



The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search

<http://ageconsearch.umn.edu>

aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

No endorsement of AgEcon Search or its fundraising activities by the author(s) of the following work or their employer(s) is intended or implied.



ELSEVIER

Agricultural Economics 18 (1998) 53–62

AGRICULTURAL
ECONOMICS

Seasonality and unit roots: the demand for fruits

Carlos Arnade ^{*}, Daniel Pick

*U.S. Department of Agriculture, Economic Research Service, Trade Analysis Branch, 1800 M Street, NW,
Washington DC 20036-5831, USA*

Accepted 30 July 1997

Abstract

We apply seasonal unit root tests to apple and pear price and quantity data. We then develop a method for testing shifts in amplitude and/or phase of the seasonal cycles. The results have implications to econometric specifications of models which use short-run data (quarterly, monthly). Published by Elsevier Science B.V.

Keywords: Seasonality; Unit roots; U.S. fruits

1. Seasonality and unit roots: implications to demand for U.S. fruits

Testing the properties of time series data has received much attention in the general economic literature with particular emphasis on macroeconomic variables. More recent literature has emphasized the importance of seasonality in macroeconomic time series data (Hylleberg et al., 1990, 1993; Kunst, 1993; Canova and Hansen, 1995; Franses, 1991; Franses and Paap, 1995). In particular, if seasonal unit roots cannot be rejected, then seasonal differencing of the data is appropriate. These papers have also tested the stability of seasonal trends which exist in certain macroeconomic variables. However, the implication of such tests is that if seasonal trends change over time, deterministic seasonal dummy variables do not capture the true essence of seasonal variations.

While seasonality exists in various time series data, it is readily observed in agriculture. Production

of agricultural goods in a particular region is confined to certain months of the year, thus leading to pronounced seasonal variation in observed variables such as supply, demand, prices and others.

The U.S. fruit markets exhibit characteristics of significant seasonality both in prices and quantities. Demand for certain fruits may be influenced by seasonal traditions. Prices of fruits rise during the winter season when domestic supply diminishes, while imports from other countries such as Chile and Mexico increase dramatically. The traditional approach for accounting for such seasonality in modeling agricultural markets (supply, demand, trade) has been to use seasonal dummy variables. Both the dramatic increase in availability of fruits, (particularly apples, grapes, and pears) imported from other producing countries during the off season and demographic changes within U.S. households may have changed the seasonal cyclical structure of the market. Thus, the cyclical seasonal variation in such markets may have shifted. While it may be possible to specify a model which describes such changes, there are less costly and less data-demanding techniques for

^{*} Corresponding author.

capturing seasonal trends which extend beyond the use of seasonal deterministic dummy variables.

Unit root tests are typically applied to trends in time series data. Many time series data have been shown to be integrated of order 1, thus indicating that first differencing of the data will insure stationarity. Recently, Hylleberg et al. (1993) devised a formal method of testing for seasonal unit roots. If seasonal unit roots exist, then seasonal trends are present in the data requiring seasonal differencing as a first step in time series analysis. While this point has been the focus of seasonal root studies, a related point is that evidence of seasonal trends indicate that either the amplitude of the seasonal cycles have shifted, and/or the timing of the season has shifted. This latter case is referred to by Hylleberg et al. (1993) (p. 328) as the case of ‘winter becomes spring.’

There are number of examples as to why amplitude and phase shifts may arise in economic data. Increase harvest from year to year and growing demand for heating fuel are just two examples for why amplitude shift can be observed in economic data. Phase shifts in seasonal cycles can occur when new seed varieties are introduced or new areas are brought under cultivation. Hylleberg et al. (1993) cite the example of growing irrigation as a contributor to phase shift, and Franses (1991) mentions the shift of vegetables production to greenhouses as a reason for changing seasonal agricultural cycles.

Phase shifts in consumption patterns may result from demographic changes such as changes in the number of school age children, changes in the percentage of working women, or changes in the ratio of manufacturing to service workers. Changes in relative regional population levels, each with its own seasonal consumption habits, can cause a phase shift in the aggregate demand for some products. Franses

(1991) states that the timing of winter and summer clearance sales has shifted over time. While many of the examples provided in the literature are agriculture in nature, most studies go on to test for seasonal unit roots in macroeconomic variables.

