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Compensated optimal response under uncertainty in agricultural household models

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Abstract

This study derives the qualitative properties of a household's optimal consumption, family labour, hired labour and non-labour input choices under price and/or output risk through a Slutsky-type compensation without imposing any restriction on risk preference structure or production technology. These compensated responses provide the underpinning for welfare analysis in agricultural household models under risk. The framework for the evaluation of welfare effects of product and factor price interventions in a setting of output and price risk is developed. The paper also outlines an empirical model for estimation of the compensated demand and supply responses and for validation of the paper's analytical results.

Agricultural pricing remains a principal mode of government intervention in most less developed countries (LDCs) [1]. A vast majority of rural households in these regions are semi-subsistence farmers, i.e., they produce staple food grains partly for their consumption and partly for sale. This interdependence between consumption and production decisions has significant effects on households' optimal responses to price interventions. For example, an increase in the price of food crop that the household produces has two distinct effects: a pure price effect that renders consumption more expensive, and an income effect arising from the increased value of farm output. The interaction between the two effects yields ambiguous consumption and marketed surplus responses. A change in the rural wage rate has similar price and income effects for households who are sellers as well as buyers of labour. In the agricultural household (AH) literature, unambiguous comparative static properties of a household's optimal choices are typically derived in a 'compensated' framework [2]. Compensation removes the income effect of a price change to isolate the pure price effect yielding qualitative results.

The purpose of this study is to extend the compensated framework of comparative static analysis to AH models in a setting of risk. The

[1] See, for example, Binswanger and Scandizzo (1982), Timmer (1986), World Bank (1986), Anderson and Hayami (1986), Mellor and Ahmed (1988), Pinstrup-Andersen (1988), and the references therein.

[2] See Strauss (1986), Ellis (1988), and the references therein.

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analysis shows that qualitative properties of a household's optimal choices under price and/or output risk can be derived through a Slutsky-type compensation scheme in a relatively simple manner without imposing any restriction on the household's risk preference structure, production technology or the distribution of the random variable(s).

The compensated framework of analysis proposed in this paper owes its underpinning to a number of insightful studies on compensated optimal response under uncertainty. They include Epstein (1975), Chavas and Pope (1985) and Paris (1989). In expected utility maximization models – unless risk neutrality or constant absolute risk aversion is assumed – any price change has a pure price effect and an income effect similar to ones in AH models. Removal of the income effect through compensation secures qualitative results without relying on risk preference restrictions like decreasing or constant absolute risk aversion.

The absence of risk preference restrictions is particularly important in the context of AH models where, typically, the household is assumed to have a multi-attribute utility function. Extension of the Arrow-Pratt measures of risk-aversion to multi-attribute utility complicates analysis considerably. This is evident from Epstein (1975), Karni (1979), and more recently from Finkelshtain and Chalfant (1991). Also, most comparative static results in AH models rely on restrictions on the signs and magnitudes of the second derivatives of the utility function with respect to own and cross arguments (see, for example, Dawson, 1988; Bar-Shira and Finkelshtain, 1992). [3] These restrictions are virtually impossible to validate through empirical analysis. Compensated comparative static analysis circumvents these problems by not relying on the assumption of risk aversion to derive qualitative results; in fact, compensated comparative static expressions do not involve any second derivative of the utility function with respect to own or cross arguments.

The motivation for compensated comparative static analysis under uncertainty arises from a number of other considerations. First, any meaningful welfare analysis – crucial in evaluating the effects of policy measures – relies on compensated or 'Hicksian' demand and supply schedules (Chavas and Pope, 1981; Pope et al., 1983). This, obviously, is true for AH models as well. However, despite its importance, there exists little research on welfare evaluations in the context of AH models under risk. This paper proposes a framework of welfare analysis that relies on the household's compensated response functions.

Second, comparative static results in the uncompensated framework often rely on risk preference restrictions such as decreasing or constant absolute risk aversion. While there is a substantial body of evidence to suggest that the typical farmer in less developed countries is risk averse, [4] whether risk aversion decreases, remains constant or increases with wealth is still a moot issue (Pope, 1982)).

Third, qualitative results that rely on risk preference restrictions are difficult to validate econometrically. Consider, for example, the analytical result: non-labour input demand schedule is downward sloping in own price under decreasing absolute risk aversion (DARA). This comparative static result cannot be tested using the parameter estimates of an input demand equation because risk preference is not directly observable. More importantly, DARA is sufficient but not necessary for downward sloping input demand. Consequently, if an own-price elasticity estimate is found to be positive, one is left in doubt whether the empirical rejection of the comparative static result arises from rejection of the underlying analytical model or from rejection of DARA preferences. Complications such as these have led some

[3] In an insightful extension of Dawson's results, Bar-Shira and Finkelshtain (1992) analyze the comparative static properties of family labor supply and hired labor demand under price risk. However, based on Dawson's model, they assume that the family's welfare function is additively separable in income and leisure. This restricts the cross-derivative between income and leisure in the welfare function to be zero.

[4] See Binswanger (1980), Walker and Ryan (1990), and other relevant references therein.

analysts to conclude that "the implications of risk-averse behavior are empirically intractable" (Pope, 1978, p. 619). In contrast, compensated responses, being risk preference restriction-free, are amenable to econometric analysis and validation.

The paper's empirical section shows that the compensated demand and supply elasticities can be computed from observable uncompensated elasticity estimates. In fact, all comparative results derived in the paper can be empirically validated using data on household consumption, income, prices, and input demands.

Finally, parsimony in assumptions has its own appeal – the weaker the assumptions the more general are the results. In the interest of tractability, risk models often assume either price or output uncertainty, rarely both. In this paper, the compensated framework of analysis allows us to derive a number of qualitative results in the general framework of output *and* price risk.

