

## A COMPARISON OF GLOBAL TIMBER MODELS

YOON HYUNG KIM\*

### **Keywords**

global timber model, constant elasticity of transformation, carbon sequestration cost

### **Abstract**

This study identifies differences of global timber models in its three different versions by comparing marginal abatement cost. The forest-only model has a relatively linear marginal cost curve; the other two models show concave cost curves, indicating that the marginal cost for carbon sequestration increases faster than in the forest-only model. Such basic differences among the models may be caused by the characteristics of the CET(Constant Elasticity of Transformation) model. The differences between the forest-only model and CET model are more narrowed when CET results are converted to physical units. This study contributes to the enhancement of the understanding of GTM development and provides foundation for future studies to improve global timber modeling.

## I. Introduction

Forests play traditionally important roles in economic life as a source of wood and other products. Recently, their environmental usefulness is enjoying the spotlight, as forests can provide a carbon sink via forest carbon sequestration as a part of reduction in greenhouse gas. This, in turn, should reduce global warming. In particular, carbon sequestration using forests increased in policy significance because it presents a method for both a relatively low-cost means

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\* Research Fellow, Korea Rural Economic Institute, Seoul, Korea

of removing carbon dioxide from the atmosphere (US Environmental Protection Agency, 1995) and long-term effectiveness for carbon abatement with other policies (Sohngen & Mendelsohn, 2007). In this context, forest economic models grew out of traditional supply-demand ones to link with various environmental problems.

Given that greenhouse gas emission from land use change and deforestation accounted for 17.4% of the total (IPCC 2007), land-use models for understanding relations between agriculture and forest sectors are increasingly important in the analysis of environmental problems. Therefore, there have been various attempts in modeling approaches: Scopes of models of forest economy expanded from wood products in the past to soil, land-use changes, and integrated existing economic models. Such forest models can be roughly divided into computable general equilibrium models (CGE) and partial equilibrium models (PEM).

Computable general equilibrium models include general sectors of production and consumption in economy, and interactions and feedback effects across economic sectors can be considered in these models in cases of various economic environments and changes in structures. On the other hand, partial equilibrium models are for analyzing specific economic sectors of concern without considering relations with other economic sectors. These allow for analysis of concerned sectors in the detailed sectoral model. Examples of computable general equilibrium models are GTAPEM (OECD, 2003) and GTAPE-L (Burniaux & Lee, 2003). Both models are modified versions of the standard Global Trade Analysis Project (Hertel, 1997) and often are used to analyze the impacts of environmental problems on the entire economy. The Global Timber Model (Sohngen et al., 1999) and the Forest and Agricultural Sector Optimization Model (Adams et al., 2005) are examples of partial equilibrium models. Meanwhile, despite the same theoretical backgrounds (or modeling approaches), there are considerably various models based on scale of model, period of analysis, and static or dynamic analysis. For instance, optimization problems solve for each period in static models, while dynamic models are for maximizing the net present value of welfare defined over the whole period by optimal distribution of resources over time through intertemporal optimizing with perfect foresight. The Timber Assessment Market Model (Adams & Haynes, 1980) is an example of a static model, whereas GTM and FASOM are dynamic models. Also, there are regional models and global modes divided by scale.

Forest Carbon Budget Model (Smith & Health, 2001), which is the model to estimate carbon in U.S. forests, and the forest carbon flux model FORUG (Verbeeck et al., 2006), which was used to analyze NEE (net ecosystem exchange) for the Hesse Forest in France, are examples of regional models, whereas a GTM is a global model. Recently, model developers often combine or couple existing economic models. Usually, a detailed bottom-up model is linked to a comprehensive top-down model. For example, Tavoni et al. (2007) linked the World Induced Technical Change Hybrid Model (WITCH) to a Global Timber Model built by Sohngen et al. (1999) to investigate the potential contribution of forest management to climate stabilization.

Among these various forest models, a GTM has several appropriate features that are differentiated from others. First, a GTM is a dynamic model. Because forests take time to grow and participate in a dynamic process affected by long-term decision making, a time path should be explicitly considered in a model. For instance, accumulation of carbon in the atmosphere results in rising carbon prices over time and consequently increases incentives, inducing carbon sequestration using forests. Static models have a demerit in that they treat forests as a steady-state stock rather than as a dynamic stock and then fail to explain intertemporal adjustment in response to changing incentives. However, in a GTM, dynamic optimization does account for important adjustments in timber inventories, which are composed of trees at various ages, and the dynamics of forest carbon sequestration are associated with that adjustment.

