The Structure of South African Milk Production Technology: A Parametric Approach to Supply Analysis

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THE STRUCTURE OF SOUTH AFRICAN MILK PRODUCTION TECHNOLOGY: A PARAMETRIC APPROACH TO SUPPLY ANALYSIS.

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A parametric approach was followed in this study to analyse milk production and supply systems based on farm level production cost data from a cross-section of dairy farms in South Africa for the 1997/1998 production year. Both single equation and system estimation techniques were applied to Normalised Quadratic, Normalised Translog and standard Translog specifications of the profit and derived output supply and input demand functions. Estimated functions were evaluated for adherence to structural properties. Results showed that convexity of the profit function in all prices holds in South African milk production. Uncompensated and compensated price elasticities of supply and demand were calculated. The results indicated that milk production and livestock trading activities are complements in the production activities of the observed multi-input, multi-output dairy farms. Both activities were intensive in the use of purchased and self-produced feed inputs, with a higher intensity in purchased feed use. The variable inputs are gross complements in the long-run and net substitutes in the short term. The long-term expansion effects overshadow short-term substitution between inputs. The results from this study suggest that dairy producers in South Africa are rational profit maximisers who use resources efficiently to the point where the marginal returns are zero. They allocate bought and self-produced feed components as substitutes in the short-run but treat both inputs as complements in the long-run.

INTRODUCTION

Livestock husbandry and the production of food and by-products from animal origin, is an important part of the highly diverse agricultural sector of the South African economy. In 1996 South Africa produced 2.22 million tons of milk (equivalent to 2 150.49 million litres) by approximately 6 220 commercial farmers. With a gross value of production (1997/1998) of R2 620 million, fresh milk and dairy products represents the fourth largest agricultural sector in South Africa (7,4% of gross value of all agricultural products). The country has the potential to be self-sufficient in milk production, but it currently exports and imports fluid milk and processed dairy products. If the dairy sector strives to attain a larger world market share, it has to be competitive. A proper understanding of the structure of the underlying dairy production technology is the key to improving the sector's competitiveness. Thus, analysis of the supply of fluid milk (the focus of this study) will provide useful information to managers and policy makers for predicting how the dairy sector would respond to shifts in world and local market prices of production inputs and fluid milk. Appropriate and timely adjustment mechanisms can hence be designed to improve the dairy sector's local and international competitiveness.

Analysis of milk supply response continues to be an important part of the international agricultural economics literature. However, the same cannot be said for South Africa [Beyers, 2000]. Contemporary estimation of supply and demand elasticities, structure and efficiency of production and input substitutability in dairy farming is lacking. Such analysis for the industry as a whole and for different production groups within the industry is imperative for a better understanding of the impacts of changes in the business environment and for developing effective policy measures.

The main objective of this study is to analyse the underlying structure of milk production technology in South Africa. Available cross-sectional farm survey data is used to analyse the supply response to policy changes. In addition, the appropriateness of the dual approach to supply response analysis, as applied to South African cross-sectional milk production cost data, is evaluated. This is a positive, rather than a normative approach, which should ideally form the basis for further, more expanded studies. Production cost data was obtained from the Milk Producer Organisation (MPO). This cross-sectional set, of 49 observations for the 1998 production year, contained a substantially detailed record of input use and corresponding prices.

FOCUS OF THE STUDY

SUPPLY ANALYSIS AND ITS APPLICATION

Supply analysis encompass the larger set of techniques that evaluate production responses to output-, input prices and other measurable policy and environmental changes. The theory of the firm is the basis from which analysis is conducted [Colman, 1983]. Analysis of the supply behaviour of firms, aims to improve the understanding of how producers combine inputs in the production process. Agricultural producers are both users of resources and suppliers of agricultural products. The production technology underlying this process can be described through production elasticities, input substitution possibilities, returns to scale and the bias in technology [Thijssen, 1992]. Description of production technology is necessary [Nerlove and Bachman, 1960] and the determinants of agricultural production functions are important as far as they influence the supply of agricultural products and affect the use of inputs (under varying technological conditions) [Thijssen, 1992].

Approaches to supply analysis can be classified into two main groups: normative (programming) and positive (econometric) approaches [Day, 1963; Shumway and Chang, 1977; Colman, 1983; Sadoulet and De Janvry, 1995]. In this study, a positive (econometric) approach is followed.

THE PRIMAL AND DUAL APPROACHES TO SUPPLY ANALYSIS

When the choice falls on a positive approach, the next consideration pertains to the two sub-groups of positive analysis: the *primal approach* and the *dual approach* [Colman, 1983; Sadoulet and De Janvry, 1995]. The *primal approach* involves estimation of the structural production function or stochastic frontier [Coelli, *et al.*, 1998] from cross-sectional or time-series data

[Blackorby, *et al.*, 1978; Fuss and McFadden, 1978]. Amongst other problems with the primal approach, simultaneity bias occurs between inputs and outputs unless experimental data is used [Colman, 1983]. The problem arises from the joint and simultaneous determination of levels of inputs and resulting levels of outputs [Lau and Yotopoulos, 1972; Colman, 1983]. In addition, partial adjustments and adaptive expectations are not taken into account, resulting in over-estimation of short run elasticities, also, the supply elasticities are very sensitive to the chosen functional form [Wipf and Bawden, 1969; Burgess, 1975; Anderson, *et al.*, 1996].

The *dual* or *reduced form approach* involves estimation of a profit function from either cross-sectional data (that shows inter-farm variation in effective prices) or from long run time series that show variation in fixed factors, or from a combination of the two data types [Sadoulet and De Janvry, 1995]. Supply and factor demand are derived analytically. Alternatively, complete systems of supply response can be estimated from the underlying profit function. In systems, cross-equation restrictions are imposed on parameters so that the system derives rigorously from a profit or cost function [Ray, 1982; Colman, 1983; Higgins, 1986; Thijssen, 1992; Sadoulet and De Janvry, 1995; Griffiths, et al., 2000].

Duality between production-, cost- and profit functions implies existence of a correspondence between these functions, such that one or both of the latter can be used to derive the properties of the former [Pope, 1982; Beattie and Taylor, 1993]. This approach is mainly used in cases with limited information on the relevant primal variables and where possible estimation problems are associated with the production function approach [Blackorby, *et al.*, 1978; Fuss and McFadden, 1978; Chambers, 1988; Sadoulet and De Janvry, 1995]. Cost and revenue data is usually more readily available than data on physical input quantities. Therefore, parameters of the production functions can be estimated indirectly through application of duality (via a cost or profit function approach) [Leontief, 1969; Shephard, 1970; Diewert, 1971; Lau and Yotopoulos, 1972; Binswanger, 1974; Varian, 1978; Colman, 1983; Debertin, 1986]. This is where the dual approach attempts to isolate those circumstances under which purely economic phenomena can be used to reconstruct the underlying technology, given that technological restrictions are disclosed in the economic behaviour of agents. Duality implies that well-behaved cost- and profit functions are equivalent to well-behaved production functions [Lopez, 1984; Chambers, 1988].

