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ELSEVIER

Agricultural Economics 24 (2000) 87–100

AGRICULTURAL  
ECONOMICS

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# The importance of structure in linking games

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## Abstract

A common approach in modeling negotiations is to apply game theory to single issues. Recent work has suggested that the complexity of international negotiations can be better modeled by linking independent games. Successful linking is possible when the linked issues have compensating asymmetry of similar magnitude. An important result of linked games is that such games produce a greater feasible set of choices relative to the aggregated isolated games. In this paper, we demonstrate that achieving strict dominance of the linked game is not trivial and that results and implications depend on the structures of the isolated games. © 2000 Elsevier Science B.V. All rights reserved.

**Keywords:** Linking games; Cooperation; Partial cooperation; Prisoner's dilemma game

## 1. Introduction

Negotiations over water among sovereign nations are typically difficult. Often geographic and topographic attributes of international water resources cause dramatic differences in political boundaries and economically efficient alignment of water use. Issues of equity typically cause negotiations to stall or agreements to reach outcomes seemingly inconsistent with economic efficiency. The case of Israeli–Palestinian negotiations over water is an example. Frustration has often been expressed in international circles with the pace of the peace process and Israeli–Palestinian negotiations in general because the international community believes large gains can be made with full cooperation that are currently not being captured. This paper investigates explanations of such cases based on equity concerns.

The model presented here suggests that the problem may be that full cooperation, while satisfying economic efficiency considerations, does not satisfy concerns of the sovereigns involved. This problem is shown to arise when the payoffs from negotiations are highly asymmetric and equity is of great concern. Both of these conditions appear to apply in the Israeli–Palestinian case. Payoffs from water projects seem to be highly asymmetric favoring the Israelis because of structural differences in the economies and hydrological circumstances, and equity concerns are great because the potential payoffs are highly inequitable.

When negotiations address an issue with strong asymmetry, grouping issues with compensating asymmetry can be advantageous. Viewing the negotiation of individual issues as games, linking two asymmetric games can be advantageous because countries are more likely to (1) exchange in-kind side payments than monetary side payments and (2) sustain self-enforceable agreements that facilitate credible threats against defection. The conditions under which

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linked games dominate the aggregated results of playing the respective isolated games are the focus of this paper.

In many international negotiations, the characteristics of a prisoner's dilemma are present, i.e., the most attractive actions from the standpoint of one country are detrimental to a neighboring country. Participation in voluntary agreements in such situations is unlikely because both parties face incentives to defect. On the other hand, binding agreements enforced by a third party, while possible within sovereign nations, are not effective at the international level. Fortunately, many such situations occur repeatedly over succeeding time periods. When a game is repeated, incentives for cooperation are greater. However, even when cooperative agreements are reached, countries often face incentives to defect. Particularly with large differences in preferences, full cooperation is less likely to be sustained. The problem without enforcement infrastructure is how to create mutual incentives so that all players prefer continued cooperation. One way is to expand the set of strategies available to players by linking independent asymmetric issues.

Interestingly, situations are observed in practice where a country is a signatory to an agreement even though its welfare appears to decline with cooperation. This observation could suggest that some countries behave irrationally. Alternatively, it is possible that by focusing on a single issue or agreement that the full complexity of international negotiations is not realized. More likely, countries are willing to lose on one agreement in return for a larger gain from another agreement. This paper addresses cases of international negotiations involving multiple agreements.

We consider problems with two 2-strategy, 2-player games in general form and analyze the outcomes of the aggregated isolated games compared to the case of a linked game. A generalized framework is presented for analyzing such problems under different structures using the prisoner's dilemma case as an example. The relevance of linked games is emphasized for the case where equity is important, a typical problem in international negotiations. Linking can offer advantages for international cooperation in principle because full cooperation is often not feasible. However, we show that important conditions must be satisfied for gains to be attained.

## 2. Structures of games

Before discussing the potential of linking games, a delineation of game structures is useful. Although the principles apply to multi-player and multi-strategy games, the discussion here is kept simple and intuitive by examining 2-strategy, 2-player games using the framework of Fig. 1. In each cell, the first entry gives the payoff to Player A and the second entry gives the payoff to Player B if strategies of both players correspond to that cell. Generally, Players A and B each have two strategies, cooperate or defect, in each game. Without loss of generality, the payoffs of the cooperate–cooperate strategies in Game 1 can be represented by (1, 1) and the payoffs of defect–defect in both games can be represented by (0, 0) by simply re-scaling and translating the origin. Payoffs of other various strategies in Games 1 and 2 are depicted in Fig. 1.

Depending on the structure of the games, the Nash equilibrium can be determined as incompletely summarized in Table 1. Four particular structures are emphasized here because they have defection equilibria (see, e.g., Barrett, 1994; Folmer et al., 1993). Games with equilibria consisting only of cooperation are not interesting because the intent here is to explain less than full cooperation.

Linking games of different structures has been investigated by Folmer et al. (1993), Ragland (1995),

		Player B			
		Cooperate		Defect	
Player A	Cooperate	1	1	$q$	$r$
	Defect	$s$	$t$	0	0
(a)					

		Player B			
		Cooperate		Defect	
Player A	Cooperate	$u$	$v$	$w$	$x$
	Defect	$y$	$z$	0	0
(b)					

Fig. 1. A generic case for: (a) Game 1; (b) Game 2.

Table 1  
Alternative game structures

Structure	Conditions for Game 1	Conditions for Game 2 ( $u > 0$ and $v > 0$ )	Nash equilibrium <sup>a</sup>
PD game	$q < 0, r > 1, s > 1, t < 0$	$w < 0, x > v, y > u, z < 0$	D–D
Assurance game	$q < 0, r < 1, s < 1, t < 0$	$w < 0, x < v, y < u, z < 0$	C–C, D–D
Iterated dominance game	$q < 0, r > 1, s < 1, t < 0$	$w < 0, x > v, y < u, z < 0$	D–D
Chicken game	$q > 0, r > 1, s > 1, t > 0$	$w > 0, x > v, y > u, z > 0$	C–D, D–C

<sup>a</sup> The defect strategy is denoted by D and the cooperate strategy is denoted by C.

and Bennett et al. (1998). Folmer et al. consider linking a pair of asymmetric prisoner's dilemma (PD) games. Ragland considers linking two isolated PD games, a PD and a chicken game, two chicken games, and a PD and an iterated dominance game. Bennett et al. consider linking a PD game and an iterated dominance game. By examining the convex hull of the feasible sets of the aggregated isolated games and the feasible set of the linked game, they show that strict dominance of the linked game over the aggregated isolated games can be achieved. Only Ragland gives examples where strict dominance is not obtained.

