AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

No endorsement of AgEcon Search or its fundraising activities by the author(s) of the following work or their employer(s) is intended or implied.

ELSEVIER

Agricultural Economics 10 (1994) 233–243

AGRICULTURAL
ECONOMICS

Value of information on crop response function to soil salinity in a farm-level optimization model

Eli Feinerman

Department of Agricultural Economics, The Hebrew University of Jerusalem, P.O. Box 12, 76100 Rehovot, Israel

(Accepted 25 August 1993)

Abstract

The study investigates the value of additional information on the response function to soil salinity of a given crop (potatoes), with regard to a stochastic long-run optimization model for utilization of saline water in a single-farm framework. The analysis provides a conceptual and methodological framework for investigating the expected value of sample information (EVSI), as well as an efficient tool for empirical application. Although a few approximations have been used, the results provide an estimate of EVSI and indicate the need for additional information.

1. Introduction

The response function of a given crop yield to soil salinity is an important factor in every optimization model concerning irrigation with saline water. The true value of the response function parameters are usually unknown to the decision maker, and therefore he or she uses their estimates and may become a victim of a suboptimal solution. The damage may be measured by a loss function and the calculation of its expectation. The parameters' estimates (which are arguments in the loss function) are based on a priori information available to the decision maker. But he or she can generally acquire additional information, which will decrease the uncertainty and reduce the loss expectation. The expected value of sample information (EVSI) is defined as the difference between the reduction of the expected value of the loss function due to the additional information and the cost of its acquisition. The optimal number of observations to be acquired is the one that maximizes EVSI.

A broad theoretical presentation of decision theory, value of information and the Bayesian approach can be found in the textbooks of Pratt et al. (1965) and DeGroot (1970). A number of studies deal with the value of information in farm management (Ryan and Perrin, 1974; Maddock, 1973; Bie and Ulph, 1972; Mjelde et al., 1988; Preckel et al., 1987; Antonovitz and Roe, 1984) and in management of water resources (Davis and Dvoranchik, 1972; Duckstein et al., 1977; Klemes, 1977). It should be pointed out that most of these articles did not deal explicitly with the optimal size of the additional information. Moreover, the articles that dealt with the management of irrigation systems ignored water quality.

Feinerman and Yaron (FY, 1983a) have developed and applied a methodology to estimate the expected profitability to farmers from acquiring additional information on the biological crop re-

0169-5150/94/$07.00 © 1994 Elsevier Science B.V. All rights reserved
SSDI 0169-5150(93)E0044-N

sponse function to soil salinity. Based on a switching regression technique to estimate a piecewise linear response function and on a *short-run* economic optimization model which includes a *single-crop* and a *single saline water resource*, they formulated a loss function and calculated the expected value of additional information and the optimal number of additional observations. The present study generalizes and expands the analysis of FY to a stochastic *long-run farm-level* optimization model (hereafter referred to as an SLRO-model) that includes *several crops* and *several sources of irrigation water* – differing in availability salinity level and price – which can be mixed. The switching regression technique to estimate the crop response function to soil salinity and a few methodological components of the present study are similar to those presented in FY. They are repeated here for the sake of completeness. The current study is aimed at providing a conceptual and methodological framework for investigating the expected value of sample information on the crop response function to salinity in a complex farm framework as well as a tool for empirical application.

The organization of the rest of the paper is as follows. Section 2 presents a switching regression model to estimate the response function parameters and their statistical properties. Section 3 presents a brief review of the SLRO model. Considering the optimum values of the SLRO model and the response function parameters, a loss function is formulated and its possible situations are presented in Section 4. The loss function is approximated by a Taylor series expansion. Then, in Section 5, the expected value of additional sample information – EVSI – is calculated and the optimal sample size is determined. The analysis in the section utilizes the empirical results of the SLRO-model and, following reasonable approximations, shows how the possible situations of the loss function may be reduced to only one. Empirical findings for potatoes are presented in Section 6. Finally, Section 7 contains a brief summary and concluding comments.

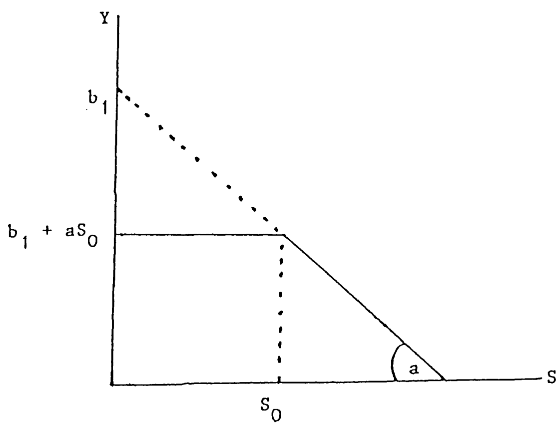

Fig. 1. Response function.