The next section presents an AH model in a risk-free setting and derives the compensated comparative static properties of optimal choices. These results serve as a benchmark for comparison with the corresponding results from the AH model in a setting of risk. Also, compensated responses, defined in the risk-free setting, provide the groundwork for their extension to the framework of uncertainty.

1. Compensated optimal responses in AH models under certainty

In the following it is assumed that the agricultural household is a price taker in the factor and goods markets, faces no risks, and considers on- and off-farm labour as perfect substitutes. The household maximizes the following utility function that is continuously differentiable in its arguments:

$$U = U(c,y,l) \tag{1}$$

where c is the consumption level of farm output, l denotes leisure, and y is the numéraire composite commodity 'all other goods'. The household faces the following income constraint:

$$y = z + I \equiv \pi + wF - pc + I \tag{2}$$

where z denotes the household's 'total income' net of expenditure on food crop. The rural wage rate is denoted by w, F is the family's total labour supply, i.e., it is the sum of on-farm and off-farm family labour; p denotes the crop price, and I is exogenous income. π, farm profit, is defined as follows [5]:

$$\pi = pq - r^{\mathrm{T}}A - wL \tag{3}$$

where q is farm output, $A \in \Re^n_+$ is a vector of non-labour farm inputs, L is total labour (family plus hired) used in production, and r denotes the $n \times 1$ vector of non-labour input prices. The farmer's production function is:

$$q = G(A,L) \tag{4}$$

where $G: \Re^n_+ \times \Re_+ \rightarrow \Re_+$ is a twice continuously differentiable function, that is concave in A and L. In addition to the income constraint (2), the household faces the following time constraint:

$$F + l = T^0 \tag{5}$$

where T^0 denotes the household's total time endowment. Substitution of (2), (3), (4) and (5) into (1) yields the following optimization problem:

$$\underset{S}{\mathrm{Max}}\, H \equiv U(c,\ p(G(A,L) - c) - w(L - F) - r^{\mathrm{T}}A + I,\ T^0 - F) \tag{6}$$

where $S \equiv \{c, F, L, A\}$ is the choice vector. Observe that $(L - F)$, if positive, implies the household is a net buyer of labour, and if negative, a net seller. Let the parameter vector of prices be $\alpha \equiv (p, w, r)$. It is readily verified that the set of first-order conditions of (6) with respect to the inputs:

$$\{H_L, H_A\} \equiv \mathbf{0} \tag{7}$$

determine the optimal input levels, $A^*(\alpha)$ and $L^*(\alpha)$, independently of the optimal consumption

[5] The following notations are used throughout this paper. If $h(x, y)$ is a real-valued function of two vectors x and y, then h_x denotes the vector of partial derivatives of $h(\cdot)$ with respect to x, and h_{xy} the matrix whose ijth element is $\partial^2 h(\cdot)/\partial x_i \partial y_j$. The transpose of a matrix M is denotes by M^{T}, I_n indicates the $n \times n$ identity matrix, and $\mathbf{0}$ the null vector of the appropriate dimension.

and household labour supply choices, c^* and F^*. This is the familiar 'separability' result prevalent in AH models under certainty. Given concavity of $G(\cdot)$ in A and L, the following comparative static properties of optimal input choices and output supply follow:

A_r^* is a negative semi-definite and symmetric matrix (8a)

$$L_w^* \leq 0 \tag{8b}$$

$$q_p^* \geq 0 \tag{8c}$$

The property of 'separability' noted above, allows the household's choice problem in (6) to be restated as:

$$\operatorname*{Max}_{x} HH \equiv U(c, \pi^*(\alpha) + I + wF - pc, T^0 - F) \tag{9}$$

where $x \equiv \{c, F\}$, and $\pi^*(\alpha)$ denotes profit evaluated at A^* and L^*. Let the vector of optimal consumption and family-labour choices be denoted by $x^* \equiv \{c^*(\alpha, I), F^*(\alpha, I)\}$ [6]. The effect of a change in the parameter vector a on these optimal choices is summarized in the following result.

Claim 1. $x_\alpha^* \equiv x_z^* \cdot z_\alpha + (-HH_{xx})^{-1} z_x \alpha \cdot U_y$

The proof of this, as well as other Claims, Lemmas and Propositions, are presented in the Appendix. It can be verified that $c_i^* \gtrless 0$ and $F_i^* \gtrless 0$, $i = p, w, r$ [7]. Unambiguous responses, however, can be secured in a compensated framework without additional assumptions. As a step in that direction, we adopt the following definition based on Claim 1:

Definition 1. The compensated optimal response to a change in α is:

$$(x_\alpha^*)_{\text{comp}} \equiv x_\alpha^* - x_z^* \cdot z_\alpha$$

where the subscript 'comp' denotes 'compensated'.

Definition 1 states the familiar Slutsky equation for the AH model. To elaborate, re-write the optimal choice vector for (9) as follows:

$$x^* \equiv x^{**}(\alpha, z(\alpha)) \tag{10a}$$

where x^{**} solves the first-order condition of (9). Differentiation of the above identity with respect to α yields:

$$x_\alpha^* \equiv x_\alpha^{**} + x_z^{**} \cdot z_\alpha \tag{10b}$$

Thus, the total effect of a change can be decomposed into a substitution effect (first term in the RHS of identity 10b) and an income effect (second term in the RHS).

The following Lemma summarizes all the relevant compensated comparative static properties and reciprocity relations of the optimal choice vector x^*.

Lemma 1. $(z_{x\alpha})^{\mathrm{T}} \cdot (x_\alpha^*)_{\text{comp}}$ is a positive semi-definite and symmetric matrix.