In the present study, availability of data on primal variables is a problem. Only farm level budget information (cost, profit, prices) was available for this study (as opposed to regional or national aggregated data [Higgins, 1986]]. Accordingly, data available for this study could only support the use of a dual approach to supply analysis.

Milk production in South Africa is not subject to quotas or any other form of centrally determined output level specification. Therefore, farmers are assumed to be maximising profits, subject to technological and environmental constraints. It could be argued that farmers minimise cost for an “expected” level of output. Since the data does not contain a time dimension, the “expected” output cannot be calculated for each farm. The underlying assumption is that the results from a profit maximising as opposed to a cost minimising approach will not differ significantly. The choice in this study falls, therefore, on the profit function approach. Higgins [1986] states that the basic behavioural assumptions required when modelling production possibilities with a profit function approach are that farmers are profit maximisers and that markets are competitive. Both are realistic assumptions in the South African context.

Choice of the Functional Form

The third step in selecting an appropriate approach involves a choice of functional form to represent the data. Quoting Anderson *et al.* [1996], “... the choice of functional form is not a trivial matter”. Anderson, *et al.* [1996] point out three functional forms that seem to dominate in empirical production economics literature. Those forms are the translog, the normalised quadratic and the generalised Leontief functions. They concede that economic theory is not sufficient to determine the suitable functional form, although it does aid in identifying relevant variables and homogeneity restrictions. The preferred functional form is both data and method specific, thus making testing of alternative forms imperative to the selection process [Anderson *et al.,* 1996; Ornelas, *et al.*, 1993].

Correct specification of a functional form is important in so far as it impacts on predicted responses of modelled policy interventions [Anderson *et al.,* 1996]. Bairam [1994], Fuss [1978] and Christensen [1973] emphasised the importance of flexible functional structures that require a minimum of maintained hypotheses. Consequently, Christensen *et al.* [1971, 1973] introduced the transcendental logarithmic ("translog" for short) production function – a non-homogenous function with varying scale elasticity [Berndt, *et al.,* 1973], following on the work of Cobb and Douglas [1928]. Flexible functional forms allow for objective testing of structural properties before imposing the properties on the technology, as is done with restrictive functional forms (e.g. linear functions) [Andersen, *et al.*, 1996].

In this study, the translog function [Christensen, *et al.*, 1973] is hypothesised as an appropriate approximation of the true profit function, as well as a good representation of the underlying production function, due to its flexibility and (consequent) wide use. This study also adopts the normalised quadratic (NQ) profit functions for comparative analysis. Both forms are estimated and the results compared to establish which form is most appropriate, given the data. The normalised quadratic form has the advantage of linearity in the parameters and simple expressions for the elasticities (evaluated at any level of prices and quantities) [Bouchet, *et al.*, 1989].

THE THEORETICAL DUAL PROFIT MODEL

Under the profit function approach, the production function is specified as $h(q, x, z) = 0$, implying that $q = f(x,z)$ where *h* is the technology function with *q* as the vector of output quantities and *x* and *z* the vectors of variable and fixed factors, respectively. Denoting output (milk) price by *p* and the price of inputs by the vector *w*, the restricted profit function (in which only variable costs are deducted from gross revenues) is written as $\pi_r = p'q - w'x$ (with *p*' and *w*' the respective transposed vectors of *p* and *w*). A producer thus chooses levels of inputs and output that will maximise restricted profit (π_r), subject to the technological constraints. Algebraically, the profit maximisation problem can be specified as:

$$\underset{x,q}{Max} \quad \pi_r = (p'q - w'x), \quad s.t. \quad h(q,x,z) = 0 \qquad \textbf{\{Eq.1\}}$$

The solution of the latter optimisation problem is a set of input demand and output supply equations:

$$x = x(p,w,z), \rightarrow \text{Input demand and}$$
$$q = q(p,w,z) \rightarrow \text{Output supply.}$$
$$\text{Therefore}: \quad \pi_r = p'q(p,w,z) - w'x(p,w,z)$$

Ideally, the profit function should satisfy the regularity conditions that would make it a "well-behaved" profit function. These properties are: non-negativity; continuity; twice differentiability; monotonicity - increasing (decreasing) in output (input) prices; non-decreasing in fixed inputs; convexity in prices; concavity in fixed inputs; and homogeneity of degree zero in all prices and homogeneity of degree one in all fixed factors (if the production functions exhibits constant returns to scale, CRS) [Chambers, 1991; Higgins, 1986]. When these properties are satisfied, or imposed, the profit function will be the dual function of the transformation function. Then the parameters of the profit function contain adequate information from which to infer the properties of the underlying technology (e.g. elasticities of substitution, homogeneity, etc) [Higgins, 1986].

When dealing with a single output, the normalised variable profit function is estimated. It represents the ratio of profit to the output price, as a function of relative prices of variable inputs and of quasi-fixed factors. In the case of a multi-output [Färe, *et al.*, 1995] normalised profit function, the numéraire is the output price of the n^{th} commodity. Normalisation has the purpose of removing any money illusion – in other words, firms respond to relative price changes. Normalisation also reduces the demand on degrees of freedom, by effectively reducing the number of equations and parameters to estimate. Algebraically:

$$\pi^* = \pi^*(p^*, w^*, z), \;\; with \;\; w_i^* = \frac{w_i}{p_n} \;\; and \;\; p_i^* = \frac{p_i}{p_n} \qquad \textbf{\{Eq.2\}}$$

where w_i denotes the price of input-i (x_i) and where p_n is the price of the single output, or the price of the nth commodity in the case of multi-output. From the profit function a system of output supply and input demand equations are derived by using Hotelling' s Lemma and the first order conditions (F.O.C) for profit maximisation:

$$\frac{\partial \pi^*}{\partial p_i^*} = q_i, \quad and \quad \frac{\partial \pi^*}{\partial w_i^*} = -x_i \quad \textbf{\{Eq.3\}}$$

The symmetry in outputs and inputs is further exploited by treating inputs as negative outputs, thus simplifying notation:

$$q = \begin{bmatrix} q \\ -x \end{bmatrix}, \quad p = \begin{bmatrix} p^* \\ w^* \end{bmatrix} \Rightarrow q_i = \frac{\partial \pi^*(p,z)}{\partial p_i} \qquad \textbf{\{Eq.4\}}$$

The derived supply and demand functions satisfy the symmetry (of the second order derivatives of the profit function) property, implying that $\frac{\partial q_i}{\partial p_j} = \frac{\partial q_j}{\partial p_i}$, or, stated in terms of elasticities: $\frac{\partial q_i}{\partial p_j} \times \frac{p_j}{q_i} = \frac{\partial q_j}{\partial p_i} \times \frac{p_i}{q_j}$. In addition to these partial elasticities, the total effect of price changes is comprised of a substitution and expansion effect [Higgins, 1986]. Substitution in the input case implies a movement along the isoquant, and a compromise between outputs on the transformation frontier. Expansion implies a movement from one isoquant to another, as output expands along the expansion path to the new transformation frontier [Higgins, 1986]. The corresponding types of elasticities are the Marshallian (uncompensated, long-run) and Hicksian (compensated, short-run) elasticities of substitution. Based on these elasticities, outputs and inputs can be classified into categories of gross or net substitutes or complements in the production process [Higgins, 1986; Deaton and Muellbauer, 1980].