2. Estimates of the response-function parameters

An accepted theory among soil researchers states that crop yield is dependent of average soil salinity below a certain critical threshold, and thereafter decreases linearly (Maas and Hoffman, 1977). On the basis of this specification, the following switching regression model is used (see Fig. 1):

$$Y_i = \begin{cases} b_1 + aS_0 + U_{1i} & \text{if} \quad S_i \le S_0 \\ b_1 + aS_i + U_{2i} & \text{if} \quad S_i > S_0 \end{cases} \quad i = 1, \ldots, T \tag{1}$$

where Y is yield per hectare (in physical units); i the observation index; T the a priori number of available observations; S average soil salinity level in the root zone during the growing season (meq Cl/l); S_0 the critical threshold (meq Cl/l); U_1, U_2 are independent random variables normally distributed with zero means and variances equal to σ_1^2, σ_2^2, respectively; and b_1, a, S_0 the (unknown) parameters of the response function.

Let $\hat{\beta} = [\hat{S}_0, \hat{a}, \hat{b}_1]$ be the maximum likelihood (ML) estimate of $\beta = [S_0, a, b_1]$. From the properties of ML estimates, under fairly general conditions (e.g., Zacks, 1971), $\hat{\beta}$ is asymptotically normally distributed with mean β and variance-covariance matrix:

$$\Sigma_\beta = \left\{ \mathrm{E}\left(-\frac{\partial^2 \ln L}{\partial \beta_i \, \partial \beta_j} \right)^{-1} \right\}$$

$$\Sigma_\beta = \begin{pmatrix} V(\hat{S}_0) & \text{cov}(\hat{S}_0, \hat{a}) & \text{cov}(\hat{S}_0, \hat{b}_1) \\ \text{cov}(\hat{a}, \hat{S}_0) & V(\hat{a}) & \text{cov}(\hat{a}, \hat{b}_1) \\ \text{cov}(\hat{b}_1, \hat{S}_0) & \text{cov}(\hat{b}_1, \hat{a}) & V(\hat{b}_1) \end{pmatrix} \tag{2}$$

$$= \begin{pmatrix} \dfrac{\sigma_2^2\left((T-t)S_0^2 + \sum_j S_j^2 - 2S_0\sum_j S_j\right)}{a^2A} + \dfrac{\sigma_1^2}{ta^2} & \dfrac{\sigma_2^2\left(\sum_j S_j - S_0(T-t)\right)}{aA} & \dfrac{\sigma_2^2\left(S_0\sum_j S_j - \sum_j S_j^2\right)}{aA} \\ \dfrac{\sigma_2^2\left(\sum_j S_j - S_0(T-t)\right)}{aA} & \dfrac{\sigma_2^2(T-t)}{A} & \dfrac{-\sigma_2^2\sum_j S_j}{A} \\ \dfrac{\sigma_2^2\left(S_0\sum_j S_j - \Sigma S_j^2\right)}{aA} & \dfrac{\sigma_2^2\Sigma S_j}{A} & \dfrac{\sigma_2^2\Sigma S_j^2}{A} \end{pmatrix}$$

where ln L is the logarithm of the relevant likelihood function. By substituting the explicit partial derivatives, we get the asymptotic results as represented in (2) above, where t is the number of observations that satisfies $S_i \leq S_0$; V is variance; and cov is covariance:

$$\sum_j \equiv \sum_{j=t+1}^{T}$$

and

$$A \equiv \left((T-t)\sum_j S_j^2 - \left(\sum_j S_j\right)^2\right)$$

As mentioned in the introduction, the empirical findings relate to potatoes. The estimates of response function parameters are based on experimental results of Sadan and Berglas (1980). Their one-year experiment conducted in the Northern Negev of Israel yielded a total of 17 observations (S_i, Y_i). By applying a switching regression technique (Quandt, 1972), the following ML estimates were derived:

$$\hat{S}_0 = 6.054[\text{meq Cl/l}]$$

$$\hat{a} = -1.09\left[\frac{\text{t/ha}}{\text{meq Cl/l}}\right]$$

$$\hat{b}_1 = 52.55\ [\text{t/ha}]$$

$$\hat{\sigma}_1^2 = 0.14\ [\text{t/ha}]^2$$

$$\hat{\sigma}_2^2 = 15.78\ [\text{t/ha}]^2$$

and

$$\hat{t} = 3$$

Thus, the estimated response function is:

$$\hat{Y}_i = \begin{cases} 45.95 & \text{if } S_i \leq 6.054 \\ 52.55 - 1.09S_i & \text{if } S_i > 6.054 \end{cases} \quad R^2 = 0.776 \tag{1'}$$

By substituting these estimates into (2), a consistent estimator of Σ_β is achieved.