This paper shows that strict dominance of the linked game over the aggregated isolated games is not obtained in a wide variety of circumstances and embellishes understanding of when dominance is obtained. Given recent advocacy for linking games (e.g., Folmer et al., 1993; Hauer and Runge, 1997), a clear determination of conditions under which linking is advantageous is needed. In general, linking can be advantageous (1) when it expands the feasible choice set for the parties and (2) when it makes new strategies possible that are not permitted in the isolated games.

### 3. Linking prisoner's dilemma games: an example

Consider first the case where each of two isolated games are PD games. In the Nash equilibrium of PD games, both players defect. Figs. 2a and b illustrate the feasible sets of two PD games. Fig. 2a (Fig. 2b) is constructed using the payoffs in Fig. 1a (Fig. 1b) according to conditions in the first row of Table 1. Only the relevant positive quadrant is represented. To consider the problem where cooperation has merit, suppose  $u > 0$  and  $v > 0$ . The convex hull of feasibility in Fig. 2a is defined by points  $(0, 0)$ ,  $(0, Y_1)$ ,  $(1, 1)$ ,  $(X_1, 0)$  where  $X_1 = (s - t)/(1 - t)$  and  $Y_1 = (r - q)/(1 - q)$ . The convex hull of feasibility in Fig. 2b is defined

by points  $(0, 0)$ ,  $(0, Y_2)$ ,  $(u, v)$ ,  $(X_2, 0)$  where  $X_2 = u - v(y - u)/(z - v)$  and  $Y_2 = v - u(x - v)/(w - u)$ .

Figs. 2a and b are constructed for the case where cooperation is superior to convex combinations of partial cooperation, i.e.,  $s_1 = 1 - Y_1 > s_2 = 1/(1 - X_2)$  and  $s_3 = (v - Y_2)/u > s_4 = v/(u - X_2)$  where  $s_1$ ,  $s_2$ ,  $s_3$ , and  $s_4$  are slopes of the respective segments as shown. The alternative case of inferiority is illustrated in Fig. 2c. Inferiority of cooperation corresponds to  $(r - 1)/(1 - t) > (1 - q)/(s - 1)$  in Game 1 and  $(x - v)/(v - z) > (u - w)/(y - u)$  in Game 2. In other words, cooperation is not preferred if Player B's gain from defecting (when Player A cooperates) relative to his loss from Player A's defecting (when Player B cooperates) exceeds Player A's loss from Player B's defecting (when Player A cooperates) relative to his gain from defecting (when Player B cooperates).

Now consider constructing the feasible set of the two aggregated PD games, i.e., the convex hull of the aggregate payoffs for all combinations of play in Figs. 2a and b. Table 2 displays the relevant points and Figs. 3a–d show the exercise graphically depending on whether none, both, or one of the individual PD games have cooperation inferior to convex combinations of partial cooperation, respectively. Note that vertices of the aggregate convex hull correspond to playing pure strategies in each game. Other points on the convex hull correspond to playing mixed strategies, i.e., each of two strategies part of the time in one or both of the games.

#### 3.1. Superiority of cooperation to partial cooperation in both games

Several alternative characterizations define the alternative cases. One distinction is whether cooperation is preferred to combinations of partial cooperation in the separate games. This is the case when the feasible

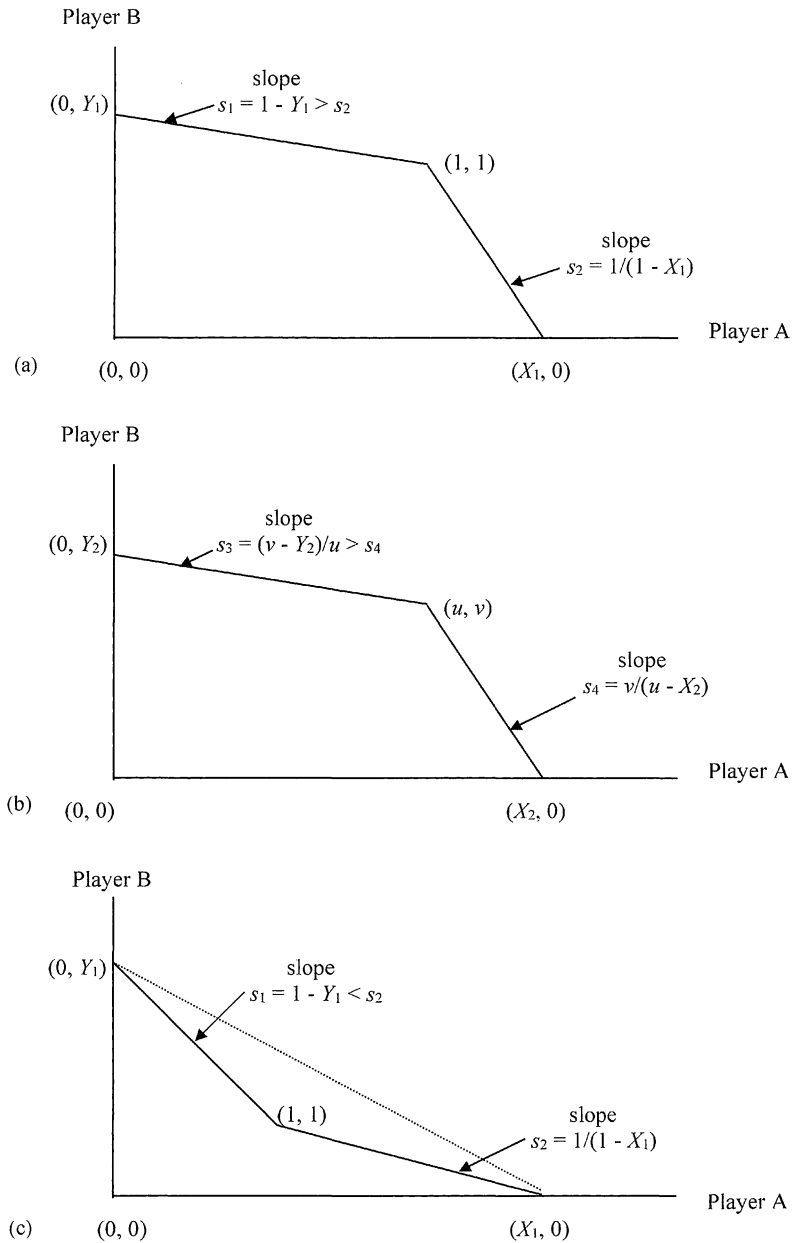


Fig. 2. (a) The feasible payoff set of: (a) Game 1 and (b) Game 2 in the PD Case; (c) Game 1 when cooperation is inferior.

sets of both isolated games have convex frontiers as in Fig. 3a.