ECONOMETRIC SPECIFICATION

SELECTION OF VARIABLES FOR THE EMPIRICAL ANALYSIS

The data set used for this study contains detailed production cost for fifty dairy farmers. On average, total feed cost constitutes sixty percent (60%) of the total annual cost outlay. A breakdown of self-produced (grazing and administered feed) and purchased feed types, quantities and cost is given in the data. However, data is lacking for different variables on different farms, in other words, the gaps in the data are not consistent within or between farms. The composition of labour remuneration (in terms of cash, rations, farm produce and other benefits) is detailed, but the number of hours allocated to milk production and livestock, respectively, is not recorded. Capital constitutes investment in livestock, as one group, and investment in land, fixed improvements and equipment, as the second group. The herd structure is also reported.

The lack of a time dimension precluded evaluation of technological change. In addition, the small sample size (48 farms with positive profits) places serious limitations on the size of the supply system to be estimated. Therefore, the initial choice of explanatory variables fell on two variable inputs (self-produced feed, and purchased feed) and on three quasi-fixed variables (a proxy for management, livestock capital and labour input). This choice was supported by the records of unit prices for the variable inputs for the majority of the farms. The distinction between self-produced and purchased feed is drawn, based on the assumption that the ability to produce feed lowers the input cost and thus increases profit. Similarly, the ability to purchase feed effectively expands the farm size in the short term [Burton, 1984]. Furthermore, most of the input substitution in dairy production occurs between these two input groups. It is assumed that management determines the efficient allocation of resources. A proxy for managerial ability is used, namely, restricted profit per unit of fixed cost. This measure gives an indication of a farmer’ s ability to generate short-run profit sufficiently to cover medium and long-term costs.

Restricted profit was calculated as gross product value (quantity of milk multiplied by the milk price plus the value of animal trade income) minus the variable inputs (self-produced and purchased feed). The shares of the variable inputs are calculated as the total value of the input (price times quantity) divided by the restricted profit. Only those farms with positive restricted profit are considered (hence only 48 farmers out of the available fifty observations). The aggregated price of traded animals is used as the numéraire for profit normalisation.

TRANSFORMATION OF VARIABLES

When estimating the translog profit system the independent variables are expressed in their natural logarithmic form. This poses problems when some observational units report zero values for a specific input. Given the already small sample size, it was decided not to exclude these observations from the system. As a solution, the relevant input is scaled by a constant, making all observations on that input larger than zero.

The income from animal trade (as the second output activity) is comprised of the sales of calves, heifers, dry cows, bulls and oxen at different free market prices. Similarly, purchased feed constitutes a mix of bought feed components; and self-produced feed constitutes a diverse crop mixture. Due to the small sample size, the dynamics of each of these three processes could not be modelled. The alternative is to construct three price indices for an aggregated unit of a traded animal, an aggregate unit of purchased feed and an aggregate unit of self-produced feed per farm [Higgins, 1986]. Following the methodology of Higgins [1986] and Caves *et al.* [1982], a cross-section type Divisia index is constructed for each of the three cases. The form of the index is:

$$\ln P_j^k = \frac{1}{2}\sum_i^g (r_{ij}^k + \bar{r}_{ij})(\ln P_{ij}^k - \overline{\ln p}_{ij}) \qquad \textbf{\{Eq.5\}}$$

The variables in {Eq.5} are defined as: $\ln P_j^k$, the price index for the aggregate-j (e.g. purchased feed) on the farm-k; r_{ij}^k, the share of good-i (i.e.

licks in the purchased feed aggregate) in the aggregate-j on the farm-k; $\bar{r}_{ij}$, the average share of good-i in aggregate-j over all farms; $\ln P_{ij}^{k}$, the natural logarithm of the unit price for good-i in the aggregate-j on farm-k; and $\overline{\ln p}_{ij}$, the average of the natural logarithm of the price of good-i in aggregate-j over all farms. The index has a base of zero (the average of the sample) and for the farms that report zero values for an input or output, the average sample price was used in calculating the index.

The set of dependent and independent variables included in the supply system is given in Table 1.

Table 1: Variable definition

Variable	Description	Unit of measurement	Variable	Description	Unit of measurement
Q_{mlk}	Total quantity of fluid milk produced	Litres	S_{mlk}	Share of milk revenue in the restricted profit	
Q_{trd}	Total aggregate units of animals traded	Aggregated unit	S_{trd}	Share of trade revenue in restricted profit	
Q_{fb}	Total aggregated units of purchased feed demanded	Aggregated unit	S_{fb}	Share of purchased feed in restricted profit	
Q_{fs}	Total aggregated units of self-produced feed demanded	Aggregated unit	S_{fs}	Share of self produced feed in restricted profit	
P_{mlk}	Unit price of a litre of milk	Rand / litre	MPRX	Index for management efficiency	Profit / Fixed cost
P_{trd}	Price index for an aggregated animal	Rand / unit	LCAP	Livestock capital (average of	Rand

	unit traded			beginning and ending stock)	
P_{fb}	Price index for an aggregated unit of feed purchased	Rand / unit	LABR	Labour cost (cash, rations, payment in kind and grants)	Rand
P_{fs}	Price index for an aggregated unit of self-produced feed	Rand / unit	Profit (π)	Restricted profit (Gross value of production – variable input cost)	Rand

THE NORMALISED QUADRATIC PROFIT FUNCTION

In the multi-output case, profit and prices are normalised by the price of the n[th] commodity. Algebraically it is stated as follows:

$$\pi^* = \frac{\pi}{p_n} = \alpha_0 + \sum_i \alpha_i p_i + \frac{1}{2}\sum_{ij} \beta_{ij} p_i p_j + \sum_{im} \beta_{im} p_i z_m \quad \begin{bmatrix} i, j = 1, \cdots, n-1 \\ \beta_{ij} = \beta_{ji} \end{bmatrix} \quad \textbf{\{Eq.6\}}$$

where $p = \begin{bmatrix} p^* \\ w^* \end{bmatrix}$ is the vector of normalised output and input prices. This type of profit function is homogenous in prices, but not in the fixed factors (z). The derived system of output supply and input demand equations is:

$$q_i = \alpha_i + \sum_j \beta_{ij} p_j + \sum_m \beta_{im} z_m \quad and \quad q_n = \pi^* - \sum_i p_i q_i = \alpha_0 - \frac{1}{2}\sum_{ij} \beta_{ij} p_i p_j, \quad \textbf{\{Eq.7\}}$$

for commodity n, whose price was the numéraire

Price elasticities are computable at any value of prices and quantities, such that:

$$E_{ij} = \frac{\beta_{ij} p_j}{q_i}, \quad i, j \neq n, \quad E_{nj} = \frac{1}{s_n}\sum_i s_i E_{ij}, \quad and \quad E_{nn} = -\sum_i E_{ni}. \textbf{\{Eq.8\}}$$

This function has the distinct advantages of linearity in the parameters and simple equations for the elasticities [Bouchet *et al.*, 1989; Thijssen, 1992; Sadoulet and De Janvry, 1995].