3. Stochastic long-run (SLRO) optimization model – a brief review

The planning model considers a single kibbutz farm in southern Israel and incorporates in one endogeneous system both physical/biological relationships (such as response functions and salt

t, metric tonne = 1000 kg.

distribution in the soil profile) and economic relationships. The farm has three water sources, differing in availability, quality (salinity level 5, 10, 25 meq Cl/l, respectively); and price (0.10, 0.07, 0.06 US$/$m^3$, respectively). Water from different sources may be mixed, providing for additional quality options. The farm has at its disposal five plots (Plot A–Plot E) of land, differing in area: (50, 50, 50, 60 and 60 ha, respectively); and initial salinity level of the soil solution (5, 10, 15, 20 and 25 meq Cl/l, respectively).

The cropping alternatives of the farm are as follows: fall potatoes, fall carrots, cotton, and a mature grapefruit grove. The land area of the grapefruit grove is 50 ha, 10 ha in each soil plot. The farm faces yearly quotas of potatoes (100 ha) and carrots (60 ha). The yields of these crops (except cotton) are sensitive to soil salinity (the critical soil salinity thresholds, S_0, for carrots and grapefruits are 2.78 and 10.28 meq Cl/l, respectively). The parameters for the yield response functions to salinity for potatoes and grapefruits were estimated by a switching regression approach whereas the estimated parameters for carrots were taken from Maas and Hoffman (1977). An irrigation season is defined as one year and is subdivided into two subseasons: spring/summer ($t = 1$) and autumn/winter ($t = 2$).

The SLRO-model refers to the water-soil-crop-farm system over a sequence of four irrigation seasons and considers rainfall uncertainty. Conceptually, it is an extension of the two-stage LP model under uncertainty (Dantzig and Madansky, 1961; El Agizy, 1967). The objective function of the risk-neutral farmer is to maximize the present value of the expected net profits from the crops' net returns over the time horizon subject to total water and land supplies, the quotas for potatoes and carrots, and linear balance equations which describe the evolution of the soil-related state variables over time. The farm's decision variables include crop mix on each soil plot and for every irrigation season, quantities and qualities of irrigation water for the various crops, and quantities and qualities of leaching water for the soil plots. The results provide priorities in the allocation of water and soil plots of varying salinity levels as well as empirical estimates of the shadow prices and the rates of substitution among the limited sources.

A detailed description of the SLRO-model and its empirical application can be found in Feinerman and Yaron (1983b). For the sake of completeness, however, a few selected results for the first year of the planning horizon are presented here in Tables 1 and 2. Feinerman and Yaron (1983b) have demonstrated that the three crops sensitive to salinity can be ranked in order of decreasing profitability as follows: potatoes, grapefruits, and carrots. This ranking clarifies the priorities with respect to the allocation of the limited land (Table 1) and water (Table 2) resources to the crops. Most high-quality water from source 1 was allocated by the program to

Table 1
Land allocation to the crops and shadow prices of land restrictions in the first year of the planning horizon

Soil plot	Crop				Shadow price ($/ha)
	Potato (ha)	Grapefruit (ha)	Carrot (ha)	Cotton (ha)	
A	40	10	0	0	9620
B	40	10	0	0	9040
C	17.5	10	22.5	0	8370
D	2.5	10	3.5	44	8090
E	0	10	0	50	8090
Total	100	100	26	94	

Table 2
Water allocation to the salinity-sensitive crops and shadow prices of water restrictions in the first year of the planning horizon

Water source		Crop			Shadow price ($/$m^3$)
		Potato (%)	Grapefruit (%)	Carrot (%)	
($t = 1$)	Source 1	61.3	0	14.8	0.42
	2	38.7	96.3	61.8	0.33
	3	0	3.7	23.4	0.19
Total		100	100	100	
($t = 2$)	Source 1	23.3	0	1.9	0.26
	2	76.7	95	15.4	0.14
	3	0	5	82.7	0.0
Total		100	100	100	

the most profitable crop, potatoes, while the remaining water needed for this crop was supplemented from source 2. Grapefruits should be irrigated mainly by water from source 2, while carrots, the least profitable out of the three, was irrigated mainly by water from sources 2 (in $t = 1$) and 3 (in $t = 2$), utilizing residual quantities from source 1.

Since the grapefruit grove was restricted to 10 ha on each soil plot, land allocation was relevant only with respect to the fields crops. The least saline plots (in terms of initial soil salinity) A and B were allocated by the program to the most profitable crop, potatoes. Carrots were planted on the residual of plots C and D. As expected, the most saline plots – D and E – were allocated mostly to cotton, which is not sensitive to salinity.

