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# Applying search theory to determine the feasibility of eradicating invasive populations

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## Abstract

The detectability of invasive organisms influences the feasibility of eradicating an infestation. Search theory offers a framework for defining and measuring detectability, taking account of searcher ability, biological factors and the search environment. In this paper, search theory concepts are incorporated into a population model and the costs of search and control are calculated as functions of the amount of search effort (the decision variable). Simulations are performed on a set of four weed scenarios in a natural environment, involving different combinations of plant longevity, seed longevity and plant fecundity. Results provide information for the design of efficient search strategies.

Keywords: invasive species, population dynamics, bioeconomics, weeds, operations research

## Introduction

Invasive plant species contribute to losses in natural ecosystem function and agricultural production; and they may also impact human and animal health (Groves, 2002). It is difficult to value the full impact of weeds, nevertheless partial estimates of the damage they cause in various countries are available. In South Africa the 161 serious weed pests are estimated to cause annual losses of at least US\$16 billion; in the United States the spread of invasive weeds causes estimated losses of US\$34 billion in damage and control costs each year; and in New Zealand weeds cost at least NZ\$100 million in control costs and lost agricultural production (Pimentel, 2002).

In Australia, weed impacts on agricultural production are estimated to cost at least A\$4 billion per year (Groves, Boden and Lonsdale, 2005; Sinden et al., 2004). This cost can be expected to increase as the proportion of naturalised alien plants in the Australian flora continues to grow, despite many decades of quarantine laws regulating the entry of alien plant species (Groves and Hosking, 1997). It is estimated that 290 plant taxa naturalised in the 25-year period between 1971 and 1995, with the rate of naturalisation increasing in the latter half of this period to around 14 plants per year (Groves and Hosking, 1997).

Given the prevalence and cost of weed invasions it is important to have an understanding of how resources should be used to search for, control, and possibly eradicate weed populations. One important factor influencing the feasibility of eradication is weed detectability, which is a function of biological characteristics of the weed, the environment in which it is located and searcher ability.

It is possible that organisms will be overlooked when searching for weeds, and the overlooked organisms may influence invasion dynamics. For example, organisms existing at low densities may not be observed until after they have set seed, causing an increase in the seedbank that may require decades to eradicate. Reasons for overlooking weeds include features related to the individual plants, spatial arrangement of weeds, environmental conditions, or searcher ability. Even if all above ground organisms could be detected, seeds in the seedbank would remain undetected, thus searches must be repeated over time.

Search theory (Koopman, 1980) offers a technique to determine the probability of detection as a function of effort expended in different search environments. Search theory was developed originally to improve success rates in detecting military targets. The theory has been applied to a wide range of problems, including the search for enemies and allies by the Military (Koopman, 1946, 1980), the search for explosive mines (Gage, 1993), and search and rescue problems (Cooper et al., 2003; Frost and Stone, 2001). One of the central

features of search theory is the development of quantitative measures relating effort to the probability of detection of an object or objects of interest, with the ultimate concern being optimal effort allocation. This paper presents a brief explanation of search theory and the associated formulas for detection, coverage and mortality that are later incorporated into a population simulation model. The population dynamics are described based on a stage matrix that drives the spread of the weed invasion under alternative search and control strategies. The eradication effort is analysed in terms of expected years to eradication and present value of costs.

## Method

Cacho et al (2006) developed a combined search and population dynamics model to study the feasibility of eradicating an invasion. Their model is extended in this study by incorporating labour and herbicide input equations and calculating costs. A brief review of search theory is presented followed by derivation of the cost component of the model.

### Search and control

Search theory is based on the concept of coverage ( $c$ ), defined as the ratio of the area actually searched over the total area of the invasion:

$$c = \frac{STR}{A} \quad (1)$$

Where  $A$  is the total area ( $m^2$ ) at risk of invasion,  $S$  is the speed of search ( $m h^{-1}$ ),  $T$  is time spent searching (h) and  $R$  is the effective sweep width (m).  $R$  is a measure of the *detectability* of the plant taking into account target characteristics and environmental conditions. The numerator of equation (1) represents the area effectively searched ( $m^2$ ) as the product of search effort in terms of distance traversed ( $ST$ ) times the detectability of the plant ( $R$ ).

