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## The cost of agricultural production risk <sup>☆</sup>

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Accepted 27 March 1995

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### Abstract

We examine the relative influence of preferences and technology on producers' ex ante willingness to pay for a reduction in production risk. A risk averse producer pays both an Arrow–Pratt risk premium to stabilize income and a 'production premium' to stabilize yield. Using soil-nitrate risks as our motivating example, we demonstrate that the production premium accounts for 40–85% of producers' willingness to pay for risk reduction. These results demonstrate the relative importance of technology over risk preferences when estimating the costs of agricultural production risk.

### 1. Introduction

Agricultural economists typically analyze the effects of production risk in terms of the Arrow–Pratt risk aversion concept, which shows that risk is costly to risk-averse producers because the utility of expected income exceeds expected utility (Friedman and Savage, 1948). This tendency to focus on income variability reveals itself most strikingly in the analysis of input decisions under risk. In basic models (for example, Babcock et al., 1987; SriRamaratnam et al., 1987; Love and Bucola, 1991), the introduction of risk affects optimal input levels according to how yield (and income) variance changes with input use. In a more general context, Pope and Kramer defined inputs as risk-reducing or risk-increasing accord-

ing to how use affects the marginal risk premium, and Ramaswami (1992) followed up by finding the weakest condition on technology sufficient to sign the marginal risk premium for all risk-averse agents. Loehman and Nelson (1992) used a multiple input framework to determine how optimal input use under risk aversion differs from risk neutral optimal input use. The primary message from this literature is that risk matters because producers are risk averse. That is, risk is costly to producers, and they will adjust their input decisions away from expected profit-maximizing levels, because of a declining marginal utility of income.

The focus on income variability as the source of the cost from agricultural production risk is warranted in the special case where risk affects profits linearly. But risk often affects profits nonlinearly, thereby affecting the decisions of risk-neutral firms. With nonlinear production risk the willingness to pay to avoid risk and the effects of risk on optimal decisions are not captured solely by measuring how risk affects the variability of

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<sup>☆</sup> Journal Paper No. J-16312 of the Iowa Agriculture and Home Economics Experiment Station, Ames, IA. Project No. 3048.

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income. Rather, the effect of risk on expected profits also influences willingness to pay to avoid risk. Previous studies that have made this point include Just (1975), who showed that risk affects decisions of risk-neutral firms when actual yields deviate from planned yields; Antle (1983), who argued that risk affects risk-neutral farmers when they make sequential production decisions subject to random shocks; and Taylor (1986), who showed that risk affects optimal decisions aimed at maximizing expected after-tax income when there is a nonlinear tax schedule. Just (1975) and Taylor (1986) differentiate between ‘proper’ risk aversion and ‘pseudo’ or ‘apparent’ risk aversion, the latter describing the effects of risk on expected profits. Other studies have shown that random input availability can affect optimal input usage under risk neutrality. For example, Letey et al. (1984) showed that risk can increase optimal irrigation water applications by up to 50%. Babcock and Blackmer (1992) showed that risk can increase optimal nitrogen fertilizer rates by up to 40%.

What the existing literature clearly shows is that risk affects the decisions of both risk-neutral and risk-averse firms. What has not been shown, and what is critical to our understanding of how risk affects behavior, is the importance of nonlinear utility relative to nonlinear profits in determining the willingness to pay for risk reductions. There are three possible outcomes. First, if the primary source of willingness to pay for risk reductions is derived from the effects of risk on expected profits, then risk analyses should focus on correctly modeling the magnitude of the relevant risk and how risk enters the profit function. This would unburden the researcher from measuring risk aversion levels or selecting appropriate risk aversion levels to use in simulation. Second, if the primary source of willingness to pay for risk reductions comes from the disutility of variable income, then it is less important to determine how risk enters profit functions, and more important to determine risk magnitudes and relevant risk aversion levels. And finally, if both sources are important, then consideration of both tastes (the utility function) and technology (how risk enters the production function) is essential.