THE TRANSLOG PROFIT FUNCTION

The translog specification is a second-degree flexible function in prices and fixed inputs, with variable elasticities of substitution and is considered as a second order approximation of any functional form. Algebraically, the translog is specified as follows [Christensen, *et al.*, 1973; Capalbo *et al.*, 1988]:

$$\ln\pi = \alpha_0 + \sum_i \alpha_i \ln p_i + \sum_m \beta_m \ln z_m + \frac{1}{2}\sum_{ij}\beta_{ij}\ln p_i \ln p_j + \frac{1}{2}\sum_{mn}\gamma_{mn}\ln z_m \ln z_n + \sum_{im}\gamma_{im}\ln p_i \ln z_m \qquad \textbf{\{Eq.9\}}$$

Certain restrictions are required when the properties of homogeneity with respect to prices and fixed factors are imposed. The necessary restrictions are symmetry, additivity and homogeneity, respectively:

$$\beta_{ij}=\beta_{ji}; \quad \gamma_{mn}=\gamma_{nm}; \quad \sum_i \alpha_i = 0; \quad \sum_m \beta_m = 1; \quad \sum_i \beta_{ij} = \sum_m \gamma_{mn} = \sum_i \gamma_{im} = \sum_m \gamma_{im} = 0.$$

The system of derived factor demand and output supply is:

$$q_i = \frac{\pi}{p_i}\left[\alpha_i + \sum_j \beta_{ij}\ln p_j + \sum_m \gamma_{im}\ln z_m\right] \qquad \textbf{\{Eq.10\}}$$

Elasticities are calculated as: $E_{ij} = s_j + \frac{\beta_{ij}}{s_i};\ and\ \ E_{ii} = -1 + s_i + \frac{\beta_{ii}}{s_i}$ **{Eq.11}**

The translog function has an additional beneficial property. Differentiation of the profit function with respect to input or output price (Hotelling' s Lemma) yields the profit-share equation for that specific input or output [Christensen, *et al.*, 1973]. Higgins [1986] clearly shows that:

$$\frac{\partial \ln\pi}{\partial \ln p_i} = \frac{\partial\pi}{\partial p_i}\times\frac{p_i}{\pi} = \frac{q_i p_i}{\pi} = S_i = \alpha_i + \sum_j \beta_{ij}\ln p_j + \sum_m \gamma_{im}\ln z_m \quad i = 1,\ldots,n \qquad \textbf{\{Eq.12\}}$$

The profit shares are the basis from which to compute price elasticities of inputs and [Christensen, *et al.*, 1973; Thijssen, 1992; Debertin, 1986; Sadoulet and De Janvry, 1995; Binswanger, 1974]. For the translog model, Higgins [1986] defines the Marshallian and Hicksian elasticities of substitution as follows:

$$Marshallian: \eta_{ii} = \frac{\beta_{ii}}{S_i} + S_i - 1 \quad and \quad \eta_{ij} = \frac{\beta_{ij}}{S_i} + S_j \quad i,j = 1,\ldots,n \qquad \textbf{\{Eq.13\}}$$

where η_{ii} and η_{ij} represent the own-price elasticity of supply (and demand), and the cross-price elasticity of supply, respectively. The system of compensated elasticities is represented in matrix form as follows (following the notation used by Higgins, [1986]) and this applies to the Normalised Quadratic specification' s compensated elasticities as well:

$$Hicksian: \quad \begin{aligned} \{\eta_{lk}^{S}\} &= \{\eta_{lk}\} - \{\eta_{lt}\} \times \{\eta_{th}\}^{-1} \times \{\eta_{tl}\} \\ \{\eta_{th}^{S}\} &= \{\eta_{th}\} - \{\eta_{tl}\} \times \{\eta_{lk}\}^{-1} \times \{\eta_{lt}\} \end{aligned} \qquad \textbf{\{Eq.14\}}$$

The matrices and symbols are defined as: $\{\eta_{lk}^{S}\}$, Hicksian *input demand* elasticities with respect to *input prices*; $\{\eta_{lk}\}$, Marshallian *input demand* elasticities with respect to *input prices*; $\{\eta_{lt}\}$, Marshallian *input demand* elasticities with respect to *output prices*; $\{\eta_{th}\}$, Marshallian *output supply* elasticities with respect to *output prices*; $\{\eta_{tl}\}$, Marshallian *output supply* elasticities with respect to *input prices*; and $\{\eta_{th}^{S}\}$, Hicksian *output supply* elasticities with respect to *output prices*.

SINGLE EQUATION, SYSTEM- OR FULL INFORMATION ESTIMATION PROCEDURES

Supply response from a profit function approach can be estimated through two procedures. Firstly, by estimating the profit function itself from data on farm profits, exogenous explanatory variables, prices and fixed factors. Secondly, since observations on inputs and outputs or their respective shares are readily available, direct estimation of the output supply and input demand equations can be done without imposing assumptions of cost minimising or profit maximising behaviour.

When following the second approach, all equations (assuming a profit-share approach) are jointly estimated except for the profit function itself, because of the linear dependency of the profit function on the coefficients of the share equations (identification requirement). In the translog case, the dependent variables are the profit shares, which add to one. Therefore, one share equation is dropped from the system to avoid singularity of the variance-covariance matrix. The parameters of the dropped equation are derived from the estimated parameters. An additional advantage of the system approach is the avoidance of multicollinearity problems. In the profit function, square and cross product terms introduce multicollinearity, but these terms are not present in the share- or input demand and output supply equations of the system.

For the system of input demand and output supply functions to be compatible with profit maximisation, monotonicity and convexity of the underlying profit function, as well as homogeneity and symmetry must hold. The unrestricted system could be estimated and then the theoretical constraints could be formally tested, both locally and globally [Capalbo *et al.*, 1988]. The latter effectively provides a test for profit maximising behaviour [Lopez, 1980].

The econometric estimation of a system of equations can be done with various techniques[1] [Johnston, 1984; Zellner, 1987; Pindyck and Rubinfield, 1991; Johnston and DiNardo, 1997; Greene, 1997]. For this study, Seemingly Unrelated Regression (SUR) and Ordinary Least Squares (OLS) estimation techniques are considered and compared to determine the appropriate estimation technique, given the data set. SUR estimation (also known as the multivariate regression or Zellner' s method) [Zellner, 1962] accounts for both heteroskedasticity and contemporaneous cross-equation error correlation. This technique is appropriate when all the right hand side variables are assumed exogenous, and when some common factors, which are not explicitly modelled, influence the disturbances across equations [Zellner, 1962; Johnston and DiNardo, 1997]. Using the Iterative-SUR makes the system indifferent to the choice of the dropped share equation. In addition, the cross-equation symmetry restrictions and possible contemporaneous correlation between the errors of the various share equations, justify the choice of this method [Higgins, 1986; Pindyck and Rubinfeldt, 1991; Kotsoyannis, 1981; Johnston and DiNardo, 1997].

