



**AgEcon** SEARCH  
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

*The World's Largest Open Access Agricultural & Applied Economics Digital Library*

**This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.**

**Help ensure our sustainability.**

Give to AgEcon Search

AgEcon Search

<http://ageconsearch.umn.edu>

[aesearch@umn.edu](mailto:aesearch@umn.edu)

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

*No endorsement of AgEcon Search or its fundraising activities by the author(s) of the following work or their employer(s) is intended or implied.*



ELSEVIER

Agricultural Economics 24 (2000) 73–85

AGRICULTURAL  
ECONOMICS

www.elsevier.com/locate/agecon

## R&D policies for desalination technologies

Yacov Tsur<sup>a,b,\*</sup>, Amos Zemel<sup>c,d</sup>

<sup>a</sup> Department of Agricultural Economics and Management, The Hebrew University of Jerusalem, PO Box 12, Rehovot 76100, Israel

<sup>b</sup> Department of Applied Economics, The University of Minnesota, St. Paul, MN 55108, USA

<sup>c</sup> Center for Energy and Environmental Physics, The Jacob Blaustein Institute for Desert Research,  
Ben Gurion University of the Negev, Sede Boker Campus 84990, Israel

<sup>d</sup> Department of Industrial Engineering and Management, Ben Gurion University of the Negev, Beer-Sheva 84105, Israel

### Abstract

In many arid and semi-arid regions whether or not to desalinate seawater has long been a non-issue and policy debates are focused on the timing and extent of the desalination activities. We analyze how water scarcity and demand structure, on the one hand, and cost reduction via R&D programs, on the other hand, affect the desirable development of desalination technologies and the time profiles of fresh and desalinated water supplies. We show that the optimal R&D policy is of a non-standard most rapid approach path (NSMRAP) type, under which the state of desalination technology — the accumulated learning from R&D efforts — should approach a pre-specified *target process* as rapidly as possible and proceed along it thereafter. The NSMRAP property enables a complete characterization of the optimal water policy. The renewable nature of the fresh water stock permits a non-monotonic behavior of the optimal stock process: under certain conditions, the stock is depleted, to be (fully or partly) refilled at a later date. © 2000 Elsevier Science B.V. All rights reserved.

*JEL classification:* Q16; Q25

*Keywords:* Water scarcity; R&D; Desalination; Renewable resources; MRAP

### 1. Introduction

Whether or not to desalinate water has long been a non-issue in many arid and semi-arid regions and policy debates focus instead on the timing and extent of desalination. At stake here is not water needed for basic subsistence (this relatively small quantity can be supplied from local fresh sources in most cases), but rather water used as input to agricultural, industrial

and environmental production, for which the usual economic considerations apply. Currently, desalinated water is expensive — estimates range between \$0.6 and \$1 per cubic meter (Glueckstern and Priel, 1998) — hence attracts only small demand. However, the various technologies considered, such as distillation, Reverse Osmosis and Electrodialysis (Spiegler and Laird, 1980) leave a large room for cost reduction, pending appropriate investment in R&D.

As R&D programs consume resources and take time to bear fruits, their scheduling vis-à-vis the temporal exploitation of available fresh water sources entails delicate intertemporal tradeoffs. The present paper investigates these tradeoffs. We assume that technological progress due to R&D evolves continuously in time, as the R&D efforts accumulate through learning

\* Corresponding author. Present address: Department of Agricultural Economics and Management, The Hebrew University of Jerusalem, PO Box 12, Rehovot 76100, Israel.

*E-mail addresses:* tsur@agri.huji.ac.il,  
ytsur@dept.agecon.umn.edu (Y. Tsur),  
amos@bgumail.bgu.ac.il (A. Zemel).

in the form of knowledge, which in turn affects to reduce the unit cost of desalination in a continuous fashion. The problem, then, is to set the optimal time profiles of the supply of fresh (primary) and desalinated (backstop) water resources and of the R&D efforts.

Most studies of resource exploitation with a potential backstop substitute deal with non-renewable resources and assume that the backstop resource becomes competitive at a particular date (e.g., as a result of a technological breakthrough). The backstop technology arrival date may be known or uncertain (Dasgupta and Heal, 1974, 1979; Heal, 1976; Dasgupta and Stiglitz, 1981), and it may be influenced by R&D activities (Kamien and Schwartz, 1971, 1978; Dasgupta et al., 1977; Deshmukh and Pliska, 1985; Hung and Quyen, 1993). Just et al. (1996) considered renewable water resources, investigating the adoption of desalination technologies whose uncertain arrival dates depend on exogenous R&D activities.

Departing from the discrete-event nature of the new technology arrival date, Tsur and Zemel (1998, 2000a) analyzed the development of solar technologies as a backstop substitute to fossil energy, by considering a technological process that advances continuously in time rather than in abrupt major improvements. They found that gradual technological progress tends to motivate intensive early engagement in R&D programs — a feature not shared by R&D programs under the discrete event framework.

The present effort modifies Tsur and Zemel's framework to the case of renewable resources. Recharge processes change the optimal policy in a number of ways. In particular, they allow the stock to be depleted during some period and refill at a later date, following desalination cost reduction. With a non-renewable resource, this option is not available. Nonetheless, the underlying structure of the optimal R&D programs in both cases is otherwise similar.

We find that the optimal desalination R&D process admits a *non-standard* most rapid approach path (MRAP) (Spence and Starrett, 1975): the state of desalination technology (the net accumulation of learning from R&D) approaches *as rapidly as possible* a pre-specified *target process* (rather than a *stationary state*) and proceeds along it thereafter. The optimal supply policy is tuned so as to ensure a continuous transition from fresh to desalinated water, avoiding

sudden cuts in the fresh water supply rate due to a premature depletion of the fresh water stock.

The next section sets up the dynamic decision problem and defines a feasible water policy in terms of three control variables (supply rates of fresh and desalination water and R&D investments) and two state variables (fresh water stock and desalination knowledge). In Section 3 we provide an explicit characterization of the optimal policy in terms of simple policy rules. Section 4 concludes and Appendix A contains the technical derivations.

## 2. Formulation of the decision problem

Water can be derived from two sources: a renewable fresh water stock of finite size, and desalinated seawater. The use of the latter source is practically limited only by its cost, which can be reduced with the technological progress associated with R&D. To focus attention on the tradeoffs associated with R&D, we simplify and consider a single fresh water stock and a single desalination technology, leaving aside such extensions as multiple primary and backstop stocks with different water qualities.

*Demand.* Beyond basic subsistence needs (the quantity of which has been set aside and is not part of the water demand), water is an input to production processes of various sectors, e.g., household, agricultural, industrial and environmental. Let  $G_j(q_j)$  represent sector  $j$ 's output, measured in monetary flow rates, when it uses water input at the rate  $q_j$ ,  $j = 1, 2, \dots, J$ , where  $J$  is the number of sectors. The usual properties are assumed for  $G_j$ , namely,  $G_j(0) = 0$ ,  $G'_j > 0$  and  $G''_j < 0$ . At a price  $p$ , sector  $j$  will demand water at the rate that maximizes  $G_j(q) - pq$ , i.e., the rate  $D_j(p)$  defined by  $G'_j(D_j(p)) = p$ . Thus, the derived demand for water for sector  $j$  is given by  $D_j(p) = G'^{-1}_j(p)$ . Given the assumed properties of  $G_j$ , the demand  $D_j(p)$  is decreasing. The total demand for water is obtained by summing the sector demands, i.e., at a price  $p$ , the total demand for water is  $D(p) = \sum_{j=1}^J D_j(p)$ . The aggregate benefit (output) function  $G$  is defined by  $G'(q) = D^{-1}(q)$  and  $G(0) = 0$ ; thus  $G(q) = \int_0^q D^{-1}(s) ds$ . Allowing the derived demand to vary with time, e.g., due to population growth, complicates the analysis but does not change the basic features of the solution (see Tsur and

Zemel, 1998), hence this extension will not be further considered here.

*Supply of fresh water.* Let  $C(q^c)$  represent the instantaneous cost of supplying fresh water at the rate  $q^c$  (covering pumping, conveyance, etc.). We assume that  $C(q^c)$  is increasing and strictly convex, hence the marginal cost  $M_c(q^c) \equiv dC(q^c)/dq^c$  increases with the supply rate.