In this paper, we apply seasonal unit root tests to price and quantity data for apples and pears. These fruits were chosen because of observed changes in their consumption and price and because of data availability. One of our goals is to demonstrate that seasonal unit roots appear in data that are sector specific. Previous studies have primarily demonstrated that macroeconomic data have seasonal roots. While this test has the usual implications for time series analysis, we argue that it also has implications for constructing structural models. In effect, we argue that these tests can determine whether traditional seasonal dummy variables capture all aspects of the seasonality in a structural model. We then illustrate how trigonometric variables in conjunction with trend variables can be used to determine whether a seasonal trend arises from changes in the magnitude of a seasonal cycle or a shift in a seasonal cycle. We apply this approach to a simple specification of demand equations. Our objective is not to provide a complete description of the demand for apples and pears but to demonstrate that seasonal unit roots can be found in data beyond the standard seasonal macroeconomic variables and to demonstrate how to distinguish between trends in seasonal amplitude and seasonal phases.

The rest of the paper is organized as follows. Section 2 uses three simple specifications to estimate the demand for apples and pears. Results are presented and compared. In Section 3, tests for seasonal unit roots are outlined and discussed. The results of the seasonal unit roots are listed and analyzed in Section 4. To account and test for shifts in the

Table 1
Apple and pear demand equations with no seasonal dummies

	Apples	Elasticity	Pears	Elasticity
Constant	–5278.8 (–6.35)		–1335.6 (–5.31)	
P(apples)	–2697.0 (–5.97)	–0.97	(170.6) (1.17)	0.06
P(pears)	170.6 (1.17)	0.05	–837.1 (–4.41)	–0.26
Income	5.47 (9.48)	4.50	1.30 (7.17)	1.08
Adjusted R^2	0.29		0.21	

t-values are listed in parentheses.

Table 2
Apple and pear demand equations with seasonal dummies

	Apples	Elasticity	Pears	Elasticity
$P(\text{apples})$	–1217.9 (–2.98)	–0.43	294.7 (2.76)	0.11
$P(\text{pears})$	294.7 (2.76)	0.09	–534.5 (–3.89)	–0.16
Income	4.13 (8.25)	3.44	0.86 (6.34)	0.72
D_I	–3628.6 (–5.10)		–849.6 (–4.55)	
D_{II}	–3959.9 (–5.62)		–1070.0 (–5.88)	
D_{III}	–4960.9 (–7.15)		–852.8 (–4.71)	
D_{IV}	–3847.8 (–5.33)		–615.0 (–3.20)	
Adjusted R^2	0.51		0.60	

t -values are listed in parentheses.

seasonal cycles, the demand equations are specified with seasonal trends in Section 5. Results of the estimation and tests are analyzed. Section 6 concludes the paper.

2. Simple specifications of fruit demand and results

Hylleberg et al. (1990) developed seasonal unit root tests which can aid in univariate modeling. Osborn (1993) argues that these tests are generated by statistical concerns and need to have more economic rationalization. We review and apply these tests and show an alternative method for capturing seasonal trends using a simple specification of demand equations for apples and pears.

To facilitate the above objective, several specifications of the demand for apples and pears are estimated. This is done using monthly data spanning the 1976–1993 period.¹ The first model does not account for any seasonality:

$$q_{i,t} = a_0 + a_1 P_{i,t} + a_2 P_{j,t} + a_3 I + \varepsilon \quad (1)$$

where $q_{i,t}$ is the quantity consumed of fruit i in period t ($t = 1, \dots, T$), $P_{i,t}$ is the own price, $P_{j,t}$ is the price of substitute, I is income and ε is an error term with zero mean. The parameters in Eq. (1) for apples and pears were estimated jointly with each fruit serving as a substitute for the other. Note that these equations serve as a base for building in seasonal effects. To maintain focus on seasonality the

specification of the demand equations are kept as simple as possible. Extensions to more formally derived demand systems could be explored in future studies.

The results of estimating Eq. (1) and implied elasticities are summarized in Table 1. Symmetry restrictions on the cross price variables could not be rejected and therefore were imposed on the two equations.² All the coefficients carry the correct signs with the own price and income variables being highly significant. The cross-price coefficient in the apple equation was positive but not significant and negative and significant in the pear equation. While the adjusted R^2 is relatively low at 0.29 for apples and 0.21 for pears, each equation is significant at the 0.01 confidence level using an F -test (see Chow (1983) p. 58).

The second specification of the demand equations is similar to Eq. (1), but contains four seasonal (quarterly) deterministic dummy variables. Thus, Eq. (1) becomes:

$$q_{i,t} = a_1 P_{i,t} + a_2 P_{j,t} + a_3 I + \sum_{i=1}^4 \beta_i D_i + \varepsilon \quad (2)$$

where D_i s are the quarterly dummy variables taking a value of one for observations occurring during quarter i and zero otherwise.