Second, a GTM is a global model. Regional studies may be useful in that relatively elaborate modeling is possible by using detailed data of a specific region. However, problems such as global warming and changes in land use are not limited to a national but worldwide phenomenon, and environment policies therefore should be conducted on a global basis. In this context, global models can show significant policy implications for environmental problems. For instance, when a nation imposes carbon tax for carbon sequestration, it may induce forest owners to release carbon in other nations. A GTM can present useful information for policymakers both by considering such leakage and by showing different reactions of nations on an environment policy. Or, when international prices of grain increase due to an international shortage of food, forests or pastures of a nation can be converted to compensate for the shortage. Global models are needed to analyze such land conversion and contention. Finally, a GTM includes forest products that are not handled by other models,

indicating that it can present a more accurate analysis of environmental problems.

The purposes of this study are to present and illustrate the current status of GTM researches, to compare and contrast various models so far, and then to present useful information for further study. The results might contribute to improvement of models in the future. In this study, three different versions of global timber models are introduced. The marginal abatement cost of each model is compared and analyzed for differences in the models. The paper is organized as follows: A GTM including forests only is introduced, followed by a GTM including agriculture and livestock. Models in which the results of the GTM including agriculture and livestock are converted to physical units are explained, and differences in marginal abatement costs of the models are described. A conclusion will follow.

## II. Model and Data

### 1. The Global Timber Model (Forest only)

The global timber model (forest only) was developed by Sohngen and Mendelson (2003, 2007) to analyze forest carbon sequestration in a global framework. The GTM is an optimal control model that solves for the maximum present value of net welfare in the forest sector. Net welfare is defined as the difference between the timber demand function and the costs of producing timber. Annual welfare is given as

$$(1) \quad W_t = \int_0^{Q^*(t)} \left\{ D(Q_t, Z_t) - \sum_{i,j,k} CH^{i,j,k}(\cdot) \right\} dQ(t) - \sum_{i,j,k} p_m^{i,j,k} m_t^{i,j,k} G_t^{i,j,k} - \sum_{i,j,k} (R_t^{i,j,k}) (X_t^{i,j,k})$$

$$\text{where} \quad Q_t = \sum_{i,j,k} \left( \sum_a H_{a,t}^{i,j,k} V_{a,t}^{i,j,k} (m_{t0}) \right)$$

In equation (1),  $D(Q_t, Z_t)$  is a global demand function for industrial

wood products given the quantity of wood  $Q_t$ , and income,  $Z_t$ . The quantity of wood harvested depends on  $H_{a,t}^{i,j,k}$ , the area of land harvested in the timber types in  $i$ ,  $J$  and  $k$ , and  $v_{a,t}^{i,j,k}(m_{t0})$ , the yield per hectare of timber in each age class. The yield per hectare depends upon the species, the age of the tree ( $a$ ), and the management intensity at the time of planting ( $m_{t0}$ ).  $CH^{i,j,k}(\cdot)$  is the cost function for harvesting and transporting logs to mills from each of the timber types. Marginal harvest costs for temperate and plantation forest types are constant, while marginal harvest costs for inaccessible forests rise as additional land is accessed. The costs of planting forests are given as  $\sum_{i,j,k} p_m^{i,j,k} m_t^{i,j,k} G_t^{i,j,k}$ , where  $G_t^{i,j,k}$  is the area of land planted,  $m_t^{i,j,k}$  is the management intensity of planting, and  $p_m^{i,j,k}$  is the per unit cost of a unit of management intensity. Units of management intensity enhance yield when the timber is harvested. The yield function has properties typical of ecological species, e.g.,  $V_a > 0$  and  $V_{aa} < 0$ . In addition, the following two conditions hold for trees planted at time  $t_0$  and harvested “a” years later  $(a + t_0) = t_{ai}$ :

$$(2) \quad \frac{dV^{i,j,k}(t_{ai} - t_0)}{dm^{i,j,k}(t_0)} \geq 0 \quad \text{and} \quad \left( \frac{d}{dm} \right) \frac{dV^{i,j,k}(t_{ai} - t_0)}{dm^{i,j,k}(t_0)} \leq 0$$

## 2. The GTM Incorporating Agriculture and Livestock Sector (CET Approach)

The model is extended by incorporating the crop and livestock sectors into the previous forestry-only model. The model maximizes the net present value of consumers’ plus producers’ surplus in the forestry, crop, and livestock markets and uses the constant elasticity of substitution production (CES) function to model the interaction between agriculture and forestry. Each dataset is aggregated into 18 regions with 18 agro-ecological zones (AEZs). There are many constraints in the model. However, one of the most important constraints is the forest land supply function because it captures the interaction between agri-

culture and forestry. The area of land in each forest type in each age class is given as  $QFCET_t^{i,j,k}$  and  $R_t^{i,j,k}$  is the annual rental value of the land. The area of land is the sum of the area of land in each age class.