The fresh water stock, denoted  $X_t$ , evolves over time according to

$$\dot{X}_t \equiv \frac{dX_t}{dt} = R(X_t) - q_t^c \quad (1)$$

where  $R(X) = \xi(\bar{X} - X)$  is the fresh water rate of replenishment, which vanishes at a full stock, when  $X = \bar{X}$ . Integrating (1) gives

$$X_t = \bar{X} + (X_0 - \bar{X})e^{-\xi t} - \int_0^t q_s^c e^{-\xi(t-s)} ds \quad (2)$$

*Supply of desalinated water.* The unit cost of desalination is independent of the supply rate  $q^s$  of desalinated water but depends on the state of desalination technology, which we call *knowledge* and denote by  $K_t$ . Given  $K_t$ , the desalination technology at time  $t$  admits constant returns to scale and can be described by the unit (or marginal) cost function  $M_s(K_t)$  which decreases with knowledge. The latter, in turn, accumulates due to the learning associated with the R&D investments  $I_\tau$ ,  $\tau \leq t$ , that had taken place up to time  $t$ .

The balance between the rate of R&D investment,  $I_t$ , and the rate at which existing knowledge is lost or becomes obsolete due to aging or new discoveries determines the rate of knowledge accumulation

$$\dot{K} \equiv \frac{dK}{dt} = I_t - \delta K \quad (3)$$

where the knowledge level  $K$  is measured in monetary units and the constant  $\delta$  is a knowledge depreciation parameter. Integrating (3), we obtain

$$K_t = \int_0^t I_\tau e^{\delta(\tau-t)} d\tau + K_0 e^{-\delta t} \quad (4)$$

*Benefit.* The gross surplus generated by using water at the rate  $q$  is given above in terms of the area below the inverse demand curve to the left of  $q$ :  $G(q) = \int_0^q D^{-1}(s) ds$ . The cost of supplying water at the rates

$q^c$  and  $q^s$  is given by  $C(q^c) + M_s(K_t)q^s$ . The net surplus generated by  $q = q^c + q^s$  is thus  $G(q^c + q^s) - [C(q^c) + M_s(K_t)q^s]$ . Accounting also for the R&D cost  $I_t$ , the instantaneous net social benefit at time  $t$  is given by

$$G(q_t^c + q_t^s) - C(q_t^c) - M_s(K_t)q_t^s - I_t \quad (5)$$

*Water policy.* A water policy consists of the time profiles of  $q_t^c$  (fresh water supply rate),  $q_t^s$  (desalinated water supply rate) and  $I_t$  (R&D investment rate). A policy  $\Gamma = \{q_t^c, q_t^s, I_t | t \geq 0\}$  determines the evolution of the state variables  $X_t$  (fresh water stock) and  $K_t$  (desalination knowledge) via (1)–(4) and gives rise to the instantaneous net benefit process (5). The optimal policy is the solution to

$$V(X_0, K_0) \equiv \text{Max}_\Gamma \int_0^\infty [G(q_t^c + q_t^s) - C(q_t^c) - M_s(K_t)q_t^s - I_t] e^{-rt} dt \quad (6)$$

subject to (1), (3),  $q_t^c, q_t^s \geq 0$ ,  $0 \leq I_t \leq \bar{I}$ ,  $X_t \geq 0$  and  $X_0, K_0$  given. In (6),  $r$  is the time rate of discount and  $\bar{I}$  is an exogenous bound on the affordable R&D effort that implies, in view of (3), the upper bound  $\bar{K} = \bar{I}/\delta$  on desalination knowledge.

### 3. Characterization of the optimal policy

It is expedient to characterize the optimal policy in two steps. First, the optimal supply rates of fresh ( $q_t^c$ ) and desalinated ( $q_t^s$ ) water are specified in terms of the state of desalination knowledge ( $K_t$ ) and of the fresh water scarcity price  $\lambda_t$ . In the second step, the optimal R&D policy (i.e., the investment rate  $I_t^*$  and the corresponding  $K_t^*$  process) is determined together with the scarcity rent ( $\lambda_t^*$ ) process. The derivation of the optimal policy is rather involved and is, therefore, relegated to Appendix A. Here we present the main characteristics and discuss their policy implications.

*Step 1: The optimal rates of fresh and desalinated water supplies*

The effective marginal cost of fresh water supply consists of the direct supply cost  $M_c(q^c) \equiv dC(q^c)/dq^c$  plus the shadow price (or scarcity rent)  $\lambda_t$  of the remaining stock of fresh water. The shadow price reflects the value of avoiding fresh water shortage in the future. The marginal cost of desalination at

a given state of knowledge  $K$  is  $M_s(K)$ . At each point of time, an additional unit of water should be supplied from the cheapest available source. Thus, fresh water is supplied up to the rate  $q_t^c$  defined by

$$M_c(q_t^c) + \lambda_t = M_s(K_t) \tag{7}$$

At this rate, desalination is competitive and any additional supply comes from desalination plants. Thus, given  $K_t$  and  $\lambda_t$ , the water supply curve (i.e., the marginal cost of water supply) is given by (see right panel of Fig. 1).

$$M(q|K_t, \lambda_t) = \text{Min}\{M_c(q) + \lambda_t, M_s(K_t)\} \tag{8}$$

The total rate (from both sources) of water supply at time  $t$ , denoted  $q(K_t, \lambda_t)$ , is determined by the intersection point of supply and demand, as the right panel of Fig. 1 depicts. If this point falls on the flat portion of the supply curve (where the marginal cost equals  $M_s$ ), then water is supplied from both sources, yielding

$q^c(K_t, \lambda_t)$  and  $q^s(K_t, \lambda_t)$  for the fresh and desalinated water supply rates, respectively:

$$q^c(K_t, \lambda_t) + q^s(K_t, \lambda_t) = D(M_s(K_t)) \tag{9}$$

otherwise, only fresh water is used. Indeed, the supply rule (7)–(9) resembles static economic optimization by maximizing the area ABCD of Fig. 1, which represents the sum of the consumers and suppliers surplus. The dynamics of the problem enter via the incorporation of the dynamic shadow price into the effective cost of fresh water supply.

A difficulty with implementing the supply rule (7)–(9) may arise if the fresh water stock is empty and the fresh water supply rate  $q^c(K_t, \lambda_t)$  required by (7) exceeds the recharge rate. Fortunately, this situation cannot occur under the optimal policy. This is so because the optimal processes  $K_t^*$  and  $\lambda_t^*$  are so chosen that at the time of depletion  $T^*$ , and while the fresh water stock is empty thereafter, it is not desirable

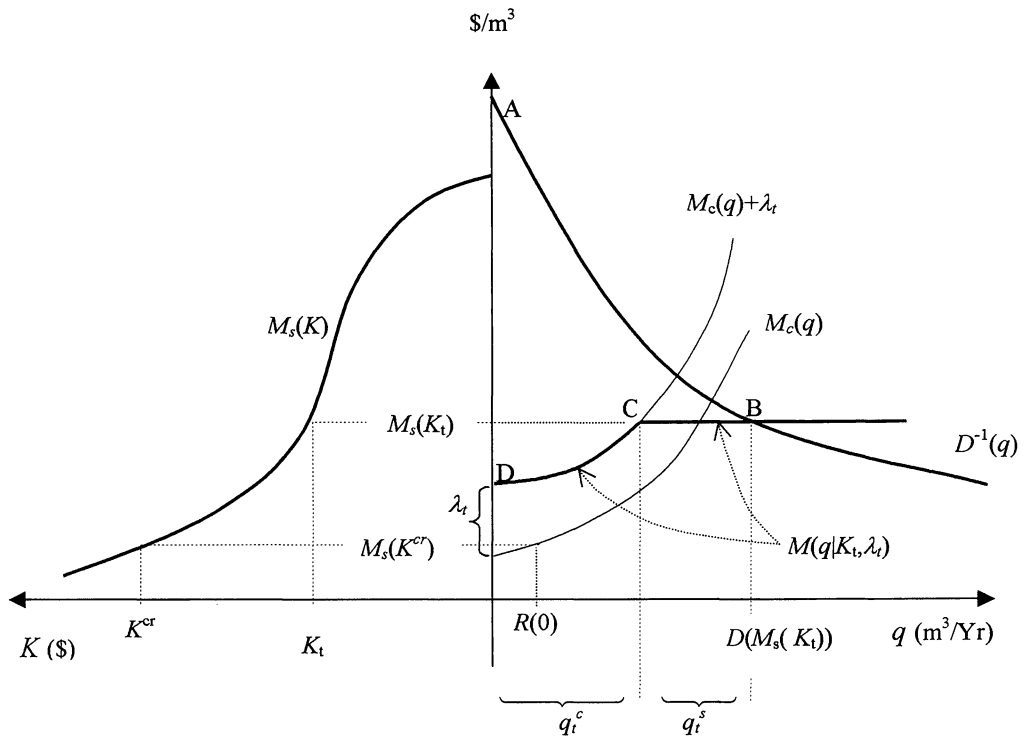


Fig. 1. Right panel: water supply and demand at time  $t$ , given  $K_t$  and  $\lambda_t$ . The area ABCD represents the sum of consumer and producer surpluses. Left panel: marginal cost of desalination as a function of knowledge.

to supply fresh water beyond the recharge rate  $R(0)$ . This property is formulated as follows.

