Modeling inter-sectoral growth linkages: An application to U.S. agriculture

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Abstract

General equilibrium and open economy trade theory are used along with time series data on the U.S. agricultural sector to provide insights into the structure of agricultural supply, factor returns and linkages to the rest of the economy. Output expansion and factor returns are found to vary depending on relative factor intensities, which we refer to as Rybczynski and Stolper–Samuelson like effects. The effect of the rest of the economy, particularly the increase in price of services, is found to have relatively large negative impacts on agriculture. The short-run effects of prices and factor endowments on growth in agricultural supply and factor returns are dominated by the long-run effects of technological change. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

This paper focuses on the structure of U.S. agricultural supply, corresponding factor returns and linkages to the rest of the economy. Previous empirical studies (see Capalbo, 1988 for a survey) of sectoral factor productivity and supply response have tended to ignore agriculture's linkages with the rest of the economy with which it must compete for resources. In a general equilibrium – open economy framework, the productivity of resources specific to agriculture, such as land, are affected by the evolution of the services and manufacturing sectors which have increased the demand for economy-wide resources. For instance, the favorable changes in the domestic terms of trade for the services sector have almost doubled its share of GDP and increased the share of labor from 59 to over 65%, since 1949 (US Department of Commerce Bureau of Economic Analysis National Income and Product Accounts of the United States, 1929–1992).[1] Moreover, the price index of intermediate factors of production produced by the manufacturing sector have tended to fall relative to the price index of services, but not relative to agriculture.

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[1] See Schultz (1953) and Barkley (1990) for previous arguments and empirical support for the claim that the net effect of economy-wide productivity growth has been higher wages, capital intensification, and a shrinking share of agriculture.

Since labor is an economy-wide factor, these changes may lead to the substitution of intermediate factors of production for the labor departing agriculture, while raising unit returns to those remaining in the sector. However, without growth in agricultural total factor productivity (TFP), these changes may not be sufficient to increase overall production. Moreover, as some subsectors of agriculture are likely to be more labor intensive than others, these changes also affect the subsectoral demand for factors specific to agriculture differentially, depending on relative factor intensities and TFP growth in each subsector.

The methodology used to study these linkages have tended to rely on computable general equilibrium models. These models typically are calibrated to a single year data expressed in a social accounting matrix (Robinson, 1989). While this approach has a number of advantages over the application of statistical models to time series data, several authors including Chipman et al. (1991) and Hansen and Heckman (1996) have criticized the over-reliance on the calibration approach because it has not made use of time series information.[2]

This paper utilizes the latter approach. We specify a sectoral GDP function following Diewert (1980) and Woodland (1982), and make use of its envelope properties along the lines of Kohli (1994) to obtain agricultural supply, factor returns and linkages to the rest of the economy. Parameters of these functions are estimated using time series data for the period 1949–1991 from Ball et al. (1997). A distinction is made between short and long-run on growth in agricultural supply and factor returns. Changes in economy-wide and sectoral output prices, and endowments are generally one-time effects and hence, referred to as short-run effects, whereas the effects of technological change are on-going and long-run in nature.

The results show that the rapid growth in non-farm economy increased the wages of hired labor, which placed considerable cost pressures on agriculture. These pressures had large negative effects on dairy relative to the grains sector. Fortunately, growth in the rest of the economy also increased the supply of material (intermediate) inputs, thus easing the substitution of these inputs for the ever more expensive labor. The rise in the price of services relative to the prices of agricultural commodities has had a relatively large negative impact on the supply and returns to specific factors in agriculture, while that of the industrial sector is opposite, but small. Productivity growth is found to have helped agriculture retain hired labor, purchase material inputs, and in the presence of declining terms to trade, to be the sole factor accounting for growth in output. Of the factors relatively specific to agriculture, the returns to family labor have benefitted the most from growth in productivity. While all own price supply elasticities are positive, they are relatively small, which is consistent with other general equilibrium estimates. Estimates of cross price effects suggest competition for specific resources, contrary to other studies (Ball, 1988; Luh and Stefanou, 1993).[3] The factor return responses to output prices (Stolper–Samuelson like effects) and output responses to factor endowments (Rybczynski like effects) are found to reflect the relative factor intensities of each subsector. In general, the long-run effects of technological change outweigh short-run effects.

2. The model

Consider the two element vector of outputs (vectors) Y_j, j = Agriculture (A) and Non-agriculture (N) and three inputs (v_A, v_N, v_E) where input vector v_j, j = A, N is specific to sector j and v_E is a vector of economy-wide factors which can be allocated among sectors. Following Woodland (1982), the economy-wide GDP function can be stated as:

$$G(p_A, p_N, \bar{v}_A, \bar{v}_N, \bar{v}_E)$$
$$= \max_\chi \{p_A Y_A(v_A, v_E^A) + p_N Y_N(v_N, v_E^N)\} \quad (1)$$

where,

$$\chi = \{(v_A, v_N, v_E^A, v_E^N) : v_A \leq \bar{v}_A, v_N \leq \bar{v}_N, v_E^A + v_E^N \leq \bar{v}_E\} \quad (2)$$

[2] Chipman et al. (1991) suggest that Jorgenson (1986) and Jorgenson et al. (1987) were among the first to show how econometric methods can be used in place of calibration.

[3] Non-parametric methods are typically partial equilibrium in nature and tend to provide relatively large bounds on supply and factor demand elasticities (Chavas and Cox, 1995).

and Y_A and Y_N are vintage production functions which exhibit constant returns to scale at the firm level (Diewert, 1980). Note that the Lagrangian multipliers of this maximization problem $(\lambda_A, \lambda_N, \lambda_E)$ are the shadow prices for the three categories of inputs. The envelope properties of $G(p_A, p_N, \bar{v}_A, \bar{v}_N, \bar{v}_E)$ (Woodland, 1982) imply the net output supply function:

$$\frac{\partial G}{\partial p_j} = y_j(p_A, p_N, \bar{v}_A, \bar{v}_N, \bar{v}_E), \; j = A, N \qquad (3)$$

and the factor rental rate or inverse demand function:

$$\frac{\partial G}{\partial v_i} = w_i(p_A, p_N, \bar{v}_A, \bar{v}_N, \bar{v}_E), \; i = A, N, E \qquad (4)$$

Eqs. (3) and (4) provide supply response to inputs (Rybczynski like effects) and factor rental rate response to output prices (Stolper–Samuelson like effects).[4]

Given λ_E from the solutions to the problem in Eq. (1), redefine it as:

$$\max\{p_A, Y_A(v_A, v_E^A) + p_N Y_N(v_N, v_E^N)$$
$$- \sum_j \lambda_E v_E^j : \; v_A \leq \bar{v}_A, \; v_N \leq \bar{v}_N\} \qquad (5)$$

