ELSEVIER

Agricultural Economics 16 (1997) 55–65

AGRICULTURAL
ECONOMICS

Technical change and irreversible investment under risk

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Accepted 19 August 1996

Abstract

A stochastic dynamic model was constructed to analyze investment decisions of an individual farmer under risk in the presence of irreversibilities, embedded technical change and indivisible capital. An analytical solution was obtained and its local behavior studied by numerical methods. Optimal investment is obtained by regulating the difference between the desired and actual capital stocks between two barriers that define an inaction interval. While the desired capital drifts between the barriers, no action is taken. If the desired capital touches the upper barrier, the farmer invests pushing the average efficiency of the actual capital stock up. This in turn raises the desired capital even higher and contracts the inaction interval. If these effects are strong enough, the farmer will invest again until the potential gains of the technological package are exhausted. If the desired capital falls enough, the farmer disinvests, pushing down the average productivity and expanding the inaction interval. Disinvestment continues until it stops either because the inaction interval becomes so wide that it is no longer optimal to disinvest or because the actual capital stock is so small that it is no longer profitable to produce.

1. Introduction

Most physical capital in agriculture has specific technological characteristics. In such cases investment implies the adoption of embedded technical change; conversely, embedded technical change can only be adopted through investment in physical capital. For example, a new 100 hp tractor is more efficient than an older one with the same power; also, it is not equivalent to two 50 hp tractors.

Often agricultural technologies are complex packages of capital, inputs and techniques that yield the maximum benefits when adopted completely. Partial adoption is possible but renders a lower rate of return. For example, improved seeds for grain production can be used with old machinery but they yield their full potential only when combined with precision seeders, modern agrochemicals and powerful tractors and combines. While the technological package can be adopted partially, most agricultural machinery is indivisible; in other words, tractors and combines can only be purchased in whole units.

As noted in Pindyck (1991), investment in physical capital is partially irreversible for two reasons. First, most capital is sector specific; since negative economic conditions affect all firms in the sector in a similar way, the market value of used machinery falls for all firms simultaneously. The second reason is asymmetric information about the quality of used capital (Akerlof, 1970).

A stochastic dynamic model was constructed to analyze investment decisions under risk in the presence of irreversibilities, embedded technical change and indivisible capital. When decisions are irre-

PII S0169-5150(96)01206-6

versible and risky, investment by individual firms is known to be sporadic. In such cases, optimal investment and its timing are defined by regulating the difference between the desired and actual capital stocks between an upper and a lower barrier. These barriers are defined by each farmer based on his initial conditions (wealth, human capital, etc.), expectations about markets, risk, the gains to be obtained from adopting improved technologies, and the cost associated with making the wrong investment decision if market behavior turns out to be significantly different from what was expected (Dixit, 1992).

While the difference between actual and desired capital drifts between these barriers, no action is taken. Assume that a one period shock raises expectations about future profits enough to trigger investment. With constant technology, all the consequences of the shock disappear immediately after investment. Embedded technical change, however, modifies the dynamic stochastic process that governs the farmer's decisions. By investing, he adopts new technologies that increase the average efficiency of his capital stock. If the rise in productivity is strong enough, it induces further investment in future time periods. The effects of the shock that triggered the first bout of investment last until all potential benefits of the technological package are exhausted.

The study of irreversible investment has attracted considerable interest over time. Johnson (1956) and Edwards (1959) associated irreversibilities with asset fixity and sunk investment cost. Baquet (1978) analyzed the influence of sunk costs on investment in a dynamic framework under perfect knowledge. Recently, the analysis of irreversible investment under uncertainty has been the subject of a large number of works (for reviews of the literature see Pindyck, 1991; Dixit and Pindyck, 1994).

Several sources of risk have been studied in the literature on irreversible investment; the most common has been revenue instability (Dixit, 1989; Pindyck, 1991; Smith, 1994). Other works analyzed uncertain regulations (Teisberg, 1993), stochastic costs (Pindyck, 1993), random interest rates (Dixit and Pindyck, 1994) and stochastic production functions (El-Gamal, 1994). This paper includes both random capital prices and a stochastic production function. Agricultural production is stochastic due to a number of factors out of the farmer's control (weather, pests infestations, variability in seeds' quality, etc.). In many cases, improved technologies allow better control of the production process, reducing output variability and consequently the production risk. Investment, then, involves a trade-off between two sources of risk; as capital expands, exposure to the volatility of the capital value grows but production risk falls.

Most studies on irreversible investment specify constant production technologies. Different types of technical change, though, are considered in a few works. He and Pindyck (1992) analyzed investment when the firm chooses among three types of production processes; once the investment decision is made, however, technology is fixed. Parente (1994) and Dixit and Pindyck (1994) analyzed investment when firms accumulate expertise about the technologies after investment. The better knowledge of the technology allows firms to use it more efficiently but the technical coefficients of the production function are constant.