**Claim 1** (Continuity of fresh water supply at depletion). If it is optimal to deplete the fresh water stock, then  $q^c(K_t^*, \lambda_t^*) \rightarrow R(0)$  as  $t \rightarrow T^*$ .

Claim 1 implies that the transition from fresh to desalinated water is a gradual process, so that the supply of desalinated water starts well before the depletion event. This behavior is in contrast to the policy advocated by the standard theory of a resource exploitation industry facing a backstop technology, namely to abruptly abandon the primary resource when its price reaches that of the backstop. A similar continuity property has been derived by Hung and Quyen (1993) and by Tsur and Zemel (1998) in the context of non-renewable resources.

*Step 2: The optimal R&D policy ( $I_t^*$  and  $K_t^*$ )*

The optimal R&D policy is a non-standard variant of the so-called MRAP of Spence and Starrett (1975). A *standard* MRAP is defined by the process that approaches as rapidly as possible some pre-specified steady state level  $\hat{K}$  and remains there. Formally, let  $K_t^m$  denote the process initiated at  $K_0$  and driven by the R&D policy that invests in learning at the maximal feasible rate  $I_t = \bar{I}$ . Recalling (4),

$$K_t^m = (1 - e^{-\delta t})\bar{K} + K_0 e^{-\delta t} \quad (10)$$

The standard MRAP initiated below the steady state level  $\hat{K}$  is given by  $K_t = \text{Min}\{K_t^m, \hat{K}\}$ .

A *non-standard* MRAP (NSMRAP) involves a pre-specified *target process*, rather than a fixed steady state. Initiated below the target process, the NSMRAP begins, like the MRAP, as  $K_t^m$ . As soon as it arrives at the target process, the NSMRAP switches to the target process and cruises along it from that time on. A NSMRAP, therefore, is specified in terms of  $K_t^m$  and some target process such that the most rapid approach is to the target process rather than to a target steady state. Of course, if the target process settles at its own steady state  $\hat{K}$  before being crossed by  $K_t^m$ , the NSMRAP reduces to the standard MRAP. We now introduce the target process corresponding to the optimal R&D policy. We refer to it as the *root process* for a reason soon to become obvious.

Define

$$L(K, \lambda) \equiv -M'_s(K)q^s(K, \lambda) - (r + \delta) \quad (11)$$

This function (which is a generalization of the *evolution function* used to determine steady states of infinite-horizon dynamic problems by Tsur and Zemel (1996, 2000b)) can be viewed as the derivative (with respect to  $K$ ) of some utility to be maximized by the optimal R&D process (see Appendix A). Thus, we seek the root  $K(\lambda)$  of  $L(K, \lambda)$ , i.e., the solution of  $L(K(\lambda), \lambda) = 0$ , in the domain in which  $L(K, \lambda)$  decreases in  $K$ . To rule out corner solutions, we assume that the root is unique in this domain, and that  $K_0 < K(\lambda) < \bar{K}$  for all  $\lambda$ . Indeed, the evolution functions displayed in Fig. 2 (based on some simple specifications of the relevant demand and supply functions) have double roots. However, the evolution functions increase at the smaller roots. Thus, only the larger roots are relevant to our discussion.

$K(\lambda_t)$  is the *root process* corresponding to the scarcity rent process  $\lambda_t$  and it bears a simple economic interpretation: Increasing knowledge by the infinitesimal amount  $dK$  reduces the cost of desalination at the rate  $q^s(K, \lambda)$  by  $-M'_s(K)q^s(K, \lambda) dK$  but incurs an extra cost of  $(r + \delta) dK$  due to interest payment on the investment and the increased depreciation. The root  $K(\lambda)$  represents the optimal balance between these conflicting effects.

Initiated at  $K_0 < K(0)$ , the NSMRAP with respect to  $K(\lambda_t)$  is given by  $K_t = \text{Min}\{K_t^m, K(\lambda_t)\}$ . The associated R&D investment rate is

$$I_t = \begin{cases} \bar{I} & \text{if } K_t < K(\lambda_t) \\ K'(\lambda_t)\dot{\lambda}_t + \delta K(\lambda_t) & \text{if } K_t = K(\lambda_t) \end{cases} \quad (12)$$

(It is assumed that  $\bar{I}$  is large enough to support the rate required by (12) along the root process, so that the NSMRAP is feasible for the optimal  $\lambda_t^*$ .) We can now state

**Claim 2** (The NSMRAP property). The optimal R&D policy is the NSMRAP with respect to the root process  $K(\lambda_t^*)$ .

Claim 2 bears an important policy implication: If R&D is at all worthwhile, then the R&D program must be initiated immediately, at the highest feasible rate. The Claim also implies that the characterization of the

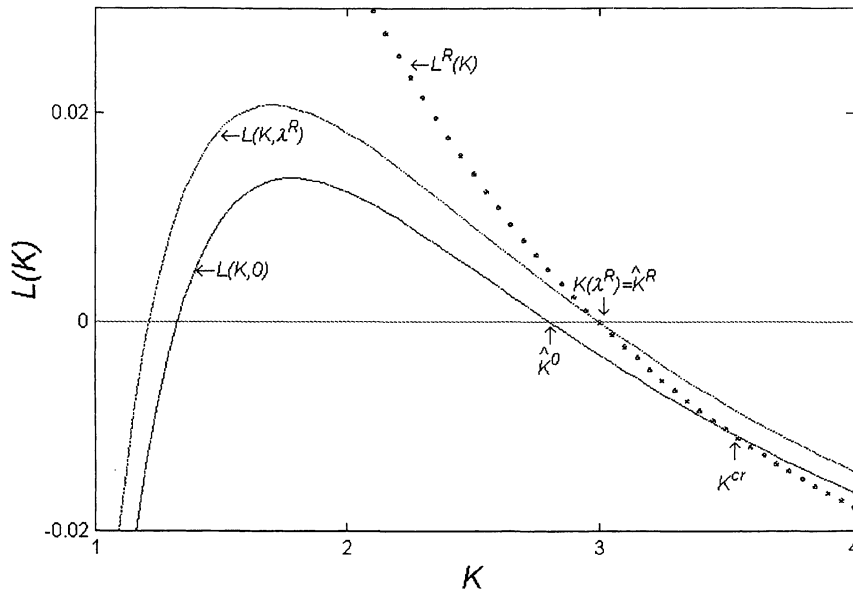


Fig. 2. The evolution functions  $L(K, \lambda)$  (Eq. (11)) and  $L^R(K)$  (Eq. (15)) vs. the knowledge level  $K$  when  $K^{cr} > \hat{K}^0$ .  $\hat{K}^0$  is the root of  $L(K, 0)$ ,  $K^{cr}$  is the critical knowledge level in which  $M_s(K^{cr}) = M_c(R(0))$  and is also the intersection of  $L^R(K)$  and  $L(K, 0)$ . Both  $L^R(K)$  and  $L(K, \lambda^R)$  vanish at  $\hat{K}^R$ .

optimal R&D policy requires to specify the optimal scarcity process  $\lambda_t^*$ . The derivation is presented in Appendix A, where we find that the dynamic behavior depends on the initial fresh water stock and on the relative position of the following knowledge levels:

1.  $\hat{K}^0 = K(0)$  which is the root of  $L(K, 0)$ , i.e., the solution of

$$-M'_s(\hat{K}^0)q^s(\hat{K}^0, 0) - (r + \delta) = 0 \tag{13}$$

2. the solution  $K^{cr}$  of

$$M_s(K^{cr}) = M_c(R(0)) \tag{14}$$

is the critical knowledge level above which the fresh water stock becomes inessential, since extraction above the recharge rate is more expensive than desalination.