4. Loss function and its possible situations

Based on the results of the SLRO-model, the following loss function, h, is defined:

$$\mathrm{h}(\hat{\beta}-\beta) = Z(\beta/\beta) - Z(\hat{\beta}/\beta) \tag{3}$$

where

$Z(\beta/\beta)$ the optimal value of the objective function, given that the true values of the parameters (β) are known to the decision makers with certainty; and

$Z(\beta/\hat{\beta})$ the optimal value of the objective function when β is unknown and the decision makers use instead (in the SLRO-model) its ML estimates ($\hat{\beta}$).

The optimal solution determines the following values:

$\hat{S}^*_{ng}(i, k)$ the average soil salinity level (in meq Cl/l) of soil plot g, associated with crop n, in year i with winter type (rainfall level) k; ($n = 1, \ldots, N$; $g = 1, \ldots, G$; $i = 1, \ldots, I$; $k = 1, \ldots, K$); and

$\hat{\Pi}^*_{ng}(i, k)$ the net return (in US\$/ha) of crop n, associated with plot g, in year i with rainfall level k.

The net return function is given by:

$$\hat{\Pi}^*_{ng}(i, k) = \begin{cases} R_{1n}\left(a\hat{S}_{0n} + b_1\right) - R_{2n} \\ \quad \text{if} \quad \hat{S}^*_{ng}(i, k) \le \hat{S}_{0n} \\ R_{1n}\left(a\hat{S}^*_{ng}(i, k) + b_1\right) - R_{2n} \\ \quad \text{if} \quad \hat{S}^*_{ng}(i, k) > \hat{S}_{0n} \end{cases} \tag{4}$$

where

R_{1n} net income (\$/t) of crop n, as a function of the yield (revenue less yield-dependent variable costs such as harvesting, grading, packing and transportation);

R_{2n} variable cost (\$/ha), independent of yield;

$\hat{S}_{0n}$ ML estimate of crop n's average threshold-salinity level of the soil.

It is assumed that R_{1n}, R_{2n} and $\hat{S}_{0n}$ are independent of g, i and k.

Let $S^*_{ng}(i, k)$ and $\Pi^*_{ng}(i, k)$ be the optimal values of the average soil salinity and the net income, respectively, given that the true values of parameters (β) are known to the decision maker.

Let us now define the following sets (for convenience the index n is omitted from now on):

$$\hat{\mathrm{E}}_1: \left\{\hat{S}^*_g(i, k) \mid \hat{S}^*_g(i, k) > \hat{S}_0\right\}$$

$$\mathrm{E}_1: \left\{S^*_g(i, k) \mid S^*_g(i, k) > S_0\right\}$$

$$\hat{\mathrm{E}}_2: \left\{\hat{S}^*_g(i, k) \mid \hat{S}^*_g(i, k) \le \hat{S}_0\right\}$$

$$\mathrm{E}_2: \left\{S^*_g(i, k) \mid S^*_g(i, k) \le S_0\right\}$$

One may distinguish among four alternatives associated with the possible values of the loss function – hereafter alternatives a, b, c, d – based on all possible combinations of the relationships between $\hat{S}_0$ and S_0, S^*_g and S_0, and $\hat{S}^*_g$ and $\hat{S}_0$:

$$\hat{S}_0 \le S_0,\ S^*_g(i, k) > S_0,\ \hat{S}^*_g(i, k) > \hat{S}_0$$
$$\forall S^*_g(i, k) \in \mathrm{E}_1 \qquad \hat{S}^*_g(i, k) \in \hat{E}_1 \tag{5a}$$

as (5a) but with $\hat{S}_0 > S_0$ (5b)

$$\hat{S}_0 \le S_0,\ S^*_g(i, k) \le S_0,\ \hat{S}^*_g(i, k) \le \hat{S}_0$$
$$\forall S^*_g(i, k) \in \mathrm{E}_2 \qquad \hat{S}^*_g(i, k) \in \hat{\mathrm{E}}_2 \tag{5c}$$

as (5c) but with $\hat{S}_0 > S_0$ (5d)

There are four additional possibilities, but they can be disregarded since the ML estimates $\hat{\beta}$ are consistent and tend to β, so that asymptotically:

$$\Pr\left[\left(S_g^*(i, k) > S_0\right) \cap \left(\hat{S}_g^*(i, k) \le \hat{S}_0\right)\right] \to 0 \tag{6a}$$

$(\forall i, j, k)$

and

$$\Pr\left[\left(S_g^*(i, k) \le S_0\right) \cap \left(\hat{S}_g^*(i, k) > \hat{S}_0\right)\right] \to 0 \tag{6b}$$

where Pr stands for probability.