### 3.1.1. Aggregation when games are symmetric

The results of aggregation depend on the relationship of the separate games. With PD structure,

$s_1 > s_2$  and  $s_3 > s_4$ . This leaves six possible orderings of the slopes in Figs. 2a and b:  $s_1 > s_2 > s_3 > s_4$ ,  $s_3 > s_4 > s_1 > s_2$ ,  $s_1 > s_3 > s_4 > s_2$ ,  $s_1 > s_3 > s_2 > s_4$ ,  $s_3 > s_1 > s_2 > s_4$ , and  $s_3 > s_1 > s_4 > s_2$ . For purposes of discussion, the last four of these can be characterized as “sufficiently

Table 2  
Aggregation of two isolated games

Point	Payoffs in Game 1 (Fig. 2a)	Payoffs in Game 2 (Fig. 2b)	Aggregate payoffs
1	$(0, Y_1)$	$(0, Y_2)$	$(0, Y_1 + Y_2)$
2	$(0, Y_1)$	$(u, v)$	$(u, Y_1 + v)$
3	$(0, Y_1)$	$(X_2, 0)$	$(X_2, Y_1)$
4	$(1, 1)$	$(0, Y_2)$	$(1, 1 + Y_2)$
5	$(1, 1)$	$(u, v)$	$(1 + u, 1 + v)$
6	$(1, 1)$	$(X_2, 0)$	$(1 + X_2, 1)$
7	$(X_1, 0)$	$(0, Y_2)$	$(X_1, Y_2)$
8	$(X_1, 0)$	$(u, v)$	$(X_1 + u, v)$
9	$(X_1, 0)$	$(X_2, 0)$	$(X_1 + X_2, 0)$

symmetric” because the payoff sets of the isolated games have similar shapes in the positive quadrant of rationality. Manipulation of the points in Fig. 3a reveals that the convex hull will include points  $(0, 0)$ ,  $(X_1 + X_2, 0)$ ,  $(1 + u, 1 + v)$ , and  $(0, Y_1 + Y_2)$  and exclude points  $(X_1, Y_2)$  and  $(X_2, Y_1)$ . Which of the other points will be on the frontier is determined as follows.<sup>1</sup> If  $s_1 < (>)s_3$ , then  $(u, Y_1 + v)$  is (is not) on the frontier and  $(1, 1 + Y_2)$  is not (is). Note that  $s_1 > s_3$  corresponds to  $(1 - r)/(1 - q) > (v - x)/(u - w)$ . In other words, Player A’s gain relative to Player B’s loss associated with Player A’s defection (when Player B cooperates) is greater than Player A’s loss relative to Player B’s gain associated with Player B’s defection (when Player A cooperates). Similarly,  $(1 + X_2, 1)$  is on the frontier when  $s_2 \leq s_4$ , and  $(X_1 + u, v)$  is on the frontier when  $s_2 \geq s_4$ . Thus, the convex hull is completely characterized by conditions on the slopes of the frontiers in Figs. 2a and b.

### 3.1.2. Aggregation when games are asymmetric

Consider next the case when slopes are ordered as  $s_1 > s_2 > s_3 > s_4$  or  $s_3 > s_4 > s_1 > s_2$ . We characterize these cases as sufficiently asymmetric, because the payoff sets of the isolated games have very different shapes in the positive quadrant of rationality. That is, one mixed strategy highly favors one player

in one game and another mixed strategy highly favors the other in the other game. Graphically, this is the case where the entire frontier of one game is steeper than the entire frontier of the other game. The result is that full cooperation is in the interior of the feasible set of the aggregated games. For example, Fig. 3d corresponds to the case where  $s_3 > s_4 > s_1 > s_2$  and the convex hull is defined by  $(0, 0)$ ,  $(X_1 + X_2, 0)$ ,  $(1 + X_2, 1)$ ,  $(X_2, Y_1)$ ,  $(u, Y_1 + v)$  and  $(0, Y_1 + Y_2)$ . If  $s_1 > s_2 > s_3 > s_4$ , then the convex hull is defined by  $(0, 0)$ ,  $(X_1 + X_2, 0)$ ,  $(X_1 + u, v)$ ,  $(X_1, Y_2)$ ,  $(1, 1 + Y_2)$  and  $(0, Y_1 + Y_2)$ . These are interesting cases because full cooperation is not an equilibrium. This situation cannot apply where full cooperation produces an economically desirable outcome.

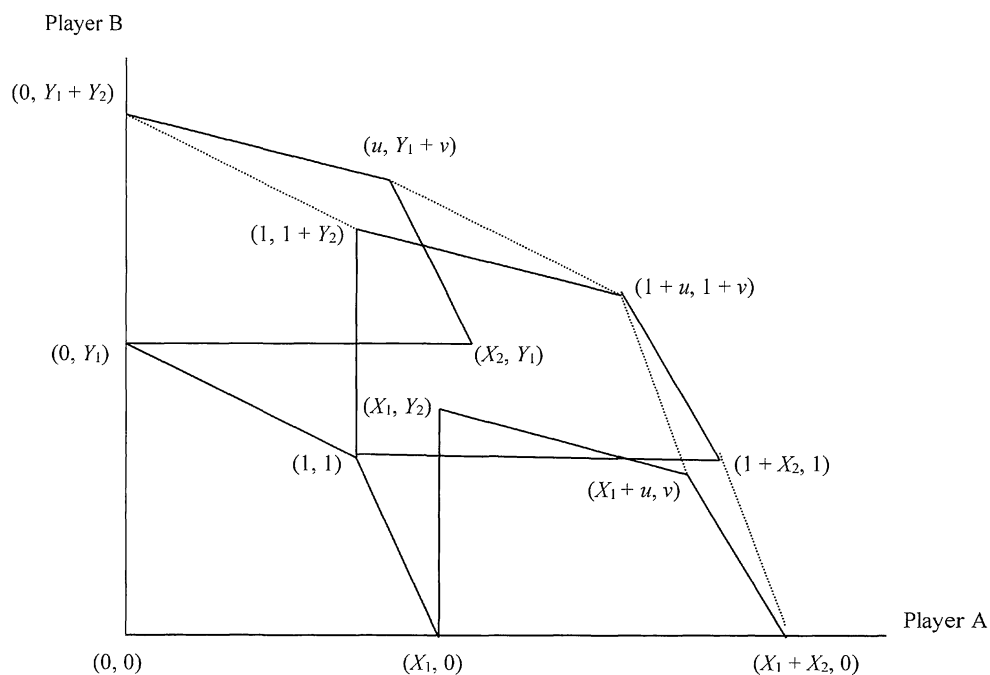
### 3.2. Inferiority of cooperation to partial cooperation in both games

Similar manipulation in the case of Fig. 3b reveals that the convex hull includes  $(0, 0)$ ,  $(X_1 + X_2, 0)$ , and  $(0, Y_1 + Y_2)$  and excludes  $(1 + u, 1 + v)$ ,  $(u, Y_1 + v)$ ,  $(1 + X_2, 1)$ ,  $(X_1 + u, v)$  and  $(1, 1 + Y_2)$ . Which of the other two points are on the frontier can be determined as follows. If  $Y_1/X_1 > (<)Y_2/X_2$ , then  $(X_2, Y_1)$  is (is not) on the frontier and  $(X_1, Y_2)$  is not (is). These conditions depend on how much each player benefits relative to the other player’s loss in cases of partial cooperation compared to no cooperation.