While the systems approach allows for cross-equation restrictions and takes account of cross-equation error correlation, it does come at a cost. Misspecification of an equation within the system may contaminate estimates

[1] Seemingly Unrelated Regressions (SUR); Full Information Maximum Likelihood (FIML); Generalised Method of Moments (GMM); Two-stage Least Squares (2SLS); Three-stage Least Squares (3SLS), Weighted Least Squares (WLS), Ordinary Least Squares (OLS), etc.

of the other parameters. When employing single equation estimation, only the parameters of the misspecified equation are affected. Thus, OLS provides an intuitive test for the correct specification of the different equations in the system approach. If the system estimation yields unsatisfactory results, the single equation OLS results may indicate which equation(s) causes the problems.

Each specified equation for both methods contains an additive error term that captures the unexplained difference between the profit maximising levels of input and output versus the realised levels [Higgins, 1986]. The error term will inexorably capture the effect of all variables that are not explicitly specified as well as some quality differences in inputs and outputs. No quality distinctions are reported and could thus not be incorporated. In addition, the cross-sectional nature of the data leads to the use of White's Heteroskedasticity Consistent Variance Co-variance Estimator [White, 1980] to account for possible heteroskedasticity of unknown form.

A NOTE ON PRICE VARIABILITY IN CROSS-SECTIONAL STUDIES

For estimation of a profit function or a system, sufficient price variation is necessary. While this is seldom a problem in time series data, the nature of price variation in cross-sectional studies draws attention [Higgins, 1986; Quiggin and Bui-Lan, 1984; Dawson and Hubbard, 1987]]. Difference in market prices can be the result of transaction cost such as transport or marketing cost due to differences in farms' proximity to markets. Monopoly power of processors, input suppliers or co-operatives and discounts for bulk sales or purchases may also cause variation in effective prices paid and received. The use of a profit function approach is not invalidated by these causes of price variation. However, if price variation can be ascribed to quality differences in inputs and outputs, the source of variation should be modelled, thereby removing the required variability in prices.

When dealing with aggregated inputs and outputs the price differences could be a result of the difference in the composition of the aggregates, causing the price index to be correlated with the error term (resulting in biased estimates). Similarly, difference in managerial efficiency (in interpreting market signals, timing production actions, etc.), which is not accounted for explicitly or correctly, will bias the parameter estimates [Higgins, 1986]. The assumption is made that the observed price variability is due to proximity to markets and to bulk discounts, as well as preferential contracts with processors. Furthermore, the proportion of price variability that is due to managerial differences is assumed to be captured by the management proxy.

TESTING THE PROPERTIES OF THE PROFIT FUNCTION

Under the assumptions of profit maximising behaviour with a continuous and a twice-differentiable profit function, the parameters of the estimated equations must satisfy symmetry, convexity, monotonicity and homogeneity conditions. The estimated input demand and output supplies and profit should be zero or positive (property of non-negativity). This was evaluated at the level of each farm. By definition, quantities are non-negative values. Similarly, a profit maximising farmer will rather produce zero output than to incur negative profits.

Monotonicity requires that the profit function strictly increases in output prices and strictly decreases in input prices [Chambers, 1988; Capalbo *et al.*, 1988; Higgins, 1986]. This property is tested through evaluation of the first derivatives of the profit function with respect to input and output prices. In the translog case, this implies evaluation of the profit shares. For inputs, the first derivatives of the profit function with respect to the input price should be non-positive. The first derivatives of the profit function with respect to the output prices should be non-negative. Since the functions approximate the true profit function and the first derivatives are expressions in the levels of the variables, the evaluation is done at the point of approximation[2] [Capalbo *et al.*, 1988]. In the normalised quadratic case, this implies setting the values of the variables to zero and for the translog function the values are set to one.

[2] The values of the variables in the expression of the elasticities are set to one so that the parameter values provide a proxy of the point elasticities.

The necessary condition for convexity is that the Hessian matrix of second order derivatives of the profit function with respect to all prices be positive semi-definite. This implies that all the principal minors must have non-negative determinants {Capalbo *et al.*, 1988]. This follows from the fact that $\partial^2\pi/\partial p_i\partial p_i = \partial Q_i/\partial p_i > 0$ and $\partial^2\pi/\partial w_i\partial w_i = -\partial X_i/\partial w_i > 0$, making the profit function convex in input and output prices (i.e. output supply is upward sloping and input demand is downward sloping).

The Wald-test was used to test for the homogeneity restrictions. The Normalised Quadratic profit function is homogeneous of degree zero in prices (by construction), but non-homogenous in fixed factors. For the translog profit function to be homogeneous, the symmetry condition ($\beta_{ij}=\beta_{ji}$ and $\gamma_{mn}=\gamma_{nm}$,)' the additivity restriction ($\Sigma_i\ \alpha_i=0$ and $\Sigma_m\beta_m=1$), as well as the condition that the sum of the coefficients of the squared and interaction terms are zero ($\Sigma_i\beta_{ij}=\Sigma_m\gamma_{mn}=\Sigma_i\gamma_{im}=\Sigma_m\gamma_{im}=0$) must hold. However, homogeneity in prices can also be imposed by normalising the translog profit function. Symmetry is imposed due to the restricted sample size. Without the symmetry condition, there are not sufficient degrees of freedom in order to estimate all the parameters of the specified equations.

RESULTS[3]

THE NORMALISED QUADRATIC

Application of Zellner' s Iterative Seemingly Unrelated Regression method (ISUR) yielded more coefficients with higher significance levels than coefficients from the OLS estimations. Estimation of the full system (see {Eq.7}) indicated that the quasi-fixed variables had a very significant relation with restricted normalised profit. Since this could be the cause for observed unexpected signs of the price variables, it was decided to drop the livestock capital and labour variables, but keep the management proxy. The modified supply system was estimated with the SUR estimator [Zellner, 1962] and the

[3] To avoid unnecessary repetition, the term "not rejected" is used without specification of the level of acceptance when the tested hypothesis cannot be rejected at the 1%-, 5%-, 10%- and 15%-level of acceptance. In all the other circumstances, the level of acceptance will be specified.

results are presented in

Table 2. More degrees of freedom were available due to the reduced number of parameters to be estimated.

The results indicate a substantial improvement: the t-ratios improved and the coefficients of the milk price variables have the expected sign (α_1 and β_{11}). Milk supply responds positively towards its price and negatively to increased feed prices (β_{12} and β_{13}). In addition, higher levels of management has an increasing effect on milk supply (β_{1M}).

From the results of the estimated quantity of purchased feed equation (Q_{FB}), it follows that the price of milk affects the demand for purchased feed negatively (β_{12}) and the price of purchased feed has a decreasing effect on the demand for the input (α_2 and β_{22}). The coefficient of the price of self-produced feed has the expected sign (for the hypothesis of substitution), but is statistically insignificant (β_{23}). The management proxy shows a positive, yet insignificant, influence on the demand for purchased feed: improved management is associated with more intense use of purchased feed (β_{2M}).

In the case of the self-produced feed demand, the price of milk is neither significant nor has the expected sign - purchased feed responds negatively to an increase in milk prices. An *a priori* hypothesis was that purchased and self-produced feeds are substitutes. This is confirmed by these results: despite the statistical insignificance of the β_{23}-coefficient, the sign is positive. Self produced feed demand responds positively to an increase in the price of purchased feed, and negatively to its own price (α_3 and β_{33}). A higher level of management is in this case is associated with lower levels of self-produced feed use (β_{3M}).