Using indicator functions the loss function in (3) can be written as:

$$\mathrm{h}(\hat{\beta} - \beta) = \mathrm{h_a}(\hat{\beta} - \beta) I_{\{a\}} + \mathrm{h_b}(\hat{\beta} - \beta) I_{\{b\}} + \mathrm{h_c}(\hat{\beta} - \beta) I_{\{c\}} + \mathrm{h_d}(\hat{\beta} - \beta) I_{\{d\}} \tag{7}$$

where I takes values of 1 or 0 as follows:

$$I_{\{l\}} = \begin{cases} l & \text{if alternative 1 holds} \\ 0 & \text{otherwise} \end{cases} \qquad l = \mathrm{a, b, c, d}$$

Assume that:
(A1) $\mathrm{h}_l(0) = 0$, $l =$ a, b, c, d
(A2) $\mathrm{h}_l(\hat{\beta} - \beta)$ is an increasing function of $|\hat{\beta} - \beta|$; and
(A3) the first and the second, right and left derivatives of $\mathrm{h}_l(\hat{\beta} - \beta)$ exist at the point $\hat{\beta} = \beta$ (since $\mathrm{h}_l(\hat{\beta} - \beta)$ has a minimum at the point $\hat{\beta} = \beta$, the first derivatives are zero).

Approximating Eq. (7) by a second-order Taylor series expansion around $\mathrm{h}_l(0)$ and utilizing assumptions (A1)–(A3) yields:

$$\begin{aligned} \mathrm{h}_l(\hat{\beta} - \beta) &\approx \frac{1}{2} \frac{\partial^2 \mathrm{h}_l(\ )}{\partial \hat{S}_0^2 |_{\hat{\beta}=\beta}} \left(\hat{S}_0 - S_0\right)^2 + \frac{1}{2} \frac{\partial^2 \mathrm{h}_l(\ )}{\partial \hat{a}^2 |_{\hat{\beta}=\beta}} (\hat{a} - a)^2 \\ &+ \frac{1}{2} \frac{\partial^2 \mathrm{h}_l(\ )}{\partial \hat{b}_1^2 |_{\hat{\beta}=\beta}} \left(\hat{b}_1 - b_1\right)^2 \\ &+ \frac{\partial^2 \mathrm{h}_l(\ )}{\partial \hat{S}_0\, \partial \hat{a} |_{\hat{\beta}=\beta}} \left(\hat{S}_0 - S_0\right)(\hat{a} - a) \qquad (8) \\ &+ \frac{\partial^2 \mathrm{h}_l(\ )}{\partial \hat{S}_0\, \partial \hat{b}_1 |_{\hat{\beta}=\beta}} \left(\hat{S}_0 - S_0\right)\left(\hat{b}_1 - b_1\right) \\ &+ \frac{\partial^2 \mathrm{h}_l(\ )}{\partial \hat{a}\, \partial \hat{b}_1 |_{\hat{\beta}=\beta}} (\hat{a} - a)\left(\hat{b}_1 - b_1\right) \equiv \mathrm{TAI}(\mathrm{h}_l) \end{aligned}$$

The observations Y_i are normally distributed and the approximated loss function (8) is proportional to the squared errors. It is therefore asymptotically true that Bayes estimates (i.e., the parameter estimates that minimize the expected loss) are equivalent to the ML estimates (e.g., Bickel and Yahav, 1969).

As $\hat{\beta} = (\hat{S}_0, \hat{a}, \hat{b}_1)$ is a random vector, the loss function is also random. For given values of β, σ_1^2, σ_2^2 and a given scatter $\widetilde{\mathrm{sc}}(T)$ of the T observations $S_1, S_2, \ldots, S_T$, the conditional expectation of the loss function is:

$$\mathrm{E}\left[\mathrm{h}(\hat{\beta} - \beta)/\beta, \sigma_1^2, \sigma_2^2\right] \equiv \mathrm{H}\left[T, \widetilde{\mathrm{sc}}(T)\right] \approx \sum_l \mathrm{E}\left[\mathrm{TAI}(h_l) \cdot I_{\{l\}}/\beta, \sigma_1^2, \sigma_2^2\right] \tag{9}$$

$l =$ a, b, c, d

Since the true parameter values of $(\beta, \sigma_1, \sigma_2)$ are unknown, their ML estimates $(\hat{\beta}, \hat{\sigma}_1^2, \hat{\sigma}_2^2)$ are used instead, respectively. Thus the best attainable estimate of (9) is:

$$\hat{\mathrm{E}}\left[\mathrm{h}(\hat{\beta} - \beta)/\hat{\beta}, \hat{\sigma}_1^2, \hat{\sigma}_2^w\right] \equiv \hat{\mathrm{H}}\left[T, \widetilde{\mathrm{sc}}(T)\right] \tag{10}$$

Since $(\hat{\beta}, \hat{\sigma}_1^2, \hat{\sigma}_2^2)$ tend asymptotically to $(\beta, \sigma_1^2, \sigma_2^2)$, $\hat{\mathrm{H}}[\cdot]$ might be regarded as a good approximation to $\mathrm{H}[\cdot]$ if the number of observations, T, is large enough.