Proposition (see Appendix A for proof). The solution to problem (5) is given by:

$$G = (p_A, p_N, \bar{v}_A, \bar{v}_N, \bar{v}_E) = g_A(p_A, \lambda_E, \bar{v}_A)$$
$$+ g_N(p_N, \lambda_E, \bar{v}_N) + \lambda_E \bar{v}_E \qquad (6)$$

where the envelope properties of Eq. (1) imply the function

$$\lambda_E = \lambda_E(p_A, p_N, \bar{v}_A, \bar{v}_N, \bar{v}_E) \qquad (7)$$

which is homogeneous of degree one in prices and zero in factor endowments. In the empirical model, labor is treated as the economy-wide resource which is assumed to be non-traded in international markets. In this case, Eq. (7) is used as an instrumental or reduced form equation. The underlying economy implied by Eq. (1) can be viewed as being in a short-run Walrasian equilibrium. At any point in time, the supply of factor endowments can be taken as given although their

supply may be variable in the long run (Kohli, 1994). The function $g_j(.)$ is referred to as a sectoral GDP function which, under certain regularity conditions, completely characterizes the underlying technology set (following Diewert, 1974). This product function is homogeneous of degree one in each of (p_j, λ_E) and $(\bar{v}_i)$, and has the same envelope properties as the economy-wide GDP function. Functions g_j and λ_E provide the basis for our parametric analysis of the responses of supply and factor returns in the jth sector. At time t, it follows that,

$$\text{GDP}_A^t = g_A^t(p_A, \lambda_E, \bar{v}_A) = \sum_{k=1}^{K} p_{Ak}^t y_{Ak}^t - \lambda_E^t v_E^{At}$$
$$= \sum_{l=1}^{L} w_{Al}^t \bar{v}_{Al}^t \qquad (8)$$

where returns to specific factors are represented by w_A^t. We assume that g_A^t can be represented by a translog functional form. See Appendix B for explicit specification of the translog sectoral GDP function with restrictions related to its homogeneity and symmetry properties.

The envelope properties of g_A^t applied to the translog form imply the output share equations, for $k = 1, \ldots, K$

$$S_k^t = \alpha_k^t + \sum_{r=1}^{K} \alpha_{rk} \ln p_{Ak}^t + \gamma_{Ek} \ln \lambda_E^t$$
$$+ \sum_{l=1}^{L} \delta_{kl} \ln \bar{v}_{Al}^t + \mu_{kt} \qquad (9)$$

and the input share equations, for $l = 1, \ldots, L$

$$S_1^t = \beta_1^t + \sum_{s=1}^{L} \beta_{As} \ln \bar{v}_{As}^t + \gamma_{El} \ln \lambda_E^t$$
$$+ \sum_{r=1}^{K} \delta_{Ar} \ln p_{Ar}^t + \mu_{lt} \qquad (10)$$

where, (μ_{kt}, μ_{lt}) are error terms. The time dependent constant terms (α_k^t, β_1^t) in the above Eqs. (9) and (10) are replaced by $(\alpha_k^0 + \alpha_k^1 t, \; \beta_1^0 + \beta_1^1 t)$, where t denotes a trend variable, time. Note that the derivative with respect to the price of hired labor is the negative share of hired labor (since λ_E^t is endogenous). This implies that the output shares and the share of hired labor sum to unity, as do the shares of sector specific inputs. The response of net supplies and factors rental rates to changes in output prices and levels of primary inputs can be computed from the parameter estimates of Eqs. (9) and (10) (Takayama, 1985 pp. 147–149). These are referred to as short-run effects on growth. In addition, following Kohli (1994), define the semi-elasticities of

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[4] We refer to these as 'like' effects, since Rybczynski and Stolper–Samuelson theorems do not necessarily apply to the general case (Woodland, 1982).

supply of outputs and returns to factors with respect to the time index as:

$$\epsilon_{kt} = \frac{\partial \ln y_{Ak}}{\partial t}; \quad \epsilon_{lt} = \frac{\partial \ln w_{Al}}{\partial t} \qquad (11)$$

These semi-elasticities indicate the effects of the passage of time (as a surrogate for technical change) on output supplies and factor returns which, we refer to as long-run effects on growth.[5] For the case of our translog sectoral GDP function, these semi-elasticities translate into:

$$\epsilon_{kt} = \frac{\alpha_k^1}{S_k} + \frac{\partial \ln g_A}{\partial t}; \quad \epsilon_{lt} = \frac{\beta_l^1}{S_1} + \frac{\partial \ln g_A}{\partial t} \qquad (12)$$

We make use of a discrete measure to approximate $\partial \ln g_A / \partial t$ as suggested by Jorgenson (1986), and evaluate these elasticities at average shares.

3. Data

The data that made this study possible are from Ball et al. (1997). The data on U.S. agriculture for 1949–1991 were aggregated into four outputs and five inputs. The output categories are meat animals, rest of livestock referred to as dairy, grain (food and feed grains), and crops (other than grains). The input categories are family labor, hired labor, real property, capital, and materials. Prices and quantities for outputs, are derived as Tornqvist indices. Price indices reflect market prices inclusive of deficiency payments, other commodity programs and net Commodity Credit Corporation loans (Ball et al., 1997).

The data show that the share of grain and crops in agriculture's GDP increased marginally, and at the expense of the livestock sectors (meat and dairy) over 1949–1991. Grains account for an average share of 18% of GDP. The average share of crops in agriculture's GDP is relatively large (31%) and stable, experiencing an annual growth rate of only 0.3% while the grains grew at an annual rate of 1.1%. The meat and

dairy sectors account for 27 and 24% of GDP, respectively. On average, the share of dairy has fallen more rapidly than the meat sector (at rates of −0.7 and −0.4%, respectively).

Among the five inputs, hired labor is treated as an economy-wide input while the other four are treated as specific to the agricultural sector. On average, the share of material inputs in the total cost is the largest at 40% followed by the shares of family labor (21), real property (18), capital (13) and hired labor (8).[6] The share of real property and capital have increased relative to other inputs mostly at the expense of labor (family and hired) over the period. The share of real property has grown at an average annual rate of 6.1% followed by capital at 1.1%. The share of family and hired labor declined at annual average rates of 2.5 and 2%, respectively. The decline in the share of materials is relatively small (0.4% per year). Thus, unlike the relatively constant share composition of agricultural output, the composition of costs have changed appreciably since 1948 with labor's share falling and real property and capital rising.