The results of estimating Eq. (2) and the implied elasticities are summarized in Table 2. As in Eq. (1), symmetry restrictions could not be rejected and

¹ See Appendix A for data sources.

² This result was determined by the data. In economic terms, this implies that the income effects of both Marshallian demand functions are either negligible or approximately equal.

Table 3

Apple and pear demand equations with trigonometric variables

	Apples	Elasticity	Pears	Elasticity
Constant	–3980.4 (–6.08)		–676.9 (–5.31)	
$P(\text{apples})$	–978.3 (–2.57)	–0.35	239.2 (3.27)	0.09
$P(\text{pears})$	239.2 (3.27)	0.07	–336.8 (–3.43)	–0.10
Income	3.98 (8.54)	3.32	0.72 (7.61)	0.60
f_1	695.5 (10.41)		–111.3 (–10.66)	
f_2	–252.3 (–4.02)		–78.0 (–7.57)	
f_5			29.4 (2.90)	
g_1	394.9 (6.02)		261.9 (19.44)	
g_2	135.0 (2.17)		–112.4 (–10.98)	
g_5			36.2 (3.57)	
Adjusted R^2	0.58		0.81	

t -values are listed in parentheses.

therefore were imposed. Including seasonal dummy variables improved the results in several ways. First, the cross price effects are now significant and indicate that apples and pears are substitutes. Second, the seasonal dummy variables are all significant. Last, the fit of each equation as indicated by the adjusted R^2 has improved to 0.51 for apples and 0.60 for pears.

A third specification of the demand equation includes trigonometric variables and is specified as follows:

$$q_{i,t} = a_0 + a_1 P_{i,t} + a_2 P_{j,t} + a_3 I + \sum_{m=1}^6 a_{4m} f_m + \sum_{k=1}^6 a_{5k} g_k + \varepsilon \quad (3)$$

where $f_m = \cos((m/Z)\pi t)$, $g_k = \sin((k/Z)\pi t)$, $Z = s/2$, and t is the observation number. Different values of m and k correspond to different seasonal frequencies of the data. For quarterly data, $s = 4$ and for monthly data, $s = 12$. In our case, since we use monthly data, $Z = 6$.³ By construction, the elements of f_m and g_k are cyclical processes at the seasonal frequencies $((m/6)\pi)$ and $((k/6)\pi)$. Each frequency variable is orthogonal to all other frequency variables. The coefficients (a_{4m}, a_{5k}) represent the contribution of each cycle to the seasonal process s .

Results of Eq. (3) and the calculated elasticities are listed in Table 3. Given the monthly data fre-

quency, we could potentially have 11 trigonometric coefficients (note that $\sin(1) = 0$). However, only significant trigonometric variables were included in the final regression. Both in the apple and pear equations the own price, cross price, and income variables carry the expected signs and are highly significant. In the apple and pear equations, f_1 and g_1 represent one cycle per year, while f_2 and g_2 represent two semi-annual cycles. These variables were included in both equations. The variables f_5 and g_5 represent five cycles per year and are included only in the pear equation. The adjusted R^2 measure improved further to 0.58 for apples and 0.81 for pears.

The above equations were estimated without any exploration of seasonal trends and are used only to illustrate how seasonality, which is often portrayed through dummy variables, influences the relationships in our data. Tests for seasonal trends are reported in Section 3.

3. Seasonal unit roots

In an article on seasonal trends, Hylleberg et al. (1990) have devised a method for exploring seasonal roots in the data. Noting that recent literature on time series analysis of data with unit roots has affected the practice of econometric applications, they conclude that current tests of unit roots assume a zero-frequency peak in the spectrum of the data. They go on to claim that there has been little effort to develop

³ For more complete explanation of the trigonometric specification, see Chow (1983) and Canova and Hansen (1995).

tests for other unit roots that may be inherent in the data. Hylleberg et al. (1990) (p. 216) state, “because many economic time series exhibit substantial seasonality, there is a definite possibility that there may be unit roots at other frequencies such as the seasonals.”

To determine whether there are any seasonal roots in a univariate time series data, Hylleberg et al. (1993) decompose the polynomial which describes seasonal differenced data. For example, consider quarterly data for variable X and seasonal difference it : $X_t - X_{t-4}$. This expression can be represented in a backshift notation: $(1 - B^4)X_t$, where B is the backshift operator. The polynomial $(1 - B^4)$ can be decomposed into four components:

$$\begin{aligned}(1 - B^4) &= (1 - B)(1 + B)(1 + B^2) \\ &= (1 - B)(1 + B)(1 - iB)(1 + iB) \quad (4)\end{aligned}$$

This decomposition highlights that there are four roots to differenced quarterly data. Two real roots $(-1, 1)$ and two imaginary roots $(-i, +i)$. By expanding the polynomial about these roots, a test for seasonal unit roots can be devised. Several studies (Hylleberg et al., 1993; Franses, 1991; Joutz et al., 1995) have used monthly macroeconomic data to test for seasonal unit roots.