This paper demonstrates that the cost of risk from nonlinear profits can be greater than the cost from nonlinear utility under reasonable levels of risk aversion. We first focus on the common situation of uncertain input availability and develop a measure of a producer’s willingness to pay to eliminate that uncertainty. Our measure has two parts. The first, which we call the ‘production premium’ measures the effect of risk on expected profit. The second is the traditional Arrow–Pratt risk premium which measures willingness to pay to reduce income variability. Using the application of nitrogen in corn production as a motivating example, our results demonstrate that the production premium is responsible for between 48 and 85% of total willingness to pay. Also we consider the importance of the production premium when there is an additional exogenous production risk. For risk averse producers who face this linear production shock, the production premium still accounts for over 20% of the willingness to pay to eliminate all sources of production risk. Our results suggest that the existence of a positive willingness to avoid risk does not necessarily imply that an individual is risk averse in the traditional sense of wanting to avoid income variability. Rather, the individual may be risk neutral and responding to nonlinear production risk. Our results reinforce the often ignored point made by Just (1975) (p. 351): “To verify reaction to changing risk is not sufficient to refute profit maximization.”

## **2. Nonlinear production risk and the production premium**

The Arrow–Pratt risk premium is the payment that makes a decision maker indifferent between receiving an income risk and receiving the mean value of income. The risk premium is an appropriate measure of willingness to pay to avoid risk when analyzing risks that affect income (or wealth) linearly. Alternative measures of willingness to pay to fix a random variable at its mean value are required for other types of risk. For example, Shogren (1991) developed the protection premium to measure the willingness to pay for certainty when the efficacy of protection

against loss is stochastic. Lence and Babcock (1995) developed conditions under which the protection premium depends upon both the utility function and the curvature of the protection probability function. Under these conditions the protection premium has two components because stabilizing protection efficiency has implications on both protection and income.

In agriculture the Arrow–Pratt risk premium measures the total cost of production shocks when the shocks affect yields linearly. But when the production risk affects yields nonlinearly, the Arrow–Pratt risk premium does not capture the full willingness to pay to fix the risk at its mean level. Production shocks affect yields nonlinearly, rather than proportionately or additively, when they occur inside the production function. A well-known example of a nonlinear production shock is when service flows from applied factors of production are random (Ratti and Ullah, 1976).

Consider a producer investing in an input,  $x$ , costing  $p$  per unit, to produce an output,  $q$ , which can be sold at a price,  $r$ . Let production be stochastic such that  $q$  depends on a stochastic input  $\theta$

$$q = f(\theta) \quad (1)$$

where we assume  $f' > 0$  and  $f'' \leq 0$ . The producer cannot control  $\theta$  directly because of random events, but applications of  $x$  can influence the distribution of  $\theta$ . Define the conditional density of  $\theta$  as

$$g(\theta | x) \quad \underline{\theta} \leq \theta \leq \bar{\theta} \quad (2)$$

An example of this type of production relationship is when  $\theta$  is available soil moisture and  $x$  is applied irrigation water. Because of variability in moisture-holding capacities of soils across a field,  $\theta$  is a random variable with moments that can be controlled by the amount of irrigation water applied. The type of production relationship given by Eqs. (1) and (2) is based on identifiable physical relationships relating expected production to both available and applied inputs.

The producer derives utility from the profits received

$$U(\pi) = U(rf(\theta) - px) \quad (3)$$

where  $U' > 0$  and  $U'' \leq 0$ . The producer's expected utility is now written as

$$\int_{\underline{\theta}}^{\bar{\theta}} U(rf(\theta) - px) g(\theta | x) d\theta \quad (4)$$

We define a premium that represents the producer's willingness to resolve the uncertainty regarding  $\theta$  for a given  $x$ . Let  $\beta(x)$  be the premium such that

$$U(rf[E(\theta)] - px - \beta(x)) = \int_{\underline{\theta}}^{\bar{\theta}} U[rf(\theta) - px] g(\theta | x) d\theta \quad (5)$$

Now define a risk premium,  $RP$ , such that the producer pays to stabilize yield at its mean level

$$U[rEf(\theta) - px - RP] = \int_{\underline{\theta}}^{\bar{\theta}} U[rf(\theta) - px] g(\theta | x) d\theta \quad (6)$$