Since:

$$(z_{x\alpha})^{\mathrm{T}} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \\ \mathbf{0} & \mathbf{0} \end{pmatrix}$$

the comparative static results contained in Lemma 1 can be written more explicitly as the matrix:

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} c_p^* & c_w^* & c_r^* \\ F_p^* & F_w^* & F_r^* \end{pmatrix}_{\text{comp}} \equiv \begin{pmatrix} -c_p^* & -c_w^* & -c_r^* \\ F_p^* & F_w^* & F_r^* \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}_{\text{comp}}$$

which is positive semi-definite, implying:

$$(c_p^*)_{\text{comp}} \leq 0 \tag{11a}$$

$$(F_w^*)_{\text{comp}} \geq 0 \tag{11b}$$

And by symmetry of the above matrix:

$$(F_p^*)_{\text{comp}} = -(c_w^*)_{\text{comp}} \tag{11c}$$

$$(F_r^*)_{\text{comp}} = (c_r^*)_{\text{comp}} = 0 \tag{11d}$$

[6] Optimal consumption and family-labor choices are functions of T^0 as well. However, for the purpose of comparative static analysis, we will ignore the parameter T^0.

[7] Unambiguous results can be secured only through onerous restrictions. For example, the sufficient conditions for the result $c_p^* < 0$, are: $m^* = q^* - c^* < 0$, $U_{cy} > 0$ and $U_{yl} > 0$.

Expressions (11a) and (11b) restate the familiar results that the compensated consumption demand schedule is negatively sloped in own price, p, and the compensated family labour supply is upward sloping in its price, w. These results hold for all households, regardless of whether they are net sellers or buyers of labour or crop. Also, these results do not rely on the signs of utility function second derivatives such as U_{yy}, U_{yl}, U_{yc}, etc. The results in (11c) and (11d) are the reciprocity relations in the AH model under certainty.

2. Compensated optimal response in AH models under uncertainty

To anticipate some of the results in this section observe that, in models of expected utility maximization, a change in a parameter of the model, say α_i, affects optimal choices not only directly, but also indirectly through the α_i-induced change in total income. Unless risk neutrality or constant absolute risk aversion is assumed, the change in income affects the household's degree of risk aversion. As a result, the optimal response remains ambiguous without additional assumptions specifying how the household's aversion to risk changes with wealth. However, the confounding income effect can be eliminated through an ex post compensation that keeps the household's income constant by offsetting the α_i-induced income change. This isolates the 'pure substitution' or the compensated effect that yields unambiguous optimal responses without imposing risk preference restrictions.

The first of the two AH models presented in this section derives properties of the compensated optimal input choices in a general setting of price and output risk.

2.1. Model A: Optimal input demand under price and output risk

The household, a price-taker in the goods and factor markets, faces the following expected utility maximization problem:

$$\underset{X}{\text{Max}} J \equiv E[U(c,y,l)] \tag{12}$$

subject to

$$y = \tilde{\pi} + wF - \tilde{p}c + I \tag{13a}$$

$$\tilde{\Pi} = \tilde{p}\tilde{q} - r^{\mathrm{T}}A - wL \tag{13b}$$

$$\tilde{q} = G(A,L)\tilde{\epsilon} \tag{13c}$$

$$T^0 = F + l \tag{13d}$$

where $X \equiv \{c, F, L, A\}$ is the choice vector, U the household's Von Neumann-Morgenstern utility function, and E the expectation operator; $\tilde{p}$ denotes random price, and $\tilde{\epsilon}$ the random output coefficient. All other variables and parameters have the same definitions as in the preceding section. We impose the following structure to random output and price:

$$\tilde{\epsilon} = \theta + \phi\tilde{e}_1 \tag{14a}$$

$$\tilde{p} = \bar{p} + \gamma\tilde{e}_2 \tag{14b}$$

where $E[\tilde{e}_1] = E[\tilde{e}_2] = 0$. Note, no restriction on the covariance between random output and random price has been imposed. Under (14), the constraints in (13a)–(13c) can be collapsed to a single constraint:

$$\begin{aligned} y &= Z + \tilde{R}(\cdot) + I \\ &\equiv \bar{p}(\theta G(A,L) - c) - w(L-F) \\ &\quad - r^{\mathrm{T}}A + \tilde{R}(\cdot) + I \end{aligned} \tag{15}$$

where $\tilde{R}(\cdot) \equiv \bar{p}\phi\, G(\cdot)\,\tilde{e}_1 + \gamma\theta\, G(\cdot)\,\tilde{e}_2 - c\gamma\tilde{e}_2 + \phi\gamma\, G(\cdot)\,\tilde{e}_1\tilde{e}_2$ is the stochastic component of $\tilde{y}$. Note, here Z, the household's expected income net of food consumption expenditure, is the counterpart to z in the risk-free setting (Eq. 2). In the remainder of the paper, unless explicitly stated otherwise, the household's 'expected income' will mean Z. Using (15) and (13d), the optimization problem in (12) can be reformulated as:

$$\begin{aligned} \underset{X}{\text{Max}} J \equiv E\big[U\big(c,\ &\bar{p}(\theta G(A,L) - c) - w(L-F) \\ &- r^{\mathrm{T}}A + \tilde{R}(\cdot) + I,\ T^0 - F\big)\big] \end{aligned} \tag{16}$$

Assuming interior solutions, the first-order conditions of (16) are given by:

$$J_c = E\big[U_c - \tilde{p}U_y\big] = 0 \tag{17a}$$

$$J_F = E\big[wU_y - U_l\big] = 0 \tag{17b}$$

$$J_L = E\left[U_y\{(\bar{p}\theta G_L - w) + \tilde{R}_L(\cdot)\}\right] = 0 \quad (17c)$$

$$J_A = E\left[U_y\{(p\theta G_A - r) + \tilde{R}_A(\cdot)\}\right] = 0 \quad (17d)$$

Clearly, the optimal values of L and A cannot be solved independently of c and F as was done in the case of certainty. 'Separability' holds in AH models in a setting of risk only under onerous restrictions (Fabella, 1989). These conditions are not explored here since the issue is not central to our analysis. Also, it is well known that under perfect and complete markets, certainty results, including 'separability', can be recovered. However, in most developing countries' rural areas capital markets are thin, information costly and insurance markets practically non-existent (see, for example, Walker and Ryan, 1990). Thus, the assumption of incomplete markets, implicit in the model framework, is maintained throughout the analysis.