The upper bound on search effectiveness is achieved by a “definite range” sensor that detects all objects out to a specific distance on either side of the sensor, and no objects beyond that distance. In this case, the probability of detection increases linearly with coverage until both reach 1.0 (dotted line in Figure 1). A definite range sensor can detect all targets once the entire searchable area is swept perfectly (when  $c=1$  as would result with parallel search tracks and no overlap). Thus, a single sweep of the area with a definite range sensor is all that is required to solve the search problem.

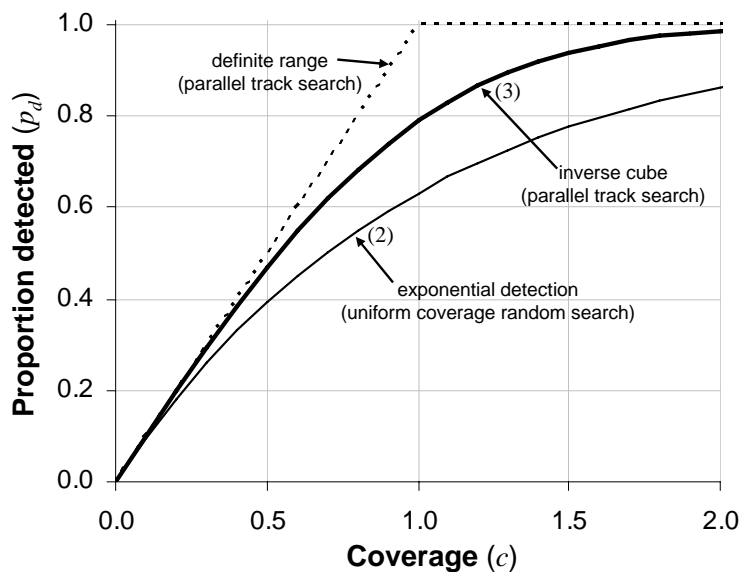


Figure 1. Detection functions obtained with a definite range sensor (dotted line), a parallel-track search (equation 3) or a random search pattern (equation 2).

At the other extreme from the definite range sensor, the exponential (random) detection function (lower curve in Figure 1) provides a conservative estimate of search effectiveness. In random searching there is no pattern to the search process, some areas will be searched repeatedly while others may not be searched at all. The proportion of objects detected ( $p_d$ ) for random sweeping is given by:

$$p_d = 1 - e^{-c} \quad (2)$$

Random searching would be expected to produce the fewest detections unless there were systematic biases in the search process. An intermediate detection function (dark solid curve in Figure 1), known as the inverse cube function, is obtained with parallel, equidistant search tracks. The proportion of objects detected with this search mode is:

$$p_d = \theta\left(\frac{\sqrt{\pi}}{2} c\right) \quad (3)$$

where  $\theta$  represents the error function:

$$\theta(y) = \frac{2}{\sqrt{\pi}} \int_0^y \exp(-u^2) du \quad (4)$$

This function was derived by Koopman (1946, 1980) based on the geometry of sighting opportunities in the search for ships from aircraft.

The effective sweep width ( $R$ ) in equation (1) can be calculated based on the lateral range curve (LRC) showing the probability that the target will be detected as a function of its lateral distance from the searcher (Figure 2). The efficiency of search per unit of distance covered is given by the area under the LRC.

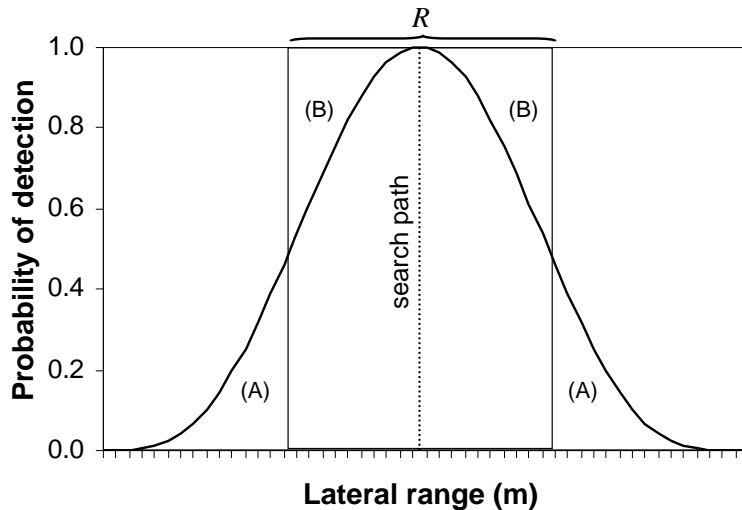


Figure 2. A typical lateral range curve (LRC) illustrating the estimation of effective sweep width.