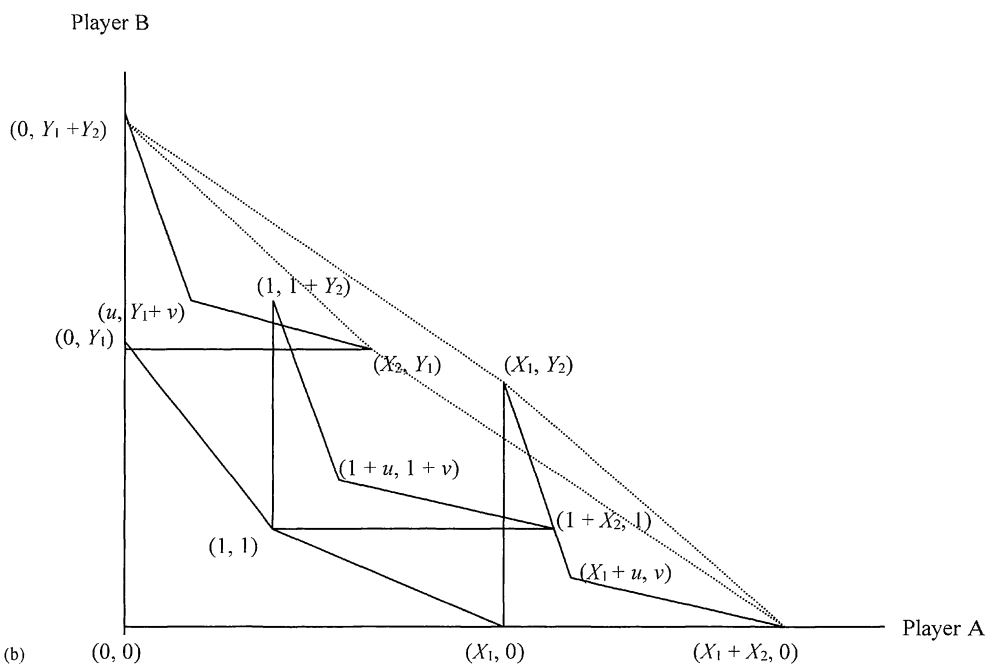
### 3.3. Inferiority of cooperation to partial cooperation in one game

Suppose Game 1 is the game where cooperation is inferior to combinations of partial cooperation as

<sup>1</sup> In this case, as in many others throughout the paper, which points fall on the frontier can be determined by observing a parallelogram. For example, in this case the points  $(0, Y_1 + Y_2)$ ,  $(u, Y_1 + v)$ ,  $(1 + u, 1 + v)$ , and  $(1, 1 + Y_2)$  form a parallelogram in which all the segments have slopes  $s_1$  or  $s_3$ . Because the two end points of the parallelogram fall on the frontier, which of the other two points falls on the frontier is determined by comparing the slopes of the two segments.



(a)



(b)

Fig. 3. (a) The feasible payoff set of aggregated PDs; (b) aggregated PDs when cooperation is inferior; (c) aggregated PDs when cooperation is inferior in one game; (d) an alternative feasible set for aggregated PDs.

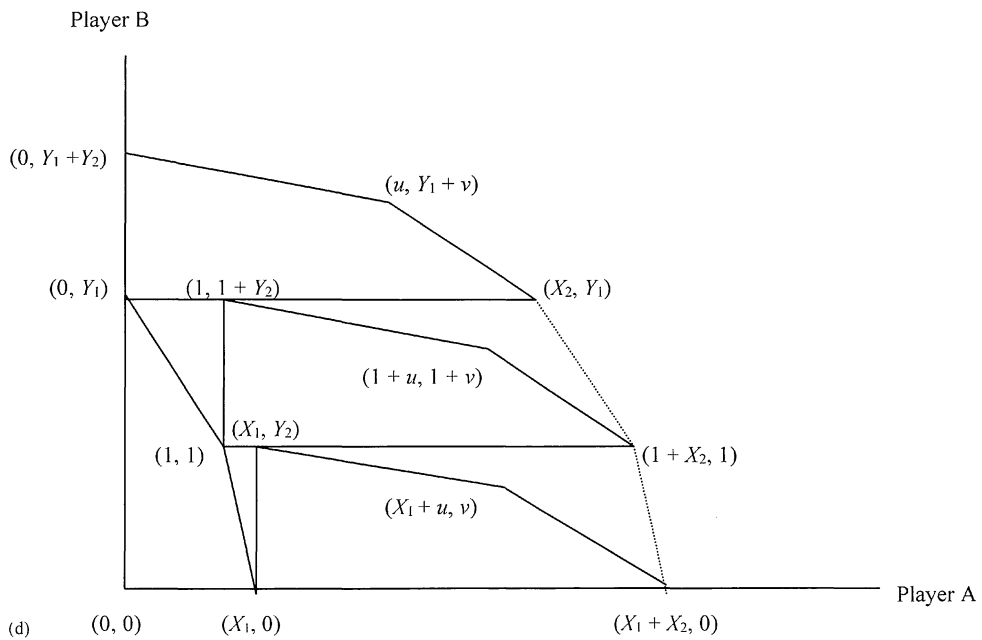
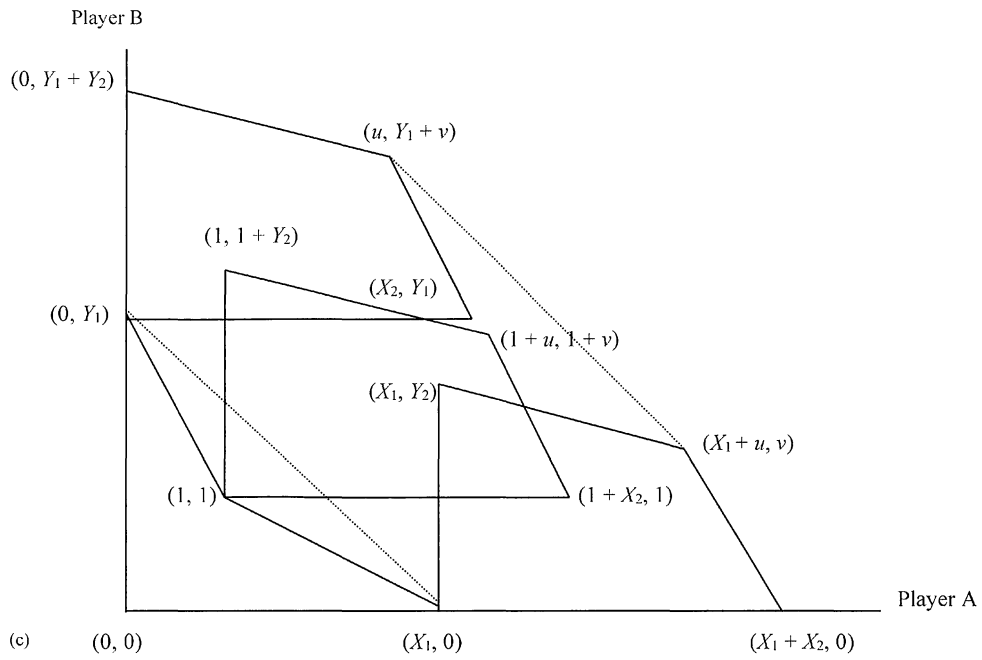