Another implicit assumption of the model structure in (16) is the absence of ex post flexibility in a household's consumption decisions. It may be argued that, when a household's consumption choices are made, prices are known with certainty. However, the same argument does not hold for output risk because of the time lag between input decisions and output realization. Thus the assumption that all decision variables, including consumption, are chosen simultaneously, though not wholly innocuous, is maintained for analytical tractability. The temporal sequence of a household's decisions can be fully captured only in a multiperiod dynamic model that allows for intemporal income transfer. Such an analysis is beyond the scope of this paper.

We assume that the second-order necessary condition of (16) is satisfied, i.e., J_{XX}, evaluated at the optimum, is negative semi-definite. Let the optimum choice vector be: $X^*(\beta, \mu, I) \equiv \{c^*(\beta, \mu, I),\ F^*(\beta, \mu, I),\ L^*(\beta, \mu, I),\ A^*(\beta, \mu, I)\}$, where $\beta \equiv \{w, r\}$ and $\mu \equiv \{\bar{p}, \theta, \gamma, \phi\}$. The parameter vector of the model has been separated into the two parts, β and μ, to distinguish the parameters that enter the stochastic part of the household's income from those that do not. β, unlike μ, changes only the first moment of the household's random income, leaving its higher moments unchanged since $\tilde{R}_\beta(\cdot) = \mathbf{0}$. This feature of β proves crucial in deriving the comparative static properties of the optimal choices under uncertainty.

Claim 2. $X_\beta^* \equiv X_I^* Z_\beta + (-J_{XX})^{-1} \cdot Z_{X\beta} E[U_y]$

The expression in Claim 2 is identical in structure to Claim 1, its certainty analogue, with the exception that $E[U_y]$ replaces U_y. As before, the two terms in the right hand side of the expression are the income and substitution effects of a change in β [8]. We use Claim 2 to define the compensated optimal response to a change in β.

Definition 2. The compensated optimal response to a change in β is given by:

$$(X_\beta^*)_{\text{comp}} \equiv X_\beta^* - X_Z^* \cdot Z_\beta \quad (18)$$

The following result summarizes all the relevant compensated comparative static properties of X^* with respect to the parameter vector β.

Lemma 2. $(Z_{X\beta})^{\mathrm{T}} \cdot (X_\beta^*)_{\text{comp}}$ is a positive semi-definite and symmetric matrix.

This Lemma implies that the following matrix of compensated responses is positive semi-definite and symmetric:

$$\begin{pmatrix} 0 & 1 & -1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\boldsymbol{I}_n \end{pmatrix} \begin{pmatrix} c_w^* & c_r^* \\ F_w^* & F_r^* \\ L_w^* & L_r^* \\ A_w^* & A_r^* \end{pmatrix}_{\text{comp}}$$

$$\equiv \begin{pmatrix} F_w^* - L_w^* & F_r^* - L_r^* \\ -A_w^* & -A_r^* \end{pmatrix}_{\text{comp}} \quad (19)$$

Since the matrix in (19) is positive semi-definite, it follows that:

$$(ML_w^*)_{\text{comp}} \geq 0 \quad (20a)$$

[8] The decomposition of the total effect is feasible only in the case of β and not μ since the latter enters $\tilde{R}(\cdot)$; as a result, X_μ^* cannot be expressed in terms of the income and substitution effects.

where $ML^* \equiv (F^* - L^*)$ is the household's optimal level of marketed labour surplus. Thus, we have:

Proposition 1. Under output and price risk, the household's compensated response to an increase in the rural wage-rate is to increase its optimal level of marketed labour surplus.

Recall, in the certainty case, we had $(F_w^*)_{\text{comp}} \geq 0$ and $L_w^* \leq 0$, since 'separability' allowed L^* to be determined independently of F^*. Here, in the absence of separability, it is the *difference* between the two compensated responses, $(\partial(F^* - L^*)/\partial w)_{\text{comp}}$, that provides the qualitative result. Since $(F^* - L^*) > 0$ means that the household is a net seller in the labour market, the result implies that the household's compensated marketed labour surplus schedule is upward sloping in own price, w. Positive semi-definiteness and symmetry of the matrix in (19) also implies that the matrix $(A_r^*)_{\text{comp}}$ is negative semi-definite and symmetric, which yields:

Proposition 2. Under output and price risk, the compensated non-labour input demand curves are downward sloping in own prices and are symmetric, i.e.

$$\left(\frac{\partial A_i^*}{\partial r_i}\right)_{\text{comp}} \leq 0 \text{ and } \left(\frac{\partial A_i^*}{\partial r_j}\right)_{\text{comp}} = \left(\frac{\partial A_j^*}{\partial r_i}\right)_{\text{comp}}$$
$$\forall i,j = 1,\ldots,n$$

Proposition 2 shows that the certainty result in (8a) extends to the case of output and price risk in the compensated framework. Finally, the matrix in (19), being symmetric, provides the reciprocity relation:

$$(ML_r^*)_{\text{comp}} = (-A_w^*)_{\text{comp}} \tag{20b}$$

The following remarks summarize the salient features of the foregoing propositions:

Remark 1. Propositions 1 and 2 do not rely on any restriction on the household's risk preference structure or on the probability distribution of the random variables. The result in Proposition 2 stands in contrast to the analogue of the same result in the uncompensated framework (Pope, 1980), where unambiguous results do not follow even when only price risk is assumed and restrictions on risk preference are imposed.