$R$  is computed by constructing a rectangular box of height 1 and lateral range equal to that at which the number of missed detections (B) within this range equals the number of detections (A) outside the range (Figure 2). In this case, we could replace area B with area A, and have the same number of total detections. Thus, a standard rectangle can characterise detectability for a given search method applied in a given environment. Effective sweep width is the width of the box in Figure 2.

The LRC, and therefore  $R$ , will be affected by the speed and mode of search. But for simplicity we assume  $S$  and  $R$  in equation (1) to be fixed, based on the characteristics of the plant and the environment and using a standard search procedure. Thus coverage is controlled by adjusting search time ( $T$ ).

We assume that all weeds found are killed, or an attempt is made to kill all of them subject to the effectiveness of the control method used. The mortality caused by the search and control effort is:

$$m = p_d \times p_k \quad (5)$$

Where  $p_k$  is the probability that a target organism will die each time control is applied and  $p_d$  is given by equation (2) or (3) depending on the search mode.

### **Population Dynamics**

The spread of the weed invasion was modelled using a stage matrix. This is a standard technique for population dynamics modelling and is explained in detail in Caswell (2001). The matrix population model used in this study is described by Cacho et al. (2006).

The stage matrix has dimensions  $n \times n$ ; where  $n$  is the number of stages in the life cycle of the plant. The minimum size of  $n$  is 3 (new seeds, seedlings and mature plants) for annual plants. The value of  $n$  increases for perennial plants according to the number of years required to reach maturity, with juvenile stages inserted between the seedbank and the adult stages.

The population growth rate ( $\lambda$ ) is given by the dominant eigenvalue of the stage matrix, and  $\lambda$  is related to the intrinsic rate of increase ( $r$ ) by the function  $\lambda = e^r$  (Caswell 2001, p. 86). When the population reaches a steady state,  $r=0$  and therefore  $\lambda = 1$ . This fact was used to implement density dependence in the model by finding the value for germination ( $G$ ) that results in a value  $\lambda = 1$  for a given weed. This germination rate was denoted  $G_\infty$ .

### **Inputs and Costs**

Costs associated with weed control are assumed to consist of labour costs for searching, labour costs for applying herbicide or other treatments to kill the plants, and the cost of herbicide. The labour input for searching is determined by search time ( $T$ ), regardless of density and frequency of weed sightings, but the labour input for killing plants (control) depends on the time required to treat each plant and therefore it is a function of density and size of the weed. The annual cost function ( $C_t$ ) is:

$$C_t = p_L(T + L_t) + p_H H_t \quad (6)$$

Where  $L_t$  and  $H_t$  are labour and herbicide inputs respectively, and  $p_L$  and  $p_H$  are their corresponding prices.

Published studies that have measured the amount of labour required to search for and control weeds at different densities and sizes are difficult to find. One recent study, however, measured the effort required to search for and control quinine (*Cinchona pubescens*) at different densities and stem sizes on the Galapagos islands (Buddenhagen and Yañez, 2005). Weed control involved uprooting small plants, and applying herbicide to cut stumps or to machete cuts in the bark of larger trees. The authors found that effort (person-hours) required to control an area was positively related to both density of all size classes and to density of stems >150 cm tall. The authors fitted a quadratic function to the data, where the intercept term can be interpreted as the (constant) cost of searching for the weed.

