .MAS	-0.1604718	-0.0548150	0.5636361	0.0308481
3 .ORZ	-0.0010041	-0.0004893	0.0002990	1.7138346

4 .BRB	0.1498236	0.0792003	0.0117342	0.1266127
4 .TEN	0.1421042	0.1922074	0.1629968	0.1342106
4 .MAS	0.0346885	0.2685535	0.5708824	0.0734581
4 .ORZ	0.0135593	0.0080106	0.0026611	0.1010721
5 .BRB	0.7519677	0.0203591	0.0631472	-0.2061992
5 .TEN	0.0517109	0.2941533	0.0128389	0.0472601
5 .MAS	0.1593730	0.0127575	0.9632467	0.0125159
5 .ORZ	-0.0321378	0.0029000	0.0007729	0.7184072
6 .BRB	0.4892820	-0.0248342	0.0476229	0.0560784
6 .TEN	-0.1112182	0.4073913	0.0782588	0.0545393
6 .MAS	0.3154131	0.1157368	0.3487402	0.2633955
6 .ORZ	0.0479386	0.0104106	0.0339966	0.1655064
7 .BRB	0.7604396	-0.0754531	0.0374950	-0.0220037
7 .TEN	-0.2078675	0.5057651	0.0471786	0.0747142
7 .MAS	0.1227837	0.0560792	0.4079067	0.3460141
7 .ORZ	-0.0052627	0.0064864	0.0252720	0.1289352
8 .BRB	0.1484611	0.1016614	0.0562885	0.1524102
8 .TEN	0.2049137	0.3023802	0.0864834	0.2117115
8 .MAS	0.0954277	0.0727399	0.5328994	0.1045020
8 .ORZ	0.0217211	0.0149692	0.0087849	0.2594045
9 .BRB	2.8143152	-0.1846698	-0.3077856	-0.4447153
9 .TEN	-0.3300430	0.4755956	0.0456149	0.0534597
9 .MAS	-0.7435700	0.0616603	0.6624602	0.1377950
9 .ORZ	-0.1341901	0.0090259	0.0172107	0.5700326
10.BRB	0.2662527	0.2414317	-0.0249606	0.0532964
10.TEN	0.4390854	0.5262271	-0.0181933	0.1436754
10.MAS	-0.0337400	-0.0135222	0.3702642	0.2416150
10.ORZ	0.0085020	0.0126023	0.0285139	0.0706232
11.BRB	0.5801062	0.0249263	0.1232958	-0.5694353
11.TEN	0.0869422	0.4413164	-0.0020388	-0.0348018
11.MAS	0.2588757	-0.0012273	0.8486678	-0.2021372
11.ORZ	-0.1435328	-0.0025150	-0.0242667	0.5203708
12.BRB	0.1577737	0.1364173	0.1310066	0.0799211
12.TEN	0.1900428	0.2621488	0.1517444	0.1097347
12.MAS	0.1233306	0.1025435	0.7652580	0.0597780
12.ORZ	0.0190317	0.0187576	0.0151210	0.2762043

13.BRB	0.2102844	0.0085146	0.0593984	0.0776281
13.TEN	0.0192303	0.4979895	0.0029319	0.0064071
13.MAS	0.1932327	0.0042231	0.4784756	0.1878072
13.ORZ	0.0355694	0.0012999	0.0264523	0.3017488
14.BRB	0.6404801	0.1106366	0.0834764	-0.0440072
14.TEN	0.3130847	0.3691093	0.0535008	0.0071644
14.MAS	0.3714208	0.0841200	0.4214809	0.0664628
14.ORZ	-0.0110310	0.0006346	0.0037443	0.3282892

PARAMETER $\hat{\mathbf{w}}$ - expected revenue weights

	BRB	TEN	MAS	ORZ
1	0.0406137	0.0290924	0.0295132	0.0164307
2	0.1334411	0.0937356	0.0489096	0.0628467
3	0.0526939	0.0446014	0.0698368	0.0072908
4	0.0999728	0.0893119	0.1383142	0.0539749
5	0.0326467	0.0412868	0.0385614	0.0256519
6	0.0371438	0.0828246	0.1151340	0.1600763
7	0.0501907	0.0688463	0.0769208	0.0605182
8	0.1287999	0.1292641	0.1021937	0.0925409
9	0.0376563	0.0335089	0.0425760	0.0572831
10	0.1026666	0.0929675	0.0649489	0.0825661
11	0.0424099	0.0736526	0.0416738	0.0538919
12	0.1555074	0.1078650	0.0685144	0.1866885
13	0.0298559	0.0335738	0.0454558	0.0689665
14	0.0564011	0.0794691	0.1174474	0.0712735

The exogenous own-price supply elasticities for the region (state) were taken as sugar beets = 0.5; soft wheat = 0.4; corn = 0.6; barley = 0.3. For a sample of 14 areas (representative large farms), the endogenous disaggregated elasticities are

Supply elasticities

Sugar Beets	Soft Wheat	Corn	Barley	
0.5	0.4	0.6	0.3	exogenous regional (state) own-price elasticities

PARAMETER $\hat{\eta}$ - disggregated own-price supply elasticities

	BRB	TEN	MAS	ORZ
1	0.4370057	0.5989745	1.7409425	0.7757036
2	0.9275876	0.3248811	1.1421314	0.4816843
3	0.3535859	0.9746280	0.5636361	1.7138345
4	0.1498236	0.1922074	0.5708824	0.1010721
5	0.7519677	0.2941533	0.9632466	0.7184071
6	0.4892820	0.4073913	0.3487402	0.1655063

7	0.7604396	0.5057651	0.4079066	0.1289352
8	0.1484611	0.3023802	0.5328994	0.2594045
9	2.8143151	0.4755956	0.6624602	0.5700326
10	0.2662527	0.5262271	0.3702642	0.0706232
11	0.5801061	0.4413163	0.8486677	0.5203707
12	0.1577737	0.2621488	0.7652579	0.2762043
13	0.2102844	0.4979895	0.4784756	0.3017488
14	0.6404800	0.3691093	0.4214809	0.3282892

References

Fisher, I. (1906). *The Nature of Capital and Income*. MacMillan Company, London

Freund, R.J. (1956). The introduction of Risk Into a Programming Model. *Econometrica* 24:253-263.

Hicks, J.R. (1933). The Application of Mathematical Methods in the Theory of Risk, Lecture presented at The Meeting of the Econometric Society in Leyden, September-October 1933, summarized by J. Marschak, *Econometrica* 1934, 2:187-203.

Markowitz, H. (1952). Portfolio Selection. *Journal of Finance* 7:77-91.

Meyer, J. (1987), Two-Moment Decision Models and Expected Utility Maximization. *American Economic Review,* June:421-430.

Saha, A. (1997). Risk Preference Estimation in the Nonlinear Mean Standard Deviation Approach. *Economic Inquiry* 35:770-782.

Tintner, G. (1941). The Theory of Choice Under Subjective Risk and Uncertainty. *Econometrica* 9:298-304.

Tobin, J. (1958). Liquidity Preference as Behavior Toward Risk. *Review of Economic Studies* 67.

Relevant GAMS file:

PMP_GenRisk_MSD_Leont_SuppElastict_OneSolve.gms