Stefanou (1987) modeled investment with quadratic adjustment costs, stochastic input prices and uncertain evolution of technology. Rather than considering a few well defined techniques to be available in the future, he defines the technology index as a continuous state variable evolving in a stochastic manner. Specification of technology in this paper follows a similar approach.

The purpose of this paper is to analyze the dynamics of investment by an individual farmer when decisions are partially irreversible, technical change is embedded and capital is indivisible. A heuristic exposition of the decision process is presented in Section 2; Section 3 formalizes this process in the framework of a stochastic dynamic model of investment with endogenous technical change. Section 4 contains the analytical solution that characterizes the optimal investment policy.

This solution is a linear combination of two power series whose coefficients are non-linear functions of the parameters of the decision process. Even tough an analytical solution was found, it was not possible to study analytically its properties. These are studied locally in Sections 5 and 6. Section 5 presents a static numerical analysis of the solution while Section 6 reviews the dynamics of investment in the light of the results obtained in the previous sections. Section 7 summarizes the results and provides insights on the implications of uncertainty, investment irreversibilities and embedded technical change on policy analysis and economic development.

2. The dynamics of investment with embedded technical change

The traditional Marshallian rule states that firms should invest when output price just exceeds long run average cost. When a project is partially irreversible and the future uncertain, this policy is not optimal. If production or market conditions turn out to be different from what was expected, decisions can only be reversed at a cost; frequent adjustments, then, cannot be optimal because costs accumulate fast (Dixit and Pindyck, 1994).

Harrison (1985) showed that in this context, the optimal policy is to regulate the difference between the desired and actual capital stocks in such a way that it is always contained between a lower and upper barrier that minimize the need for intervention. These barriers are determined by the firm based on the technical and economic characteristics of investment and the firm's expectations about future market conditions (Dixit, 1991; Kushner and Dupuis, 1992). The firm continuously updates its desired capital stock according to the evolution of several variables, among which are: expected profitability, potential gains from technical change, production risk, and the firm's expectations of economic instability. In contrast, the firm adjusts its actual capital only when it differs substantially from the desired capital stock (Dixit and Pindyck, 1994).

Fig. 1 shows a sequence of events that eventually triggers investment under the assumptions that capital does not depreciate and technology is constant. Between time 0 and time t_1, the desired capital drifts between the external barriers (k_- and k^-); since the difference between the desired and actual stocks is contained in the optimal inaction interval, the actual stock (k_0) remains constant. At time t_1 the upper barrier is hit and the firm invests.

When capital is perfectly divisible, firms invest exactly enough to move the actual capital stock instantly to the external barrier. Harrison (1985) showed that when capital is indivisible, the firm invests (or disinvest) less than with perfectly divisible units because it is never optimal to have an actual stock larger than the desired. When the firm invests (disinvests), instead of increasing its actual capital up to k^- (the desired level), the firm invests only until K^- (K_-). The partial adjustment represents a real cost to the firm. At the upper barrier, it has less capital than desired and output is lost; at the lower barrier, it has too much capital and costs are too high.

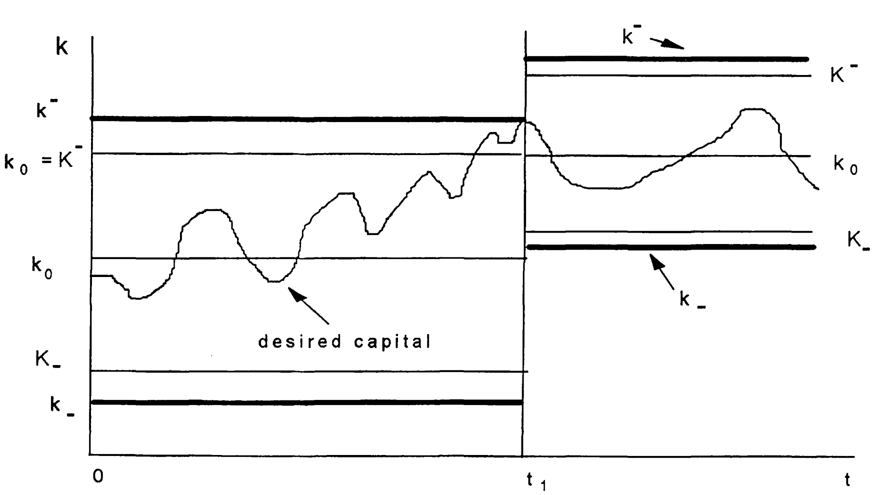

Fig. 1. Optimal investment policy.