The data from Buddenhagen and Yañez (2005) were reproduced using a data capture program, OCapture (Cacho, 2005) to further analyse the relationships between effort and plant density, and between herbicide use and plant density. A power function was fitted to the relationship between effort and density of stems:

$$L = \alpha N^\beta \quad (7)$$

Where  $L$  is the labour input (h) required to control each weed,  $N$  is the density of the weed (stems ha<sup>-1</sup>), and  $\alpha$  and  $\beta$  are parameters. Dummy-variable analysis indicated that the value of  $\alpha$  varied with plant size. Although the exact plant sizes were not known, as Buddenhagen and Yañez report only all plants and plants > 150 cm tall, a plausible relationship between  $\alpha$  and relative plant size was estimated using the two coefficients derived from the analysis, and was substituted into (7). Despite a lack of additional data points to assist in the estimation process, this relationship was considered to be an acceptable first approximation:

$$\alpha = \exp(\alpha_0 + \alpha_1 X) \quad (8)$$

Where  $X$  is the relative size of the plant being killed ( $X=1.0$  for a mature plant) and  $\alpha_0$  and  $\alpha_1$  are parameters.

Data on herbicide and weed density showed a positive linear relationship, commencing at the origin, for both stems >150 cm tall (adults) and for all plant sizes. As expected, the larger the plant, the more herbicide must be applied to kill it, and when no weeds exist no herbicide is required. The regression between the amount of herbicide ( $H$ ) required to kill one plant and stem density ( $N$ ) was found to be linear, but the slope varied according to plant size ( $X$ ). The following power function was estimated based on this analysis:

$$H = \gamma X^\varphi \quad (9)$$

Where  $H$  is the amount of herbicide (litres) required to kill one plant, and  $\gamma$  and  $\varphi$  are parameters.

## Numerical Assumptions

The assumptions of the numerical model are presented in Table 1. The starting number of organisms in each life stage was calculated based on the number of adults when the invasion is first discovered ( $x_{0n}$ ). This required solving the population dynamics model iteratively to determine the number of seeds and juveniles that would be consistent with the observed number of adults in the exponential portion of the growth curve.

Table 1. Parameter values used in simulations.

Parameter	Value	Description
Search and control parameters		
$T$	0.1-2.0	time searching (h/ha)
$S$	1,000	speed of search (m h <sup>-1</sup> )
$R$	20	effective sweep width (m)
$pk$	0.95	efficiency of control agent
$A$	100	area of invasion (ha)
Economic Parameters		
$r$	0.06	discount rate
$\beta$	1.3	labour input parameter
$\alpha_1$	4.38	labour input parameter
$\alpha_0$	-11.32	labour input parameter
$\gamma$	0.001567	herbicide parameter
$\varphi$	2.423	herbicide parameter
$p_L$	35	price of labour (\$ h <sup>-1</sup> )
$p_H$	5	price of herbicide (\$ l <sup>-1</sup> )

Some initial analysis of the eradication effort was undertaken by running deterministic simulations of the model. Of particular interest are the isoquants of years required for eradication depending on weed detectability and search effort shown in Figure 3.

This plot was obtained by solving the model repeatedly for various combinations of  $R$  and  $T$ , using the base parameter values in Table 1 and weed number 2 in Table 2. Each isoquant represents a given number of years as indicated by labels next to the curves. Two important features are illustrated here. The first is that, as the visibility of the weed decreases the search time must be increased at increasing rates to achieve eradication in the designated number of years (moving left along a curve in Figure 3). The second important feature is that the gaps between isoquants become wider as the time to eradication decreases (moving up between curves in Figure 3), this reflects the diminishing returns to the search effort.

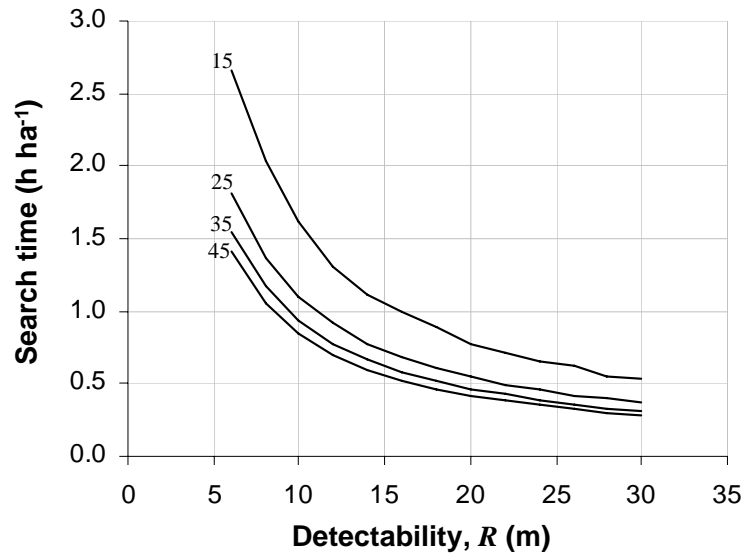


Figure 3. Isoquants of detectability against search time. Each line represents the set of detectability and search-time values that result in a fixed eradication time (years); labels (15, 25, 35, 45) indicate the number of years represented by each line.