We follow the Franses (1991) version of the seasonal unit root test and seasonally difference the monthly data, $Y_{8,t} = (1 - B^{12})X_t$. $Y_{8,t}$ is the seasonal first difference (the difference between an observation in period t and corresponding observation in the previous year). First, factor $(1 - B^{12})$ into 12 polynomials, each representing a different frequency. Note that each Y_i , represents a filtered version of the variable X . We then apply the Hylleberg et al. (1990) procedure to linearize these polynomials around the zero frequency unit root and 11 seasonal unit roots. The resulting equation can be used to test for monthly seasonal roots using monthly data:

$$\begin{aligned}Y_{8,t} &= \Pi_1 Y_{1,t-1} + \Pi_2 Y_{2,t-1} + \Pi_3 Y_{3,t-1} + \Pi_4 Y_{3,t-2} \\ &+ \Pi_5 Y_{4,t-1} + \Pi_6 Y_{4,t-2} + \Pi_7 Y_{5,t-1} \\ &+ \Pi_8 Y_{5,t-2} + \Pi_9 Y_{6,t-1} + \Pi_{10} Y_{6,t-2} \\ &+ \Pi_{11} Y_{7,t-1} + \Pi_{12} Y_{7,t-2} + \mu_t + \varepsilon_t \quad (5)\end{aligned}$$

where Π_i are parameters to be estimated and:

$$\begin{aligned}Y_{1,t} &= (1 + B)(1 + B^2)(1 + B^4 + B^8) X_t \\ Y_{2,t} &= -(1 - B)(1 + B^2)(1 + B^4 + B^8) X_t \\ Y_{3,t} &= (1 - B^2)(1 + B^4 + B^8) X_t \\ Y_{4,t} &= -(1 - B^4)(1 - 3^{1/2}B + B^2) \\ &\quad \times (1 + B^2 + B^4) X_t \\ Y_{5,t} &= -(1 - B^4)(1 + 3^{1/2}B + B^2) \\ &\quad \times (1 + B^2 + B^4) X_t \\ Y_{6,t} &= -(1 - B^4)(1 - B^2 + B^4)(1 - B + B^2) X_t \\ Y_{7,t} &= -(1 - B^4)(1 - B^2 + B^4)(1 + B + B^2) X_t \\ Y_{8,t} &= (1 - B^{12}) X_t; \mu_t = \text{constant}\end{aligned}$$

The various representations of the data are derived by taking a polynomial decomposition of $(1 - B^{12})$ to represent different seasonal cycles. Y_1 represents data where all seasonal cycles have been eliminated. A failure to reject the null hypothesis $H_0: \Pi_1 = 0$ implies that we cannot reject the presence of a non-seasonal unit root. If this holds and seasonal unit roots are rejected then an autoregressive model in first differences can adequately represent the time series of interest. The variables $Y_2 \dots Y_7$ represent data where all but one frequency has been eliminated. Descriptions of these frequencies are provided by Hylleberg et al. (1990) and Beaulieu and Miron (1993).

A data series may reflect a mix of both seasonal and non-seasonal trends. One of the advantages of the multivariate test in Eq. (5) is that it isolates seasonal cycles from each other and from non-seasonal trends. Beaulieu and Miron (1993) (p. 307) state that “the goal is to test hypotheses about a particular unit root without taking a stand on whether other seasonal or zero frequency unit roots are present.”

To test for seasonal unit root, we test the significance of the parameters $\Pi_2 \dots \Pi_{12}$ which correspond to each Y_i . The null hypothesis is such that the data have a unit root at the zero frequency and the seasonal frequencies. Rejection of a non-seasonal unit root is based on the estimated standard error on Π_1 . For seasonal unit root testing, joint tests of the Π coefficients, representing the same transformed variables Y_i s (for example, Π_3 and Π_4 are coeffi-

cients on Y_3), are performed. If a test result is not significant, then we cannot reject unit root at the seasonal frequency represented by the respective Y_i .⁴ Beaulieu and Miron (1993) note that unlike past approaches to exploring seasonal trends, this test allows one to distinguish processes that may be integrated at only some of the frequencies.