4. Econometric model

As our interest lies in estimating the structural production-side equations for the agricultural sector only, the econometric model is based on the share Eqs. (9) and (10). From the parameter estimates of share equations, sectoral supply and factor rental rate elasticities with respect to output prices and input quantities are computed. Following Jorgenson (1986), we refer to the parameters (α_k^1, β_l^1) in Eqs. (9) and (10) as biases of technical change (productivity growth), although other factors, such as efficiency gains from process innovations, may well be captured by these parameters. For α_k^1 positive, technical change is referred to as output-augmenting (see footnote 5). For β_l^1 positive (negative), technical change is referred to as input-saving (input-using). While it is straight forward to see the effect of technical change on output, technological biases on factor returns and factor use are not as apparent. A traditional cost function

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[5] Note that product-augmenting technical change and producer subsidies are formally equivalent in Walrasian general equilibrium (Jorgenson and Griliches, 1967; Jones, 1965; Ferguson, 1969). Since output prices take into account of producer subsidies in the data used by this study, the effect we capture using time trend reflect product-augmenting technical change.

[6] Note that the largest change has occurred in the share of hired labor which declined from over 20% of the total cost of production in 1949 to under 6% in 1991.

approach to measuring Hicksian bias (B_i) with respect to ith input at constant factor prices is given by (Binswanger, 1974):[7]

$$B_i = \frac{\partial \ln S_i}{\partial t}\Big|_{w_i} = \frac{\partial \ln v_i}{\partial t} - \frac{\partial \ln C}{\partial t} \qquad (13)$$

where, C is total cost, $S_i = w_i v_i / C$ is the share of ith input. Technical change is input i-using (saving), if $B_i > 0$ (<0) suggesting that the conditional demand (i.e., holding output constant) for input i falls less (more) rapidly than the extent to which production costs are lowered by growth in the agricultural TFP. An equivalent measure is to determine the change in the marginal rate of factor substitution at constant input ratios (Brown, 1968). At constant input ratios, in general equilibrium where factor rental rates are endogenous, the rental rate of the factor being saved rises relative to the rate of the factor being used.[8] In this situation, the sectoral GDP function, holding factor input ratios constant, Eq. (13) translates into:

$$B_i = \frac{\partial \ln S_i}{\partial t}\Big|_{v_i} = \frac{\partial \ln w_i}{\partial t} - \frac{\partial \ln g_A}{\partial t} \qquad (14)$$

In Eq. (14), if $B_i > 0$, then the growth in returns to factor i exceed the extent to which growth in the agricultural TFP increases sectoral GDP. Thus, $B_i > 0$ (<0) suggests that technical change is input i-saving (using).

It should be noted that the parameters (α_k^1, β_l^1) indicate rates of output and factor return augmentation relative to the average rate $\partial \ln g_A / \partial t$ for the entire agricultural sector. These measures are provided by the semi-elasticities $(\in_{kt}, \in_{lt})$. These elasticities indicate how some quantities may increase faster than others, and how some factors may benefit more than others from technological progress (shifts in production and factor-price possibilities frontiers). However, they cannot distinguish whether progress occurs

because it is inputs which become more productive, or because it is outputs which become easier to produce (Kohli, 1994, p. 12).

As the rate of return to hired labor, λ_E^t, (Eq. (7)) is endogenous it should be estimated along with the share equations as an instrumental variable. This equation is specified as:

$$\ln \lambda_E^t = \eta_0 + \sum\nolimits_{j=1}^{3} \eta_j \ln p_j^t + \sum\nolimits_{i=1}^{5} \eta_i \ln \bar{v}_i^t + \mu_t \qquad (15)$$

where, the variables corresponding to p_j are prices of aggregate agricultural goods, industrial goods and services. The variables $\bar{v}_i$ denote sector specific variables; land, materials and capital in agriculture, and economy-wide endowments; capital in non-agriculture and aggregate labor.[9]

Eqs. (9) and (10) suggest that the prices of outputs (p_A^t) evolve contemporaneously. However, this is not necessarily the case since agricultural production typically involves a time lag. Accordingly, the (p_A^t)'s in the right as well as left hand side (as shares are functions of prices) of the Eqs. (9) and (10) are replaced by (p_A^{t-1}). Since the shares sum to one, one output share (crops) and one input share (capital) equation are omitted from the system. Hence, the share equations of meat, dairy and grain on the output side and hired labor, family labor, materials and real property on the input side along with the equation for the rental rate of hired labor were fit to the data. The restrictions pertaining to homogeneity and symmetry properties of the sectoral GDP function were imposed on the system and used to obtain parameter estimates of the omitted equations. The unexplained variation in the dependent variables, as depicted by the residual terms $(\mu_{kt}, \mu_{lt}, \mu_t)$ for Eqs. (9), (10) and (15), respectively, were assumed to be random and normally distributed with zero mean and constant variance. The United States may be large in the world agricultural economy and so, its agricultural trade likely affects output prices, thus potentially creating a problem of serially correlated residuals. Our initial results also suggest that the residuals were correlated across equations and time periods as a first order vector auto-

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[7] Note that B_i equals β_l^1 / S_1 in Eq. (12).

[8] The underlying principle remains unchanged. In a two commodity-CES technology world, if technical change increase the marginal product of input i relative to input j, then the supply of services from input i are growing relative to the supply of services from input j. In general equilibrium, the rental rate of the factor whose supply of services are rising falls relative to the other factor, all else constant. Technical change is using factor j and saving factor i. Detailed exposition is available from the authors upon request.

[9] The non-agricultural sector constitutes both the industrial goods and services. Source of data: National Income and Product Accounts, BEA, U.S. Department of Commerce.