When the actual stock changes, the interval of inaction shifts accordingly. After t_1, the desired capital drifts between the new barriers until an action is again triggered. If the desired capital touches the lower barrier, the firm disinvests.

Introduction of embedded technical change and endogenous risk modifies the firm's behavior. When new capital is purchased, the average efficiency of the actual capital stock increases and production risk shrinks. As a consequence, expected profits rise, pushing up the desired capital stock. The same reasons reduce the inaction interval; a smaller difference between the desired and actual capital stocks now is required to trigger investment. The process continues while there are gains to be made from expanding the capital stock with the same technological package; at some point, however, the potential production gains from the technological package are exhausted. Decreasing marginal efficiency of capital is sufficient for investment to be bounded.

If the capital stock contracts, the average efficiency falls and the production risk increases; consequently, the firm is induced to further reduce its capital. Additionally, the inaction interval is enlarged. Reduction of the actual capital stock continues until it stops either because the inaction interval becomes so wide that it is no longer optimal to disinvest or because the actual capital stock is so small that it is no longer profitable to produce. [1]

In synthesis, irreversibility of investment and risk are the causes of the inaction interval or, in other words, of the external barriers. The position of these barriers is determined by the technical and economic characteristics of investment and the firm's expectations about future market conditions. Indivisibility of investment does not allow perfect adjustment and is the reason for the existence of the internal barriers. Endogenous technical change and endogenous risk add to the inertia of the model and, in this way, affect the dynamic behavior of investment through their effect on the desired capital stock and the position of the barriers.

[1] In the technical literature this is called an absorbing barrier while the previous case is known as a reflecting barrier.

3. A model of the firm

Assume that the firm uses capital and a vector of costlessly adjustable inputs to produce a non storable good. At any moment, the firm chooses the amount of costlessly adjustable inputs to maximize the difference between its revenues and its costs in these inputs. Let π denote the value of this instantaneous profit per unit of output. In order to simplify the model, assume that it is constant.

Since capital is indivisible, investment is made in lump sums. Denote by a^- the purchase value of new capital and by a_- the sell price of used capital. The size of a^- determines the severity of the indivisibilities; the difference between a^- and a_- creates the irreversibility. Because of the indivisibility, adjustment is not complete; instead of investing up to k^-, the firm invests only up to K^-. The cost of investment, then, is

$$\bar{C}(t) = a^- + b^-(k^-(t) - K^-(t)) \qquad (1)$$

where b^- is the cost arising from the indivisibility of capital and it is equal to the expected [2] capitalized profits lost by the firm because its capital stock is K^- instead of k^-. [3]

When the lower barrier is hit, the firm sells enough capital to adjust the actual capital stock to the optimal level (K_-), rendering an income of

$$\underline{C} = -a_- + b_-(K_- - k_-) \qquad (2)$$

The problem is meaningful only if

$$\bar{C} + \underline{C} > 0$$

otherwise the firm could make infinite profits buying and selling used capital.

Since capital is the only input that cannot be adjusted immediately and without cost, all the dynamic properties of the model are derived from the evolution of the capital stock. To simplify the model, the production function is approximated by a

[2] The expectation is conditional on the information available at time t.

[3] To simplify notation, the time index is dropped wherever possible.

quadratic polynomial in K. [4] Assuming the presence of capital augmenting technical change, the production function is

$$y = q + r\alpha K + s(\alpha K)^2 + \sigma_y K^{-1}\epsilon \tag{3}$$

where y is output, K is the desired level of capital, α is an index denoting the stock of accumulated technical knowledge associated with capital augmenting technical change, σ_y is the standard deviation of output and ϵ is normally distributed white noise. The addition of new capital allows for a better control of the production process reducing the risk associated with production; consequently, the expected variance of output is inversely related to the desired stock of capital. [5] Convergence to a solution requires that the coefficient s be negative, i.e. the marginal product of capital is decreasing.

While the expected variance and the technology index drift with the desired capital stock, the actual level of technical change and the actual variance are functions of the existing stock of capital (K_0) and change discretely with investment

$$\begin{aligned} &\frac{\partial\alpha}{\partial K_0} > 0 \quad \frac{\partial^2\alpha}{\partial K_0^2} < 0 \\ &\frac{\partial\sigma_y}{\partial K_0} < 0 \quad \frac{\partial^2\sigma_y}{\partial K_0^2} > 0 \end{aligned} \tag{4}$$

While drifting in the inaction interval defined by the external barriers, capital changes only by depreciation at a constant rate

$$dK = -\zeta K dt \tag{5}$$

Improved technologies can only be adopted through investment in advanced machinery; as a consequence, the efficiency of capital is determined by the actual stock. The rate of growth of capital efficiency evolves in a continuous deterministic manner

$$d\alpha = \left(\beta_1 K + \beta_2 K^2 + \vec{\beta} Z\right) dt \quad \alpha(0) = \alpha(K_0) \tag{6}$$

where Z is a vector of erogenous shifters such as expected changes in output or input prices.