Table 2. Parameter values used in simulations for four weed scenarios: (1) annual; (2) perennial; (3) perennial with extended seed longevity; (4) perennial with extended plant longevity.

		Scenario			
		1	2	3	4
		annual	perennial	perennial	perennial
Biological parameters:					
$G$	germination rate	0.0013	0.0259	0.0207	0.017
$P_J$	juvenile survival	0	0.05	0.05	0.1
$F$	fecundity	1500	1500	1500	15000
$L_S$	seed longevity (y)	5	5	10	5
$L_P$	plant longevity (y)	1	10	10	20
$M_T$	years to maturity	0	1	1	3
Calculated parameters:					
$P_S$	seed survival	0.25119	0.25119	0.50119	0.25119
$P_A$	adult survival	0	0.46416	0.46416	0.66608
$G_\infty$	$G$ at steady state	0.00052	0.00534	0.00356	0.00119
$\lambda$	growth rate	1.5	1.5	1.5	1.5
Initial population:					
$x_{01} + x_{02}$		3,804	4,471	10,894	87,616
$x_{03}$		10	176	234	190
$x_{04}$		-	10	10	27
$x_{05}$		-	-	-	11
$x_{06}$		-	-	-	10

## Simulations

Four weed scenarios were designed (Table 2) and deterministic simulations were undertaken for a range of search efforts and assuming parallel search tracks. Scenario 1 represents an annual weed and scenarios 2, 3 and 4 represent perennial weeds. Scenario 2 represents the base perennial plant; Scenario 3 represents a plant with longer-lived seeds; and Scenario 4 represents a weed with higher fecundity but which requires three years to reach maturity. The demographic parameters (Table 2) were selected to ensure that all scenarios had the same expected population growth ( $\lambda=1.5$ ), thus preventing differences in intrinsic growth rate from confounding the effects of differences in demographic characteristics.

### Time to Eradication

Time to eradication was found to decrease at a decreasing rate as search time increased (Figure 4). The drop in years to eradication is rapid as search time increases from 0.3 to 0.5 hours per hectare. Further increases in search effort lead to a slower drop in years to eradication; beyond a search effort of one hour per hectare there is little improvement in time to eradication.

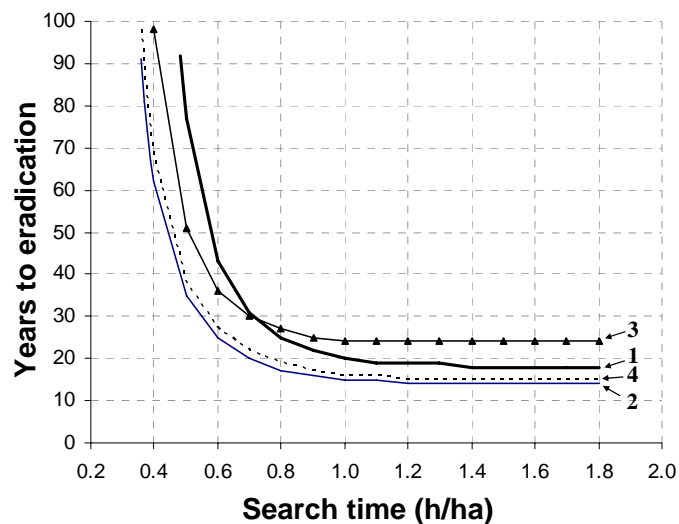


Figure 4. The relationship between time spent searching for weeds and the total number of years to eradication for each of the four scenarios described in Table 2.

With constant search effort, doubling seed longevity from 5 to 10 years (scenario 3) causes the years to eradication to increase; and delaying the onset of maturity from 1 to 3 years (scenario 4) causes the years to eradication to decrease. The weed in scenario 4 takes longer to reach maturity and has a lower germination rate than the weed in scenario 2, but it also has larger fecundity and juvenile survival (see Table 2). The former two parameters make the plant in scenario 4 easier to eradicate, but the two latter parameters make it harder to eradicate. Hence, the changes between scenarios 2 and 4 are relatively small.