Comparing (5) and (6) we find that

$$\beta(x) = r\{f[E(\theta)] - Ef(\theta)\} + RP \quad (7)$$

As shown in Eq. (7),  $\beta(x)$  has two parts—the production premium and the risk premium. The production premium is the change in expected profits obtained by fixing  $\theta$  at its mean level. By Jensen's inequality, the production premium is positive if  $f$  is concave in  $\theta$ . That is, a risk-neutral producer will pay a premium to resolve uncertainty if expected profits are greater after the uncertainty about  $\theta$  is resolved. The production premium can play a significant role in how producers value new technologies. For example, improved sprinkler technologies can reduce spatial variability of applied irrigation water. If yields are concave in irrigation water then expected yield holding mean application constant is higher under the more uniform sprinkler technology (Bernardo, 1988). In the dairy industry new technologies tailor rations to individual cows so that more aggressive cows do not overfeed at the expense of their more timid sisters. Production is thereby increased with no increase in total feed consumption. Recent work on soil fertility (Babcock and Blackmer, 1992) indicates that producers can increase expected yields by reducing year-to-year variability in available soil nutrient

levels at the time of rapid plant uptake. In broiler production, equalization of marginal products by tailoring feed rations to the biological potential of homogeneous groups of chickens can result in increased production without an increase in total feed use (Han and Baker, 1991). Similar production responses have been found in hog production (Stahly, 1993).

The second part of  $\beta(x)$ ,  $RP$ , measures the willingness to pay to fix income at its mean level. That is,  $RP$  is the Arrow–Pratt risk premium. To see the equivalence, take a Taylor series expansion around both sides of Eq. (5), which yields a second-order approximation to the premium

$$\beta(x) = r\{f[E(\theta)] - Ef(\theta)\} - \frac{1}{2} \frac{U''}{U'} r^2 E\{f(\theta) - f[E(\theta)]\}^2 \quad (8)$$

Risk preferences or tastes influence  $\beta(x)$ , as is evident in Eq. (8), but risk aversion is not a necessary condition for a positive  $\beta(x)$ . Under risk neutrality the producer's premium to avoid risk is still positive if  $f(x)$  is concave. The total premium is driven solely by tastes only if production is linear in the risk. Which effect, tastes or the production technology, plays a larger role in determining the size of  $\beta(x)$  is an empirical question.

Eq. (8) defines the willingness to resolve uncertainty about the stochastic input  $\theta$ . Define this risk as input-risk.  $\beta(x)$  is the appropriate measure of the value that would be derived from investments in risk reductions targeted at  $\theta$ . However, when there is more than one source of risk, Eq. (8) does not measure the value of eliminating all risk. Suppose that the stochastic production function is

$$q = f(\theta) + \epsilon \quad (9)$$

where  $\epsilon$  is a mean-zero random variable that captures all sources of risk other than  $\theta$ . Define this additive production risk as  $\epsilon$ -risk. Assume that  $\theta$  is independent of  $\epsilon$ . Let  $\beta_\epsilon(x)$  be the willingness to pay to resolve both input-risk and  $\epsilon$ -risk

$$\beta_\epsilon(x) = r\{f[E(\theta)] - Ef(\theta)\} + RP_\theta + RP_\epsilon \quad (10)$$

where  $RP_\theta$  is the risk premium caused by input-risk and  $RP_\epsilon$  is the risk premium caused by  $\epsilon$ -risk. A Taylor series expansion around the point  $f[E(\theta)] + E(\epsilon) - px$  results in

$$\begin{aligned} \beta_\epsilon(x) = & r\{f[E(\theta)] \\ & - Ef(\theta)\} \frac{1}{2} \frac{U''}{U'} r^2 E\{f(\theta) - f[E(\theta)]\}^2 \\ & - \frac{1}{2} \frac{U''}{U'} r^2 \sigma_\epsilon^2 \end{aligned} \quad (11)$$

where  $\sigma_\epsilon^2$  is the variance of  $\epsilon$ . Note that if  $\epsilon = h(x)u$ , where  $u$  is a mean zero random variable, then Eq. (9) is similar to the commonly used heteroscedastic production function  $q = g(x) + h(x)u$  (Just and Pope, 1978). If this is the case, then in Eq. (11)  $\sigma_\epsilon^2 = h^2 \sigma_u^2$ . However, the standard Just–Pope specification assumes that all sources of risk affect yield from outside the production function, whereas in Eq. (9)  $\theta$  affects yields from inside the production function. Maintaining the standard Just–Pope functional form is tantamount to assuming that the production premium is zero; that is, the only effects of risk are caused by aversion to variable income because  $q$  is linear in  $u$ .