Note, the appropriateness of applying the standard seasonal filter $(1 - B^{12})$ implies that a series is integrated at the zero frequency and at all seasonal frequencies. Thus, the above procedure is primarily used as a mean to test the validity of seasonal differencing. However, emphasis should also be placed on the fact that the presence of seasonal unit roots indicates that either amplitudes of seasonal cycles are changing over time, and/or that there are phase shifts in the seasonality of the data. Phase shifts indicate that the seasons themselves are not constant and may shift over time. If the test is performed with seasonal deterministic dummy variables included, then the results can be used to determine whether dummy variables capture all aspects of seasonality. If seasonal dummy variables are included and unit root cannot be rejected, then standard dummy variables will not capture a changing seasonal trend.

4. Results

We tested the monthly data for seasonal unit roots in each of the five variables of the model. These are pear price and quantity, apple price and quantity, and income. Three versions of Eq. (5) were estimated, and the test results are summarized in Table 4. The first version is identical to the presentation of Eq. (5) earlier. Second, a trend variable was added. The last version included a trend variable as well as seasonal dummy variables.

The results must be viewed in light of the fact that the traditional unit root tests, such as the Dickey Fuller test, are known to have low power. Franses (1991) (p. 205), realizing this point then states that “that the power to the test statistic may be low,

Table 4
Unit root test results

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7
<i>Apples price</i>							
No Trend	-0.10	-2.37	0.35	0.85	0.23	0.76	6.39*
Trend	4.16	-2.58	1.26	0.74	2.13	0.63	6.85*
Trend, dummies	3.78	-2.71*	0.65	0.24	1.33	0.09	6.17*
<i>Apples quantity</i>							
No Trend	2.73	-1.18	10.92*	4.62	2.41	7.24*	5.30
Trend	6.76	0.86	8.16*	6.41*	7.90*	9.20*	0.16
Trend, dummies	2.55	1.05	5.97*	6.16*	9.45*	8.28*	0.68
<i>Pears price</i>							
No Trend	-0.20	-1.25	0.69	3.30	3.12	2.84	0.97
Trend	-0.57	-1.25	0.68	3.15	3.22	2.96	1.14
Trend, dummies	-1.47	-1.53	0.89	3.27	3.05	2.88	1.71
<i>Pears quantity</i>							
No Trend	1.53	-0.74	2.17	0.11	0.50	0.26	0.34
Trend	-0.63	0.65	1.48	0.13	0.28	0.06	0.07
Trend, dummies	-0.26	-0.62	1.42	0.13	0.28	0.06	0.08
<i>Income</i>							
No Trend	0.98	-2.11	3.99	1.13	1.08	1.12	1.11
Trend	1.40	-2.15	4.23	1.24	1.19	1.23	1.22
Trend, dummies	0.76	-2.15	4.16	1.72	1.17	1.20	1.19

* Significant at the 5% level.

except for the joint F -tests for all complex Π_i , and hence that significance levels of 10%, or even higher may be more appropriate.”

Using the approach suggested by Hylleberg et al. (1993) and Franses (1991), we test for unit roots and seasonal roots at each frequency as implied by the Y_i s.⁵ The results indicate that in general, unit roots as well as seasonal roots cannot be rejected for most variables and seasonal frequencies. Both price and quantity data for pears as well as income are characterized by unit and seasonal roots at all frequencies. For apple prices, we reject unit root at the frequency 4/12 (8, 12) represented by Y_7 . With seasonal dummies and trend included in the model, we also re-

⁴ See Hylleberg et al. (1993) for expanded discussion of testing unit roots at different seasonal frequencies.

⁵ Joutz et al. (1995) employ a joint test to all frequencies by testing $\Pi_1 = \Pi_2 = \dots \Pi_{12} = 0$. We also applied this test and could not reject seasonal unit roots for the dependent variables, apple and pear quantities. Note that such an approach eliminates the ability to test hypothesis about a particular seasonal unit root which is a major advantage of the Hylleberg et al. (1993) approach.

jected seasonal root at the 6/12 frequency implied by Y_2 .

For the apple quantity data, results suggest rejection of seasonal unit roots at several frequencies when trend or trend and dummy variables are included in the model. These frequencies are represented by Y_3 , Y_4 , Y_5 , and Y_6 . Overall, these results indicate the presence of trends in the seasonality of the data. Notice, however, that these results do not provide information as to the nature of the seasonal trend. In the next section, we develop a test which identifies the source (phase vs. amplitude) of the shift for the dependent variables in the demand equations.