Fig. 3 (Continued).



in Fig. 3c. Note that Fig. 3c is drawn for the case where  $s_4 < -Y_1/X_1 < s_3$ . The convex hull is defined by  $(0, 0)$ ,  $(0, Y_1 + Y_2)$ ,  $(u, Y_1 + v)$ ,  $(X_1 + u, v)$  and  $(X_1 + X_2, 0)$  with all other points from Table 2 in the interior. If  $s_4 < s_3 < -Y_1/X_1$ , then the convex hull is defined by  $(0, 0)$ ,  $(0, Y_1 + Y_2)$ ,  $(u, Y_1 + v)$ ,  $(X_1 + u, v)$  and  $(X_1 + X_2, 0)$  with all other points from Table 2 in the interior. If  $-Y_1/X_1 < s_4 < s_3$ , then the convex hull is defined by  $(0, 0)$ ,  $(0, Y_1 + Y_2)$ ,  $(u, Y_1 + v)$ ,  $(X_2, Y_1)$ ,  $(X_1 + u, v)$  and  $(X_1 + X_2, 0)$  with all other points from Table 2 in the interior. Intuitively, these outcomes correspond to adding the possibilities of Game 2 onto the combinations of partial cooperation in Game 1.

### 3.4. Payoff possibilities with linked games

Next, consider the linked game obtained by adding the payoffs of the two isolated games for all combinations of strategies as depicted in Fig. 4a. The difference in the linked case from the aggregated case is that strategies associated with the individual games need not be individually rational. Only group rationality is required of the aggregate payoffs. This difference is illustrated by comparing the diagrammatic representation of linked strategies in Fig. 4b with the aggregated strategies available in Fig. 3a. By considering strategies with negative payoffs for one player in one game, strategies with higher positive payoffs for the other player may be available.

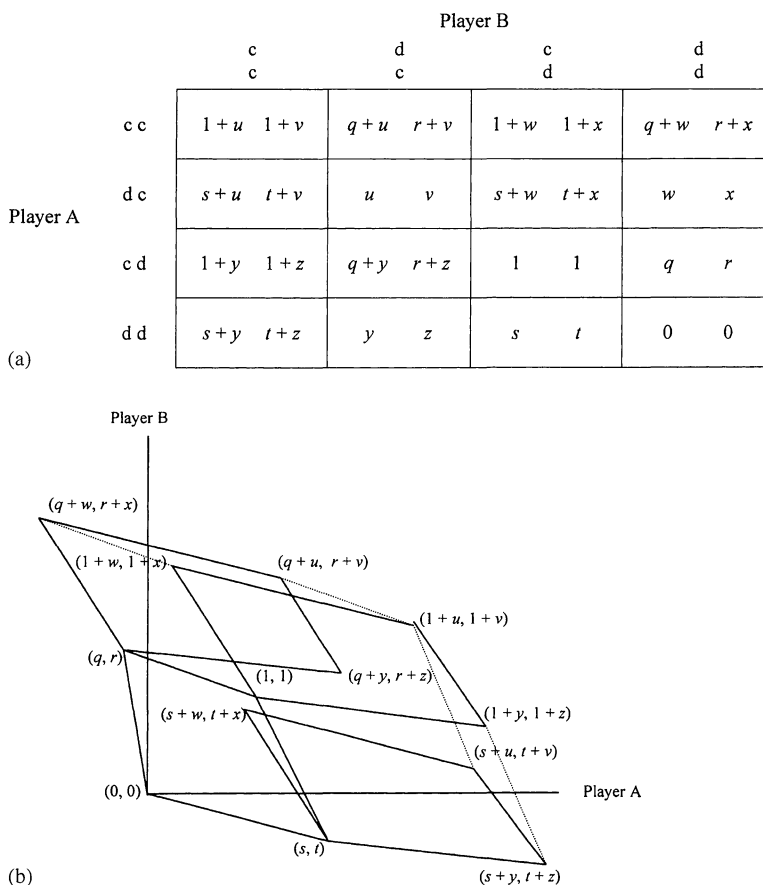


Fig. 4. (a) The general linked game; (b) the feasible set of the linked game; (c) comparison of feasible sets of the aggregated and linked games; (d) the feasible linked set when cooperation is inferior in one game.