Remark 2. The results in the two Propositions do not involve second derivatives of the utility function with respect to own or cross arguments, U_{ii} and U_{ij}, $i, j = c, y, l$. This implies that the results do not require the decision maker to be risk averse.

Remark 3. The Propositions nest corresponding results from models of only output risk or only price risk as special cases.

To analyze a household's optimal supply response, it is necessary to sign the household's optimal non-labour and labour input responses to a change in expected price, $\bar{p}$. However, these responses are ambiguous even in the compensated framework. This is because $\bar{p}$, unlike w and r, enters both the non-stochastic and the stochastic part of the household's income. As a result, a change in $\bar{p}$, by changing the mean *and* the variance of income, has a 'risk effect' in addition to the income effect. The household's optimal response to these effects cannot be determined without assumptions specifying its attitude towards risk. Since the primary object of this paper is to derive qualitative results without imposing risk preference restrictions, we assume in the next model that the household faces only output risk or price risk, but not both.

2.2. Model B: Marketed surplus response under output risk or price risk

The model analyzed in this subsection is identical to the one set out in (12) and (13) with the only exception that (14) is changed as follows: (a) in the case of output risk only, $\gamma = 0$ and $p = \bar{p}$ is non-stochastic, and (b) in the case price risk only $\phi = 0$ and $\theta = 1$. All other variables and parameters have the same definition as in Model A. Let the optimum choice vectors in the present model be denoted by $\tilde{X}(\eta^{p}, I)$ and $\tilde{X}(\eta^{o}, I)$, where η^{p} and η^{o} are the parameter vectors corresponding respectively to the cases of price or output risk only.

It is clear from Remark 3 in the preceding sub-section that Propositions 1 and 2 extend to the present model and therefore the corresponding results are not derived here. To analyze the compensated supply response, we proceed by presenting the following result:

Lemma 3a. Under output risk:
$(Z_{X\theta})^{T} \cdot (\tilde{X}_{\theta})_{comp}$

Lemma 3b. Under price risk:
$(Z_{X\bar{p}})^{T} \cdot (\tilde{X}_{\bar{p}})_{comp}$

Since $(Z_{X\theta})^{T} = (0, 0, pG_L, pG_A)$, Lemma (3a) implies:

$$p\left\{G_L \cdot \left(\tilde{L}_{\theta}\right)_{comp} + G_A \cdot \left(\tilde{A}_{\theta}\right)_{comp}\right\} > 0 \tag{21a}$$

where $\tilde{L}$ and $\tilde{A}$ are the optimal input choices in Model B. Since $\tilde{q}_j = G_L \cdot \tilde{L}_j + G_A \cdot \tilde{A}_j, j = \theta, p$, where $\tilde{q}$ denotes the optimal output level, Lemma 3a and (21a) yield $(\tilde{q}_{\theta})_{comp} > 0$, which suggests:

Proposition 3. Under output risk, the household's compensated response to a change in production technology that increases expected output without changing its riskiness is to increase the optimal output level.

For the case of price risk only, $(Z_{x\bar{p}})^{T} = (-1, 0, G_L, G_A)$; therefore, by Lemma (3b):

$$-\left(\tilde{c}_{\bar{p}}\right)_{comp} + G_L \cdot \left(\tilde{L}_{\bar{p}}\right)_{comp} + G_A \cdot \left(\tilde{A}_{\bar{p}}\right)_{comp} > 0 \tag{21b}$$

which yields:

$$\left(\tilde{q}_{\bar{p}}\right)_{comp} - \left(\tilde{c}_{\bar{p}}\right)_{comp} = \left(\tilde{m}_{\bar{p}}\right)_{comp} > 0 \tag{22}$$

where $\tilde{m} = \tilde{q} - \tilde{c}$ denotes the household's optimum marketed surplus. Thus, (22) implies:

Proposition 4. Under price risk, the household's compensated marketed surplus schedule is upward sloping in expected output price.

Observe that in a risk-free setting, the property of 'separability' yielded the result: $q_p^* \geq 0$, implying optimal output supply is not downward sloping in price. In contrast, under price risk, in the absence of separability, it is the *difference* between the compensated production and consumption responses that is positive.

3. Welfare analysis in AH models under uncertainty

In the analysis so far, the compensated *response* has been analyzed, but the explicit structure of the compensation scheme has not been developed. This structure, however, is essential for welfare analysis in the context of AH models. Recall, by Definition 2, the compensated optimal choice is one where any β-induced change in expected income is just offset by a suitable compensation. This compensation can be achieved by a change in one or more parameters of the model. However, the compensation scheme is probably best understood by changing the exogenous wealth parameter, I.