5. Demand equations with seasonal trends

Failure to reject seasonal unit roots implies that a data series is generated by integrated seasonal process which can be described by:

$$(1 - B)(1 - B^{12})X_t = e_t \quad (6)$$

where e_t is a random error. Such result is useful for univariate modeling of time series data, but provides little guidance for economic modeling.

Several authors (Kunst, 1993; Boswijk and Franses, 1995) discuss multivariate extensions of the above specification. Kunst (1993) estimated a seasonal error correction model (ECM) using quarterly data.⁶ Quarterly differences were regressed on lagged quarterly differences and levels of each variable transformation. A monthly seasonal ECM with four variables would require 48 explanatory variables of which four variables would represent lagged differences and 44 variables would represent the 11 transformations of each of the four variables. Viewing the seasonal ECM as an extension of the stan-

dard ECM, it should be clear that a simplifying two-step procedure is available. One would estimate level models for each transformation and then use the error terms as explanatory variables in the seasonal ECM which include the seasonal differences. In a four variable model, this would reduce the number of explanatory variables to only 15. Yet, it would still be difficult to provide meaningful interpretation of such results.

An alternative approach is to argue that there exists a model which is deterministic about seasonal trends. One can create a variable that consists of an interaction term between a trend variable and a trigonometric variable representing a particular frequency. This variable can then be used as an exogenous variable in the economic model. Though the approach to handling seasonality is quite distinct from time series analysis or estimation of a seasonal error correction model, it is motivated from the observation that there are seasonal trends in the data. In a complex model, interaction terms could be created for every frequency.

In this section, we keep matters simple and create an example of using an interaction term for the frequency represented by $\sin((1/6)\pi t)$ and $\cos((1/6)\pi t)$.⁷ This interaction term between data which trends annually and seasonal trigonometric variables can be used to determine the source of seasonal trend. For example, it may be of interest if seasonal trends changes are occurring from changes in the amplitude of the seasonal cycle, from a phase shift of a cycle, or both. Though seasonal unit root tests can measure if there are seasonal trends, it cannot determine whether they arise from amplitude or phase changes.⁸

Consider the following function (Chow, 1983):

$$\beta_1 \cos(\cdot) + \beta_2 \sin(\cdot)$$

The dot inside the parentheses represents the argu-

⁶ With quarterly data, Kunst (1993) shows that a seasonal extension of the Johansen's co-integration test is possible. Seasonal co-integration testing requires that a canonical correlation problem is solved between each filtered series and the seasonally-differenced data. Thus rather than one test, as in a standard cointegration problem, there would be 11 tests, one for each of the Y_i transformations in a monthly seasonal model. To our knowledge, no one has tried seasonal co-integration tests with monthly data.

⁷ Interaction terms can be created at any frequency where unit roots are shown to exist.

⁸ Recall that a data series with a unit root will have a permanent memory of an error. Similarly, a data series with seasonal roots indicates that an error in a certain season will be reflected in that season's observations in the future. Thus, a seasonal shock can cause permanent changes in the amplitude of a cycle and a series of adjacent shocks can permanently shift the cycle.

ments of the function. The amplitude of the function is given by:

$$(\beta_1^2 + \beta_2^2)^{1/2}$$

Changes in amplitude, at a particular frequency, can be monitored by observing the changes in the coefficients of the trigonometric variables representing that frequency. This can be done using a trend variable interacting with the trigonometric variables.⁹ A phase shift is more complex. Such shift changes the starting and ending points or the location of a seasonal cycle. The location of a cycle at a particular frequency depends on a weighted average of the cosine and sine variables. These weights are determined by the parameter values of the trigonometric variables in the estimated model. If the location of the seasonal cycle is displaced by the amount τ , then the relative weights must change as well. Formally, $\tan(\tau)$ which is the tangent of the phase displacement, can be shown to be proportional to (β_1/β_2) . Therefore, changes in the relative coefficients of the trigonometric variables can represent a phase shift in the season.¹⁰

In short, if there are seasonal unit roots, then there are either changes in amplitudes and/or phase shifts. However, seasonal unit root tests cannot determine which changes are occurring. If one creates a structural model that is deterministic about seasonal trends, there is a way of isolating these changes by including the following term in the estimated model:

$$\beta_1 * \cos(\cdot) + \alpha_1 * TR * \cos(\cdot) + \beta_2 * \sin(\cdot) + \alpha_2 * TR * \sin(\cdot) \quad (7)$$

where TR is a variable that exhibits a trend like effect over the time period under consideration. If either α_1 and/or α_2 are significant, then from the definition of the amplitude, we can conclude that the

amplitude of the seasonal cycle is changing over time.¹¹

The test for a phase shift is slightly different. If the coefficient restriction $\alpha_2 = \beta_2 \alpha_1 / \beta_1$ does not significantly change the fit of the model, then we cannot reject the hypothesis of no phase shift. This comes from recognizing that no phase shift occurs only if the tangent of displacement (τ) without dummy variables (β_2/β_1) equals the tangent of displacement in the presence of dummy variables. Thus, when no phase shift occurs in the seasonal cycle, the following holds:

$$\frac{\alpha_2 * TR + \beta_2}{\alpha_1 * TR + \beta_1} = \frac{\beta_2}{\beta_1} \quad (8)$$