Total expected profits are defined as

$$\Pi = \pi y$$

Since π was assumed constant, the randomness in the dynamics of total profits (Π) arises only from the stochastic changes in output. [6]

Its total differential is

$$\begin{aligned} d\Pi = \pi dy = \pi\Big[& r\alpha dK + 2s\alpha^2 K dK - \sigma_y \epsilon K^{-2} dK \\ & + rK d\alpha + 2sK^2\alpha d\alpha + \sigma_y K^{-1}\epsilon dt\Big] \\ = \pi\Big[& \left(\vec{\beta} Z - \alpha\zeta\right) rK \\ & + \left(\beta_1 r + 2s\alpha\left(\vec{\beta} Z - \alpha\zeta\right)\right) K^2 \\ & + (r\beta_2 + 2s\alpha\beta_1) K^3 \\ & + 2s\alpha\beta_2 K^4 + (1+\zeta)\sigma_y K^{-1} dz_y\Big] \end{aligned} \tag{7}$$

where dz_y is a Wiener process. Assume that the firm derives utility from wealth. Then, the representative firm maximizes the infinite stream of expected discounted utilities derived from wealth (W) net of investment costs

$$\begin{aligned} J(W_0) = \text{Max } E\Big\{ & \int_0^\infty e^{-\mu t} U \\ & \times (W - \text{investment costs})\, dt | W(0) \\ & = W(_0)\Big\} \end{aligned} \tag{8}$$

Wealth is equal to the stock of capital valued at

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[4] It is possible to use a production function that allows for substitution among inputs. However, since all inputs except capital adjust immediately and without cost, such specification would not change the dynamic properties of the solution; it would only increase the complexity of an already complex mathematical expression.

[5] A reviewer noted that investment can actually increase the variability of output; such a situation is easily handled by making the power of K in Eq. (3) positive. The case in which investment does not affect the variance of output is represented by setting the exponent to 0. Even though the solutions in these two cases will be different from the one presented here, they can be obtained by the technique used in this paper.

[6] The variability in output prices can be introduced by specifying Eq. (3) as a profit function instead of a production function. Since investment does not affect output prices, the effect of investment on the variance of total profits would still arise from its influence on the output variance. Introduction of the variance of output prices would not affect the dynamic behavior of the model, it would only increase its mathematical complexity.

its market price p [7] plus non-invested profits from production

$$W_t = p_t K_t + \int_0^t \Pi_t ds \tag{9}$$

Wealth is stochastic because capital prices and profits are; in the financial literature it is typically assumed that price expectations are log-normally distributed, that is

$$dp/p = (\delta + \sigma_K \epsilon) dt = \delta dt + \sigma_K dz \tag{10}$$

where δ denotes the expected rate of change in the price of capital, σ_K is the standard deviation of the forecast, ϵ is normally distributed white noise and dz is a Wiener process.

Differentiating Eq. (9) using Ito's rule gives (Hertzler, 1991; Dixit and Pindyck, 1994)

$$dW = dpK + [p + dp]dK + \Pi dt + d\Pi \tag{11}$$

Substitute Eqs. (5), (7) and (10) into Eq. (11) to obtain an expression for dW

$$\begin{aligned} dW &= dpK - [p + dp]\zeta K dt + \Pi dt + d\Pi \\ &= \left\{(\delta - \zeta) pK + \Pi + \pi\left[\left(\vec{\beta} Z - \alpha\zeta\right) rK \right.\right. \\ &\quad + \left(\beta_1 r + 2s\alpha\left(\vec{\beta} Z - \alpha\zeta\right)\right) K^2 \\ &\quad \left.\left. + (r\beta_2 + 2s\alpha\beta_1) K^3 + 2s\alpha\beta_2 K^4\right]\right\} dt \\ &\quad + pK\sigma_K dz + (1 + \zeta)\pi\sigma_y K^{-1} dz_y \end{aligned} \tag{12}$$

The total change in wealth is equal to the expected net capital gains from the initial stock of capital (the first term in the second line), plus the operating margin (Π), plus the expected increase in future operating profits due to changes in technical change and capital stock (the term in square brackets), plus stochastic changes in the price of capital (the first term after dt), plus the variation in the total operating margin due to stochastic changes in production (last term in the equation). Investment affects total risk in two opposite ways: (a) it reduces variability in production (Eq. (4)) but increases exposure to asset price oscillations.

[7] In some cases, economic policies (i.e. a subsidy) make p different from the acquisition cost (a^-).