To this end we define exogenous wealth as a function of the parameters, i.e.

$$I = I^{**}(\beta) \tag{23}$$

such that $I^{**}(\cdot)$ adjusts to offset the change in $Z(\cdot)$ induced by the β-change. Substitution of (23) in the objective function allows us to define compensated optimal choices as solutions to the following optimization problem:

$$X_c^* \equiv \arg\max_X \left\{E\left[U(c, Z(X,\beta,\mu) + R(\cdot) + I^{**}(\beta), T^0 - F)\right]\right\} \tag{24}$$

where $X_c^* \equiv \{c_c^*, F_c^*, L_c^*, A_c^*\}$ denotes the optimal choice vector under compensation. The indirect utility associated with (24) is given by:

$$V(\beta,\mu) \equiv E\left[U\left(c_c^*, y_c^*, T^0 - F_c^*\right)\right] \tag{25}$$

where $y_c^* \equiv \{Z(X_c^*,\beta,\mu) + (X_c^*,\mu) + I^{**}(\beta)\}$. Differentiation of (25) with respect to β and application of the envelope theorem yields:

$$V_\beta(\cdot) \equiv E\left[U_y \cdot (Y_\beta^*)_{comp}\right] \equiv E\left[U_y\{Z_\beta(X_c^*,\beta,\mu) + I_\beta^{**}(\beta)\}\right] \equiv 0 \tag{26}$$

The identity $V_\beta \equiv 0$ in (26) follows from the fact that a β-induced change in $Z(\cdot)$ is just offset by a change in I^{**} and that c_c^* and F_c^* have been already optimally chosen. Since $E[U_y] \neq 0$ and $\{Z_\beta^* + I_\beta^{**}(\beta)\}$ is non-stochastic, the identity in (26) implies:

$$Z_\beta(X_c^*, \beta, \mu) + I_\beta^{**}(\beta) \equiv 0 \tag{27}$$

Identity (27) provides the crucial expression in the formulation of welfare evaluations for changes in β. Let β^0 and β^1 be the two values of the parameter vector before and after a change, respectively. It follows from (27) that the compensating variation (CV) of the parameter change is given by: [9]

$$I^{**}(\cdot) \equiv -\int_{\beta^0}^{\beta^1} Z_\beta(X_c^*, \beta, \mu)\,\mathrm{d}\beta \tag{28}$$

The measure of CV in (28) corresponds to the reference level of indirect utility $V^0(\cdot)$ where:

$$V^0 \equiv E\big[U\big(c^*(\beta^0),\, Z^*(X^*(\beta^0), \beta^0) + I,\, T^0 - F^*(\beta^0)\big)\big] \tag{29}$$

Thus, V^0 corresponds to the pre-change level of indirect utility.

An example to illustrate the welfare measure outlined above may prove useful. Suppose the agricultural household faces uncertain output price and the initial level of expected price, denoted by $\bar{p}^0$, increases to $\bar{p}^1$. Since in most LDCs government interventions in agricultural product markets often assume the form of price supports or price stabilization schemes which alter the mean price, it is of interest to understand the required lump-sum payments which will keep the level of welfare at the pre-intervention level. In accordance to (28), the CV of the expected price change is given by:

$$I^{**}(\cdot) \equiv -\int_{\bar{p}^0}^{\bar{p}^1} Z_{\bar{p}}(X_c^*)\,\mathrm{d}\bar{p} \equiv -\int_{\bar{p}^0}^{\bar{p}^1} (m_c^*)\,\mathrm{d}\bar{p} \tag{30}$$

where $(m_c^*) \equiv \{G(A_c^*, L_c^*) - c_c^*\} \equiv (q_c^* - c_c^*)$ is the expected compensated marketed surplus (or deficit) schedule. The negative sign in the right hand side of the (30) suggests, for households that are net sellers in the product market, i.e. $m_c^* \equiv (q_c^* - c_c^*) > 0$, the compensation is negative. But for net buyers, i.e. $m_c^* < 0$, compensation is positive since a crop-price increase reduces the welfare of deficit households.

Turning now to the magnitude of the compensation, note that the CV of expected price change is given by the area over (under) the compensated marketed surplus (deficit) curve. By the same token, the CV of a wage-rate change is given by the area over (under) the compensated labour surplus (deficit) schedule, ML_c^*.

A potential problem of the welfare measure set out in (28) is that the compensated response schedule on which the measure is based is not observable. The welfare measure cannot be approximated by the area under the observable uncompensated schedule since, unless risk neutrality or constant absolute risk aversion is assumed, this area will provide a biased estimate of the CV. However, using Definition 2, the compensated *response* schedules can be expressed in terms of observable uncompensated response measures (i.e., elasticities), quantities and prices. In particular, the relation between compensated and uncompensated elasticities are given by:

$$\left(\eta_{\beta_i}^{x_j}\right)_{\text{comp}} = \eta_{\beta_i}^{x_j} - \eta_Z^{x_j}\left\{\frac{\bar{\beta}_i}{\bar{Z}}\frac{\partial Z}{\partial \beta_i}\right\} \tag{31}$$

where $\eta_{\beta_i}^{x_j}$ denotes the elasticity of the jth choice variable, $j = c, F, L, A$, with respect to the ith element of the parameter vector β. The components in the RHS of (31) can be empirically estimated, thereby recovering the unobservable 'compensated' elasticities. These then can be utilized to compute the relevant CV measures set out in (28).

4. Implications for empirical analysis

This section provides an outline of the empirical model for estimating compensated response

[9] In the context of multiple parameter changes, the issue whether the welfare measure exhibits path dependence becomes relevant. It can be verified that the measure set out in (28) does not because of the symmetry of compensated responses. See Chavas and Pope (1981) for fuller discussion on this issue.

coefficients that are necessary for testing the restrictions implied by the comparative static results and for welfare analysis. The analysis, in part, relies on a paper by Chavas and Pope (1985).