Table 1
Parameter estimates of share equations[a]

Shares	Price of					Endowment of				Time
	Meat	Dairy	Grain	Crops	H. Lab	F. Lab	Matrl	R Prop	Capl.	
Meat	0.229	−0.110	−0.047	−0.091	0.018	0.013	0.016	−0.013	−0.016	−0.0027
	(17.5)	(−8.9)	(−3.1)	(−10.5)	(3.1)	(1.1)	(0.9)	(−1.1)	(−3.0)	(−6.0)
Dairy	−0.110	0.230	−0.063	−0.053	−0.005	0.035	−0.046	−0.004	0.015	−0.0010
	(−8.9)	(15.3)	(−4.1)	(−5.1)	(−0.7)	(2.5)	(−2.3)	(−0.3)	(2.2)	(−2.1)
Grain	−0.047	−0.063	0.198	−0.111	0.023	−0.046	0.030	0.005	0.011	0.0011
	(−3.1)	(−4.1)	(9.6)	(−10.4)	(3.2)	(−2.9)	(1.5)	(0.4)	(1.7)	(1.9)
Crops	−0.091	−0.053	−0.111	0.262	−0.008	−0.005	0.023	0.006	−0.024	0.0022
	(−10.5)	(−5.1)	(−10.4)	(14.2)	(−1.2)	(−0.4)	(1.2)	(0.6)	(−2.1)	(2.4)
−H.Lab	0.018	−0.005	0.023	−0.008	−0.028	0.002	−0.023	0.006	0.015	0.0004
	(3.1)	(−0.7)	(3.2)	(−1.2)	(−5.2)	(0.2)	(−1.7)	(0.8)	(3.5)	(1.4)
F. Lab	0.013	0.035	−0.046	−0.005	0.002	0.043	0.004	−0.036	−0.011	0.0112
	(1.1)	(2.5)	(−2.9)	(−0.4)	(0.3)	(1.5)	(0.1)	(−1.5)	(−1.3)	(12.6)
Matrl	0.016	−0.046	0.030	0.023	−0.023	0.004	0.077	−0.086	0.005	0.0019
	(0.9)	(−2.3)	(1.5)	(1.2)	(−1.7)	(0.1)	(1.1)	(−2.9)	(0.4)	(2.5)
R Prop	−0.013	−0.004	0.005	0.006	0.006	−0.036	−0.086	0.111	0.011	−0.0094
	(−1.1)	(−0.3)	(0.4)	(0.6)	(0.8)	(−1.5)	(−2.9)	(4.0)	(1.7)	(−10.2)
Capl.	−0.016	0.015	0.011	−0.024	0.015	−0.011	0.005	0.011	−0.006	−0.0037
	(−3.0)	(2.2)	(1.7)	(−2.1)	(3.5)	(−1.3)	(0.4)	(1.7)	(0.36)	−(2.47)

[a] *t* ratios in parentheses

regressive (VAR) process, which we correct for in the results reported here.[10]

The correction proceeds as follows. Residuals $(\mu_{kt}, \mu_{lt}, \mu_t)$ obtained from the system are regressed on all $(\mu_{kt-1}, \mu_{lt-1}, \mu_{t-1})$ to obtain the matrix of parameters for the VAR process. The dependent and independent variables are transformed using this matrix, and then estimated using three stage least squares. This correction alleviates the need, if any, to form instruments for the other variables.

5. Results

Eq. (15) links the agricultural sector (share Eqs. (9) and (10)) to the rest of the economy through the competition for the economy-wide resource (labor). Table 1 presents the estimates of the share equations. The econometric model appears to fit the data surprisingly well, as indicated by the high t ratios in Table 1 and the system R^2 is 92%. Most of the restrictions pertaining to homogeneity and symmetry properties of the sectoral GDP function are accepted by the data. For the translog sectoral GDP function to be convex in output and input prices, it has to be the case that the Hessian matrix formed by rows 1 to 5 and columns 1 to 5 of Table 1 should be positive semi-definite which implies non-negative eigen values. Our results confirm this condition.[11] Output supply and factor rental rate response to output prices, factor endowments and technological progress (time) are computed using these parameter estimates evaluated at average shares. These results appear in Table 2. Specific attention is given to the short- and long-run effects on output supply and factor returns (and thereby, factor productivity).

5.1. Rybczynski and Stolper–Samuelson like effects

Output responses to factor endowments appear in rows 1 to 4 and columns 6 to 9 of Table 2. Note that these elasticities sum to one as the supply function is homogeneous of degree one in sector specific inputs.

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[10] See Bowden and Turkington (1984) for the estimation procedure. See also Berndt and Savin (1975).

[11] Note that the negative sign on the left hand side of the hired labor share equation (Table 1) should be passed on to the parameter estimates before deriving the eigen values.

Table 2
Supply and factor return elasticities

Elasticity with respect to										
	Price of					Endowment of				Time
	Meat	Dairy	Grain	Crops	H. Lab	F. Lab	Matrl	R Prop	Capl	
Supply										
Meat	0.082	−0.119	0.038	0.026	−0.028	0.276	0.484	0.144	0.096	0.0136
Dairy	−0.132	0.145	−0.043	0.138	−0.108	0.365	0.253	0.176	0.206	0.0191
Grain	0.056	−0.055	0.192	−0.215	0.024	0.003	0.580	0.215	0.203	0.0284
Crops	0.022	0.106	−0.127	0.111	−0.113	0.216	0.498	0.208	0.079	0.0294
H. Lab (demand)	0.091	0.312	−0.052	0.426	−0.778	0.208	0.682	0.127	−0.016	0.0185
Factor returns										
F. Lab	0.348	0.412	0.002	0.319	−0.081	−0.585	0.447	0.033	0.104	0.0716
Material	0.327	0.153	0.270	0.394	−0.143	0.239	−0.392	−0.009	0.162	0.0273
R Prop	0.221	0.241	0.226	0.372	−0.060	0.039	−0.02	−0.228	0.208	−0.0266
Capital	0.185	0.357	0.270	0.179	0.009	0.160	0.463	0.264	−0.887	−0.0018

Parameter estimates of the effects of endowments on supply (Rybczynski like effects) are also reflected in the estimated effects of output prices on factor returns (Stolper–Samuelson like effects).

All the sectors use material inputs relatively intensively except dairy, whose response to family labor is relatively high (an elasticity of 0.365). Family labor appears to be relatively more important to the meat and crops sectors (0.276 and 0.216, respectively) than to the grain sector.[12]

The result that the grain and crops output response to real property (which is predominantly land) are larger than the elasticities of two livestock sectors is consistent with their intensity of land use. The responses of meat and crops outputs to capital (0.096 and 0.079) are small, while that of dairy and grain outputs are fairly large and similar (0.206 and 0.203).

These Rybczynski like effects suggests that an augmentation in the availability of material inputs tends to favor the output of grain the most, followed by crops, meat and dairy outputs, respectively. In addition, the dairy and grain sectors use real property and capital relatively more intensively than the meat and crops sector. Meat, dairy and crops outputs remain relatively dependent on family labor.