The equation states that the current value of total wealth is known with certainty but future values are unknown and are distributed with a multivariate distribution whose variance grows with time. Even though information arrives with time, future wealth is always uncertain and the level of uncertainty grows with the forecasting horizon.

The final expression for dW is obtained by substituting for Π in Eq. (12) and rearranging terms

$$\begin{aligned} dW - \{\pi q & \\ &+ \left[(\delta - \zeta) p + \pi r\alpha + \pi r\left(\vec{\beta} Z - \alpha\zeta\right)\right] K \\ &+ \left[s\alpha^2 + \beta_1 r + 2s\alpha\left(\vec{\beta} Z - \alpha\zeta\right)\right]\pi K^2 \\ &+ (r\beta_2 + 2s\alpha\beta_1)\pi K^3 + 2s\alpha\beta_2\pi K^4\} dt \\ &+ pK\sigma_K dz + (1 + \zeta)\pi\sigma_y K^{-1} dz_y \end{aligned} \tag{13}$$

The firm's problem can be restated as choosing those k^-, k_-, K^- and K_- that maximize Eq. (8) subject to Eq. (13)

4. Solving the model for the optimal policy

In the interval of no intervention, the system evolves according to the following ordinary differential equation (Dixit, 1991)

$$\begin{aligned} -\mu J(K) + J_K\{\pi q & \\ &+ \left[(\delta - \zeta) p + \pi r\alpha + \pi r\left(\vec{\beta} Z - \alpha\zeta\right)\right] K \\ &+ \left[s\alpha^2 + \beta_1 r + 2s\alpha\left(\vec{\beta} Z - \alpha\zeta\right)\right]\pi K^2 \\ &+ (r\beta_2 + 2s\alpha\beta_1)\pi K^3 + 2s\alpha\beta_2\pi K^4\} \\ &+ \frac{1}{2} J_{KK}\left\{p^2 K^2 \sigma_K^2 + (1 + \zeta)^2 \pi^2 \sigma_y^2 K^{-2}\right. \\ &\left. + 2(1 + \zeta) p\pi\sigma_K \sigma_y \rho K\right\} + U(W) = 0 \end{aligned} \tag{14}$$

where ρ is the correlation coefficient between the price of capital and output. Eq. (14) is no longer a stochastic differential equation because the stochastic processes have been replaced by their expected values, instead it is a second order ordinary differential equation with variable coefficients. The general solution to Eq. (14) is a linear combination of two power series in K (J_1 and J_2) whose coefficients are nonlinear combinations of the parameters of the problem (Boyce and DiPrima, 1977) [8]

$$J_1(K) = K\Bigg(1 - \frac{q}{24(1+\zeta)^2\pi\sigma_y^2}K^3 - \frac{-\mu + (\delta-\zeta)p + \alpha\pi r(1-\zeta) + \vec{\beta}Z\pi r}{10(1+\zeta)^2\pi^2\sigma_y^2}K^4 - \frac{-\beta_1 r + \alpha^2 s(1-2\zeta) - 2\alpha s\vec{\beta}Z}{15(2+\zeta)^2\pi\sigma_y^2}K_5 + \dots\Bigg)$$

$$J_2(K) = 1 + \frac{\mu}{10(2+\zeta)^2\pi^2\sigma_y^2}K^4 - \frac{2\mu(q + 6p\sigma_K\sigma_y\rho) + 3p\sigma_K\sigma_y\rho}{\pi^3\sigma_y^4(1680 + 3360\zeta^2 + 840\zeta^3 + 105\zeta^4)}K^7 + \dots$$

A convenient particular solution is the expected discounted utility calculated ignoring all barriers and controls on the process (Dixit, 1991)

$$V(K) \equiv E\left[\int_0^\infty e^{-\mu t}U(K)\,dt \mid K(0) = K_0\right] = \frac{1}{\mu}U(K_0)$$

The solution of Eq. (14) is

$$J(K) = C_1 J_1(K) + C_2 J_2(K) + V(K)$$

where C_1 and C_2 are constants to be determined from the parameters of the problem. $V(K)$ is the value of the objective function when no controls are exercised and $J(K)$ is the value of the same function when controls are applied. Consequently, $C_1 J_1(K) + C_2 J_2(K)$ is the value of the controls. If these are applied optimally, their value must be non negative.

The optimality conditions are defined by the equations

$$J(K^-) - J(k^-) = a^- + b^-(k^- - K^-) \quad (15)$$

$$J(K_-) - J(k_-) = -a_- + b_-(k_- - K_-) \quad (16)$$

$$J'(K^-) = -b^- \quad (17)$$

$$J'(K_-) = -b_- \quad (18)$$

$$J'(k^-) = b^- \quad (19)$$

$$J'(k_-) = b_- \quad (20)$$

These six equations determine simultaneously the value of the parameters C_1, C_2, k^-, K^-, k_- and K_- (Dixit and Pindyck, 1994).