A constant elasticity of transformation (CET) function, which controls the transformation between land uses, is utilized for modeling land supply. The CET forest supply function can be expressed as follows:

$$(3) \quad QFCET_t^{i,j,k} = \frac{XE^{i,j,k} \left( \frac{\alpha_F^{i,j,k}}{R_t^{i,j,k}} \right)^\tau}{\left[ \alpha_{Cr}^\tau R_{Cr}^{1-\tau} + \alpha_{Lv}^\tau R_{Lv}^{1-\tau} + \sum_{k=1}^6 \left( \alpha_F^{i,j,k} \right)^\tau \left( R_t^{i,j,k} \right)^{1-\tau} \right]^{\frac{\tau}{\tau-1}}}$$

$$(4) \quad XE = \left( \alpha_{Cr} X_{Cr}^{\frac{(\tau-1)}{\tau}} + \alpha_{Lv} X_{Lv}^{\frac{(\tau-1)}{\tau}} + \sum_{k=1}^6 \alpha_{F,k} X_{F,k}^{\frac{(\tau-1)}{\tau}} \right)^{\frac{\tau}{\tau-1}}$$

where  $X_{cr}$ ,  $X_{Lv}$ , and  $X_F$  are AEZ-specific land, and the parameters  $\alpha_{Cr}$ ,  $\alpha_{Lv}$ , and  $\alpha_F$  are CET land shares for supply in crop, livestock, and forestry. The variables  $R_{cr}$ ,  $R_{Lv}$  are land rental for crop, livestock. For the purposes of this analysis, it is assumed that they are given. The parameter of the CET supply function,  $\tau$ , can be considered as land supply elasticity and represents the maximum value on the elasticity of land supply with additional rental payment (Hertel et al., 2009). The CET parameter is set to -0.516 based on the econometric work of Choi et al. (2006).

### 3. Converting CET results to Physical Units of Area

With the CET, the common assumption is that land is imperfectly substitutable between different uses within an AEZ. The value of the transformation elasticity determines the degree of land mobility. However, such a non-linear treatment of land in the CET function implies that land is measured in the value added to the production rather than physical units of area. The disadvantage of such treatment of land is that carbon calculations based on GTM results may not represent reality. In order to solve this problem, the model was modified by adding several constraints to the GTM. Three equations are needed to con-

vert CET hectares to physical ones. First, CET crop hectare is converted to physical ones by using the following equation:

$$(5) \quad Q_1(crop) - Q_0(crop) = \left(\frac{1}{0.66}\right) \times (Q_1^{CET}(crop) - Q_0^{CET}(crop))$$

where  $Q_0(Q_0^{CET})$  and  $Q_1(Q_1^{CET})$  denote physical (CET) crop area in the initial period and in the following period, respectively. It is assumed that the average productivity of crop land decreases if forest and livestock lands are converted to crop land. In this study, following Hertel et al.(2010), the productivity parameter is set to 0.66 in this paper, which implies that approximately 1.5 (=1/0.66) new physical hectares are required to produce the same amount of product that one hectare of current crop land produces.

Then, an endogenous productivity adjustment variable (A) is included to convert CET forest and livestock area to physical hectares using the following equation:

$$(6) \quad Q_1(\text{forestry or livestock}) = A * Q_1^{CET}(\text{forestry or livestock})$$

Additionally, It is assumed that the sum of crop, forestry, and livestock areas is constant over time.

$$(7) \quad \sum_i Q_0(i) = \sum_i Q_1(i) \quad (i=\text{crop, forestry and livestock})$$

Combining the three equations above yields the solution for the productivity variable (A) using the following equation:

$$(8) \quad \begin{aligned} Q_0(crop) + Q_0(\text{forestry}) + Q_0(\text{livestock}) &= Q_1(crop) + Q_1(\text{forestry}) + Q_1(\text{livestock}) \\ &= Q_0(crop) + \left(\frac{1}{0.66}\right) \times (Q_1^{CET}(crop) - Q_0^{CET}(crop)) + A \times (Q_1^{CET}(\text{forestry}) + Q_1^{CET}(\text{livestock})) \end{aligned}$$

Finally, substituting A into equation (6) yields physical hectares for forestry and livestock.