Factor return's response to output prices (Stolper–Samuelson like effects) are presented in rows 6 to 9 and columns 1 to 4 of Table 2. These results show that

an increase in the price of meat and the price of crops tends to have, all else constant, relatively larger effects on the rate of return to materials in contrast to that of the other inputs (0.327 and 0.394, respectively). This result is consistent with the factor intensities reported above. Similarly, the returns to family labor are more responsive to an increase in the prices of meat, dairy and crops (0.348, 0.412 and 0.319, respectively) than to an increase in the price of grain. Correspondingly, note from our previous discussion that the supply response of these three outputs are relatively sensitive to the changes in the levels of family labor. Thus, as family labor departs agriculture, the returns to remaining family labor tend to rise (as implied by the estimated wage-family labor elasticity of −0.585). Returns to real property tends to be relatively more responsive to the price of crops, and equally responsive to the prices of the other three outputs. The rate of return to capital appears most responsive to the price of dairy and least responsive to the price of crops. In general, the effects of output prices on factor returns (Stolper–Samuelson like effects) are found to be proportional to the effects of factor endowments on outputs (Rybczynski like effects) since both relate to the relative intensity of production.

5.2. Supply response and factor substitution

Direct and cross-price supply elasticities appear in rows 1 to 4 and columns 1 to 4 of Table 2. These elasticities along with the elasticity in column 5 (see

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[12] Recall that the crops sector does not include grains.

section on economy-wide resources) sum to zero as the supply function is homogeneous of degree zero in output and hired labor prices. In general, own price elasticities are positive, but some cross price effects are large and negative suggesting strong intra-sectoral competition for sector-specific resources. The meat sector's response to its own price is relatively inelastic (0.082). The cross price effects of grain and crops are small and positive (0.038 and 0.026, respectively) implying complementarity. Dairy output response to its own price is also relatively inelastic (0.145). The cross price effects between meat and dairy are negative suggesting that they compete for resources, while complementarities exist between the dairy and crops sectors (a positive cross price elasticity of 0.138). Grain output is relatively more responsive to its own price (0.192) than any other sector, and appears to be a complement to the meat sector (0.056). The crops sector's response to its own price is 0.111 and it is also a complement to the dairy and meat sectors (0.022 and 0.106). The cross price effects between grains and crops are large and negative implying substitutability.

Substitutability among inputs employed in the sector can be inferred from the elements in rows 6 through 9 and columns 6 through 9 of Table 2. The diagonal elements of this matrix indicate that the response of all factor rental rates to their respective quantities is negative, since these equations can also be viewed as inverse factor-demand functions for agriculture. Substitutability among inputs is implied by the positive signs of off-diagonal elements. The negative signs suggest some complementarity between material inputs and real property. This result is consistent with the complementarity between intermediate inputs and capital (includes real property) identified by Jorgenson et al. (1987) using share elasticities from their sectoral models of production. Among factor returns, the returns to capital are relatively more responsive to its own input quantity (−0.887) followed by family labor (−0.585), materials (−0.392) and real property (−0.228).

5.3. The case of the economy-wide resource

The elasticities corresponding to hired labor are reported in Table 2 (elements in row 5 and column 5). As mentioned in data section, hired labor is treated as an economy-wide factor. The specification of the rental rate of hired labor in (15) as an instrumental variable allows us to evaluate some effects of the rest of the economy on agriculture through the market for hired labor.

The elements in column 5 correspond to output and factor return responses to the changes in the rate of return to hired labor. The responses of meat, dairy and crops outputs to an increase in the rental rate of labor is negative (−0.028, −0.108 and −0.113, respectively), while the response of grain output is positive, but small. The effect of changes in the returns to hired labor on other factor returns are reported in column 5, rows 6 through 9. The results suggest that capital may be a complement (0.009) while, the other factors, family labor (−0.081), materials (−0.143), and real property (−0.060) substitute for hired labor.

The elements of row 6 represent the response of the quantity demanded of hired labor to changes in agricultural prices and changes in the levels of other inputs. As the crops sector consists of some of the relatively labor-intensive horticultural crops, it is not surprising that the quantity of hired labor is relatively more responsive to their prices (0.426). The dairy sector is also identified as using hired labor relatively intensively. The response of hired labor to a unit increase in its own price (−0.778) suggests a relatively elastic labor supply response relative to other inputs.

5.4. The pattern of productivity growth

The previous discussion concerned short-run effects on changes in output supply and factor rental rates. In this section, we focus on long-run effects on growth. The effects of technical change, to the extent captured by the time surrogate, on supply and factor returns are measured up to a factor of proportionality (co-efficients on a trend variable time in the share equations, Table 1).

All the parameter estimates of the time variable (Table 1) in the output share equations are significant. Since the shares sum to unity these results suggest that the relative effect of efficiency gains has been to favor the production of grain and crops relative to the production of meat and dairy. Parameter estimates of the time variable in the factor share equations for family labor and materials are positive and significant at 5% level of confidence. They suggest that, to the

extent time is a surrogate for technical change, technological progress has been family labor and material saving. The corresponding result is unclear for hired labor, since its estimated coefficient (0.0004) is not significantly different from zero. In addition, these results also indicate that the agricultural sector has been both real property and capital-using but, the rate of using of real property is relatively larger than that of capital (-0.0094 versus -0.0037).

To assess the effects of technical change on supply and factor returns, a discrete approximation of $\partial \ln g_A / \partial t$ is employed (Jorgenson, 1986, p.1856). Our estimate of this value is 2.29% which is similar to Ball et al. (1997). Using this value, we evaluate Eq. (12) at average shares (see last column, Table 2). The effect of technical change on supply is the largest for crops and for grain with estimated average annual rates of productivity growth of about 2.9 and 2.8%, respectively, over 1949–1991. Productivity growth in meat and dairy sectors have been lower relative to the sectoral average of 2.29%.

Holding short-run effects constant, the returns to family labor and materials have benefitted positively from productivity growth (average annual growth rates of 7.2 and 2.7%, respectively), while having a tendency to decrease the rates of return to real property and land. These relatively high rates of output and factor return augmentation are not surprising, as productivity growth in the U.S. agricultural sector is found to be four times that of the non-farm economy (Jorgenson and Gollop, 1992).[13] Moreover, the direction of bias is consistent with induced innovation hypothesis for the case of U.S. agriculture (Hayami and Ruttan, 1985).