It should be noted here that the ML estimates are based on the results of Sadan and Berglas (1980) who conducted a formal, well organized, experiment in an experiment station. Also, there might be a disparity between technical parameters estimated in a formal experiment and the same parameters on a specific farm. Therefore, the estimated parameters and their relevant statistical characteristics may be applied only to farms in the region of the experiment station, with similar technology, soil and climatic conditions.

5. Value of additional information

In this section the profitability of acquiring additional observations (S_i, Y_i) for potatoes is calculated. Let n be the number of additional observations and let $C(n)$ be the cost of their acquisition. Finding the optimal spread of the additional observations is a complicated statistical problem whose analytical solution is beyond the scope of this paper. In the absence of an analytical solution and following personal communication with statisticians, a prior uniform distribution of additional n observations $\widetilde{\text{sc}}^*(T+n)$, in a given interval around $\hat{S}_0(T)$ (the ML estimate of S_0, which is based on the a priori T observations) was assumed in the empirical application of the analysis.

With $\hat{\text{H}}[T, \widetilde{\text{sc}}(T)]$ (Eq. 10) describing the situation a priori, the expected value of additional information from n additional observations with spread $\widetilde{\text{sc}}^*(T+n)$, EVSI$(n)$, is given by:

$$\text{EVSI}(n) = \hat{\text{H}}\big(T, \widetilde{\text{sc}}(T)\big) - \hat{\text{H}}\big(T, \widetilde{\text{sc}}^*(T+n)\big) - C(n)$$

where $C(n)$ is the cost of acquiring the n observations. The optimal number of additional observations, n^*, is determined by:

$$\text{EVSI}(n^*) = \max_n \text{EVSI}(n) \tag{11}$$

As mentioned, the empirical computation of EVSI(n) relates to potatoes. It was found from the empirical application of the SLRO-model that only 5.6 ha of potatoes belong to $\hat{\text{E}}_2(\hat{S}_g^*(i, k) \le \hat{S}_0 = 6.054)$, which is only 1.4% of the total area of potatoes (400 ha) during the 4-year planning horizon. Hence, it is assumed that $I_{\{c\}} = I_{\{d\}} = 0$ (see Eq. 5). Thus Eq. (7) can be rewritten as:

$$\begin{aligned} \text{h}(\hat{\beta} - \beta) = {} & \text{h}_\text{a}(\hat{\beta} - \beta) I_{\{\hat{S}_g(i, k) \in \hat{\text{E}}_1\}} \cdot I_{\{S_g^*(i, k) \in \text{E}_1\}} \cdot I_{\{\hat{S}_0 \le S_0\}} \\ & + \text{h}_\text{b}(\hat{\beta} - \beta) I_{\{\hat{S}_g^*(i, k) \in \hat{\text{E}}_1\}} \cdot I_{\{S_g^*(i, k) \in \text{E}_1\}} \cdot I_{\{\hat{S}_0 > S_0\}} \end{aligned} \tag{12}$$

But this expression can be further simplified based on the following arguments: Since the ML estimates are consistent it is asymptotically true that:

$$\Pr\big[S_g^*(i, k) > S0\big) \cap \big(S_g^*(i, k) > \hat{S}_0\big)\big] \to 1 \quad \text{for every } i, g \text{ and } k \tag{13a}$$

Hence, the multiplication $I_{\{\hat{S}_g^*(i, k) \in \hat{\text{E}}_1\}} \cdot I_{\{S_g^*(i, k) \in \text{E}_1\}}$ can be replaced by the single indicator $I_{\{\hat{S}_g^*(i, k) \in \hat{E}_1\}}$. In addition,

$$\Pr\big[\hat{S}_0 \le S_0\big) \cap \big(\hat{S}_0 > S_0\big)\big] = 0$$

i.e.

$$I_{\{\hat{S}_0 \le S_0\}} + I_{(\hat{S}_0 > S_0\}} = 1 \tag{13b}$$

and

$$\text{h}_\text{a}(\hat{\beta} - \beta) I_{\{\hat{S}_g^*(i, k) \in \hat{\text{E}}_1\}} = \text{h}_\text{b}(\hat{\beta} - \beta) I_{\{\hat{S}_g^*(i, k) \in \hat{\text{E}}_1\}} \tag{13c}$$