Imposing Eq. (8) is equivalent to the coefficient restriction on α_2 above. If the test rejects the equality in Eq. (8) and indicates the existence of a phase shift, then it is equivalent to the earlier statement “winter becomes spring” by Hylleberg et al. (1993).

Table 5 lists the estimated parameters and calculated elasticities for the apple and pear equations which include the trigonometric variables interacting with the trend variable. We use the share of imports in the U.S. market as a proxy for the trend variable. Since the share of imports have been increasing over the years, the interaction of the proxy with the trigonometric variables amounts to a trigonometric trend variable.

All the price and income coefficients are significant with the correct signs. In the apple equation, one of the interaction terms is significant, while in the pear equation both terms are significant. The next step is to test whether the seasonal cycles have shifted over time in either amplitude and/or phase. Table 6 reports the results of these tests. The amplitude test was performed by setting both interaction terms equal to zero and tested against the unrestricted model. The reported statistics are distributed as χ^2 with two degrees of freedom, and the significance levels of the test are reported in square brackets. The null hypothesis of no amplitude shifts in the seasonal cycles is rejected at the 0.08 level in the

⁹ If both coefficients on the trend variable are positive (negative), then amplitude rises (falls). However, if one is positive and the other negative, whether the amplitude of the seasonal cycle rises or falls depends on their relative magnitude.

¹⁰ One way of viewing a phase shift is to take the trigonometric identity $\sin(\tau + \Gamma) = \sin\tau * \cos\Gamma + \cos\tau * \sin\Gamma$. Consider that τ is the size of displacement from $\sin(\Gamma)$, then estimate a model with trigonometric variables. The estimated parameter β_1 on the cosine variable is equivalent to $\sin(\tau)$, and the parameter β_2 is proportional to $\cos(\tau)$. Thus, (β_1/β_2) is proportional to $\tan(\tau)$.

¹¹ When the trend variable is included in the model as specified in Eq. (7), the amplitude is measured as: $((\beta_1 + \alpha_1 TR)^2 + (\beta_2 + \alpha_2 TR)^2)^{1/2}$.

Table 5
Apple and pear demand equations with trigonometric and trend variables

	Apples	Elasticity	Pears	Elasticity
Constant	– 4112.1 (– 6.20)		– 588.9 (– 4.87)	
$P(\text{apples})$	– 1087.6 (– 2.80)	– 0.39	210.7 (3.03)	0.07
$P(\text{pears})$	210.7 (3.03)	0.06	– 221.4 (– 2.39)	– 0.07
Income	4.11 (8.68)	3.43	0.64 (7.08)	0.53
f_1	632.7 (3.57)		– 199.3 (– 8.77)	
f_2	– 248.7 (– 3.98)		– 78.3 (– 8.02)	
f_5	29.2 (9.58)			
g_1	751.7 (4.26)		135.7 (6.00)	
g_2	133.3 (2.12)		– 113.4 (– 11.68)	
g_5			35.9 (3.74)	
$f_1 * \text{TR}$	501.8 (0.14)		893.3 (4.35)	
$g_1 * \text{TR}$	– 8203.2 (– 2.19)		854.3 (4.11)	
Adjusted R^2	0.58		0.84	

t -values are listed in parentheses.

Table 6
Test results for amplitude and phase shifts

	Apples	Pears
Amplitude	5.05 [0.08]	37.91 [0.01]
Phase	[0.37]	[0.36]

The significance levels of the tests are in square brackets.

apple model and at the 0.01 level in the pear model. Thus we clearly reject no amplitude shift for pears quantity data and also reject it for apples at lower confidence level.

The test for phase shift is more complex. The parameter restrictions as implied by Eq. (8) are non-linear, and therefore, the restricted model had to be estimated using non-linear methods. Only the significance levels of the test are reported.¹² The results indicate that we could not reject the null hypothesis that the restricted model was not different from the unrestricted model. Therefore, the non-linear restriction is valid indicating that no phase shift occurred in the seasonal cycles.