5.5. General equilibrium responses

We now turn our attention to analyzing the economy-wide linkages to the agricultural sector. The elasticities in Table 2 were computed holding the returns to hired labor constant. We now decompose the source of changes in the returns to hired labor and compute the supply and factor return responses arising

exclusively from changes in exogenous variables. To illustrate, the responses in Table 2 that are of the form:

$$\frac{\partial^2 g}{\partial p_i \partial p_j} = \frac{\partial y_i}{\partial p_j}\big|_{\lambda_E} = \theta_{ij}; \quad \frac{\partial^2 g}{\partial v_i \partial v_j} = \frac{\partial w_i}{\partial v_j}\big|_{\lambda_E} = \phi_{ij} \quad (16)$$

are recomputed according to,

$$\frac{\partial^2 g}{\partial p_i \partial p_j} = \frac{\partial y_i}{\partial p_j} + \frac{\partial y_i}{\partial \lambda_E} * \frac{\partial \lambda_E}{\partial p_j} = \theta_{ij}^*; \quad \frac{\partial^2 g}{\partial v_i \partial v_j}$$
$$= \frac{\partial w_i}{\partial v_j} + \frac{\partial w_i}{\partial \lambda_E} * \frac{\partial \lambda_E}{\partial v_j} = \phi_{ij}^* \quad (17)$$

Effectively, these computations show how activity on other sectors of the economy influence agriculture's competitiveness for the economy-wide resource, hired labor. First, it should be noted that the services sector of the U.S. economy is labor-intensive (Jorgenson et al., 1987). Hence, an increase in the price of quality adjusted services tends to bid up the rental rate of labor thereby, at the margin, raising production costs in agriculture. The results of estimating Eq. (15) along with the share equations (simultaneous system estimated using 3SLS) are:

$$\ln \lambda_E = -9.03^* + 0.26^* \ln p_A - 0.10 \ln p_I + 0.85^* \ln p_S$$
$$-0.11^* \ln v_A + 0.25^* \ln v_R + 0.26 \ln v_M$$
$$-0.10 \ln v_N - 0.29^* \ln v_L + 0.01^* t \quad (18)$$

where, (p_A, p_I, p_S) denote farm, industrial and service sector output prices and (v_A, v_R, v_M, v_N, v_L) denote capital, land and material inputs in agriculture, capital in non-agriculture and aggregate labor, respectively.[14]

The results corresponding to Eq. (17) are presented in Table 3. Table 3 replaces the wage variable of Table 2, column 5, with the price index for industrial goods and the price index for services (columns 5 and 6), and the endowments of economy-wide labor and non-agricultural capital (columns 11 and 12). While most of the direct and cross price, endowment elasticities are similar to those in Table 2, two results deserve special attention. The first is the responses of supply and factor returns to the changes in the price index of industrial goods and services. The other is the effects of increases in economy-wide labor and non-agricultural capital. The supply response to changes in the price of industrial goods is positive for all agricultural

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[13] Note that most studies of agricultural productivity (Ball et al., 1997; Luh and Stefanou, 1993) abstain from computing subsectoral productivity growth rates.

[14] Denotes significance at 5% level.

Table 3
General equilibrium supply and factor return elasticities

Elasticity with respect to													
	Price of						Endowment of						Time
	Meat	Dairy	Grain	Crops	Indus	Serv	F. Lab	Matrl	R. Prop	Capl	E. Lab	N. Cap	
Supply													
Meat	0.08	−0.12	0.04	0.02	0.00	−0.02	0.28	0.48	0.14	0.10	0.01	0.00	0.013
Dairy	−0.14	0.14	−0.05	0.13	0.01	−0.09	0.40	0.23	0.15	0.22	0.03	0.01	0.018
Grain	0.06	−0.05	0.19	−0.21	−0.00	0.02	−0.01	0.59	0.22	0.20	−0.01	−0.00	0.029
Crops	0.02	0.10	−0.13	0.10	0.01	−0.10	0.25	0.47	0.18	0.09	0.03	0.01	0.028
H.Lab (demand)	0.04	0.27	−0.09	0.36	0.08	−0.66	0.44	0.48	−0.07	−0.07	0.23	0.08	0.008
Factor returns													
F. Lab	0.29	0.37	−0.03	0.26	0.01	−0.07	−0.56	0.43	0.01	0.11	0.02	0.01	0.071
Matrl	0.27	0.11	0.23	0.33	0.01	−0.12	0.28	−0.43	−0.05	0.18	0.04	0.02	0.025
R Prop	0.17	0.19	0.19	0.31	0.01	−0.05	0.06	−0.04	−0.24	0.22	0.02	0.01	−0.027
Capl.	0.13	0.31	0.23	0.12	−0.00	0.01	0.16	0.46	0.27	−0.89	−0.00	−0.00	−0.002

outputs, except for grain (−0.002) which is small.[15] The demand for hired labor in agriculture increases as the price of industrial goods increase (0.08) and the returns to most sector specific inputs are positively affected by the price of industrial goods, as expected due to the linkages between the two sectors. However, as the price of services rise, the quantity of hired labor demanded in agriculture falls (−0.88) reflecting the service sector's capacity to pull labor from agriculture, as well as to lower the returns to sector specific factors, such as family labor and real property.

With one exception (capital), increases in the economy-wide labor and non-agricultural capital, like LeChatelier effects, tends to increase the returns to agriculture's sector specific resources. The supply response of agriculture to these endowments are relatively small, but the effects of labor tend to be larger than those of capital. The general equilibrium responses of supply, factor returns and quantity of hired labor with respect to time are similar to the results in Table 2, but the effect on hired labor is much smaller (0.8 %).

5.6. Contributions to predicted outputs and factor returns

The next step is to use the estimated elasticities (θ^*, ϕ^*) reported in Table 3 and the data to provide

insights into how the evolution of output prices, factor endowments and productivity growth affected output supply, hired labor demand and factor returns on average over 1949–1991.

To illustrate the calculations, the proportional change in supply is given by

$$\hat{y}_j^t \equiv \left(\frac{dy_j}{dt}\right)\left(\frac{1}{y_j}\right) \equiv \sum_i \theta_{ji}^* \frac{d\ln p_{ji}^t}{dt} + \sum_k \theta_{jk}^* \frac{d\ln v_{jk}^t}{dt} + \theta_{jt}^* dt$$

(19)

The average of the individual RHS components $\left(\theta_{ji}^*(d\ln p_{ji}^t/dt) + \theta_{jk}^*(d\ln v_{jk}^t/dt)\right)$ are then divided by the average predicted supply $\hat{y}_j$ to derive the contributions of prices, inputs and technological change to the average annual changes in predicted supply, hired labor demand and factor returns. The results reported in Table 4 are separated into short and long-run effects.

The Table 4 is constructed so that the total contributions (percent changes) sum to +100 (−100) if the dependent variable has increased (decreased) on average over the sample period. Positive (negative) numbers in the body of the table indicate the percent contribution of the row variable to increasing (decreasing) predicted change in supply, demand or factor returns.

The results show that the largest average annual rate of growth in supply occurred in grain (4.05%) followed by the crops sectors (2.96%). The diagonal elements of the matrix formed by rows 2 to 5 and columns 1 to 4 indicate that the own price contribution

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[15] This sector includes all the industries that add value to primary agricultural products.