The annual plant (Scenario 1) is more difficult to eradicate than the perennials for search times below 0.7 hours per hectare, taking between 30 and 100 years as search effort decreases. For search times above 0.7 hours per hectare the plant with longer-lived seeds (Scenario 3) becomes the most difficult to eradicate, taking between 24 and 27 years to eradicate.

The costs associated with a range of search time inputs that lead to eventual eradication of the weed are plotted in Figure 5. Total costs are high for low search efforts and rapidly fall as search effort increases. The minimum cost occurs at search efforts of around 0.5 h/ha, which represents a coverage of 1.0 based on equation (1) and the search parameters in Table 1. The control component of costs decreases rapidly as search time increases and flattens out at coverages greater than 1.0 (or  $T > 0.5$  h/ha). The cost of search is equal for all scenarios (Figure 5), but the cost of control varies. The annual plant exhibits the lowest cost (Figure 5A), and the perennial plant with long-lived seeds the highest (Figure 5C). Of the perennial plants, the lowest costs are for scenario 4 (Figure 5D), where the plant takes longer to reach maturity and therefore makes it easy to kill plants before they produce seeds.



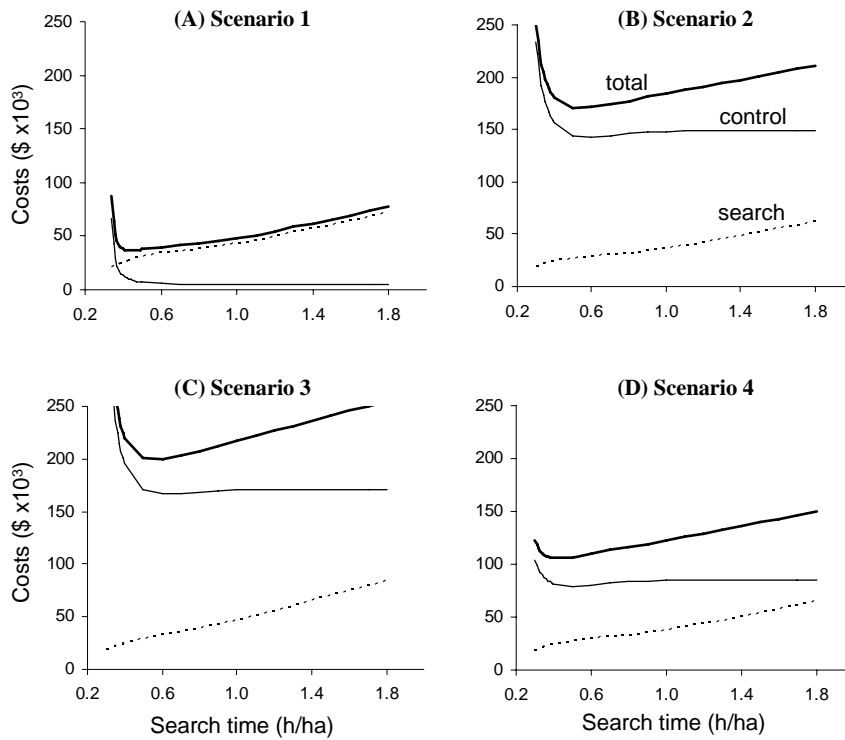


Figure 5. Costs of search and control (present values) as affected by search effort for each of the four scenarios described in Table 2.