Prior to depletion, the scarcity rent takes a simple exponential form  $\lambda_t^* = \lambda_0^* e^{(r+\xi)t}$ , where the non-negative constant  $\lambda_0^*$  depends on the initial stock as explained below. The following characterization holds

**Claim 3.** If  $\hat{K}^0 > K^{cr}$  then the optimal R&D policy is the standard MRAP  $K_t^* = \text{Min}\{K_t^m, \hat{K}^0\}$  and the steady state fresh water stock is not empty.

Claim 3 appeals to economic intuition. A steady state above the critical level  $K^{cr}$  implies a fresh water supply rate below  $R(0)$  and corresponds to a non-depleted stock. According to this claim, the initial stock does not affect the R&D policy and the equilibrium stock in this case. Yet, the time profile of the stock process can take markedly different patterns. Let  $Q^0$  be the benchmark quantity defined in Appendix A by (A.11). We arrive at the following characterization.

**Claim 4.** (a) When  $\hat{K}^0 > K^{cr}$  and  $X_0 \geq Q^0$ , the stock is never depleted and the scarcity rent vanishes at all times. (b) When  $\hat{K}^0 > K^{cr}$  and  $X_0 < Q^0$ , the stock is depleted and refills again while the optimal scarcity rent process  $\lambda_t^*$  increases exponentially until depletion and falls back to zero at the steady state.

The unique feature characterizing case (b), namely the non-monotonic behavior of the fresh water stock (and of the corresponding shadow price), is a manifestation of the renewable nature of the fresh water resource. When the initial stock is small, it is advantageous to deplete it when the desalination knowledge

is limited, then let the stock refill as knowledge accumulates and the cost of desalination decreases. With a non-renewable resource, such as fossil fuel, this behavior is not feasible (see Tsur and Zemel, 1998, 2000a). Observe, however, that the optimal R&D policy is the monotonic MRAP even in this case, as Claim 3 ensures.

When  $\hat{K}^0 = K^{cr}$  Claims 3 and 4 remain valid, except that both the stock and the scarcity rent vanish at the steady state.

We turn now to the case  $\hat{K}^0 < K^{cr}$ . Depletion is favorable in this case and the continuity condition on the depletion date (Claim 1) requires the fresh water supply rate to equal the replenishment rate  $R(0)$  on that date. It turns out that during the post-depletion period the fresh water supply remains at the rate  $R(0)$ , so that the fresh water stock remains empty and the desalinated water supply rate equals  $D(M_s(K_t)) - R(0)$ . Accordingly, define

$$L^R(K) = -M'_s(K)[D(M_s(K)) - R(0)] - (r + \delta) \quad (15)$$

and let  $\hat{K}^R$  be the root of  $L^R$ . It is verified in Appendix A that  $\hat{K}^R \in (\hat{K}^0, K^{cr})$  (see also Fig. 2) and that this root is the steady state of the optimal  $K$ -process.

The optimal  $K$ -process, by virtue of Claim 2, is of the form  $K_t^* = \text{Min}\{K_t^m, K(\lambda_t^*)\}$ . One possibility is that  $K_t^m$  lags behind  $K(\lambda_t^*)$  prior to arrival at the steady state and the optimal process is a standard MRAP to  $\hat{K}^R$ . The alternative is that  $K_t^m$  overtakes the root process at an earlier date  $\tau$ , and the optimal knowledge process is a NSMRAP, evolving along with the root process during its final stage. Which of these cases occurs depends on the initial fresh water stock vis-à-vis the benchmark quantity  $Q^m$  of (A.12) according to the following claim.

**Claim 5.** (a) When  $\hat{K}^0 < K^{cr}$  and  $Q^m \geq X_0$ ,  $K_t^* = \text{Min}\{K_t^m, \hat{K}^R\}$  is a simple MRAP to  $\hat{K}^R$ . (b) When  $\hat{K}^0 < K^{cr}$  and  $Q^m < X_0$ ,  $K_t^*$  follows a NSMRAP to  $\hat{K}^R$ .

The NSMRAP of case (b) implies that the R&D program is initiated at the highest feasible rate but slows down at the later, singular stage of the process so as to delay the arrival at the steady state. This

delay is designed to take advantage of the large initial fresh water stock. So long as this relatively cheap resource can be exploited above the recharge rate, it does not pay to arrive too early at the knowledge steady state, which is optimal only when fresh water supply is restricted to the recharge rate. Observe that even this NSMRAP is monotonic in time, although Eq. (3) can accommodate non-monotonic knowledge processes.

#### 4. Closing comments

Water scarcity can induce responses of various kinds. First, it might lead to conflicts and competition among nations, regions or sectors (see, e.g., the collection of works edited by Biswas, 1994, by Dinar and Loehman, 1995, and by Just and Netanyahu, 1998). Alternatively, it can encourage steps towards more efficient use of water via improved irrigation and distribution systems, quality-differentiated supplies and efficient pricing (see Tsur and Dinar, 1997, and works in Parker and Tsur, 1997). Finally, when the futility of the first approach is recognized and the potential of the second approach is realized, one may turn to the development of alternative sources, namely desalination technologies. This work is concerned with the third approach, focusing attention on its intertemporal aspects, particularly on the optimal scheduling of the R&D activities.

In earlier works (Tsur and Zemel, 1998, 2000a) we derived optimal rules for the development of backstop (solar) technologies in light of the environmental costs of fossil energy and its finite reserves. Here also we consider the optimal development of backstop substitutes for a limiting primary resource — fossil energy then, fresh water now. The difference between the two stems from the fact that fresh water is (typically) renewable, whereas fossil deposits are not. The presence of recharge processes renders the scarcity of the primary resource less crucial, but at the same time it changes the optimal policy quite substantially. For example, it is possible in the present case that the fresh water stock will be first depleted and eventually refill (fully or partly) as technological progress reduces the cost of desalination to the extent that fresh water exploitation decreases below the recharge rate. Such behavior is, of course, impossible



for non-renewable deposits. The time profile of the primary resource stock has far reaching implications for the optimal R&D policy, since the latter depends crucially on the scarcity (shadow) price of the former.

Sure enough, many regions around the Globe have all the water they need from local, fresh sources. But the number of water-scarce regions is growing by the year and in many desalinated seawater is (or will be) cheaper than fresh water conveyed from remote sources. As in the non-renewable case, we find that when the cost of desalination decreases with knowledge in a continuous fashion, the optimal R&D policy is of a *non-standard* MRAP type. The presence of recharge process has a substantial effect on the *target process* to which the optimal knowledge process moves as rapidly as possible. The NSMRAP property calls for early engagement in R&D efforts — well in advance times of water shortage.

For many regions assuming that demand increases with time appears more realistic. However, Tsur and Zemel (1998) showed that, although the details of the optimal policy are affected by this extension, the NSMRAP property of the optimal R&D policy is preserved. The same conclusion holds also in the present case of a renewable resource. Extensions to situations involving multiple fresh water stocks as well as the incorporation of uncertainty that affects various components of the model (water demand, the knowledge accumulation process) are important topics for future research.

## Appendix A. Derivation of the optimal policy

We present below the formal derivation of the optimal supply rule and R&D policy characterized in Section 3.