The appropriate measure of willingness to pay to resolve production uncertainty depends on the question one is asking. If one wants to determine the impacts on producer welfare from adoption of a new technology that affects the stochastic relationship between  $\theta$  and  $x$ , then one should use  $\beta(x)$  defined in Eq. (8). However,  $\beta_\epsilon(x)$  defined in Eq. (11) is the relevant measure if one wants to determine the cost of all sources of production risk. We focus on both  $\beta(x)$  and  $\beta_\epsilon(x)$  in our application below.

### 3. An application

Random production inputs that enter the production function are perhaps most common in agriculture. The remainder of this paper is devoted to estimating the relative magnitudes of the production premium and the risk premium for a producer who faces soil-nitrate risk. Soil-nitrate levels vary from year to year because of losses

from leaching and denitrification and gains from fixation of atmospheric and organic nitrogen sources (Blackmer, 1987). These loss and gain rates are random, depending (unpredictably) in part on the interaction of pre-season weather events, previous crop yields, and soil characteristics (Hanley, 1990). This is especially true in regions with highly variable climate patterns, such as the US Cornbelt. Traditional soil nitrogen tests are not widely used in the Cornbelt because the potential losses (and gains) between the time the test is conducted and the time of rapid plant uptake are large. The problem for producers is that they must apply their nitrogen fertilizer without knowing either the level of nitrates already present in the soil or the level that will be available at the time of rapid plant uptake.

To illustrate, let  $\theta = N_s$  be the amount of nitrates in the soil during the rapid growth stage of the crop, and let  $x = N_a$  be the level of applied nitrogen fertilizer so that Eq. (9) is rewritten as

$$q = f(\theta) + \epsilon = f(N_s) + \epsilon \quad (12)$$

and the conditional density equation (Eq. (2)) is rewritten as

$$g(\theta | x) = g(N_s | N_a) \quad (13)$$

The assumption that  $N_s$  and  $\epsilon$  are independent is continued. The risk  $\epsilon$  can be considered exogenous for two reasons. First, the producer has no control over its distribution. Second, optimal production decisions are unaffected by the distribution of  $\epsilon$ . That is, a mean-preserving spread in  $\epsilon$  has no effect on optimal decisions. We assume that  $f$  is an increasing concave function in  $N_s$ . The producer cannot control  $N_s$  directly because of the influence of weather events, but  $N_a$  can influence the distribution of  $N_s$ . To estimate the relative magnitudes of the components of  $\beta_\epsilon(N_a)$  as defined in Eq. (11), requires estimates of the production function, the variance of  $\epsilon$ , the density function of soil nitrates, and a level of absolute risk aversion.

A large set of experimental data has been generated to determine the relationship between applied nitrogen fertilizer, soil-nitrate levels, and corn yields in Iowa. These data are used to estimate  $f(N_s)$  and  $g(N_s | N_a)$ . The experiments in-

volve applying three replications of ten rates of nitrogen (ranging from 0 to 300 lb acre<sup>-1</sup>) shortly before planting in late April or early May, testing for soil nitrate concentrations (ppm) in early June, and measuring harvested yields (bushels). The ongoing experiments were generated on 17 sites across Iowa beginning in 1985. A single site-year of data will be used to estimate a representative corn production function. Data generated in 1987 on the Nashua site (located in north central Iowa) for the continuous corn rotation are used for estimation. This site-year is fairly typical of Iowa corn production with fairly good growing conditions. The number of observations is 30.