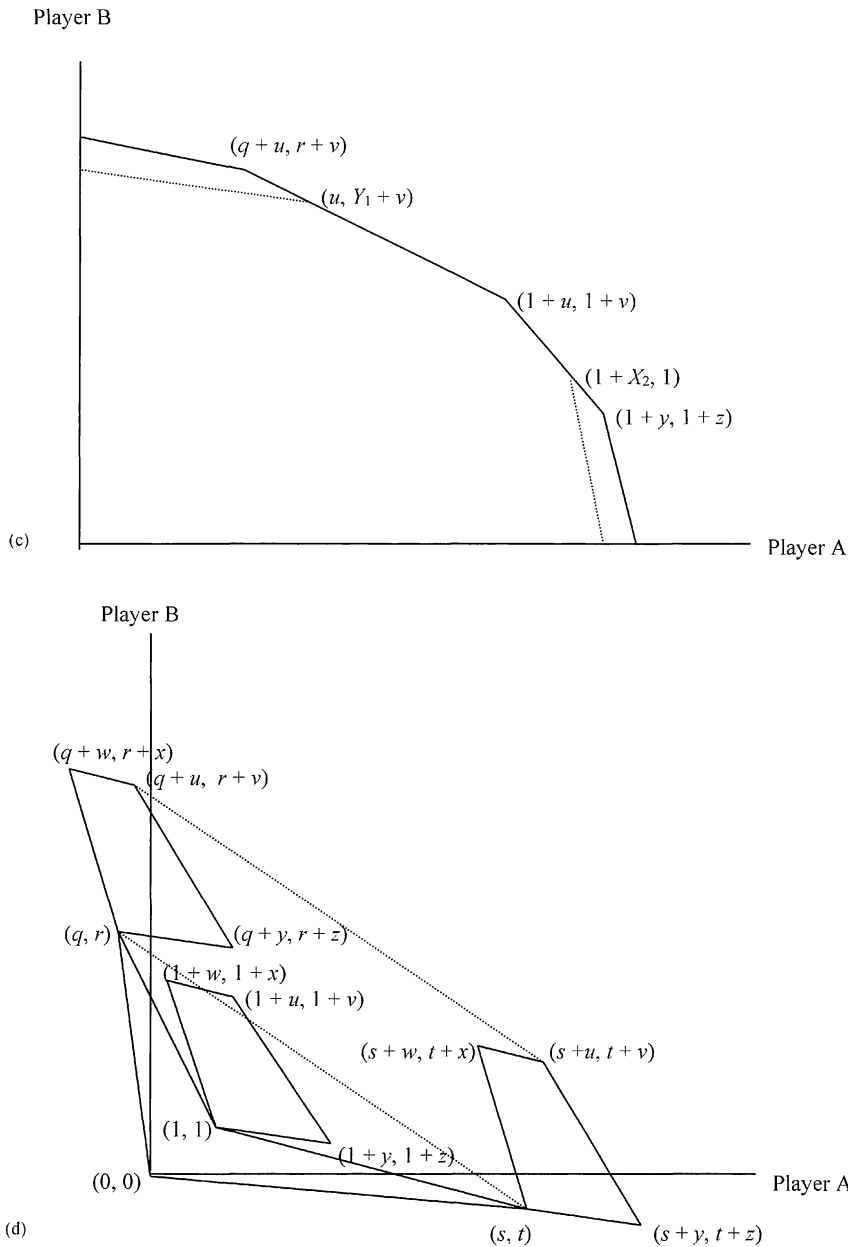


Fig. 4 (Continued).

Such cases are key to obtaining advantages from linking.

Regardless of game structure in Table 1, many of the strategies in Fig. 4a are dominated by others (lie on the interior of the convex hull) and can be eliminated from

consideration. For example, under the maintained assumption that  $u > 0$  and  $v > 0$ ,  $(1 + u, 1 + v)$  dominates  $(u, v)$ ,  $(1, 1)$ , and  $(0, 0)$ . Also,  $(1 + w, 1 + x)$  dominates  $(w, x)$  and  $(1 + y, 1 + z)$  dominates  $(y, z)$ . Strategies  $(s + y, t + z)$  and  $(q + w, r + x)$  can be elimi-

nated from consideration for the case of two PD games because they do not satisfy individual rationality, i.e.,  $t + z < 0$  and  $q + w < 0$ . Only eight of the 16 alternatives in Fig. 4a must be considered in constructing the convex hull of the linked game. The interesting cases correspond to the conditions underlying Figs. 3a–d.

### 3.5. Linking under superiority of cooperation to partial cooperation in both games

As in the case of aggregated games two possibilities deserve attention, symmetry and asymmetry.

#### 3.5.1. Linking when games are symmetric

As in the aggregated case, consider first the four possible orderings of slope in Figs. 2a and b given by  $s_1 > s_3 > s_4 > s_2$ ,  $s_1 > s_3 > s_2 > s_4$ ,  $s_3 > s_1 > s_2 > s_4$ , and  $s_3 > s_1 > s_4 > s_2$ , which correspond to sufficiently symmetry. Manipulation of the points in Fig. 4a reveals that the positive frontier of the convex hull includes  $(1 + u, 1 + v)$ . Which of the other points will be on the frontier can be determined as follows. The points  $(q + w, r + x)$ ,  $(1 + w, 1 + x)$ ,  $(1 + u, 1 + v)$ , and  $(q + u, r + v)$  form a parallelogram in which all the segments have slopes  $s_1$  or  $s_3$ , i.e., have slopes  $(1 - r)/(1 - q)$  or  $(v - x)/(u - w)$ . If  $s_1 < (>)s_3$ , then  $(q + u, r + v)$  is (is not) on the frontier and  $(1 + w, 1 + x)$  is not (is). Similarly,  $(1 + y, 1 + z)$  is (is not) on the frontier when  $s_2 \leq (\geq)s_3$ , and  $(s + u, t + v)$  is (is not) on the frontier when  $s_2 \geq (\leq)s_4$ . As for the aggregated case, the convex hull is completely characterized by conditions on the slopes of the frontiers in Figs. 2a and b. Considerable overlap of the frontiers of the aggregated and linked cases is found. The point  $(1 + u, 1 + v)$  lies on both frontiers and the adjacent frontier segments have the same slope in both cases. The only difference is that the adjacent segments extend somewhat farther in the linked case producing weak dominance of the linked frontier as demonstrated in Fig. 4c. The most striking result is that the linked outcomes dominate the aggregated outcomes only for cases where payoff combinations are substantially different than the full cooperation case. As demonstrated later, outcomes other than full cooperation are chosen in these cases when equity is a concern.<sup>2</sup>

<sup>2</sup> Hereafter, situations where payoff combinations substantially different than the full cooperation case are preferred will be called cases where equity is a concern.