#### 4. Data

Forest inventories by region, the productivity of forests, the costs of extracting and transporting timber to mills data are obtained from Sohngen et al. (2009). This analysis is based on a definition for agro-ecological zones that builds on the work of the FAO and IIASA (2000), and is described in Monfreda et al. (2009) and Lee et al. (2009). The Food and Agriculture Organization of the United Nations and the International Institute for Applied Systems Analysis have developed the Agro-ecological Zones (AEZ) methodology. The agro-ecological zones differ by growing period (6 categories of 60-day growing period intervals) and climatic zone (tropical, temperate and boreal). The length of growing period depends on temperature, precipitation, soil characteristics and topography, and the suitability of each agro-ecological zone for production of alternative crops and livestock is based on currently observed practices. In each region of this model, there are up to 18 agro-ecological zones and there are up to 6 timber types in each AEZ. Data on land rents and land areas in each agro-ecological zone are obtained from Lee et al. (2009).

To make projections in the forestry sector, I utilize a demand function of the form

$$(9) \quad Q_t = A_t \left[ \left( \frac{Y_t}{N_t} \right) \right]^h (P_t)^e$$

where  $Y_t$  is global gross domestic product,  $N_t$  is global population,  $P_t$  is the global price of timber,  $h$  is income elasticity, and  $e$  is the price elasticity. <sup>Gross</sup> domestic product per capita  $\left( \frac{Y_t}{N_t} \right)$  is assumed to grow at 2.3% per year. Income elasticity for forestry products is calculated from the AIDADS (An Implicitly Directly Additive Demand System) modeling system developed by Rimmer and Powell (1996) and estimated by Yu et al (2002). Initially, it is 0.87, and it rises to 0.93 over the century.



### III. Results and Conclusion

#### 1. A Comparison of Carbon Sequestration Cost

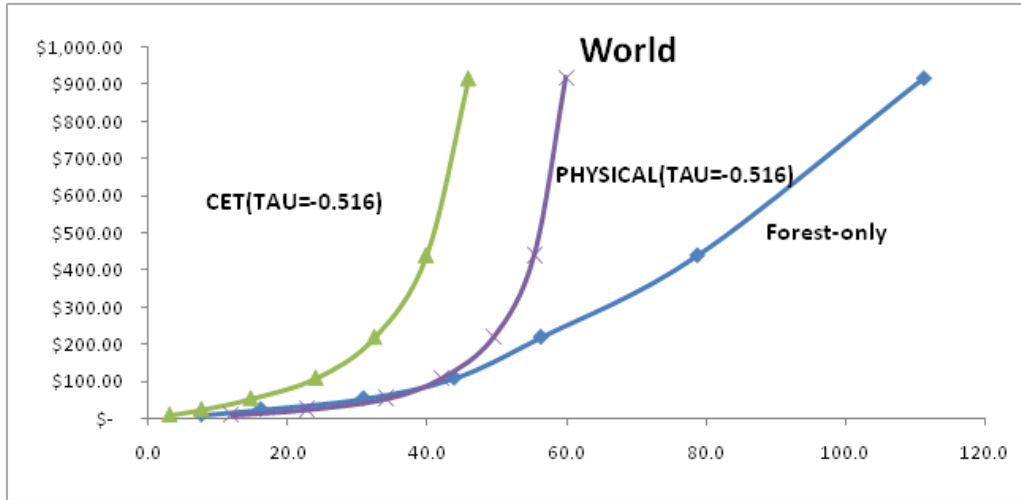
The carbon sequestration cost curves of each model explained above are compared by introducing carbon prices into the model. Carbon gains are measured as the difference between baseline carbon emissions and emissions in each price scenario, and then carbon credits are obtained by converting the net present value of carbon over different periods. The CET parameter, which controls the transformation between each of the land uses, is set to -0.516 in this simulation. Table 1 presents carbon gains worldwide in each model relative to baseline by 2055, 2075 and 2095. For example, the net present value of carbon over a 70-year period shows that for \$11.01 per Mg C, around 8 Pg of additional carbon is stored in the forest-only model, while 3 Pg and 12 Pg of additional carbon is stored in the GTM including agriculture and livestock and the GTM with physical hectares, respectively. For \$55.05 per Mg C, around 31 Pg of additional carbon is stored in the forest-only model, while 15 Pg and 34 Pg of additional carbon is stored in the GTM including agriculture and livestock and the GTM with physical hectares, respectively. For \$110.1 per Mg C, around 44 Pg of additional carbon is stored in the forest-only model, while 24 Pg and 42 Pg of additional carbon is stored in the GTM including agriculture and livestock and the GTM with physical hectares, respectively. Since the result of the forest-only model is given in terms of physical units, comparing the CET result and physical unit result with that of the forest-only model is interesting. For purposes of comparison, It is assumed that the result of the forest-only model is the true quantity and calculate the relative errors from this value. The relative error for the physical unit result is approximately 56% of that of the forest-only model at \$11.01 per Mg C, decreasing to 4% at \$110.1 per Mg C, and the differences between the forest-only and the forest model with agriculture and livestock are narrower when the CET result is converted to physical units. However, the relative error for the CET result is 59% of that of the forest-only model at \$11.01 per Mg C. The CET result remains comparatively high (45%) at \$110.1 per Mg C. This result is consistent over different periods.