This has to be true because both cases, a and b, are related only to the linear decreasing segment of the response function. Under the specific empirical results of the SLRO-model it can be assumed that $I_{\{\hat{S}_g^*(i, k) \in \hat{\text{E}}_1\}} = 1$, for every i, g, k. Noting that $\text{E}(\hat{\beta}_i - \beta_i)^2 = \text{V}(\hat{\beta}_i)$ and $\text{E}(\hat{\beta}_i - \beta_i)(\hat{\beta}_j - \beta_j) = \text{cov}(\hat{\beta}_i, \hat{\beta}_j)$, the possible situations of the loss function reduce to 1 and Eq. (10) can be written as (the letter T in parentheses represents the number of observations used to estimate β, σ_1^2 and σ_2^2):

$$\begin{aligned} \hat{\text{H}}\big[T, \widetilde{\text{sc}}[T)\big] &= \hat{\text{E}}\big[\text{h}(\hat{\beta} - \beta)/\hat{\beta}, \hat{\sigma}_1^2, \hat{\sigma}_2^2\big] \\ &= \frac{1}{2} \frac{\partial \text{h}^2(\ )}{\partial \hat{S}_0^2 \,|\, \hat{\beta} = \beta} \big[\text{V}(\hat{S}_0(T))/\hat{\beta}(T) \ldots\big] \\ &\quad + \frac{1}{2} \frac{\partial^2 \text{h}(\ )}{\partial \hat{a}^2 \,|\, \hat{\beta} = \beta} \big[\text{V}(\hat{a}(T))/\hat{\beta}(T) \ldots\big] \\ &\quad + \frac{1}{2} \frac{\partial^2 \text{h}(\ )}{\partial \hat{b}_1^2 \,|\, \hat{\beta} = \beta} \big[\text{V}(\hat{b}_1(T))/\hat{\beta}(T) \ldots\big] \\ &\quad + \frac{\partial^2 \text{h}(\ )}{\partial \hat{S}_0 \, \partial \hat{a} \,|\, \hat{\beta} = \beta} \big[\text{cov}\big(\hat{S}_0(T), \hat{a}(T)\big) /\hat{\beta}(T) \ldots\big] \end{aligned}$$

$$+\frac{\partial^2 \mathrm{h}}{\partial \hat{S}_0\, \partial \hat{b}_1 \mid_{\hat{\beta}=\beta}}\Big[\mathrm{cov}\big(\hat{S}_0(T),\, \hat{b}_1(T)\big)$$

$$/\hat{\beta}(T)\ldots\Big]$$

$$+\frac{\partial^2 \mathrm{h}}{\partial \hat{a}\, \partial \hat{b}_1 \mid_{\hat{\beta}=\beta}}\mathrm{cov}\big(\hat{a}(T),\, \hat{b}_1(T)\big)$$

$$/\hat{\beta}(T)\ldots\Bigg] \qquad (14)$$

The term $\hat{\mathrm{H}}(\mathrm{T}, \widetilde{\mathrm{SC}}(T)$ is an a priori point of reference necessary for the computation of EVSI(n) (see Eq. 11). The values of the variances and the covariances of (14) can be calculated from (2).

6. Empirical results

As mentioned, the empirical calculations of EVSI(n) were performed with respect to potatoes only. For $T = 17$ and $C(n) = \$130n$ – including the costs of soil sampling, laboratory examinations and foregone revenues to the farmer in the experimental plot (based on personal cmmunication with soil researchers from the Institute of Soil and Water, the Volcani Center, Bet Dagan) – we get (all the partial derivatives are computed at the point $\hat{\beta} = \beta$):

EVSI(n)

$$= \frac{1}{2}\frac{\partial^2 h(\;)}{\partial \hat{S}_0^2}\Big[\mathrm{V}\big(\hat{S}_0(17)\big)$$

$$-\mathrm{V}\big(\hat{S}_0(17+n)\big)/\hat{\beta}(17)\ldots\Big] \qquad (15)$$

$$+\frac{1}{2}\frac{\partial^2 \mathrm{h}(\;)}{\partial \hat{a}^2}\Big[\mathrm{V}(\hat{a}(17))$$

$$-\mathrm{V}(\hat{a}(17+n))/\hat{\beta}(17)\ldots\Big]$$

$$+\frac{1}{2}\frac{\partial^2 \mathrm{h}(\;)}{\partial \hat{b}_1^2}\Big[\mathrm{V}\big(\hat{b}_1(17)\big)$$