Previous analysis of these data supports the existence of a yield plateau in the relationship between soil nitrates and yield (see figs. 2 and 3 in Binford et al., 1992). Thus, it is appropriate to select a functional form that imposes a plateau yield. One such production function is the Mitscherlich (National Academy of Sciences, 1961). The estimated production function (with *t*-statistics in parentheses) is

$$q = 143.99[1 - \exp(-0.13656(N_s - 5.0992))] \quad (163.9) \quad (19.37) \quad (4.42)$$

$$R^2 = 0.71. \quad (14)$$

The negative estimate of the last parameter indicates that soil nitrates must be greater than approximately 5 ppm (equivalent to approximately 38 lb acre<sup>-1</sup> applied nitrogen) before a positive yield occurs.

Babcock and Blackmer (1992) determined that year-to-year variations in soil nitrate levels in the late spring for a given pre-plant application of nitrogen fertilizer are well-represented by a gamma distribution. Also, the effects of changing the fertilizer rate on the distribution of soil nitrates can be captured by allowing the parameters of the gamma distribution to respond linearly to fertilizer applications. The conditional gamma distribution is given by

$$g(N_s | N_a) = \frac{(N_s - \gamma)^{\alpha-1} \exp[-(N_s - \gamma)/\lambda]}{\lambda \alpha \Gamma(\alpha)} \quad (\alpha > 0, \gamma > 0; N_s > \gamma) \quad (15)$$

where

$$\begin{aligned}\alpha &= \alpha_0 + \alpha_1 N_a \\ \lambda &= \lambda_0 + \lambda_1 N_a \\ \gamma &= \gamma_0 + \gamma_1 N_a\end{aligned}\quad (16)$$

We use Babcock and Blackmer's (1992) estimates of  $g(N_s | N_a)$  for the continuous corn rotation to represent the year-to-year variations in soil nitrate levels in the late spring for a given pre-plant application of nitrogen fertilizer including their restriction that the lower bound of  $N_s$  is zero when no nitrogen fertilizer is applied. Their estimates are presented in Table 1 for convenience. The conditional gamma densities defined by Eqs. (15) and (16) and the parameters in Table 1 represent the amount of variation in available soil nitrates that an Iowa corn producer can expect in the first week of June at a specified pre-plant application rate of nitrogen fertilizer under a continuous corn rotation. Fig. 1 illustrates the effects of increasing  $N_a$  on the estimated densities.

The premium,  $\beta_\epsilon(x)$  is measured in monetary terms (Eq. (11)). For our example, it is more convenient to normalize output price to unity and measure  $\beta_\epsilon(N_a)$  in bushels. With the production specification given by Eq. (14) and the conditional density function given by Eqs. (15) and

Table 1

Estimated parameters and standard errors of the distribution of soil nitrate concentrations

Parameter <sup>a</sup>	Estimate
$\alpha_0$	4.920 (0.30) <sup>b</sup>
$\alpha_1$	-0.00478 (0.0010)
$\lambda_0$	1.963 (0.139)
$\lambda_1$	0.0279 (0.0016)
$\gamma_1$	0.0366 (0.0064)

<sup>a</sup> See Eqs. (15) and (16) in the text for the interpretation of the parameters. The restriction that  $\gamma_0 = 0$  was imposed.

<sup>b</sup> Estimated standard errors are given in parentheses.

Source: Babcock and Blackmer (1992).

(16), a Taylor series expansion of the willingness to pay for total resolution of uncertainty results in the following second-order approximation of  $\beta_\epsilon(N_a)$

$$\begin{aligned}\beta_\epsilon(N_a) &= f[E(N_s)] - E[f(N_s)] \\ &\quad - \frac{1}{2} \frac{U''}{U'} E\{f(N_s) - f[E(N_s)]\}^2 \\ &\quad - \frac{1}{2} \frac{U''}{U'} \sigma_\epsilon^2\end{aligned}\quad (17)$$