5. Static properties of the optimal policy

The system of Eqs. (15)–(20) is highly nonlinear and cannot be solved analytically. The solution can be analyzed locally by obtaining numerical values of the parameters. These parameters were estimated from time series of the Argentine agriculture; the system of equations was solved with the package SAS 6.7 conditional on the estimated values of the parameters.

The numerical solutions presented in this work cannot be considered as estimates or forecasts in an econometric sense. In addition to the traditional conditionalities of econometric models, numerical solutions of nonlinear models depend on a large number of assumptions about scaling and starting values. In spite of these limitations, the model shows that there is a set of numbers that can reproduce the described behavior of the desired investment and the barriers. This exercise, then, can be considered as a first step towards the building of a complete and robust empirical model. The static properties of the optimal policy can be analyzed by calculating numerical derivatives of the indirect utility function and the barriers with respect to parameters of interest.

Table 1 shows the response of the indirect utility function to a 10% increase in selected parameters. All numerical derivatives have the expected sign. Increases in the actual level of the technology index (α) increase output and future profits; as a conse-

[8] Both power series were calculated up to the 15th term with the package Mathematica. Detailed explanations of the solution to the differential equation can be requested from the author.

Table 1
Percent change in indirect utility caused by a 10% increase in the parameters

α	94.9
p	92.5
σ_y	−9.1
σ_K	−3.5
a^-	−7.4
a_-	−49.5
b^-	33.1
b_-	23.7

quence, the value of the indirect utility function rises.

In this model, p is the price at which the capital stock is valued; as it increases, both wealth and the indirect utility function rise. Greater productive risk (σ_y) increases the likelihood of economic loses and reduces the value of capital; the indirect utility function falls accordingly.

An increase in the variance of the price of capital (σ_K) raises the variability of wealth and reduces its permanent component, lowering the value of the indirect utility function. Pindyck (1991) stressed the similarities between irreversible investment and financial options; this interpretation helps to explain the numerical simulations when a component of the investment cost changes. Investment is equivalent to closing off the option of waiting for more information on the convenience of the project; by delaying investment, however, the firm loses the potential profits from increased production.

A rise in the purchase price (a^-) increases the sunk cost of investment and its opportunity cost. Since the latter now becomes larger than the revenue lost by waiting, the optimal policy is to delay investment and wait for more information. Consequently, a rise in the purchase price reduces indirect utility.

At the lower barrier the firm faces the option of reducing its capital faster than the depreciation rate. If it is no longer profitable to produce at the actual level, the firm can reduce its losses by cutting its capital stock. However, if the firm later decides to increase production, it will have to pay the sunk cost of investment. An increase in the price of used capital (a_-) has two effects: (a) it decreases the irreversibility of investment by reducing the sunk cost ($a^- - a_-$), and (b) it decreases the value of the option to maintain the capital stock for future increases in production. As a consequence, the firm cuts its actual capital stock.

Partial adjustment at the upper barrier means that the firm has a smaller capital stock compared with the case of full adjustment. An increase in the cost of not adjusting completely (b^-) makes capital more valuable and raises the value of indirect utility. Symmetrically, partial adjustment at the lower barrier means that the capital stock is larger than the case where there are no indivisibilities; then, an increase in b^- reduces the utility rendered by capital.

The barriers that define the optimal policies are also affected by changes in the parameters. Table 2 shows the changes in the relative position of the barriers when a parameter changes by 10%. Improved technologies (higher α) increase the profitability of capital and reduce the cost of adjustment relative to the return on investment. In consequence there is a reduction in the inaction interval, meaning that a smaller change in the desired capital stock is required to trigger a change in the actual capital stock.

Changes in the technology index affect not only the inaction interval but also the partial adjustment gaps. In this particular case, technical change reduced the incidence of indivisibilities by raising the revenue lost due to an incomplete adjustment.

An increase in output variance (σ_y) raises production risk and waiting becomes optimal; consequently, the inaction interval becomes larger. As capital becomes less valuable, the partial adjustment gaps also expand. The market price of capital (p) has the same influence as technical change because an increase in

Table 2
Percent change in the relevant intervals caused by a 10% increase in the parameters

	k^-/k_-	k^-/K_-	K^-/k_-
α	−4.7	−2.5	−2.1
p	−0.7	−2.5	0
σ_y	1	0.3	1.2
σ_K	0.4	1	−0.8
a^-	1	0.6	−0.3
a_-	−2.8	−1.5	−0.8
b^-	−0.4	−0.5	0.2
b_-	−0.3	0.8	0.4

p raises total wealth and consequently, the relative magnitude of the adjustment costs falls, making investment more frequent. Increases in the acquisition cost (a^-) raise the sunk cost of investment, causing the inaction interval to increase. If the sale price of capital (a_-) rises, the value of the irreversibility falls and the option of selling becomes more valuable; the inaction interval is reduced because the cost associated with a wrong decision falls. If any of the partial adjustment costs $(b^-$ and $b_-)$ increases, the cost of delaying a decision also grows; the outcome is a reduction in the inaction interval.