Table 2: Modified NQ supply system

Variable	Symbol	Coeff.	S.E.	t-Stat.	Prob.	Variable	Symbol	Coeff.	S.E.	t-Stat.	Prob.
Constant	$\square_{0}$	86773	31,928	2.72	0.01	$(P^*_{MLK})(P^*_{FB})$	$\square_{12}$	-434481	194299	-2.24	0.03
P^*_{MLK}	$\square_{1}$	622162	153990	4.04	0.00	$(P^*_{MLK})(P^*_{FS})$	$\square_{13}$	-37738	28220	-1.34	0.18
P^*_{FB}	$\square_{2}$	-423386	115939	-3.65	0.00	$(P^*_{FB})(P^*_{FS})$	$\square_{23}$	38105	25091	1.52	0.13
P^*_{FS}	$\square_{3}$	-146710	33194	-4.42	0.00	$(P^*_{MLK})(Z_{MPRX})$	$\square_{1M}$	10676	44052	0.24	0.81
$(P^*_{MLK})^2$	$\square_{11}$	373857	203643	1.84	0.07	$(P^*_{FB})(Z_{MPRX})$	$\square_{2M}$	11785	31267	0.38	0.71
$(P^*_{FB})^2$	$\square_{22}$	533709	219724	2.43	0.02	$(P^*_{FS})(Z_{MPRX})$	$\square_{3M}$	-4200	9386	-0.45	0.66
$(P^*_{FS})^2$	$\square_{33}$	40783	7372	5.53	0.00	Dependent Variables: $\square^*$, Q_{MLK}, Q_{FB}, Q_{FS}					N=48

STRUCTURAL PROPERTIES TESTS

In eight of the cases either negative profits or negative supply or demand quantities were estimated. None of these cases reported simultaneous negative profits or quantities. These results are not sufficient to classify the particular farms as non-profit maximising – small sample size bias, contamination due to aggregation and due to incorrect specification of supply or demand equations all contribute to reduced confidence in the estimation outputs. Evaluation of the first derivatives of the normalised profit function with respect to normalised input and output prices (at the point of approximation) revealed that profit is monotonically increasing in milk prices ($\alpha_1 > 0$) and monotonically decreasing in purchased feed prices (α_2, $\alpha_3 < 0$). For convexity in all prices, it is required that the determinants of the principal minors (of the Hessian matrix of normalised profit to prices - H_{PP}) are non-negative, i.e. positive semi-definiteness of the Hessian matrix. The elements of the Hessian matrix are the β_{ij}-coefficients from

Table 2. $|H_1| = 373857 > 0$, $|H_2| = 10E+09 > 0$ and $|H_3| = 4E+14 > 0$, implying that H_{PP} is positive semi-definite (convexity in prices). The latter result is in accordance with the requirements for well-behaving profit functions.

ELASTICITY CALCULATIONS

Table 3 reports the Marshallian elasticities (see {Eq.8}), calculated at the sample means.

Table 3: Marshallian elasticities calculated at sample means

$E(q_i / p^*_j)$	P^*_{mlk}	P^*_{fb}	P^*_{fs}
Q_{mlk}	0.79	-0.74	-0.09
Q_{fb}	1.62	-1.60	-0.15
Q_{fs}	0.42	-0.34	-0.48

The results indicate plausible results: higher milk prices induce higher demand for purchased feed (1.62); feed components' own-price responses are negative and cross-price responses indicate that purchased and self-produced feed inputs are complements. From the modified system' results, milk supply elasticities are consistent with *a priori* expectations. Milk supply is consistently more intensive in purchased feed use.

Using the results from

Table 2 and Table 3 and the Hicksian elasticity formulae from {Eq.14}, the Hicksian input demand elasticities with respect to input prices are calculated as follows.

$$\{\eta_{lk}^{S}\} = \{\eta_{lk}\} - \{\eta_{lt}\} \times \{\eta_{th}\}^{-1} \times \{\eta_{tl}\}$$
$$= \begin{bmatrix} -1.65 & -0.15 \\ -0.34 & -0.48 \end{bmatrix} - \begin{bmatrix} 1.62 \\ 0.42 \end{bmatrix} \times [0.79]^{-1} \times [-0.74 \quad -0.09]$$
$$= \begin{bmatrix} -0.09 & 0.02 \\ 0.05 & -0.44 \end{bmatrix} = \begin{bmatrix} \eta_{FB,FB}^{S} & \eta_{FB,FS}^{S} \\ \eta_{FS,FB}^{S} & \eta_{FS,FS}^{S} \end{bmatrix}$$

The Hicksian responses confirm that both inputs are normal goods (demand decreases when prices increase) with highly inelastic compensated elasticities as opposed to the uncompensated (long run) elasticities. The inputs are gross complements (short run), but net substitutes (long run) in the production process, with self-produced feed demand being more sensitive to purchased feed price changes than visa versa. This is in line with expectations since the price of purchased feed is determined in the open market, where the influence of self-produced feed prices play a comparatively small part. The short run (compensated) elasticities are less elastic than the long run elasticities, probably due to higher flexibility to change feeding and grazing patterns in the long-run.

The difference between uncompensated (Marshallian) and compensated (Hicksian) elasticities indicates the effect of the expansion process (movement to new production possibility frontiers) due to price changes and subsequent production shifts. The long-term (uncompensated) input demand responses are mainly a result of long-term adjustments.

Similarly, the Hicksian output supply elasticity with respect to output prices are as follows.

$$\{\eta_{th}^{S}\} = \{\eta_{th}\} - \{\eta_{tl}\} \times \{\eta_{lk}\}^{-1} \times \{\eta_{lt}\}$$
$$= [0.79] - [-0.74 \quad -0.09] \times \begin{bmatrix} -1.60 & -0.15 \\ -0.34 & -0.48 \end{bmatrix}^{-1} \times \begin{bmatrix} 1.62 \\ 0.42 \end{bmatrix}$$
$$= [0.04] = [\eta_{MLK,MLK}^{S}]$$

The short-run (compensated) elasticity of milk supply with respect to its own price is positive, yet inelastic. The long-run response (0.79) is mainly due to contraction in supply (-0.83).

The Translog

System estimation of the profit function and share equations produced overall improvements in variables' significance levels compared to single equation OLS results. However, it must be noted that the poor fits were obtained on the share of self-produced feed and on the share of trade income. This casts doubt on the reliability of the system results. Yet, since the quantity of milk supplied and the level of feed administered are determined simultaneously with profit, it is believed that the system results should represent a more realistic scenario. The most meaningful results were obtained from the system containing the profit function, the shares of milk, purchased and self-produced feed equations – estimated using Zellner' s iterative seemingly unrelated regression method (ISUR). The trade-income share equation was omitted.

Similar to the Normalised Quadratic, the highly significant quasi-fixed variables' coefficients together with unexpected signs for the price variables, obtained from single equation OLS estimation, prompted alternative specification of the system. Livestock capital and labour was dropped and homogeneity was imposed through normalisation of the profit function with the price of traded animals. The results of this process are presented in **Error! Reference source not found.**.