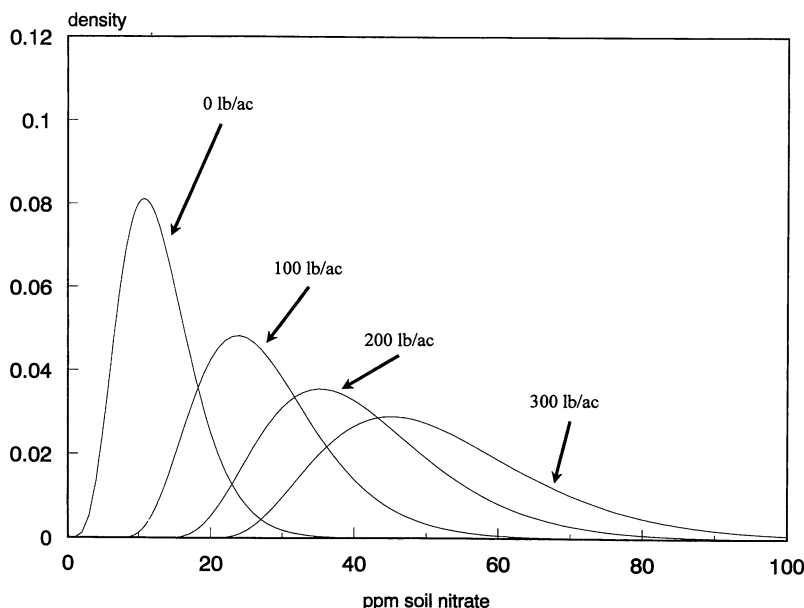


Fig. 1. Conditional density functions of soil nitrate.

As in Eq. (11),  $\beta_\epsilon(N_a)$  has three components. The first two are caused by input-risk. In the context of this example input-risk is soil-nitrate risk. The production premium is the difference between output when  $N_s$  is fixed at its mean and expected output. This component is positive with a concave production function. The second component is the Arrow–Pratt risk premium caused by the stochastic relationship between  $N_s$  and  $N_a$ . The third component is the Arrow–Pratt risk premium caused by  $\epsilon$ -risk. Note that there is no production premium associated with  $\epsilon$ -risk.

The first question we address is aimed strictly at input-risk. How large is the production premium relative to the risk premium associated with uncertain soil nitrates? This question is pertinent to understanding which factor, nonlinear production or nonlinear utility, is most important in determining the value of risk reductions aimed at input variability. The second question is, how large is the production premium relative to the sum of both risk premia? This question is aimed at determining the relative importance of the production premium when there are multiple sources of production risk.

#### 4. Results

Table 2 presents estimates of the production premium and the risk premium associated with soil-nitrate risk for three levels of  $N_a$  and four risk aversion levels. The three levels of  $N_a$  bound the average nitrogen fertilizer application in Iowa

when corn is in continuous rotation (approximately 150 lb acre<sup>-1</sup>). The four levels of risk aversion were selected to cover the likely risk attitudes of commercial grain producers in the United States (Babcock et al., 1993). These risk aversion levels were scaled to correspond to a gamble of a single acre. To shed some light on the amount of risk aversion assumed in this study, a producer with the highest selected level of absolute risk aversion, 0.031713, would be indifferent between giving up 6.83 bushels with certainty and accepting a 50% chance of gaining 20 bushels and a 50% chance of losing 20 bushels. Translating this level of absolute risk aversion into relative risk aversion requires a wealth level. Keeping the scale of the problem at 1 acre, the appropriate wealth level is 400 bushels with a land value of \$1000 and a corn price of \$2.50. Thus the four risk aversion levels imply a relative risk aversion range of between 3.17 and 12.68. This range falls in the range of risk aversion levels used in the literature as reported in table 2 of Saha et al. (1994).

The first row of estimates in Table 2 gives the producer's production premium. Because of the asymptotic yield plateau with the Mitscherlich production function, as fertilizer applications increase from 100 to 200 lb acre<sup>-1</sup>, the amount of yield uncertainty due to random nitrate levels decreases. The standard deviation of yields from random  $N_s$  is 29.7 bushels at  $N_a = 100$  lb acre<sup>-1</sup>; 20.3 bushels at 150 bushels acre<sup>-1</sup>; and 14.4 bushels at 200 lb acre<sup>-1</sup>. The production premium decreases from 9.19 bushels at 100 lb acre<sup>-1</sup> to 4.63 bushels at 200 lb acre<sup>-1</sup>. The production premium expressed as a percentage of the level of risk from  $N_s$  (represented by the standard deviation of yields) is approximately constant at just over 31%. Expected yields at the three fertilizer levels are 125.0, 133.5, and 137.6 bushels acre<sup>-1</sup>. This implies that the production premium is approximately constant when expressed as a fraction of the nitrate risk level, but declines as a fraction of expected yields.