*Preliminaries.* Let  $T$  denote the time at which the stock of fresh water is first depleted. The optimization problem (6) is recast as

$$\begin{aligned} V(X_0, K_0) = \text{Max}_{\Gamma, T} \int_0^T [G(q_t^c + q_t^s) - C(q_t^c) \\ - M_s(K_t)q_t^s - I_t] e^{-rt} dt \\ + e^{-rT} V(0, K_T) \end{aligned} \quad (\text{A.1})$$

subject to the same constraints. The current-value

Hamiltonian for (A.1) is of the form

$$\begin{aligned} H_t = G(q_t^c + q_t^s) - C(q_t^c) - M_s(K_t)q_t^s - I_t \\ + \lambda_t [R(X_t) - q_t^c] + \gamma_t (I_t - \delta K_t) \end{aligned}$$

where  $\lambda_t$  and  $\gamma_t$  are the current-value costate variables corresponding to  $X_t$  and  $K_t$ , respectively, and it is recalled that  $R(X) = \xi(\bar{X} - X)$ . Incorporating the Lagrange multipliers associated with the constraints on  $q^c$ ,  $q^s$  and  $I$ , the Lagrangian  $\mathcal{J}_t = H_t + \alpha_t^c q_t^c + \alpha_t^s q_t^s + \alpha^0 I_t + \alpha^I (\bar{I} - I_t)$  is obtained.

Necessary conditions include (see Leonard and Long (1992); all variables are evaluated at their optimal values):

1.  $\text{Max}_{q^c, q^s} \{\mathcal{J}_t\} \Rightarrow D^{-1}(q_t^c + q_t^s) - M_c(q_t^c) - \lambda_t + \alpha_t^c = 0$  and  $D^{-1}(q_t^c + q_t^s) - M_s(K_t) + \alpha_t^s = 0$ , hence

$$M_s(K_t) = M_c(q_t^c) + \lambda_t \quad (\text{A.2})$$

$$q_t^c + q_t^s = D(M_s(K_t)) \quad (\text{A.3})$$

hold along the optimal plan whenever  $q^c$  and  $q^s$  are both positive and the corresponding Lagrange multipliers vanish. This establishes the optimal supply rule given by (7)–(9), as depicted in Fig. 1. The modifications when supply from either source vanishes are straightforward.

2. Maximizing the Lagrangian with respect to  $I_t$  reveals that  $I_t$  equals 0 or  $\bar{I}$  whenever  $\gamma_t \neq 1$ . Thus,  $I_t$  can undergo a discontinuity only at the singular value  $\gamma_t = 1$ .
3.  $\dot{\lambda} - r\lambda = -\partial H/\partial X = \lambda\xi$  yielding  $\lambda_t = \lambda_0 e^{(r+\xi)t}$  prior to depletion.
4.  $\lambda_0 X_T = 0$  is the transversality condition associated with  $X_T \geq 0$ , implying that  $\lambda_0 = 0$  if the fresh water stock is never empty.
5.  $\gamma_T = \partial V(0, K_T)/\partial K$  is the transversality condition associated with the free value of  $K_T$ .
6.  $H_T = rV(0, K_T)$  is the transversality condition associated with the free choice of  $T$ .

**Proof of Claim 1.** Let  $q_-^c = \lim_{t \uparrow T} q_t^c$  and  $q_+^c = \lim_{t \downarrow T} q_t^c$  be the limiting pre- and post-depletion supply rates of fresh water (the subscripts  $-$  and  $+$  denote the corresponding pre- and post-depletion limits of other quantities as well). We need to show that

$$q_-^c = q_+^c = R(0) \quad (\text{A.4})$$

This means that the fresh water stock will *not* be depleted before the marginal cost of fresh water is high enough to exclude its supply above the natural recharge rate.

Since  $\gamma_+$  is the initial knowledge shadow price for the post-depletion problem, it follows that  $\partial V(0, K_T)/\partial K = \gamma_+$ . Moreover, for the pre-depletion problem, condition (5) above reads  $\gamma_- \equiv \gamma_T = \partial V(0, K_T)/\partial K$ . Thus, the costate variable  $\gamma_t$  evolves smoothly as the pre-depletion problem turns into the post-depletion problem at the depletion time  $T$ . In view of condition (2), the quantity  $I_t(\gamma_t - 1)$  is also continuous on that date.

The Bellman equation for the post-depletion value reads

$$rV(0, K_T) = G(D(M_s(K_T))) - M_s(K_T)[D(M_s(K_T)) - q_+^c] - C(q_+^c) - I_+ + \gamma_+(I_+ - \delta K_T) \quad (A.5)$$

where we have used again the fact that  $\partial V(0, K_T)/\partial K = \gamma_+$ . The transversality condition (6),  $H_- \equiv H_T = rV(0, K_T)$ , where

$$H_- = G(D(M_s(K_T))) - M_s(K_T)[D(M_s(K_T)) - q_-^c] - C(q_-^c) - I_- + \gamma_-(I_- - \delta K_T) + \lambda_-[R(0) - q_-^c] \quad (A.6)$$

is compared with (A.5), using the continuity of  $\gamma_t$  and of  $I_t(\gamma_t - 1)$  at  $t = T$ . We find that  $C(q_+^c) - C(q_-^c) - M_s(K_T)(q_-^c - q_+^c) + \lambda_-(q_-^c - R(0)) = 0$ , or  $C(q_-^c) - C(q_+^c) - M_s(K_T)(q_-^c - q_+^c) + \lambda_-(q_-^c - q_+^c) + \lambda_-(q_+^c - R(0)) = 0$ , which reduces, using (A.2), to  $C(q_-^c) - C(q_+^c) - M_c(q_-^c)(q_-^c - q_+^c) + \lambda_-(q_+^c - R(0)) = 0$ .

Now, to deplete the stock requires  $q_-^c \geq R(0)$  while following depletion  $q_+^c \leq R(0)$ . Thus,  $C(q_-^c) - C(q_+^c) = M_c(\tilde{q}^c)(q_-^c - q_+^c)$ , where  $q_+^c \leq \tilde{q}^c \leq q_-^c$ , hence  $[M_c(\tilde{q}^c) - M_c(q_-^c)](q_-^c - q_+^c) = \lambda_-(R(0) - q_+^c) \geq 0$ . However,  $M_c(q^c)$  increases with  $q^c$  and the left-hand side cannot be positive, implying that these conditions must hold as equalities, establishing (A.4).  $\square$

From (A.2) and (A.4) we conclude that

$$M_s(K_{T^*}^*) = M_c(R(0)) + \lambda_{T^*}^* \quad (A.7)$$

holds at the optimal depletion date  $T^*$ . Also, depletion at  $T^*$ , i.e.,  $X_{T^*} = 0$ , requires (cf. 2)

$$\int_0^{T^*} q^c(K_t^*, \lambda_t^*) e^{-\xi(T^*-t)} dt = \bar{X} + (X_0 - \bar{X}) e^{-\xi T^*} \quad (A.8)$$

Once the knowledge process is given, these two relations can be used to determine the parameters  $T^*$  and  $\lambda_0^*$ , as explained below.

**Proof of Claim 2.** We first show that given the optimal scarcity rent  $\lambda_t^*$ , the optimal R&D policy  $(I_t^*, K_t^*)$  can be obtained as the solution of the one-dimensional problem

$$V(K_0) = K_0 + \text{Max}_{\{I_t\}} \int_0^\infty \vartheta(K_t, t) e^{-rt} dt \quad (A.9)$$

subject to  $\dot{K} = I - \delta K$ ,  $0 \leq I_t \leq \bar{I}$ , and  $K_0$  given, where

$$\vartheta(K, t) = G(q^c + q^s) - C(q^c) - \lambda_t^* q^c - M_s(K)q^s - (r + \delta)K$$

and  $q^c = q^c(K, \lambda_t^*)$  and  $q^s = q^s(K, \lambda_t^*)$  are given by the optimal supply rules (7)–(9). The integrand  $\vartheta(K, t)$ , denoted the *equivalent utility*, is independent of the control  $I$  and its explicit time dependence enters through the scarcity rent  $\lambda_t^*$ . Consider first the problem

$$V(K_0) = \text{Max}_{\{I_t\}} \int_0^\infty \tilde{\vartheta}(K_t, I_t, t) e^{-rt} dt \quad (A.10)$$

subject to the same constraints and supply rule as in (A.9), where

$$\tilde{\vartheta}(K, I, t) = G(q^c + q^s) - C(q^c) - \lambda_t^* q^c - M_s(K)q^s - I$$

It is verified that the necessary conditions corresponding to (A.10) coincide with the necessary conditions associated with  $I_t, K_t$  and the costate variable  $\gamma_t$  of the original problem (6). Following Spence and Starrett (1975), we use (3) to remove  $I$  from  $\tilde{\vartheta}$ . Integrating the resulting  $\dot{K}$  term by parts, we obtain (A.9).