In the interest of notational simplicity the structure for the empirical analysis is developed in terms of the AH model of price risk only. Minor changes can incorporate output risk as well. Define the optimal (uncompensated) choice vector of the AH model under price risk as:

$$X^* \equiv X^*(\eta, \gamma, I) \tag{32}$$

where $X^* \equiv \{c^*, F^*, L^*, A^*\}$ is the $(n+3) \times 1$ optimal choice vector, $\eta \equiv \{\bar{p}, w, r\}$ is the $(n+2) \times 1$ vector of price parameters and γ is the mean preserving spread (MPS) parameter, a scalar. Total differentiation of (32) yields:

$$\mathrm{d}X^* \equiv X_\eta^* \mathrm{d}\eta + X_I^* \mathrm{d}I + X_\gamma^* \mathrm{d}\gamma \tag{33}$$

which constitutes a system of $(n+3)$ estimation equations, where X_η^* is the $(n+3) \times (n+2)$ matrix of uncompensated response coefficients (or elasticities if the necessary transformations are made), X_I^* and X_γ^* are $(n+3) \times 1$ vectors of income and MPS coefficients (or elasticities) to be estimated. $\mathrm{d}X^*$, $\mathrm{d}\eta$, $\mathrm{d}I$ and $\mathrm{d}\gamma$ are data vectors. These vectors are characterized by first-order differencing, for example, $\mathrm{d}I_t \equiv I_t - I_{t-1}$, where t denotes the tth observation. The differential specification of the system of equations in (33) is akin to the Rotterdam model. This model was originally proposed by Theil (1977) and has, since then, been used extensively in the applied literature.

The 'testable' restrictions implied by the analytical model are in terms of compensated response coefficients. But in (33), X_η^* denotes the matrix of uncompensated coefficients. Therefore, it is necessary to rewrite (33) in terms of compensated elasticities using Definition 2:

$$X_\eta^* \equiv (X_\eta^*)_{\text{comp}} + X_Z^* \cdot Z_\eta \tag{34}$$

Since $X_I^* = X_Z^*$, substitution of (34) into (33) yields:

$$\mathrm{d}X^* \equiv (X_\eta^*)_{\text{comp}} \mathrm{d}\eta + X_Z^* \{Z_\eta \mathrm{d}\eta + \mathrm{d}I\} + X_\gamma^* \mathrm{d}\gamma \tag{35}$$

Identity (35) represents a systems of equations where $(X_\eta^*)_{\text{comp}}$ and X_Z^* are, respectively, the matrix of *compensated* response coefficients and the vector of income response coefficients to be estimated, while $\mathrm{d}\eta$, $\{Z_\eta \mathrm{d}\eta + \mathrm{d}I\}$, and $\mathrm{d}\gamma$ constitute data. All the compensated comparative static results and reciprocity relations derived in the paper can now be tested through the restrictions on the elements of the compensated coefficient matrix, $(X_\eta^*)_{\text{comp}}$.

To illustrate the working of the empirical model, consider only the consumption equation of the system in (35):

$$\begin{aligned} \mathrm{d}c^* = {} & (c_{\bar{p}}^*)_{\text{comp}} \mathrm{d}\bar{p} + (c_w^*)_{\text{comp}} \mathrm{d}w \\ & + \sum_{k=1}^{n} (c_{r_k}^*)_{\text{comp}} \mathrm{d}r_k \\ & + c_Z^* \left\{ Z_{\bar{p}} \mathrm{d}\bar{p} + Z_w \mathrm{d}w + \sum_{k=1}^{n} Z_{r_k} \mathrm{d}r_k + \mathrm{d}I \right\} \\ & + c_\gamma^* \mathrm{d}\gamma \end{aligned} \tag{36}$$

The above equation can be written in the estimation form as:

$$\begin{aligned} (c_t^* - c_{t-1}^*) & \\ = {} & a_1(\bar{p}_t - \bar{p}_{t-1}) + a_2(w_t - w_{t-1}) \\ & + \sum_{k=1}^{n} b_k (r_{kt} - r_{kt-1}) \\ & + a_3 \Big\{ (q_t^* - c_t^*)(\bar{p}_t - \bar{p}_{t-1}) \\ & + (F_t^* - L_t^*)(w_t - w_{t-1}) \\ & + \sum_{k}^{n} A_{kt}^* (r_{kt} - r_{kt-1}) + I_t - I_{t-1} \Big\} \\ & + a_4(\gamma_t - \gamma_{t-1}) + \vartheta_{c_t} \end{aligned} \tag{36$'$}$$

or

$$C_t = a_1 P_t + a_2 W_t + b^{\mathrm{T}} R_t + a_3 M_t + a_4 \gamma_t + \vartheta_{c_t} \tag{36$''$}$$

where $C_t = c_t^* - c_{t-1}^*$, $P_t = \bar{p}_t - \bar{p}_{t-1}$, etc. are data vectors, $a_1 \equiv (c_{\bar{p}}^*)_{\text{comp}}$, $a_2 \equiv (c_w^*)_{\text{comp}}$, etc. are coefficients to be estimated, and ϑ_{c_t} is the error term of the consumption equation. Estimation equations similar to (36″) can be formulated for the other choice variables, F^*, L^*, and A^*, and estimated as a system. The parameter restrictions implied by the comparative static results derived earlier can then be validated using appropriate statistical tests.

Observe that (35) constitutes a system of simultaneous equations. This is because the data vector M_t in each equation of the system contains endogenous elements: $(q_t^* - c_t^*)$ and $(F_t^* - L_t^*)$. Therefore, in estimating the system, an instrumental variable approach may be adopted where the predicted values from the relevant equations are used to construct the instruments for the variables c_t^*, F_t^* and L_t^* within M_t. In addition, a Brundy-Jorgenson iterative procedure may be adopted for efficiency gain (Brundy and Jorgenson, 1971).