The next four rows of results are the risk premia associated with soil-nitrate risk (the second term in Eq. (17)). The risk premium increases as risk aversion increases and decreases

Table 2  
Willingness to pay to eliminate soil nitrate risk. Values represent the risk premium (bushels) (the second term in Eq. (17))

	Fertilizer level (lb acre <sup>-1</sup> )		
	100	150	200
	9.19 <sup>a</sup>	6.53	4.63
Arrow–Pratt risk aversion <sup>b</sup>			
0.0079	3.50	1.63	0.82
0.016	7.01	3.26	1.64
0.024	10.51	4.89	2.46
0.032	14.01	6.52	3.28

<sup>a</sup> Production premium (bushels) (the first term in Eq. (17)).

<sup>b</sup> See text for equivalent relative risk aversion levels.



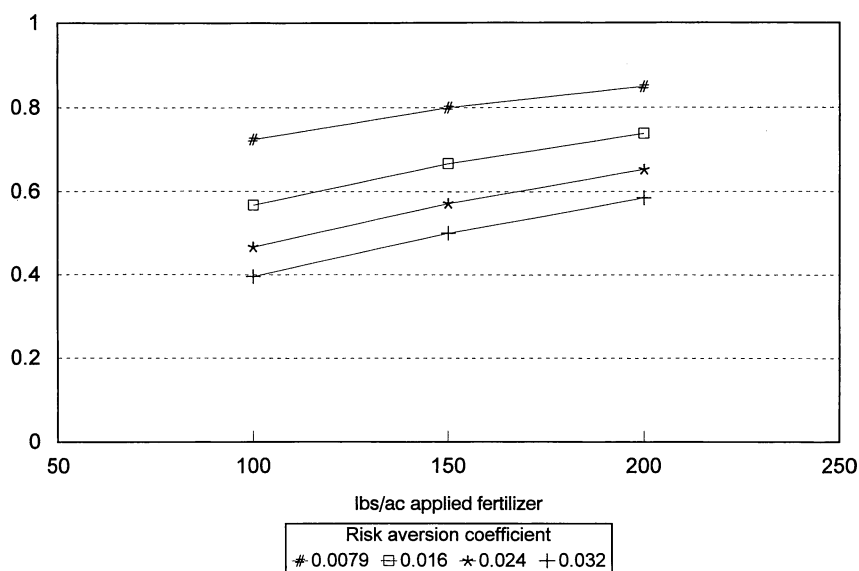


Fig. 2. Fraction of willingness to pay for input certainty accounted for by the production premium.

as the level of fertilizer increases. The risk premium ranges from less than 1 bushel at  $N_a = 100$  lb acre<sup>-1</sup> and a relatively high level of risk aversion. Fig. 2 shows the fraction of the willingness to pay to reduce soil-nitrate risk accounted for by the production premium for the four levels of risk aversion. For a given level of  $N_a$ , this fraction increases as risk aversion decreases. The production premium ranges from 40 to 72% of the

willingness to pay at  $N_a = 100$  lb acre<sup>-1</sup>, from 50 to 80% at 150 lb acre<sup>-1</sup>, and from 58 to 85% at 200 lb acre<sup>-1</sup>.

Our results illustrate the important role that the production premium plays in determining the value of reducing input risk. They suggest that nonlinear production is as important, if not more important, than nonlinear utility in determining the value of risk reduction. In addition, as risk

Table 3

Willingness to pay to eliminate all sources of risk. Values represent the risk premium (bushels) (sum of the second and third terms in Eq. (17))

	Exogenous risk (bushels) <sup>a</sup>								
	10			20			30		
	100 <sup>b</sup>	150	200	100	150	200	100	150	200
	9.19	6.53	4.63	9.19	6.53	4.63	9.19	6.53	4.63
Arrow-Pratt risk aversion <sup>c</sup>									
0.0079	3.90	2.03	1.22	5.09	3.22	2.41	7.07	5.20	4.39
0.016	7.80	4.05	2.43	10.18	6.43	4.81	14.14	10.40	8.77
0.024	11.70	6.08	3.65	15.26	9.65	7.22	21.21	15.60	13.16
0.032	15.60	8.11	4.86	20.35	12.87	9.62	28.28	20.79	17.55

<sup>a</sup> Standard deviation of  $\epsilon$  in Eq. (17).