6. The dynamics of investment revisited

The dynamic process determining optimal investment decisions of an individual firm can be sketched using the static simulations presented in Section 5. The firm continuously updates its desired capital stock according to its expectations about prices, output, risk and other relevant variables. While the desired capital drifts between the external barriers, the firm neither invests nor disinvests.

Assume that a one period positive shock to expectations occurs. If it is strong enough to increase the desired capital stock up to the upper barrier (k^-), the firm invests until the actual capital stock equals the upper internal barrier (K^-). The inaction interval is instantaneously redefined around the new actual stock. This is the result obtained with constant technology described in Harrison (1985).

With embedded technical change, though, the dynamic effects do not stop here. According to Eq. (4), the new capital reduces the actual variance of output (σ_y) and increases the efficiency of capital (α), both of which, in turn, raise the desired capital stock (Table 1), cut down the inaction interval and reduce the effect of indivisibilities (Table 2). If these effects are strong enough, the firm continues investing in subsequent periods even though the initial shock lasted only one period. As the firm invests, the inaction interval narrows, meaning that smaller increases in the desired capital are required to trigger new investments. Eq. (4), however, guarantees that the process is bounded. The process continues until the potential benefits of the technological package are exhausted.

Positive shocks to expectations can have multiple causes, including economic policies, the appearance of new technologies or favorable market conditions. As an example of a shock originated in economic policies, assume that the government subsidizes the purchase of a portion of the technological package. If the expected increase in profits is large enough, the firm will find it profitable to adopt the whole technological package even though part of it must be purchased at market prices. The subsidy required to achieve a certain level of output growth is smaller with embedded technical change than with constant technology.

Investment can also be triggered by the development of new technologies. As new production processes become available, firms have the option to adopt them. If the potential increase in profits is large enough, firms will adopt the complete technological package. Investment in this case is larger than when the firm invests in the same technology it has been using.

When the desired capital falls, there are two possible investment paths. Once the firm disinvests, its technological level falls and the production variance goes up. These two effects push the desired capital even further down (Table 1) and expand the inaction interval (Table 2). Disinvestment continues until it stops for one of two reasons.

1. For most activities there is a minimum capital stock that makes production profitable. If the desired stock falls below this threshold, the firm liquidates its capital and abandons production.
2. If the firm's actual stock is above the minimum viable capital stock, the actual capital stock settles at a lower level with a larger inaction interval. The desired capital eventually stabilizes and fluctuates between the external barriers. Because at this time the inaction interval is larger than at time 0, the required shock to trigger an action must be stronger than in the initial time. This case is symmetric with the case of a positive shock.

The presence of embedded technical change strengthens the effect of shocks to the desired capital stock. If the shock is positive, investment is larger than in the case of constant technology. If the shock is negative, the reduction of the actual capital stock is greater than otherwise; it can even lead to abandonment of production.

The efficiency of economic policy is strengthened by embedded technical change; by subsidizing only part of a technological package, the government can induce total adoption and an greater expansion of output at a lower cost. Technological policies can also trigger expansion cycles; if new and more profitable technologies become available, the desired capital increases and eventually reaches the upper barrier.

Finally, consider the simultaneous occurrence of negative and positive shocks such as an increase in inflation (measured by an increase in the variance of capital prices) and the appearance of new technologies. Investment decisions will depend on the relative strength of these shocks. If the positive shock is the strongest, the prospect of increased profits can compensate for the greater risk of investment and output will expand. Conversely, if the increased risk is large enough, firms will disinvest even though more profitable technologies are available.

7. Final remarks

This paper extended the analysis of irreversible investment under uncertainty to incorporate embedded technical change and endogenous risk exposure. In this extended framework, changes in the actual capital stock are larger than in the traditional analysis with constant technology and exogenous risk.

Harrison (1985) showed that in the presence of uncertainty and irreversibilities, the optimal investment policy for an individual firm is to regulate the difference between its desired and actual capital stocks between two barriers that minimize the need for intervention. If the desired capital touches the upper barrier, the firm invests. If it touches the lower barrier, the firm disinvests. When the actual capital changes, the barriers move accordingly to define a new interval of no intervention centered on the new stock.