#### 3.5.2. Linking when games are asymmetric

For the case of sufficient asymmetry, slopes are ordered as  $s_1 > s_2 > s_3 > s_4$  or  $s_3 > s_4 > s_1 > s_2$ . In this case, the entire frontier of one game is steeper than the entire frontier of the other game as in Fig. 3d, so full cooperation is in the interior of the feasible payoff set. For example, if  $s_3 > s_4 > s_1 > s_2$ , then the upper right frontier of the convex hull is defined by  $(q + w, r + x)$ ,  $(q + u, r + v)$ ,  $(q + y, r + z)$ ,  $(1 + y, 1 + z)$ , and  $(s + y, t + z)$ . If  $s_1 > s_2 > s_3 > s_4$ , then the upper right frontier of the convex hull is defined by  $(s + y, t + z)$ ,  $(s + u, t + v)$ ,  $(s + w, t + x)$ ,  $(1 + w, 1 + x)$ , and  $(q + w, r + x)$ . Interestingly, both of these cases generate a convex hull for the linked game that strictly dominates the aggregated games. However, as indicated above, these are cases where full cooperation cannot represent an economically desirable outcome. For example, full cooperation cannot achieve economic efficiency in the traditional sense.

### 3.6. Linking under inferiority of cooperation to partial cooperation in both games

Without presenting numerous figures, some mental geometry verifies that extending payoff representations for the individual games in Fig. 3b into negative quadrants as done in Fig. 4b generates an extended feasible payoff set for the linked case. Algebraically, the upper right frontier of the convex hull of payoffs is defined by  $(q + w, r + x)$  and  $(s + y, t + z)$ , and either  $(q + y, r + z)$  or  $(s + w, t + x)$ . The point  $(q + y, r + z)$  will be on the frontier if  $(z - x)/(y - w) \geq (t - r)/(s - q)$  and  $(s + w, t + x)$  will be on the frontier if  $(z - x)/(y - w) \leq (t - r)/(s - q)$ . Thus, the linked frontier strictly dominates the aggregated frontier. However, again, this is a case where full cooperation cannot represent an economically efficient outcome (even in the individual games).

### 3.7. Linking under inferiority of cooperation to partial cooperation in one game

Again, suppose Game 1 is the game where cooperation is inferior to combinations of partial cooperation as in Fig. 3c. Fig. 4d shows the expansion of feasible payoffs with linking. Again, with some mental geometry, the convex hull in Fig. 4d clearly dominates the

one from Fig. 3c. Alternatively, note that Fig. 4d is drawn for the case with  $s_4 < (t-r)/(s-q) < s_3$  where  $(t-r)/(s-q)$  is the slope of the broken lines in Fig. 4d. In this case, the upper right frontier of the convex hull is defined by  $(q+w, r+x)$ ,  $(q+u, r+v)$ ,  $(s+u, t+v)$ , and  $(s+y, t+z)$ . If  $s_4 < s_3 < (t-r)/(s-q)$ , then the upper right frontier of the convex hull is defined by  $(q+w, r+x)$ ,  $(s+w, t+x)$ ,  $(s+u, t+v)$ , and  $(s+y, t+z)$ . If  $(t-r)/(s-q) < s_4 < s_3$ , then the convex hull is defined by  $(q+w, r+x)$ ,  $(q+u, r+v)$ ,  $(q+y, r+z)$ , and  $(s+y, t+z)$ . Clearly, in each case, the convex hull of the linked game strictly dominates the convex hull of the aggregated games. As in other such cases, however, these are cases where the full cooperation outcome,  $(1+u, 1+v)$ , is in the interior of the feasible payoff set. And also as in other cases, these are cases where cooperation is not attractive on the basis of economic efficiency alone, i.e., according to a preference map represented by linear contours with slope  $-1$ , which would be the standard surplus criterion of welfare economics.

### 3.8. Conclusions for linking two PD games

Interestingly, the linked game associated with two isolated PD games does not necessarily strictly dominate the aggregated isolated games. In some cases, at least one point can be common to both frontiers. When the two frontiers have a common point, the point  $(1+u, 1+v)$  that corresponds to full cooperation will always lie on both frontiers. In addition, this analysis proves that the linked game and the aggregated isolated games often have common segments. In fact, when full cooperation lies on both frontiers then they have common segments on both sides of the full cooperation point. These other points along the frontiers result from mixed strategies. These mixed strategies at a given point along the frontier, however, do not necessarily represent the same set of mixed strategies in the aggregated isolated games as in the linked game because more combinations are feasible in the linked game.

Furthermore, the entire frontier of the linked game can possibly be identical to the frontier of the aggregated game. As shown by Netanyahu (1998), equivalence of the linked and aggregated frontiers requires six equations in the 10 variables introduced in Fig. 1. Thus, many cases, including cases of asymmetry, yield

identical frontiers. This result runs counter to the general assertions of Folmer et al. (1993) and Cesar and de Zeeuw (1994) that linking of asymmetric PD games enriches the set of sub-game perfect Nash equilibria or that linked games strictly dominate aggregated isolated games. Depending on circumstances, linking may or may not expand the feasible set and may expand it only in ways that do not affect the equilibrium outcome.

## 4. Linking of various game structures

The approach of the previous section can be applied to each pair of game structures suggested by Table 1.<sup>3</sup> Suppose a PD game is linked to an assurance game. Assurance games have two Nash equilibria associated with the strategies cooperate–cooperate and defect–defect. The linked game is obtained by adding the payoffs of all strategy combinations. Our results sharply contrast with the assertions of Hauer and Runge (1997). When a PD game and assurance game are linked, Hauer and Runge claim that negotiations become more complex, decreasing the chances of an agreement relative to the single issue case. We show that a significant portion of the frontiers of the aggregated isolated games and the linked game overlap, suggesting that chances of coming to an agreement of full cooperation are similar in both aggregated and linked games. Furthermore, when full cooperation is reached by isolated games, there is no advantage in playing the linked game. Finally, while Hauer and Runge focus either on full cooperation or full defection, our results show that mixed strategies are possible in the linked game. In fact, the feasible payoff set of the linked game weakly dominates the aggregated isolated games only because of these mixed strategies.

Linking two assurance games leads to a trivial outcome of full cooperation where no added benefits accrue to linking. This case is significant because it identifies conditions where both players choose the most preferred outcome of cooperation without negotiation. Linking a PD game with an iterated-dominance (ID) game reveals that an ID game yields less oppor-

<sup>3</sup> For the sake of brevity, we only summarize the results obtained under many game structures. For more detail, see Netanyahu (1998).