In summary, it appears from our simple example that the source of seasonal trends observed in the

unit root tests stem from shifts in amplitude rather than shifts in the phase of the seasonal cycles.

6. Conclusions

Recently, there has been a significant number of studies which investigated seasonal trends in macroeconomic data. In this literature, authors have developed sophisticated statistical techniques for searching for unit roots at all possible seasonal frequencies. A major drawback is that these methods focus on statistical techniques and leave little in the way of economic interpretation. One byproduct of these studies that has implications for structural economic models is the point that the presence of seasonal trends implies either that amplitudes of seasonal cycles and/or phases of seasonal cycles are shifting. Therefore, it would be useful to have a test that can distinguish between the two effects.

Developments in this literature may be of particular interest to those who analyze agricultural data which often follow seasonal cycles. With new off-season supply sources, particularly in fruits and vegetables, it is possible that the seasonal cycles are changing. Thus, seasonal unit root tests could become important in future modeling of agricultural products as well as a necessary step for time series analyses with seasonal data.

In this paper, we apply seasonal unit root tests to several agricultural data series which exhibit season-

¹² We use SAS to estimate the non-linear system. The value of the generalized χ^2 statistics is not reported by the software. Therefore, only the significance levels of the test are provided in the table.

ality. We then develop a method for incorporating seasonal trends into an economic model when the dependent variable may follow seasonal trends. Our method permits determining whether changes in seasonal cycles are occurring because of a change in the amplitude of the cycles and/or because of a change in the timing of the cycle (phase shift).

We use simple demand equations as a way of illustrating how seasonal trends can be incorporated into a structural model. Extensions to formally derived systems of demand equations should be obvious. The progression of calculated price and income elasticities from Tables 1–5 shows that the elasticities could be sensitive to standard seasonal variables and seasonal trend variables. Future research should explore the impact of incorporating seasonal trends into systems of demand equations, attempt to apply seasonal co-integration tests to monthly (rather than quarterly) data, and explore whether a monthly seasonal error correction model can provide meaningful insights into economic interpretation of seasonal relations between variables. Each of these issues, by themselves, requires extensive analysis.

Appendix A

The data were gathered from various sources and were adjusted to fit the model specification. The observed period was 1974–1993 for apples and 1976–1993 for pears. Consumption of each fruit was assumed to equal the sum of fruit shipped to domestic retail market in the United States. The Agricultural Marketing Service reports shipments as well as imports on a monthly basis.

Monthly price data were obtained from the Bureau of Labor Statistics. Data were missing for 3

years (1978–1980). Weekly wholesale market prices for three cities (Boston, Chicago, and Los Angeles) were gathered from Agricultural Marketing Service. Relationships between these prices and the existing BLS were then used to fill the missing observations.

International Monetary Fund (IMF) data were used to obtain real U.S. Gross National Product, which served as the income variable in the demand equations. All prices and income were deflated by the consumer price index.

References

- Beaulieu, J.J., Miron, J.A., 1993. Seasonal unit roots in aggregate U.S. data. *J. Econometrics* 55, 305–328.
- Boswijk, P.H., Franses, P.H., 1995. Periodic co-integration: representation and inference. *Rev. Econ. Stat.* LXXVII, 436–454.
- Canova, F., Hansen, B.E., 1995. Are seasonal patterns constant over time? A test for seasonal stability. *J. Business Econ. Stat.* 13, 237–252.
- Chow, G.C., 1983. *Econometrics*. McGraw-Hill, New York, NY.
- Franses, P.H., 1991. Seasonality, non-stationarity and the forecasting of monthly time-series. *Int. J. Forecasting* 7, 199–208.
- Franses, P.H., Paap, R., 1995. Seasonality and stochastic trends in German consumption and income, 1960.1–1987.4. *Empirical Econ.* 20, 109–132.
- Hylleberg, S., Engle, R.F., Granger, C.W.J., Yoo, B.S., 1990. Seasonal integration and co-integration. *J. Econometrics* 44, 215–238.
- Hylleberg, S., Jorgensen, C., Sorensen, N.K., 1993. Seasonality in macro-economic time-series. *Empirical Econ.* 18, 321–335.
- Joutz, F.L., Maddala, G.S., Trost, R.P., 1995. An integrated Bayesian vector autoregression and error-correction model for forecasting electricity consumption and prices. *J. Forecasting* 14, 287–310.
- Kunst, R.M., 1993. Seasonal co-integration, common seasonals, and forecasting seasonal series. *Empirical Econ.* 18, 761–776.
- Osborn, D.R., 1993. Discussion: seasonal co-integration. *J. Econometrics* 55, 299–303.