While data on input prices and income may be available directly, observations on the regressors $\bar{p}_t$ and γ_t require prior estimations. Since $\bar{p}$ and γ are the subjectively formed moments of the random output price, a quasi-rational expectations approach may be adopted in constructing data on $\bar{p}$ and γ. That is, use the forecasts from an ARIMA analysis of time series data on output price to generate the data on the moments of stochastic price $\bar{p}$ and γ (Nerlove et al., 1979).

Identity (35)'s coefficient estimates also present the necessary information for welfare analysis. For example, the coefficient estimate $\hat{a}_1$ in (36″) provides the slope of the compensated consumption demand schedule, with respect to expected price. The parameter estimates of the labour and other input demand equations in the system provide estimates of $(\partial L/\partial \bar{p})_{\text{comp}}$, $(\partial A_k^*/\partial \bar{p})_{\text{comp}}$, $i = 1, \ldots, n$. These estimates can then be used to compute:

$$\left(\frac{\partial q}{\partial \bar{p}}\right)_{\text{comp}} \equiv G_L\left(\frac{\partial L}{\partial \bar{p}}\right)_{\text{comp}} + \sum_k G_{A_k}\left(\frac{\partial A_k^*}{\partial \bar{p}}\right)_{\text{comp}} \tag{37}$$

where prior estimation of a parametric production function can furnish the estimated marginal products, $\hat{G}_L$ and $\hat{G}_{A_k}$, $k = 1, \ldots, n$. Finally, the estimates from (37) can be used in the expression:

$$\left(\frac{\partial \hat{q}}{\partial \bar{p}}\right)_{\text{comp}} - \left(\frac{\partial \hat{c}}{\partial \bar{p}}\right)_{\text{comp}}$$

to estimate the CV of a change in $\bar{p}$ as suggested by (30). Similar methods can be adopted in estimating the CV of a change in input price.

Though the compensated response estimates are indispensable for welfare analysis and for validating the comparative static results, in some cases the uncompensated response estimates may be of interest. The parameter estimates $(X_\eta^*)_{\text{comp}}$ in (35), transformed into elasticities, can be substituted into the relation in (31) to recover the uncompensated response elasticities from their compensated counterparts. Moreover, estimation of (35) provides *uncompensated* response coefficients with respect to income (X_z^*) and mean-preserving-spread (X_γ^*). The signs of the elements of (X_γ^*) indicate households' response to changes in the degree of price risk. Such a change in risk is often a consequence of price stabilization schemes.

5. Concluding comments

The paper proposed a framework of compensated comparative statics for agricultural household models under uncertainty. The analysis revealed that a household's unambiguous family labour, hired labour and non-labour input responses to price and other parameter changes can be derived in a relatively simple manner when the income effects of such changes are removed, and the pure substitution effects isolated, through a Slutsky-type compensation scheme. Significantly, these qualitative results do not rely on any restriction on the household's risk preference structure or production technology. The unobservable compensated responses can be expressed in terms of observable prices and quantities and therefore are amenable to empirical analysis.

The framework of compensated demand and supply are also useful in evaluating the welfare effects of price and policy interventions. A structure for welfare evaluation of such interventions in the setting of risk has also been developed.

A possible extension of this paper might be to explore the compensated comparative static properties of optimal choices in a household model that admits intertemporal income transfer in the form of storage and/or savings. This line of inquiry seems especially relevant in the light of the growing body of literature on temporal dimensions of agricultural households' optimal choices under uncertainty. Also, attempts to validate the theoretical results of this study through data analysis may provide new and useful insights.

Appendix 1

Proof of Claim 1. The first-order conditions of (9) in the text are:

$$HH_x = \{U_c - pU_y, wU_y - U_l\} = 0 \quad \text{(A1)}$$

Substitution of x^* into (A1) and differentiation with respect to α yields:

$$x_\alpha^* \equiv (HH_{xx})^{-1} \cdot HH_{x\alpha} \quad \text{(A2)}$$

Now note:

$$HH_{x\alpha} \equiv HH_{xI} z_\alpha + z_{x\alpha} U_y \quad \text{(A3)}$$

Substitution of (A3) into (A2), and noting that $x_z^* = x_I^*$ completes the proof. ●

Proof of Lemma 1. From Claim 1 and Definition 1 it follows that:

$$(x_\alpha^*)_{\text{comp}} \equiv (-HH_{xx})^{-1} \cdot z_{x\alpha} U_y \quad \text{(A4)}$$

The proof now follows from pre-multiplication of both sides of (A4) by $(z_{x\alpha})^{\text{T}}$ and noting that HH_{xx} is a negative semi-definite and symmetric matrix. ●

Proof of Claim 2. Differentiation of J_X, given by (17) in the text, with respect to β yields:

$$X_\beta^* \equiv (-J_{XX})^{-1} J_{X\beta} \quad \text{(A5)}$$

Since β enters only the non-random part of income, Z_β and $Z_{X\beta}$ are non-stochastic; therefore:

$$J_{X\beta} \equiv J_{XI} \cdot Z_\beta + Z_{X\beta} \cdot E[U_y] \quad \text{(A6)}$$

Substitution of (A6) into (A5) completes the proof. ●

Proof of Lemma 2. From Claim 2 and (18) in text, it follows that:

$$(X_\beta^*)_{\text{comp}} X_{\beta_c}^* \equiv (-J_{XX})^{-1} \cdot Z_{X\beta} E[U_y] \quad \text{(A7)}$$

Pre-multiplication of both sides of (A7) by $(Z_{X\beta})^{\text{T}}$ and the second-order necessary condition of (16), provide the required result. ●

Proof of Lemma 3. Omitted; the steps are essentially the same as in proof of Lemma 2. ●

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