The previous result was obtained with constant technology and exogenous risk; when technical change is embedded and risk is endogenous, investment is larger than in the previous case. If expectations about the future are positive (negative), the desired capital stock increases (falls) up to the upper (lower) external barrier and the firm invests (disinvests). In doing so, the firm raises the average technological level of its capital stock, reduces the production risk and expands total profits. These three outcomes push the desired capital stock even higher and reduce the inaction interval. If they are strong enough, these effects trigger investment in the following period, again raising the desired capital stock and reducing the inaction interval. The influence of the first positive shock to expectations lasts until all potential gains from the actual technological package are exhausted.

When the firm disinvests, a lower technology index and increased production risk push the desired capital down and expand the inaction interval. Two outcomes are possible: (a) the actual capital stock stabilizes at a lower level and the firm continues production at a smaller scale, or (b) the firm liquidates its capital and exits the activity.

The desired capital stock is a function of several variables, including the firm's expectations about future profits, risk and the evolution of the economy. Shocks to these variables may originate in changes in output demand, the availability of new technologies or economic policies. For example, assume a subsidy to the purchase of a portion of the technological package that reduces the purchase cost (a^-) enough to trigger investment. If the potential gains are large enough, the firm will invest in the whole technological package, not only in those parts that are subsidized.

In the presence of embedded technical change, a subsidy to a portion of the technological package may trigger an investment process that lasts until the potential of the technology in use is exhausted. The incentives required to obtain a certain output growth are smaller in this case than when technology is constant.

Similar results can be obtained with technological policies or risk reducing measures. Development of new techniques can trigger investment and output growth if expected profits from the new production processes are large enough to shift the desired capital up to the upper external barrier. Risk reduction also works by increasing the desired capital stock in the initial period. Examples of risk reduction measures are crop insurance, anti-inflationary policies or credits with fix interest rates.

The model can be extended to analyze investment in human capital, soil conservation, sustainability of exploitation of renewable natural resources and development policies. The ultimate goal is to find an econometric specification to fit these models to aggregate and disaggregate data.

Acknowledgements

I thank Michael Caputo, Avinash Dixit, Mark Evans and Richard Howitt for helpful comments.

References

Akerlof, G., 1970. The market for lemons: quality uncertainty and the market mechanism. Q. J. Econ., 89: 488–500.

Baquet, A., 1978. A theory of investment and disinvestment including optimal life, maintenance and usage rates for durables. Ph.D. Dissertation, Michigan State University.

Boyce, W. and DiPrima, R., 1977. Elementary Differential Equations and Boundary Value Problems, 3rd edn. John Wiley, USA.

Dixit, A., 1989. Entry and exit decisions under uncertainty. J. Political Econ., 97: 620–638.

Dixit, A., 1991. A simplified treatment of the theory of optimal regulation of Brownian motion. J. Econ. Dynam. Control, 15: 657–673.

Dixit, A., 1992. Investment and hysteresis. J. Econ. Perspect., 6: 107–132.

Dixit, A. and Pindyck, R., 1994. Investment under Uncertainty. Princeton University Press, Princeton, NJ.

Edwards, C., 1959. Asset fixity and farm organization. J. Farm Econ., 37: 747–759.

El-Gamal, M., 1994. A dynamic migration model with uncertainty. J. Econ. Dynam. Control, 18: 511–538.

Harrison, J., 1985. Brownian Motion and Stochastic Flow Systems. Wiley, New York.

He, H. and Pindyck, R., 1992. Investments in flexible production capacity. J. Econ. Dynam. Control, 16: 575–599.

Hertzler, G., 1991. Dynamic decisions under risk: application of Ito stochastic control in agriculture. Am. J. Agric. Econ., 73: 1126–1137.

Johnson, G., 1956. Supply functions—some facts and notions. In: E. Heady et al. (Editors), Agricultural Adjustments Problems in a Growing Economy. Iowa State University Press.

Kushner, H. and Dupuis, P., 1992. Numerical Methods for Stochastic Control Problems in Continuous Time, Applications of Mathematics, N. 25. Springer, Amsterdam.

Parente, S., 1994. Technology adoption, learning-by-doing, and economic growth. J. Econ. Theory, 63: 346–369.

Pindyck, R., 1991. Irreversibility, uncertainty and investment. J. Econ. Lit., 29: 1110–1148.

Pindyck, R., 1993. Investments of uncertain cost. J. Financial Econ., 34: 53–76.

Smith, W., 1994. Investment, uncertainty, and price stabilization schemes. J. Econ. Dynam. Control, 18: 561–579.

Stefanou, S., 1987. Technical change, uncertainty, and investment. Am. J. Agric. Econ., 69: 158–165.

Teisberg, E., 1993. Capital investment strategies under uncertain regulation. RAND J. Econ., 24: 591–604.