Table 4: Modified Normalised Translog profit system

Variable	Symbol	Coeff.	S.E.	t-Stat.	Prob.	Variable	Symbol	Coeff.	S.E.	t-Stat.	Prob.
Constant	$\square_{0}$	11.10	0.27	41.84	0.00	$Ln(Z_{MPRX})$	$\square_{M}$	1.67	0.36	4.65	0.00
$Ln(P^*_{MLK})$	$\square_{1}$	2.02	0.24	8.38	0.00	$Ln(Z_{MPRX})^2$	$\square_{MM}$	-0.89	0.19	-4.62	0.00
$Ln(P^*_{FB})$	$\square_{2}$	-0.65	0.22	-2.93	0.00	$Ln(P^*_{MLK})Ln(Z_{MPRX})$	$\square_{1M}$	-0.19	0.17	-1.09	0.28
$Ln(P^*_{FS})$	$\square_{3}$	-0.12	0.13	-0.98	0.33	$Ln(P^*_{FB})Ln(Z_{MPRX})$	$\square_{2M}$	-0.01	0.15	-0.08	0.94
$Ln(P^*_{MLK})^2$	$\square_{11}$	1.19	0.59	2.01	0.05	$Ln(P^*_{FS})Ln(Z_{MPRX})$	$\square_{3M}$	-0.30	0.11	-2.82	0.01
$Ln(P^*_{FB})^2$	$\square_{22}$	1.39	0.62	2.22	0.03	$Ln(P^*_{FB})Ln(P^*_{FS})$	$\square_{23}$	0.38	0.14	2.65	0.01
$Ln(P^*_{FS})^2$	$\square_{33}$	0.33	0.12	2.78	0.01						
$Ln(P^*_{MLK})Ln(P^*_{FB})$	$\square_{12}$	-1.25	0.58	-2.15	0.03	Dependent Variables:	$Ln(\tilde{\square}P_{TRD}), S_{MLK}, S_{FB}, S_{FS}$				
$Ln(P^*_{MLK})Ln(P^*_{FS})$	$\square_{13}$	-0.44	0.17	-2.53	0.01	N=48					

Both the α_1 and β_{11} coefficients are statistically significant, whilst also corresponding to *a priori* expectations on milk's own-price response. Both feed variables have significant β_{ii}-coefficients, however, their signs do not correspond with *a priori* expectations of own-price responses. In the derived milk share equation, purchased and self-produced feed prices induce negative supply responses ($\square_{12}$ and $\square_{13}$). The management proxy is negatively related to milk supply ($\square_{1M}$). Evaluating the purchased feed share equation reveals that increased milk prices would reduce the demand for purchased feed inputs. Increases in self-produced feed prices would increase the demand for purchased feed. Conversely, improved management practices are associated with lower demand for purchased feed inputs. Self-produced feed shares would decrease with increases in milk prices and with improved management practices. Higher purchased feed prices would increase the demand for self-produced feed – implying substitution between the two input groups.

Structural properties Tests

In four (out of forty-eight) cases, negative input quantities were estimated. None of these cases reported simultaneous negative profits or quantities and these results are not sufficient to classify the particular farms as non-profit maximising – small sample size bias, contamination due to aggregation and due to incorrect specification of supply or demand equations all contribute to reduced confidence in the estimation outputs. Evaluation of the first derivatives of the normalised translog profit function revealed that profit is monotonically increasing in milk prices ($\alpha_1 > 0$) and monotonically decreasing in purchased and self-produced feed prices (α_2, $\alpha_3 < 0$).

For convexity of the normalised profit function in all prices, the modified[4] Hessian matrix of second order derivatives of normalised profit with respect to normalised prices (H^*_{PP}) should have non-negative determinants for the principal minors. The elements of the modified Hessian matrix are ($\gamma_{ii}+\alpha_i^2-\alpha_i$) for the i^{th}-diagonal element, and ($\gamma_{ij}+\alpha_i\alpha_j$) for the off-diagonal elements [Capalbo, *et al.*, 1988]. These results show that the determinants $|H_1|$, $|H_2|$ and $|H_3|$ are positive. The profit function is thus globally convex in prices (i.e. positive semi-definite). The latter result conforms to the requirements for a well-behaving profit function.

Elasticity calculations

Table 5 reports the Marshallian elasticities calculated from the normalised translog profit system estimations. The normalised supply system poses milk as a normal good with an elastic long-run own-price response. Milk supply

[4] The Hessian matrix is modified by dividing it through the vector of ($\pi/p^*_i p^*_j$)

responds negatively to input price increases, especially towards purchased feed prices. Milk production is more intensive in the use of purchased feed: higher milk prices would induce larger demand increases for purchased feed than for self-produced feed inputs. All the own-price responses adhere to theoretical requirements for normal goods. The feed inputs are gross complements (long-run).

Table 5: Marshallian elasticities calculated at sample means

E (q_i / p_j)	P_{mlk}	P_{fb}	P_{fs}
Q_{mlk}	1.70	-1.53	-0.73
Q_{fb}	3.46	-3.41	-0.93
Q_{fs}	2.98	-1.66	-2.16

The normalised translog supply system yields the following Hicksian input demand elasticities;

$$\{\eta_{lk}^{S}\} = \{\eta_{lk}\} - \{\eta_{lt}\} \times \{\eta_{th}\}^{-1} \times \{\eta_{tl}\}$$
$$= \begin{bmatrix} -3.41 & -0.93 \\ -1.66 & -2.16 \end{bmatrix} - \begin{bmatrix} 3.46 \\ 2.98 \end{bmatrix} \times [1.70]^{-1} \times [-1.53 \quad -0.73]$$
$$= \begin{bmatrix} -0.31 & 0.56 \\ 1.00 & -0.89 \end{bmatrix} = \begin{bmatrix} \eta_{FB,FB}^{S} & \eta_{FB,FS}^{S} \\ \eta_{FS,FB}^{S} & \eta_{FS,FS}^{S} \end{bmatrix}$$

Long-run responses are dominated by expansion effects. The feed inputs are (short run) net substitutes in the production process. According to these results, the substitution effect is stronger when increases in purchased feed prices occur. In the long-run expansion in both inputs occur and the demand for the two components moves together.

The normalised translog supply system produces a Hicksian output supply elasticity of –0.08, calculated as follows:

$$\{\eta_{th}^{S}\} = \{\eta_{th}\} - \{\eta_{tl}\} \times \{\eta_{lk}\}^{-1} \times \{\eta_{lt}\} = [-0.08] = [\eta_{MLK,MLK}^{S}]$$

This implies that short-run response to milk price increases is supply reducing, albeit a very in-elastic response. The highly elastic long-run response (1.70) is a result of the substantial contraction in supply (-1.78) following the short-run response.