Table 4
Contributions to predicted outputs and factor returns (Avg. Ann. %)

	Supply of				Demand H. Lab	Factor Returns of			
	Meat	Dairy	Grain	Crops		F. Lab	Matrl	R. Prop	Capl
Growth Rate	1.61	1.05	4.05	2.96	−0.36	11.26	3.31	−0.80	1.36
Price									
Meat	17.41	−45.18	4.93	1.71	36.32	9.08	28.63	72.87	33.58
Dairy	−14.68	25.30	−2.63	6.51	143.94	6.34	6.23	47.67	44.72
Grain	4.37	−8.41	9.10	−8.43	−46.26	−0.56	13.41	45.42	32.75
Crops	3.77	30.45	−13.40	8.72	255.31	5.77	25.34	98.37	21.83
Indus	0.71	4.05	−0.24	1.53	87.02	0.29	1.74	3.04	−0.28
Serv	−7.72	−44.09	2.56	−16.63	−946.37	−3.17	−18.89	−33.02	3.01
Endowment									
F. Lab	−42.45	−88.49	0.26	−20.09	−290.74	11.95	−20.37	−17.21	−27.78
Material	48.14	33.89	23.60	25.58	216.07	6.14	−21.00	−7.24	55.63
R. Prop	1.47	2.38	0.94	1.04	−3.27	0.02	−0.23	−5.25	3.37
Capital	5.11	16.90	4.12	2.57	16.89	0.84	4.46	22.46	−54.40
E. Lab	1.00	5.72	−0.33	2.16	122.82	0.41	2.45	4.29	−0.39
N. Cap	0.63	3.63	−0.21	1.37	77.84	0.26	1.55	2.72	−0.25
Short-run effects	17.77	−63.84	28.70	6.04	−330.43	37.37	23.31	234.05	111.79
Long-run effects	82.23	163.84	71.30	93.96	230.43	62.63	76.69	−334.05	−11.79
Sum	100.00	100.00	100.00	100.00	−100.00	100.00	100.00	−100.00	100.00

to the share of each output is positive with the dairy sector benefitting the most from its own price (25%). Except Dairy, the sum of the price effects (including the effects of industrial and services sector) are small ranging from 3.86% of the growth in the meat output (1.61% per annum) to −6.59% of the growth in crop output (2.96%). In the case of dairy, the increase in the prices of meat and services sector had large negative effects on its growth in output. The larger the intensity of hired labor, the greater are the impacts of the increase in the price of services (dairy and crops sectors). The off diagonal values reflect complementarities and substitutability among outputs (as reported in the section on supply response) from cross price effects. Overall, the increase in the price of services has pulled resources out of agricultural, while the industrial sector appears to have had little effect.

Now, turn attention to rows 8 through 13, columns 1 to 4. The departure of family labor had fairly large negative effects on average annual changes in supply of meat, dairy and crops, sectors that are labor-intensive relative to grains (rows 7 to 10 and columns 1 to 4). At the same time, a high growth rate in material inputs coupled with their intensity of use in all the sectors is reflected in its large positive contributions to growth in all outputs. Except for dairy, the sum of the

columns of this sub-matrix shows that the effect of changes in resource levels alone account for about 14, 28 and 13% of the average annual growth in the outputs of meat, grains and crops, respectively. The large negative effect from the decline in family labor for dairy dominates the other positive resource effects.

The short-run effects of prices and endowments on the growth in agricultural outputs are relatively small ranging from a high of 28% for grain to about 6% for crops, except in the case dairy. Not surprisingly, the effects of technological change dominate the sum of short-run effects, a result that is also confirmed from growth accounting (Ball et al., 1997; Gopinath and Roe, 1997).

As stated earlier, these results show that as the non-farm economy grew, the wages for hired labor increased (Schultz, 1953; Barkley, 1990). All else constant, this would have placed considerable cost pressures on agriculture. Fortunately, growth in the rest of the economy has also increased the supply of material inputs to agriculture, thus easing the substitution of these inputs fort the ever more expensive labor. As the sum of short-run effects show, this process had small effects on increasing the growth of output. Instead, productivity growth was the major factor causing growth in output.

The contributions to average annual changes in the returns to factors (except that of hired labor) from short and long-run effects are presented in columns 6 to 9 of Table 4. The data show that the predicted quantity of hired labor demand has fallen by -0.36% per annum. Real property is the only sector specific factor that experienced a decline in its average annual rental rate (-0.80%) over 1949–1991. Other factor returns experienced real growth, the highest of which is the family labor (11.26%). The sum of the agricultural price effects on factor returns are generally positive, with relatively large effects on returns to real property. Once again, the negative effects from the increase in price of services on factor returns are well evident.

Average annual effects of changes in resource levels on factor returns appear in rows 8 through 13, columns 6 through 9. Own effects on factor returns are negative, except for family labor. This positive effect indicates that returns to family labor remaining in agriculture had grown as other family labor departed the sector (the number of farms declined). The decline in this resource negatively affected the returns to other factors of production. Changes in price levels dominate resource effects in all cases, except family labor.

Technical change appears to have been the major force behind the average annual growth in returns to family labor (11.26%) since the short-run effects, particularly from prices, have been relatively small. A similar picture emerges for the growth in returns to materials. On average, the effect of technological change on returns to real property and capital has been negative. Thus, as the rest of the economy had grown, family labor remaining in agriculture has benefitted from both price effects and productivity growth.

6. Summary and conclusions

General equilibrium and open economy trade theory and time series data on the U.S. agricultural sector are used to provide insights into the structure of agricultural supply, factor returns and linkages to the rest of the economy. The effects of changes in economy-wide and sectoral output prices, and endowments are referred to as short-run effects on growth. Technological change is a long-run effect. Rybczynski

like effects in agriculture are positive, suggesting that an increase in sector specific endowments causes an expansion in all sub-sectors, although relative factor intensities vary among sectors so that some expand more than others. All sectors except grain appear to use material and family labor inputs relatively intensively. The response of factor returns to increases in output prices (Stolper–Samuelson like effects) follows the pattern of relative factor intensity. Consequently, the returns to family labor are more responsive to an increase in the prices of meat, followed by dairy, crops, and lastly, the grain sector.

Direct price elasticities of supply vary from 0.192 for grains to 0.082 for meat. These results tend to reinforce Binswanger (1989) argument that large supply responses from partial equilibrium analyses can be misleading because they do not capture the constraint of sector specific resources on total output. The cross price elasticities suggest intra-sectoral competition for sector specific resources.