**Table 1.** World carbon gains from carbon price scenarios

	Carbon price(\$ per Mg C)	\$11.01	\$25.69	\$55.05	\$110.1	\$220.2	\$440.4	\$917.5
Forest-only	C gain over 50 years	7.6	16.0	30.6	43.0	54.8	76.0	106.6
CET	C gain over 50 years	3.0	7.6	14.6	24.0	32.4	40.2	44.9
	Relative Error	60.2%	52.4%	52.3%	44.1%	40.8%	47.1%	57.9%
Physical	C gain over 50 years	12.0	23.2	33.3	41.0	47.9	53.0	57.1
	Relative Error	57.3%	44.7%	8.9%	4.6%	12.5%	30.2%	46.5%
	Carbon price(\$ per Mg C)	\$11.01	\$25.69	\$55.05	\$110.1	\$220.2	\$440.4	\$917.5
Forest-only	C gain over 70 years	7.6	16.1	30.9	43.9	56.4	78.8	111.4
CET	C gain over 70 years	3.1	7.7	14.7	24.1	32.5	39.9	46.0
	Relative Error	59.0%	52.4%	52.3%	45.2%	42.4%	49.4%	58.8%
Physical	C gain over 70 years	11.9	22.7	34.1	42.1	49.5	55.4	60.0
	Relative Error	56.4%	40.6%	10.5%	4.2%	12.1%	29.7%	46.2%
	Carbon price(\$ per Mg C)	\$11.01	\$25.69	\$55.05	\$110.1	\$220.2	\$440.4	\$917.5
Forest-only	C gain over 90 years	7.6	16.2	30.9	44.0	56.8	79.7	112.7
CET	C gain over 90 years	3.0	7.5	14.5	23.5	31.5	38.5	43.6
	Relative Error	60.3%	53.7%	53.2%	46.7%	44.5%	51.8%	61.4%
Physical	C gain over 90 years	11.8	22.3	33.8	42.2	49.9	55.8	60.4
	Relative Error	55.3%	37.7%	9.2%	4.3%	12.2%	30.1%	46.4%

Since the carbon sequestration cost curve is the locus of carbon gains in each carbon price scenario, the carbon abatement cost curve can be presented, rendering the differences between the CET model and forest-only model clearer. Figure 1 presents the world abatement cost curve for each model. The forest-only model has a relatively linear marginal cost curve, while the other two models show concave cost curves, indicating that the marginal cost for carbon sequestration increases faster than in the forest-only model. Such basic differences among the models may be caused by the characteristics of the CET model. The CET model is based on incomplete substitution relations among land use, and the parameters of the CET functions determine the degrees of mobility for each land use. The differences between the two models are narrower when the CET result is converted to physical units.

**FIGURE 1.** World carbon abatement cost curves (C gain over 70 years)



## 2. Conclusion

Global timber model is a dynamic optimization model and has been used for examining global land-use and climate change mitigation policies including carbon prices. The GTM model used in this study has been developed over a number of years. The objective of this study is to present the current status of GTM researches and illustrate the differences among models so far. One concern with using a CET approach to model land supply is that land areas calculated by the CET function are given in value added terms rather than physical terms. Therefore, special attention was given to convert CET results to physical unit ones because the forest-only model considered just the forest (the results of the model do not need to be converted to physical units). On the other hand, the forest model with agriculture and livestock uses the CET function; thus, its results should be converted to physical units. This is because incomplete land substitution relations among agriculture, livestock, and forest are established in the CET model. In order to solve the problem, the model was adjusted by adding several constraint equations to it.

In this study, differences in the given models are presented by describing a GTM in its three different versions and comparing marginal abatement cost. Based on the comparison of marginal cost curves, the forest-only model

has a relatively linear marginal cost curve, whereas the other two models show concave cost curves, indicating that the marginal cost for carbon sequestration increases faster than in the forest-only model. Such basic differences among the models may be caused by the characteristics of the CET model. The CET model is based on incomplete substitution relations among land use, and the parameters of CET functions determine the degrees of mobility of each land use. The differences between the two models are more narrowed when CET results are converted to physical units. This study contributes to the enhancement of the understanding of GTM development and provides foundation for future studies to improve global timber modeling.

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