CHOICE OF MOST APPROPRIATE FUNCTIONAL FORM

Introduction of the two alternative specifications for the Normalised Quadratic and Translog supply systems (both estimated with the ISUR method), produced results that are consistent with profit maximising behaviour. The long-run responses in the modified Normalised Quadratic system are more in-elastic than the Marshallian responses from the modified Normalised Translog system. In contrast, the Normalised Translog Hicksian responses are more elastic than the Normalised Quadratic's short-run responses. A graphical comparison of the actual profit and estimated profit from the two specifications (Figure 1) shows that the Normalised Quadratic offers a closer correspondence with observed values. Yet, it is clear that the cross-sectional nature of the data does not allow a conclusive decision on functional form.

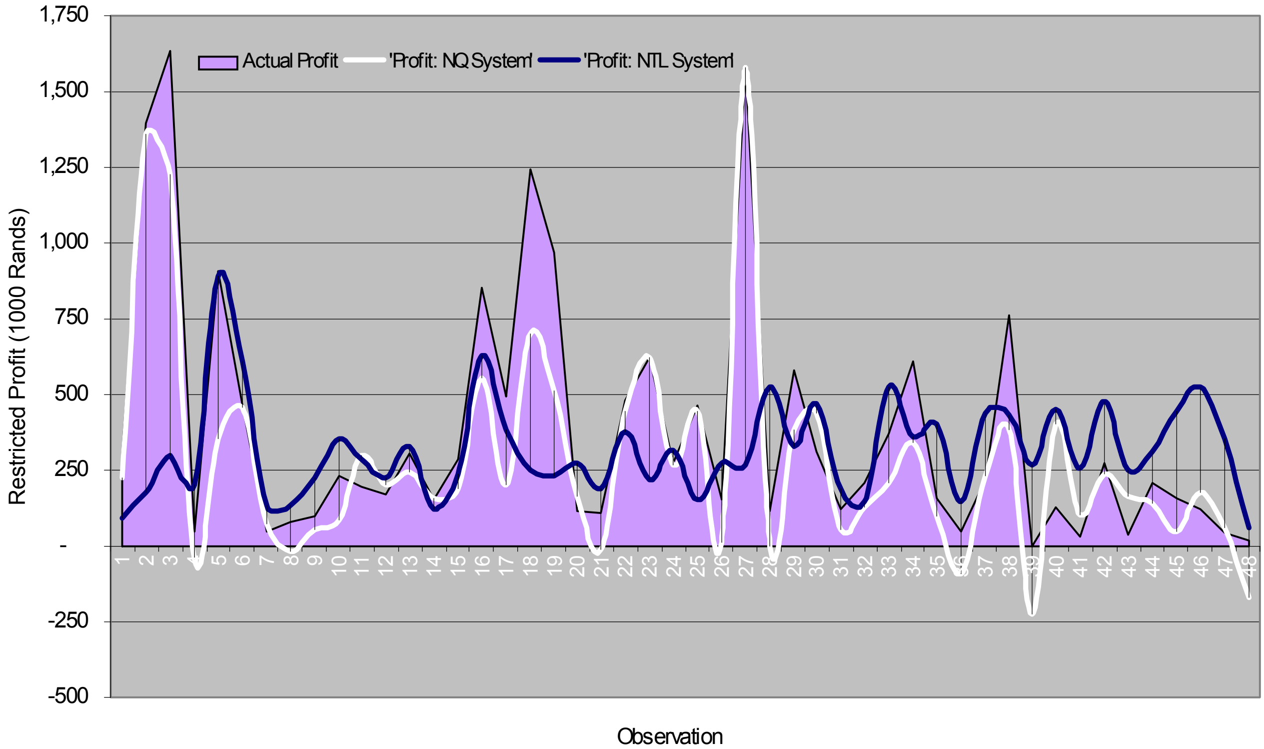

Figure 1: Comparing the Normalised Quadratic and Normalised Translog profit with actual profit levels

Conclusion

For this study, the profit function approach was chosen. Two of the most frequently used functional specifications of profit functions, the Normalised Quadratic and the Translog forms, were applied to the data. In addition, both single equation estimation techniques (OLS) and system estimation techniques (ISUR and FIML) were employed. Those structural properties that were not imposed during estimation were tested afterwards. Due to the small sample size and some missing price observations in the data, as well as the substantial demand on degrees of freedom required by the functional specifications, aggregation of inputs and outputs were necessary. The maintained hypothesis was that farmers are profit maximising agents operating in unregulated markets. Each farm was treated as a multi-input multi-output production unit. The outputs were fluid milk and an aggregate of traded livestock. Variable inputs comprised of two aggregates: aggregated purchased feed and aggregated self-produced feed. The quasi-fixed variables that were considered are livestock capital, total farm labour and a proxy for management efficiency.

Consequently, alternative specifications were tested. As a first step, the translog system was normalised by the price index of traded livestock, similar to the Normalised Quadratic. The specifications of both the Normalised Quadratic and Normalised Translog in which livestock and labour quasi-fixed variables were dropped, produced the best results. The results of both specifications conformed to theoretical requirements for well-behaved profit functions. Uncompensated- and compensated price elasticities of supply and demand were calculated. These elasticities should be interpreted as an example of the type of answers analyses and data similar to that of this study could yield.

In South African milk production, purchased and self-produced feed inputs were gross complements and net substitutes, according to both the Normalised Quadratic and Normalised Translog supply system specifications.

The normalised translog system indicated substantial expansion effects in input demand, while both systems indicated contraction effects for milk supply as a result of milk price changes. Milk production was more intensive in purchased feed use than in self-produced feed use. Finally, input demand was much more price elastic in the short- and long-run than output supply.

From the modified systems' results, improved management would reduce milk supply and input demand in the Normalised Translog case. However, the Normalised Quadratic results indicated that improved management increases milk supply and the demand for purchased feed inputs, whilst reducing the demand for self-produced feed inputs. These inconclusive results indicate that substantial improvement in specification and estimation could be achieved if the data set was expanded in terms of size and detail on prices and quantities of inputs and trading activities.

The results from this study suggest that dairy producers in South Africa are rational profit maximisers who use resources efficiently to the point where the marginal returns are zero. They allocate bought and self-produced feed components as substitutes in the short-run but treat them as complements in the long-run. The intensity of purchased feed use is higher than that of self-produced feed use. This has implications for the animal feed sector in terms of confirming dairy farmers' preferences for scientifically formulated feed components. It also suggests increased pressure on the international competition for already limited natural animal protein sources (fish meal, bone meal, etc.). Analysis of this apparent preference should be checked against time series data, along with other economic variables such as exchange rates, natural animal protein prices, maize prices and investment in animal feed research. In addition, disaggregated modelling of the self-produced and purchased components is necessary to evaluate the substitution status between components within and between the two broad classifications.

Milk supply shows an inclination to contract over time. Hence, this study's results suggest that increased milk prices will not stimulate expansion of the industry. Again, this should be checked against time series data and international patterns of herd size expansion, productivity increases and decreasing producer concentration. Very useful information can be obtained if similar analysis is conducted for different production regions (given the high geographic diversity) and different groups of producers (based on technology preferences or size of operations) to establish what effects input and output price changes might have on short and long term production dynamics.

Supply analysis, as it was performed here, provides testable hypotheses about producer behaviour, and a basis from which supply and demand elasticities for dairy products can be computed for policy simulation and analysis, thus enabling the dairy sector to be proactive in its response to international and local economic stimuli.

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