<sup>b</sup> Top value is fertilizer level (lb acre<sup>-1</sup>); bottom value is production premium (bushels) (the first term in Eq. (17)).

<sup>c</sup> See text for equivalent relative risk aversion levels.

aversion increases, the importance of the production premium relative to the risk premium remains significant.

Turning to the second question about the importance of the production premium under multiple sources of risk, Table 3 presents estimates of the willingness to pay to resolve uncertainty about both soil-nitrate risk and  $\epsilon$ -risk for three levels of  $N_a$ , four risk aversion levels, and three levels of exogenous risk (the standard deviation of  $\epsilon$ ). The three levels of exogenous risk bound the average magnitude of the standard deviation of yields in the data set after accounting for random nitrate levels (approximately 18 bushels acre<sup>-1</sup>). As shown in Table 3, increasing  $\sigma_\epsilon$  has no effect on the production premium because profits are linear in  $\epsilon$ . The estimated risk premia reported in Table 3 are the sum of the second and third terms in Eq. (17). This sum increases from a minimum of 1.22 bushels at the lowest level of risk aversion, the highest level of  $N_a$ , and the lowest level of exogenous risk to a maximum of 28.28 bushels at the highest levels of risk aversion and exogenous risk and the lowest level of  $N_a$ . It is clear from these estimates that the risk premium can be as large or as small as one wants depending on the assumed level of risk aversion and the assumed level of risk.

The production premium accounts for between 21 and 79% of the willingness to pay to resolve both input risk and  $\epsilon$ -risk. Thus, even when the total amount of production risk is large, the contribution of the production premium to total willingness to pay remains economically significant. Therefore, to gain a better understanding of producer behavior under risk requires consideration of both tastes, as represented by nonlinear utility, and technology, as represented by nonlinear production. Otherwise, well-intended predictions and prescriptions based solely on the Arrow–Pratt paradigm may give misleading and unintended results.

## 5. Conclusions

When income depends linearly on a random variable, stabilization of the random variable at

its mean level also stabilizes income at its mean level and the Arrow–Pratt risk premium captures the full willingness of an individual to pay for stabilization. If the random variable affects income nonlinearly, then stabilization of the random variable at its mean level does not stabilize income at its mean level, and the Arrow–Pratt risk premium no longer captures the full willingness to pay.

In this paper we present a measure of the willingness to pay for stabilization of nonlinear production risk. Our measure consists of two components: one due to the concavity of the utility function, one due to the concavity of the production function. The production premium measures the change in expected profits from stabilization. It is an exact measure of the welfare consequences of risk to a risk-neutral producer. The utility component is measured by the Arrow–Pratt risk premium, which is a money metric of the additional utility loss from risk to a risk-averse producer. For producers facing uncertain nitrogen levels in the soil, we estimate that the production premium can account for over 85% of the willingness to pay to eliminate input risk, and for nearly 80% of the willingness to pay to eliminate all production risk. These results strongly suggest that nonlinear production risk can play a key role in risky decision making. These results add weight to the view that appeals to risk aversion are not necessary to motivate a producer's willingness to pay a premium to resolve uncertainty.

Furthermore, our results imply that the value of adopting a risk-reducing production technology can only be estimated by a careful specification of how risk and applied inputs interact. If profits are linear in the risk, then the magnitude of risk reduction and risk aversion levels are determine the value of the technology. In this case, there is no demand for the technology by risk-neutral producers. If the targeted risk affects profits nonlinearly, then one must specify explicitly how the risk affects expected profits to estimate the full willingness to pay for risk reductions. In this latter case, standard production models with generic additive or multiplicative production errors will not lead to accurate esti-

mates of the willingness to pay for risk-reducing technologies.

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