Differentiating  $\vartheta(K, t)$  with respect to  $K$ , noting that (A.2) and (A.3) imply that the terms involving  $\partial q^c/\partial K$  and  $\partial q^s/\partial K$  vanish, gives  $\partial \vartheta/\partial K = L(K, \lambda)$  (cf. Eq. (11)), and the unique root  $K(\lambda_t)$  maximizes

$\vartheta(K, t)$  at any time  $t$ . Now, the analysis of Spence and Starrett (1975) shows that the MRAP to the maximum of the equivalent utility is the optimal process for this type of problems, characterized by utilities which do not depend explicitly on the controls. Indeed, the problem at hand is not autonomous due to the time dependence introduced by the scarcity rent  $\lambda_t^*$ . However, these authors have established (see their footnote, p. 394), that the same result applies when the MRAP process follows the root process rather than a stationary maximum. Once the root process has been reached,  $\vartheta(K, t)$  must be maintained at its maximum by tuning  $I_t$  so as to ensure that  $K_t = K(\lambda_t)$ , as specified in (12).  $\square$

Two immediate corollaries follows

**Corollary 1.** *The optimal process  $K_t^*$  cannot decrease.*

**Proof.** Initiated below the root process,  $K_t^*$  can only increase towards the root process but not exceed it. Once on the root process,  $K_t^*$  can decrease only if the latter decreases. This cannot happen before the fresh water stock is depleted, since before depletion the scarcity rent is either zero or increases exponentially, and  $K(\lambda)$  increases with  $\lambda$ . For a period of vanishing stock, with fresh water extraction at the recharge rate  $R(0)$ ,  $\lambda$  may decrease. However,  $K_t^*$  must differ from the root process during that period through which, according to (A2),  $M_s(K_t^*) = M_c(R(0)) + \lambda_t$ , and a decrease in  $\lambda_t$  implies that  $K_t^*$  must increase.  $\square$

**Corollary 2.** *The optimal process  $K_t^*$  must converge to a steady state.*

**Proof.** The corollary follows from Corollary 1 and the fact that  $K_t^*$  is bounded.  $\square$

Turning to Claim 3, we introduce the following notation:

$\hat{K}^0 = K(0)$  is the root of  $L(K, 0)$  (see Eq. (13)).

$K^{cr} = M_s^{-1}(M_c(R(0)))$  (see (14));  $K > K^{cr}$  implies  $q^c(K, \lambda) < R(0)$  for any  $\lambda$ .

$K_t^m = (1 - e^{-\delta t})\bar{K} + K_0 e^{-\delta t}$  (see (10)) is the standard MRAP initiated at  $K_0$ .

$T^{cr} = \log[(\bar{K} - K_0)/(\bar{K} - K^{cr})]/\delta$  is the time when

the process  $K_t^m$  passes through  $K^{cr}$ .

$$Q^0 = \int_0^{T^{cr}} q^c(K_t^m, 0) e^{\xi t} dt - \bar{X}(e^{\xi T^{cr}} - 1) \quad (A.11)$$

If  $Q^0 > X_0$  then  $X_{T^{cr}} < 0$  (see (2)) and  $q^c(K_t^m, 0)$  is not feasible.

**Proof of Claim 3.** To verify that  $K_t^* = \text{Min}\{K_t^m, \hat{K}^0\}$ , note that  $K(\lambda) \geq \hat{K}^0$  for any non-negative  $\lambda$ . Claim 2, then, requires that  $K_t^*$  must follow  $K_t^m$  at least up to  $\hat{K}^0$ . When  $\hat{K}^0 \geq K^{cr}$ , the fresh water stock cannot vanish when  $K_t^*$  arrives at  $\hat{K}^0$  or thereafter (with a positive  $\lambda$ ). Hence, the shadow price must vanish at the steady state, implying that the root process must converge to  $\hat{K}^0$ .  $K_t^*$  cannot exceed this state at any time, because otherwise it would violate the monotonicity property of Corollary 1, hence  $K_t^* = \text{Min}\{K_t^m, \hat{K}^0\}$ .  $\square$

Regarding the scarcity process, we establish the following proposition.

**Proposition.**  $\lambda_0^* = 0$  if and only if  $\hat{K}^0 \geq K^{cr}$  and  $X_0 \geq Q^0$ .

**Proof.** Suppose that  $\lambda_0^* = 0$ . Then, the root process reduces to the stationary point  $\hat{K}^0$  and, according to Claim 2,  $K_t^* = \text{Min}\{K_t^m, \hat{K}^0\}$  and  $q^c(K_t^*, 0)$  is the optimal fresh water supply. If  $\hat{K}^0 < K^{cr}$  then  $K_t^* < K^{cr}$  and  $q^c(K_t^*, 0) > R(0)$  at all times  $t$ . The stock will, therefore, be depleted on a finite date, at which time  $q^c$  must undergo a discontinuous drop to  $R(0)$ , violating Claim 1. Thus,  $\hat{K}^0 \geq K^{cr}$  and the optimal process must pass through  $K^{cr}$ . However, if  $X_0 < Q^0$ , then  $q^c(K_t^*, 0)$  is not feasible and the fresh water stock will be depleted prior to  $T^{cr}$ , implying again a discontinuity in  $q^c$ . Indeed, the second condition of the proposition is required to ensure that the initial stock suffices to support  $q^c(K_t^*, 0)$ . Otherwise, a positive scarcity rent is called for.

To see that the conditions of the proposition suffice, suppose that both  $\hat{K}^0 \geq K^{cr}$  and  $X_0 \geq Q^0$  hold. Then, the fresh water stock is never depleted using  $q^c(K_t^m, 0)$ . (If the stock is depleted prior to  $T^{cr}$ , then  $X_0 < Q^0$  since  $q^c(K_t^m, 0) > R(0)$  for  $K_t^m < K^{cr}$ , which holds for  $T < t < T^{cr}$ , violating the assumed condition. Similarly, if the stock is positive at  $T^{cr}$  it

will not vanish at a later date since  $q^c(K, 0) < R(0)$  for  $K > K^{cr}$ .) Moreover,  $K_t^* = \text{Min}\{K_t^m, \hat{K}^0\}$  according to Claim 3. Now, assume that  $\lambda_0^* > 0$ .  $q^c(K, \lambda)$  decreases in  $\lambda$ , hence  $q^c(K_t^*, \lambda_t^*) \leq q^c(K_t^m, 0)$  for all  $t < T^{cr}$  and the fresh water stock is never depleted under the optimal policy  $q^c(K_t^*, \lambda_t^*)$ , violating the transversality condition (4)  $X_T \lambda_0 = 0$ .  $\square$

**Proof of Claim 4.** (a) Follows directly from the proposition. (b) Suppose that  $X_0 < Q^0$ . From the proposition we know that  $\lambda_0^* > 0$ , hence the fresh water stock must be depleted at or before  $T^{cr}$  (after  $T^{cr}$ ,  $K_t^* > K^{cr}$  and depletion cannot occur). The values of  $\lambda_0^*$  and of the depletion date  $T^*$  are determined from (A.7) and (A.8).

Following depletion, the fresh water supply rate is restricted to  $R(0)$ . Eq. (A.2), then, gives  $\lambda_t^* = M_s(K_t^*) - M_c(R(0)) = M_s(K_t^*) - M_s(K^{cr})$  as long as this quantity is not negative, i.e., during the period  $T^* \leq t \leq T^{cr}$ . The supply mix is  $R(0)$  and  $D(M_s(K_t^*)) - R(0)$  for fresh and desalinated water, respectively. At  $T^{cr}$ ,  $K_t^m = K^{cr}$ , the shadow price vanishes and the third phase begins. As knowledge accumulates,  $K_t^* \geq K^{cr}$ ,  $q^c(K_t^*, 0)$  decreases below  $R(0)$  and desalination makes up the remaining demand. The fresh water stock fills up, eventually to enter a steady state at the stock level  $\hat{X} = \bar{X} - q^c(\hat{K}^0, 0)/\xi \geq 0$ , equality holding only if  $\hat{K}^0 = K^{cr}$ .  $\square$

We turn to the case  $\hat{K}^0 < K^{cr}$ . This case involves a different steady state, namely the root  $\hat{K}^R$  of  $L^R(K) = -M_s'(K)[D(M_s(K)) - R(0)] - (r + \delta)$  (cf. Eq. (15)). In terms of the root process, this steady state can be written as  $\hat{K}^R = K(\lambda^R)$ , where  $\lambda^R = M_s(\hat{K}^R) - M_c(R(0))$  is shown in Lemma 1 below to be positive. It is useful to distinguish between the date  $T^R = \log[(\bar{K} - K_0)/(\bar{K} - \hat{K}^R)]/\delta$  when the MRAP  $K_t^m$  passes through  $\hat{K}^R$  and the time  $T^{*R}$  when the optimal process  $K_t^*$  enters  $\hat{K}^R$ :  $K_{T^{*R}}^* = \hat{K}^R$ . Since no feasible process can proceed faster than  $K_t^m$ , it must be that  $T^{*R} \geq T^R$ .