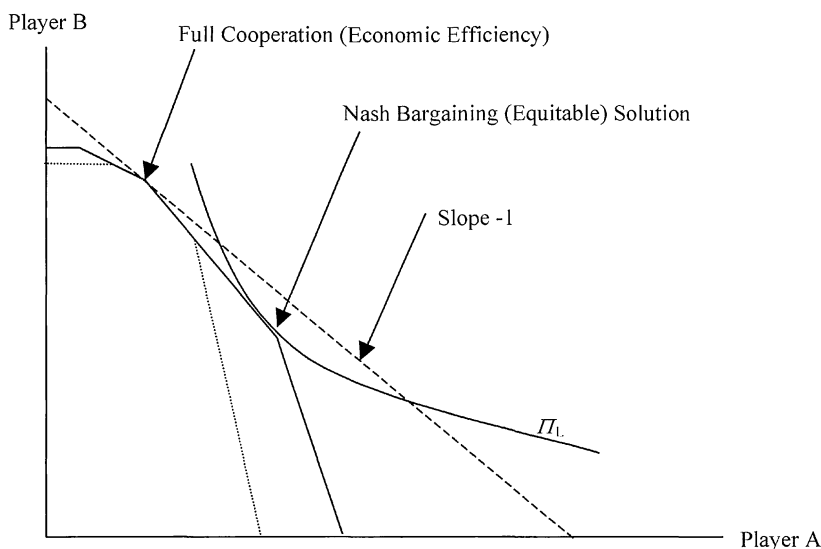


Fig. 5. The case where equity is preferred to efficiency.

tunity for linking.<sup>4</sup> Thus, linking a PD game with an ID game is not as attractive as linking a PD game with another PD game, other things equal.

Linking a PD game to a chicken game reveals that a mix of strategies is always preferred to full cooperation. Full cooperation is in the interior of the linked game when it is inferior to mixed partial cooperation in either isolated game. The frontier of the linked game and the frontier of the aggregated isolated games have considerable overlap because mixed cooperation does not generate negative payoffs for either player in the chicken game. Thus, much of the opportunity for linking (usually gained by using strategies that are not feasible under the rationality constraint of isolated play) is lost. The only points added to the feasible set with linking are those associated with mixed cooperation strategies from the PD game.

For the case of linking two chicken games, the feasible sets of the isolated games are entirely contained in the positive quadrants. Thus, linking does not add any strategies that are not feasible in the aggregated isolated games. For this reason, the feasible set of the linked game is identical and linking offers no advantages. Visual examination also reveals that if either

chicken game has cooperation dominated by mixed partial cooperation, then full cooperation cannot be on the frontier of either the linked or aggregated isolated games.

## 5. Equity considerations in negotiation

Because the dominance of linking over aggregated isolated games is attained only when full cooperation is not preferred, our analysis explains seemingly intermittent enforcement of policies. The explanation is based on equity concerns. To consider equity considerations in bargaining, it is useful to consider the Nash bargaining solution which corresponds to maximizing  $\Pi = (u_A - \bar{u}_A)(u_B - \bar{u}_B)$  where  $\bar{u}_A$  ( $\bar{u}_B$ ) corresponds to the utility derived by Player A (B) and  $\bar{u}_A$  ( $\bar{u}_B$ ) corresponds to the reservation utility of Player A (B). In the case of games such as represented in Fig. 4b, the reservation utilities are normalized to zero. Thus, contours that hold the product of utility increments constant are rectangular hyperbolas as represented by  $\Pi_L$  in Fig. 5.

Fig. 5 demonstrates why outcomes substantially different than full cooperation are sometimes preferred. When the feasible set is highly asymmetric, the parties may be reluctant to pursue full cooperation

<sup>4</sup> The payoff structure of the ID game is based on Bennett et al. (1998).

because of equity disagreements. Yet the parties may be able to identify partial cooperation strategies that obtain an outcome preferable to both non-cooperation and full cooperation given equity concerns. That is, the tangency of the rectangular hyperbola contour of the Nash bargaining criterion to the feasible set may represent a sub-game perfect equilibrium.

Interestingly, these circumstances can arise when traditional economic efficiency analysis suggests full cooperation as the preferred outcome. Suppose, e.g., that payoffs in the games represent standard surplus measures and that the feasible payoffs of the aggregated and linked games in Fig. 5 are represented by dotted and solid lines, respectively, as in Fig. 4c. The standard criterion of economic efficiency corresponds to maximizing the sum of surpluses. The associated welfare indifference contours are linear with slope  $-1$  as depicted by the broken line in Fig. 5. The resulting “optimum” is highly inequitable. So also is the Nash bargaining solution of the aggregated isolated games although less so. The Nash bargaining solution of the linked game, on the other hand, can be considerably more equitable as demonstrated by the tangency of the  $\Pi_L$  contour with the linked feasible set in Fig. 5. This phenomenon may explain why international on-lookers encourage full cooperation based on economic efficiency grounds while the parties are reluctant to accept full cooperation because of equity concerns and instead pursue seemingly intermittent cooperation (mixed strategies). For such cases, linking may serve to identify an equitable outcome that is almost efficient but yet much more acceptable to the parties.

## 6. Conclusions

This paper has introduced a conceptual framework for exploring the potential of linking games as an explanation and possible means of facilitation of international agreements. In particular, a stylized example has been developed showing that linking is preferred only in situations where one party agrees to lose in one agreement for the benefit of being a bigger winner in another agreement, i.e., when the rationality constraints of the isolated games are relaxed by linking. While linking has been explored in the literature as a way of facilitating full cooperation when it does not occur naturally, our results show that linking is

never preferred to isolated play when full cooperation is the preferred outcome under linking. Alternatively, linking is preferred when the preferred outcome under linking involves mixed strategies that are typically composed of partial and intermittent cooperation.

Despite claims elsewhere regarding strict dominance of linked games over aggregated isolated games, our results show that obtaining strict dominance is not trivial. Lack of dominance of linked strategies that uniquely satisfy equity concerns reduces the attractiveness of linking. The cases where full cooperation is not preferred under linking, i.e., the cases where linking is advantageous over isolated play, are either cases where cooperation is inferior in one of the isolated games or the two games have highly asymmetric payoff structures (so equity is likely to be a concern). However, the condition of asymmetry is typically not sufficient to make linking preferred unless one of the isolated games is a PD.

Because agreement is achieved by means of linkage to unrelated bargaining issues that have reciprocal benefits, this approach holds potential for implementing a self-enforcement mechanism. That is, a typical case is where neither party wants to defect from the joint agreement because they stand to lose more on the issue where they are a big winner than they can gain by defecting on the issue where they are a loser.

This paper shows that linking tends to be preferred in the specific circumstances that describe Israeli–Palestinian water negotiations: highly asymmetric payoffs and significant equity concerns. Thus, further investigation of linking opportunities seems to be a fruitful area for assisting further progress in negotiations and/or understanding limits to further progress. The results here, however, suggest that care is necessary in identifying linkage opportunities. Issues considered for linking to water negotiations will tend to generate more attractive opportunities from linking if they (1) have PD characteristics and (2) have payoff possibilities that highly favor the Palestinians in contrast to the asymmetry that heavily favors Israelis in water negotiation opportunities.

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