Efficiency gains appear to be the greatest in crops and grains with estimated average annual rates of growth of 3% in their supply due to technological change. Technical change, on average, has tended to be family labor and material saving, real property and capital using, and indeterminate with respect to hired labor. In particular, technological change has caused the returns to family labor to grow by an annual average of over 7% per year. The importance of technology to agricultural growth is common knowledge, but its importance to growth in returns to agricultural-specific factors appear not to have been emphasized.

A conceptual contribution of the paper lies in decomposing the GDP function for agriculture which maintains its envelope properties, and provides linkages to the broader economy. The treatment of hired labor, as an economy-wide factor for which agriculture must compete with the rest of the economy, links changes in industrial and services sector prices, economy-wide endowments and other non-agricultural sector shocks to agricultural supply and factor returns. Based on the estimated general equilibrium elasticities and the data, the effects of the observed evolution of the exogenous variables (both the short and long-run effects) on the predicted values of the endogenous variables were computed. The sum of short-run effects of prices and factor endowments on supply and factor

returns in agriculture are small. They suggest that as the non-farm economy grew, the wages of hired labor increased, which, all else constant, would have placed considerable cost pressures on agriculture. Fortunately, growth in the rest of the economy increased the supply of material (intermediate) inputs, thus easing the substitution of these inputs for the ever more expensive labor. Yet, this process had only a small effect on increasing agricultural output, while productivity growth was the major contributor.

Acknowledgements

Appendix A.

The economywide GDP function is defined as:

$$G(p_A, p_N, \bar{v}_A, \bar{v}_N, \bar{v}_E)$$
$$= \max_\chi \{p_A Y_A(v_A, v_E^A) + p_N Y_N(v_N, v_E^N)\} \quad (A1)$$

where,

$$\chi = \{(v_A, v_N, v_E^A, v_E^N) : v_A \leq \bar{v}_A, v_N \leq \bar{v}_N, v_E^A + v_E^N \leq \bar{v}_E\}$$

The following are the envelope properties of the GDP function (Woodland, 1982):

1. Supply functions for outputs: $\partial G/\partial p_j = y_j, j = A, N$
2. Factor return function for sector specific inputs: $\partial G/\partial v_i = \lambda_j, i = A, N$
3. Factor return function for economy-wide input: $\partial G/\partial v_E = \lambda_E$

Let the solution to the maximization problem in Eq. (18) be:

$$(v_A^*, v_N^*, v_E^{A*}, v_E^{N*}, \lambda_A^*, \lambda_N^*, \lambda_E^*)$$

As $v_A^* = \bar{v}_A, v_N^* = \bar{v}_N$, define $\tilde{G}$ as follows:

$$\tilde{G}(p_A, p_N, \bar{v}_A, \bar{v}_N, \bar{v}_E) = \max_\chi \{p_A Y_A(v_E^A, v_A)$$
$$+ p_N Y_N(v_E^N, v_N) - \sum_j \lambda_E v_E^j\} \quad (A2)$$

$$\chi = \{(v_A, v_N, v_E^A, v_E^N) : v_A \leq \bar{v}_A, v_N \leq \bar{v}_N, v_E^A + v_E^A \leq \bar{v}_E\}$$

Proposition

$$\tilde{G}(p_A, p_N, \bar{v}_A, \bar{v}_N, \bar{v}_E) = \tilde{G}_A(p_A, \bar{v}_A, \lambda_E)$$
$$+ \tilde{G}_N(p_N, \bar{v}_N, \lambda_E) + \lambda_E v_E$$

Proof. The Kuhn–Tucker conditions for the problem in (19) include (interior solution):

$$p_A \frac{\partial Y_A}{\partial v_E^A} - \lambda_E = 0$$

$$p_N \frac{\partial Y_N}{\partial v_E^N} - \lambda_E = 0$$

Note the 'separability' of the problem in the choice variables leads to solutions for economy-wide inputs used in $j = A, N$.

$$\tilde{v}_E^j = v(p_j, \lambda_E, \bar{v}_i)$$

and therefore,

$$\tilde{G}(p_A, p_N, \bar{v}_A, \bar{v}_N, \bar{v}_E) = p_A Y(\tilde{v}_E^A, v_A)$$
$$+ p_N Y(\tilde{v}_E^N, v_N = \tilde{G}_A(p_A, \bar{v}_A, \lambda_E)$$
$$+ \tilde{G}_N(p_N, \bar{v}_N, \lambda_E) + \lambda_E \bar{v}_E$$

Once again, Envelope Theorem applied to this $\tilde{G}_j$ gives the following:

1. Supply functions for outputs: $(\partial \tilde{G}/\partial p_j)|_{\lambda_E} = y_j, j = A, N$
2. Factor return function for sector specific inputs: $(\partial \tilde{G}/\partial v_i)|_{\lambda_E} = y_i, i = A, N$
3. Factor return function for economy-wide input: $\partial \tilde{G}/\partial \lambda_E = -v_E^j, j = A, N$

Appendix B.

A translog form for the sectoral GDP function,

$$\ln g(p_A, \lambda_E, v_A) = \alpha_0^t + \sum_{k=1}^K \alpha_k^t \ln p_{Ak}$$
$$+ \left(\frac{1}{2}\right) \sum_{k=1}^K \sum_{r=1}^K \alpha_{kr} \ln p_{Ak} \ln p_{Ar} + \alpha_E^t \ln \lambda_E$$
$$+ \sum_{l=1}^L \beta_1^t \ln v_{Al} + \left(\frac{1}{2}\right) \alpha_{E,E}(\ln \lambda_E)^2$$
$$+ \left(\frac{1}{2}\right) \sum_{l=1}^L \sum_{s=1}^L \beta_{ls} \ln v_{Al} \ln v_{As}$$
$$+ \sum_{k=1}^K \gamma_{Ek} \ln p_{Ak} \ln \lambda_E + \sum_{l=1}^L \gamma_{El} \ln v_{Al} \ln \lambda_E$$
$$+ \sum_{k=1}^K \sum_{l=1}^L \delta_{kl} \ln p_{Ak} \ln v_{Al}$$

Note that the first order parameters are time dependent. Setting $\sum_{k=1}^{K} \alpha_k^t + \alpha_E^t = 1$, $\sum_{l=1}^{L} \beta_l^t = 1$ and restricting second order parameter summations to zero (e.g. $\sum_{r=1}^{K} \alpha_{kr} = 0$, $\sum_{s=1}^{L} \beta_{ls} = 0$) imposes the homogeneity properties of this function. Symmetry conditions are also imposed by setting second order cross partials to be equal (e.g., $\alpha_{rk} = \alpha_{kr}$, $\beta_{sl} = \beta_{ls}$, $\delta_{lk} = \delta_{kl}$).

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