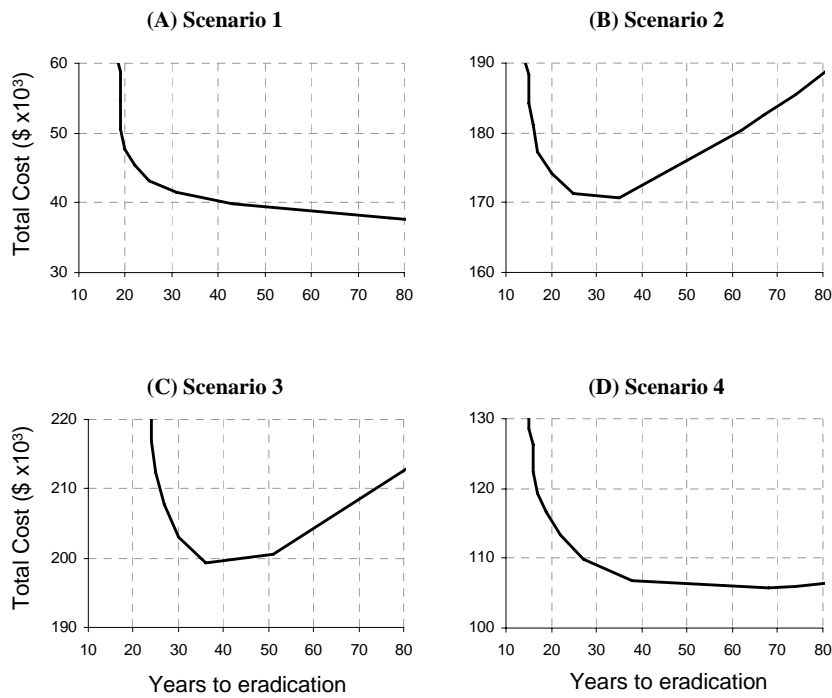


Figure 6. The relationship between total costs of eradication (present values) and years to eradication for each of the four scenarios described in Table 2. Note different ranges of values on the vertical axes to enhance readability.

Information from the previous two figures is combined in Figure 6 to show the relationship between total costs and time to eradication. There is an obvious lower bound to the number of years in which a plant can be eradicated (18, 14, 24 and 15 years for scenarios 1, 2, 3 and 4 respectively), but this lower bound is expensive to achieve as evidenced by the costs on the left side of the curves in Figure 6. Costs decrease rapidly as the desired years to eradication are increased above the lower bound. For perennial plants a minimum is reached (at 35, 36 and 69 years for scenarios 2, 3 and 4) whereas for the annual plant (Scenario 1) costs keep on decreasing as the year to eradication increases. The latter result (Figure 6A) suggests that, for the given parameter values, the minimum-cost strategy to control the annual plant is containment rather than eradication. However this is not necessarily the optimal strategy, because the benefits of early eradication have not been considered.

## Discussion

Two features of weeds that make them generally more difficult to eradicate than other pests is that seeds are not detectable until they germinate and that seeds can survive in the soil for a long time. Our model accounts for these facts by including life stages representing new seeds and a seedbank. Although the size of the invasion does not affect the probability of detecting an individual plant in our model (therefore assuming there is no clumping of plants), the size of the invasion does affect the cost of control, because both weed density and weed size affect the labour and herbicide inputs required to kill those plants that are found.

In our simulations an annual plant was found to be cheaper to eradicate than a range of perennial plants; but recall we assumed the four plant types to have the same size ( $1.0 \text{ m}^2$ ), detectability ( $R=20\text{m}$ ) and intrinsic growth rate ( $\lambda=1.5$ ). This allowed us to evaluate the effects of demographic characteristics, such as seed longevity and time to maturity, on the costs of eradication. An annual plant, however, would generally be expected to be smaller and less detectable than an older perennial plant, thus some annual plants may be more expensive to eradicate than the base case used here, because they are harder to find before they set seed.

A limitation of our analysis is that the labour and herbicide production functions were derived based on a small dataset of a woody weed in the Galapagos islands. Unfortunately there are no other data of this type available in the literature. Although the functional forms used for the production functions are standard and well accepted, the values of the parameters will vary with the situation. Therefore there is a need for more studies of the type undertaken by Buddenhagen and Yañez (2005); which allows the number and size of plants killed to be related the amount of labour and herbicide required.

An interesting finding of this study is that the demographic parameters of a weed can have a substantial effect not only on the eradication costs, but also on the shape of the cost function.

The potential to use search theory to improve the efficiency of invasive species management is promising. The model in its current form can be used for general planning and evaluation of an eradication program and preliminary allocation of resources at a general level. This paper deals only with the likely cost of the eradication effort and does not consider the benefits of eradicating the invasion earlier rather than later. The benefits of eradication depend on the value of the environment being invaded, and this is the subject of ongoing research by our group.

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