We introduce the benchmark scarcity rent  $\lambda_0^m = \lambda^R e^{-(r+\xi)T^R}$ , the corresponding process  $\lambda_t^m = \lambda_0^m e^{(r+\xi)t}$  and the benchmark quantity

$$Q^m = \int_0^{T^R} q^c(K_t^m, \lambda_t^m) e^{\xi t} dt - \bar{X}(e^{\xi T^R} - 1) \quad (\text{A.12})$$

The proof of Claim 5 is presented via a series of Lemmas.

**Lemma 1.** *When  $\hat{K}^0 < K^{cr}$ , then  $\hat{K}^R \in (\hat{K}^0, K^{cr})$ .*

**Proof.** Suppose that  $\hat{K}^R \geq K^{cr}$ , so that  $M_s(\hat{K}^R) \leq M_s(K^{cr}) = M_c(R(0))$  and  $q^c(\hat{K}^R, 0) \leq R(0)$ . Hence,  $q^s(\hat{K}^R, 0) = D(M_s(\hat{K}^R)) - q^c(\hat{K}^R, 0) \geq D(M_s(\hat{K}^R)) - R(0)$  and, since  $M_s(K)$  is decreasing,  $L(\hat{K}^R, 0) \geq L^R(\hat{K}^R) = 0 = L(\hat{K}^0, 0)$ , implying that  $\hat{K}^R \leq \hat{K}^0 < K^{cr}$  and violating the assumption that  $\hat{K}^R \geq K^{cr}$ . Indeed, with  $\hat{K}^R < K^{cr}$ , we verify that  $\lambda^R \equiv M_s(\hat{K}^R) - M_c(R(0)) = M_s(\hat{K}^R) - M_s(K^{cr}) > 0$ . Moreover, the definition of  $\lambda^R$  implies that  $q^s(\hat{K}^R, \lambda^R) = D(M_s(\hat{K}^R)) - R(0)$ , hence both  $L^R(K)$  and  $L(K, \lambda^R)$  vanish at  $\hat{K}^R$  and  $\hat{K}^R = K(\lambda^R) > K(0) = \hat{K}^0$ . The situation is depicted in Fig. 2.  $\square$

**Lemma 2.** *When  $\hat{K}^0 < K^{cr}$ , the optimal steady state is at  $X = 0$ ,  $K = \hat{K}^R$  and  $\lambda = \lambda^R$ .*

**Proof.** Assume that the fresh water steady state stock is not empty. The corresponding scarcity rent must vanish, implying that  $\hat{K}^0$  is the steady state knowledge level. But when  $\hat{K}^0 < K^{cr}$  the fresh water supply rate  $q^c(\hat{K}^0, 0)$  exceeds  $R(0)$  and the finite stock must be depleted. Thus, when  $\hat{K}^0 < K^{cr}$  the steady state occurs with an empty stock and  $\lambda_t^* > 0$ . The fresh water supply rate at the steady state must, therefore, equal  $R(0)$ , implying, in view of Claim 2, that  $\hat{K}^R$  is the knowledge steady state. Since  $\hat{K}^R = K(\lambda^R)$ , it follows from Claim 2 again that  $\lambda^R$  is the scarcity rent at the steady state.  $\square$

**Lemma 3.** *When  $\hat{K}^0 < K^{cr}$ , the optimal processes  $K_t^*$  and  $\lambda_t^*$  enter their respective steady states  $\hat{K}^R$  and  $\lambda^R$  at or after the fresh water stock depletion date, i.e.,  $T^{*R} \geq T^*$ . During  $0 \leq t \leq T^*$ ,  $\lambda_t^*$  increases exponentially. If  $T^{*R} > T^*$ , then during  $T^* \leq t \leq T^{*R}$  the scarcity rent decreases back to its steady state level  $\lambda^R$  and the process  $K(\lambda_t^*)$  is non-monotonic.*

**Proof.** Suppose  $K_t^* = \hat{K}^R$  at some  $t < T^*$  and recall that  $q_t^c > R(0)$  prior to depletion. Then, using (A.2),  $M_c(q_t^c) + \lambda_t = M_s(\hat{K}^R) = M_c(R(0)) + \lambda^R$  implying that  $\lambda_t < \lambda^R$ . It follows that  $K(\lambda_t) < K(\lambda^R) = \hat{K}^R = K_t^*$ . But the optimal knowledge cannot exceed the root process and we conclude that  $\hat{K}^R$  cannot be entered

prior to depletion, so that  $T^{*R} \geq T^*$  and  $K_{T^*}^* \leq \hat{K}^R$ . Using (A.7) we find  $M_c(R(0)) + \lambda^R = M_s(\hat{K}^R) \leq M_s(K_{T^*}^*) = M_c(R(0)) + \lambda_{T^*}^*$ , hence  $\lambda_{T^*}^* \geq \lambda^R$ . If the strong inequality holds and  $T^{*R} > T^*$ , the shadow price (and the corresponding root process) must decrease after  $T^*$  until they reach  $\lambda^R$  and  $\hat{K}^R$ , respectively, at  $T^{*R}$ .  $\square$

By Claim 2,  $K_t^* = \text{Min}\{K_t^m, K(\lambda_t^*)\}$ . One possibility is that  $K_t^m$  lags behind  $K(\lambda_t^*)$  before  $\hat{K}^R$  is reached and the optimal process is a standard MRAP to  $\hat{K}^R$ . The alternative is that  $K_t^m$  overtakes the root process at an earlier date, and the optimal knowledge process is a NSMRAP, following the root process at its final stage. To identify the conditions under which either of these cases hold, we need the following lemma.

**Lemma 4.** *Suppose  $\hat{K}^0 < K^{cr}$ . Then, (a) if  $T^* < T^R$  then  $T^{*R} = T^R$  and  $K_t^*$  follows the standard MRAP to  $\hat{K}^R$ ; (b) if  $T^* > T^R$  then  $T^{*R} = T^R$  and  $K_t^*$  follows the NSMRAP before arriving at  $\hat{K}^*$ ; (c) if  $T^* = T^R$  then  $T^{*R} = T^*$  and  $K_t^*$  follows the standard MRAP as in (a).*

**Proof.**

1. Suppose  $T^* < T^R \leq T^{*R}$ . According to Lemma 3, the root process is non-monotonic, exceeding  $\hat{K}^R$  at the depletion date and returning to it at  $T^{*R}$ . If the optimal process were to follow the root process before  $T^{*R}$ , it must also be non-monotonic, contradicting Corollary 1. Thus,  $K_t^* = K_t^m$  all the way to  $\hat{K}^R$ .
2. Suppose  $T^* > T^R$ . Then  $T^{*R} \geq T^* > T^R$  and  $K_t^*$  departs from  $K_t^m$  to follow  $K(\lambda_t^*)$  before arriving at  $\hat{K}^R$ . But the optimal process is monotonic, hence the root process must also be monotonic, which according to Lemma 3 can occur only if  $T^{*R} = T^*$ .
3. Suppose  $T^* = T^R$  but  $T^{*R} > T^*$ . According to Lemma 3, the root process is non-monotonic and cannot be followed by  $K_t^*$ , which must, therefore, proceed with the standard MRAP all the way to  $\hat{K}^R$ . It follows that  $K_t^m$  and  $K_t^*$  reach  $\hat{K}^R$  on the same date, contradicting our assumption that  $T^{*R} > T^R$ . Thus,  $T^{*R} = T^* = T^R$  and  $K_t^* = \text{Min}\{\hat{K}^R, K_t^m\}$ .  $\square$

Whether or not  $K_t^m$  overtakes  $K(\lambda_t^*)$  depends on the initial scarcity rent  $\lambda_0$ , as  $K(\lambda_t)$  increases with  $\lambda_t = \lambda_0 e^{(r+\xi)t}$ . With the benchmark process  $\lambda_t^m$  as defined above,  $K(\lambda_{T^R}^m) = K(\lambda^R) = \hat{K}^R = K_{T^R}^m$  and  $K(\lambda_t^m)$  meets  $K_t^m$  at  $t = T^R$ . The root process is assumed to be slower than  $K_t^m$  and the two processes cannot cross twice. It follows that for any  $\lambda_0 \geq \lambda_0^m$ ,  $K(\lambda_0 e^{(r+\xi)t}) \geq K_t^m$  for all  $t \leq T^R$ , whereas if  $\lambda_0 < \lambda_0^m$  the two processes must cross prior to  $T^R$ . In view of Claim 2, this observation implies the following lemma.

**Lemma 5.** *Suppose  $\hat{K}^0 < K^{cr}$ . (a) If  $\lambda_0^* \geq \lambda_0^m$  then  $K_t^* = K_t^m$  until  $T^R$ ; (b) If  $\lambda_0^* < \lambda_0^m$  then  $K_t^* = K_t^m$  for  $t \leq \tau$  and  $K_t^* = K(\lambda_t^*)$  for  $t \geq \tau$ , where  $0 < \tau < T^R$  is the date  $K_t^m$  crosses  $K(\lambda_t^*)$ .*

To establish which of the two cases in Lemma 5 applies, we need the following lemma.

**Lemma 6.** *Suppose  $\hat{K}^0 < K^{cr}$ . Then  $\lambda_0^* \geq \lambda_0^m$  if and only if  $Q^m \geq X_0$ .*

**Proof.** Assume first that  $Q^m \geq X_0$ . This implies that under the  $(K_t^m, \lambda_t^m)$  policy the stock is depleted before or at time  $T^R$ . Suppose that  $\lambda_0^* < \lambda_0^m$ . The process  $K_t^m$  is not slower than any feasible policy hence  $K_t^* \leq K_t^m$  for all  $t \leq T^R$ . Moreover, since  $q^c(K, \lambda)$  decreases in both arguments,  $q^c(K_t^m, \lambda_t^m) < q^c(K_t^*, \lambda_t^*)$  and the optimal policy yields  $T^* < T^R$ . However, according to Lemma 5,  $\lambda_0^* < \lambda_0^m$  implies that the root process is adopted before  $T^R$ , which entails, according to Lemma 4,  $T^* > T^R$ , contradicting our previous assumption. Thus,  $Q^m \geq X_0$  implies  $\lambda_0^* \geq \lambda_0^m$ .

Suppose now that  $\lambda_0^* \geq \lambda_0^m$ , hence the optimal policy is the standard MRAP  $K_t^* = K_t^m$  until  $T^R$ . Thus,  $q^c(K_t^m, \lambda_t^m) \geq q^c(K_t^*, \lambda_t^*)$  and depletion under the optimal policy cannot precede depletion under the  $(K_t^m, \lambda_t^m)$  policy. From Lemma 4 we know that  $T^* \leq T^R$ , hence depletion under the  $(K_t^m, \lambda_t^m)$  policy cannot occur later than  $T^R$ , so that  $Q^m \geq X_0$ .  $\square$

**Proof of Claim 5.** (a) When  $\hat{K}^0 < K^{cr}$  and  $Q^m \geq X_0$ , then according to Lemma 6,  $\lambda_0^* \geq \lambda_0^m$ . Lemma 5, in turn, implies that  $K_t^* = \text{Min}\{K_t^m, \hat{K}^R\}$  is a simple MRAP to  $\hat{K}^R$ . (b) When  $\hat{K}^0 < K^{cr}$  and  $Q^m < X_0$ ,

then according to Lemma 6,  $\lambda_0^* < \lambda_0^m$ . Lemma 5, in turn, implies that  $K_t^*$  follows a NSMRAP to  $\hat{K}^R$ .  $\square$

In case (b), the parameters  $\lambda_0^*$ ,  $T^*$  and the switching date  $\tau$  are determined by solving (A.7), (A.8) and  $\tau - \log[(\bar{K} - K_0)/(\bar{K} - K(\lambda_\tau^*))]/\delta = 0$ .

## References

- Biswas, A.K. (Ed.), 1994. *International Waters of the Middle East*. Oxford University Press, Oxford.
- Dasgupta, P.S., Heal, G.M., 1974. The optimal depletion of exhaustible resources. *Rev. Econ. Stud.* (Symp.) 41, 3–28.
- Dasgupta, P.S., Heal, G.M., 1979. *Economic Theory and Exhaustible Resources*. Cambridge University Press, Cambridge.
- Dasgupta, P.S., Stiglitz, J., 1981. Resource depletion under technological uncertainty. *Econometrica* 49, 85–104.
- Dasgupta, P.S., Heal, M., Majumdar, M., 1977. Resource depletion and research and development. In: Intriligator, M. (Ed.), *Frontiers of Quantitative Economics*, North Holland, Vol. 3, pp. 483–506.
- Deshmukh, S.D., Pliska, S.R., 1985. A Martingale characterization of the price of a nonrenewable resource with decisions involving uncertainty. *J. Econ. Theory* 35, 322–342.
- Dinar, A., Loehman, E.T. (Eds.), 1995. *Water Quantity/Quality Management and Conflict Resolution*. Praeger, London.
- Glueckstern, P., Priel, M., 1998. Advanced concept of large seawater desalination systems for Israel. *Desalination* 119, 33–45.
- Heal, G.M., 1976. The relationship between price and extraction cost for a resource with a backstop technology. *Bell J. Econ.* 7 (2), 371–378.
- Hung, N.M., Quyen, N.V., 1993. On R&D timing under uncertainty: the case of exhaustible resource substitution. *J. Econ. Dyn. Control* 17, 971–991.
- Just, R.E., Netanyahu, S. (Eds.), 1998. *Conflict and Cooperation on Trans-boundary Water Resources*. Kluwer Academic Publishers, Boston.
- Just, R.E., Olson, L., Netanyahu, S., 1996. Resource depletion, technological uncertainty and adoption of improved technology: the case of desalination. In: Pirgram, J.J. (Ed.), *Proceedings of the international Workshop on Security and Sustainability in Mature Water Economy: A Global Perspective*. Center for Water Policy, University of New England, Armidale, NSW.
- Kamien, M., Schwartz, N.L., 1971. Expenditure patterns for risky R&D projects. *J. Appl. Prob.* 8, 60–73.
- Kamien, M., Schwartz, N.L., 1978. Optimal exhaustible resource depletion with endogenous technical change. *Rev. Econ. Stud.* 45, 179–196.
- Leonard, D., Long, N.-V., 1992. *Optimal Control Theory and Static Optimization in Economics*. Cambridge University Press, Cambridge.
- Spence, M., Starrett, D., 1975. Most rapid approach paths in accumulation problems. *Int. Econ. Rev.* 16, 388–403.
- Spiegler, K.H., Laird, A.D. (Eds.), 1980. *Principles of Desalination*. Academic Press, New York.
- Parker, D., Tsur, Y. (Eds.), 1997. *Decentralization and Coordination of Water Resource Management*. Kluwer Academic Publishers, Boston.
- Tsur, Y., Zemel, A., 1996. Accounting for global warming risks: resource management under event uncertainty. *J. Econ. Dyn. Control* 20, 1289–1305.
- Tsur, Y., Dinar, A., 1997. The relative efficiency and implementation costs of alternative methods for pricing irrigation water. *World Bank Econ. Rev.* 11, 243–262.
- Tsur, Y., Zemel, A., 1998. Global energy tradeoffs and the optimal development of solar technologies. Working Paper 9809. Center for Agricultural Economic Research, Rehovot, Israel.
- Tsur, Y., Zemel, A., 2000a. Long-term perspective on the development of solar energy. *Solar Energy* 68, 379–392.
- Tsur, Y., Zemel, A., 2000b. The infinite horizon dynamic optimization problem revisited: a simple method to determine equilibrium states. *Eur. J. Oper. Res.* in press.