$$-\mathrm{V}\big(\hat{b}_1(17+n)\big)/\hat{\beta}(17)\ldots\Big]$$

$$+\frac{\partial^2 h(\;)}{\partial \hat{S}_0\, \partial \hat{a}}\Big[\mathrm{cov}\big(\hat{S}_0(17),\, \hat{a}(17)\big)$$

$$-\mathrm{cov}\big(\hat{S}_0(17+n),\, \hat{a}(17+n)\big)/\hat{\beta}(17)\ldots\Big]$$

$$+\frac{\partial^2 h(\;)}{\partial \hat{S}_0\, \partial \hat{b}_1}\Big[\mathrm{cov}\big(\hat{S}_0(17),\, \hat{b}_1(17)\big)$$

$$-\mathrm{cov}\big(\hat{S}_0(17+n),\, \hat{b}_1(17+n)\big)/\hat{\beta}(17)\ldots\Big]$$

$$+\frac{\partial^2 h(\;)}{\partial \hat{a}\, \partial \hat{b}_1}\Big[\mathrm{cov}\big(\hat{a}(17),\, \hat{b}_1(17)\big)$$

$$-\mathrm{cov}\big(\hat{a}(17+n),\, \hat{b}_1(17+n)\big)/\hat{\beta}(17)\ldots\Big]$$

$$-130n\,.$$

The partial derivatives of Eq. (15) were approximated and calculated numerically. The computation of each approximated derivative involves a few runs of the SLRO-model and assumptions concerning the marginal increments of the relevant parameters. The operative goal of the analysis is to provide, with a reasonable level of accuracy, an estimation of EVSI, and indicate the need for additional observations. Since considerable computer time and memory space is needed in order to calculate the partial derivatives, it was decided to compare only three combinations of marginal increments which yield three values of EVSI(n). Each of the derivatives was calculated three times; they differ from one another in the marginal increments of the relevant parameters. The derivative approximations and the numerical values of the marginal increments chosen for each alternative are left for the Appendix. It was found that the differences among the three runs are relatively small (see Table 3) so there is no significant difference in the operative conclusions.

Based on the results of Table 3, about 30 additional observation (S_i, Y_i) on potatoes should be made. The expected value of the additional information is in the range of \$10,000 to \$14,000. The magnitude of EVSI(n) is only about 1% of the total expected present value of the linear SLRO-model's objective function. However, the additional field experiments may contribute toward the reduction of the expectation of the loss function for more than a single kibbutz farm in the experimental region and therefore substantially increase the profitability of additional sampling. In other words, assuming M kibbutzim and that their soil plots, water sources, technology and

Table 3
Three computations of EVSI(n) for the response function of potatoes, with regard to the linear SLRO-model

n	EVSI(n) in US$		
	computation I	computation II	computation III
2	2338	3078	3104
4	4325	5701	5766
6	5766	7623	7714
8	6838	9071	9182
10	7649	10188	10311
12	8272	11058	11194
14	8812	11747	11890
16	9240	12286	12442
18	9390	12721	12877
20	9597	13052	13221
22	9747	13318	13494
24	9844	13519	13708
26	9909	13675	13864
28	9942	13786	13974
30	9935	13864	14052
32	9909	13916	14097
34	9864	13909	14162
36	9740	13876	14110
38	9721	13701	14084
40	9630	13610	13981

climate conditions are similar to those of the experiment station in the south of Israel which supplied our data base, the annual value of EVSI(n) should be multiplied by M.

It is important to note that since a few approximations were used in the analysis (second-order Taylor expansion, approximations of the partial derivatives [see Appendix], and the use of asymptotical statistical theory with the results based on a medium sized sample), the results must be regarded as approximate. Their main value is that they enable us to learn the order of magnitude of EVSI(n) and to draw operative conclusions about additional sampling.

7. Summary

The estimation of the response function of a given crop to soil salinity and the calculation of the expected value of additional information on the parameters of this function are important steps in the process of decision making regarding irrigation with saline water under conditions of uncertainty.

Considering the optimum value of the linear SLRO-model and the piecewise linear response function parameters, a loss function was constructed and its possible states were defined. The loss function was approximated by a second-order Taylor expansion (after some suitable assumptions) and its approximated expectation was derived. Then, the expected value of additional information on the response function parameters and the optimal sample size were calculated for potatoes.

The main advantage in the analysis provided in this paper is that it provides a conceptual and methodological framework with which to investigate the value of sample information in a long-run farm-level analysis as well as creating an efficient tool for empirical application. Although some approximations were used, the results provide an estimate of EVSI and indicate the need for additional observations.

As previously mentioned, the determination of the optimal spread over the field area of the additional observations is a complicated statistical problem and is an important direction in which the framework here presented might profitably be extended. In irrigated agriculture, the uneveness with which water infiltrates the root zone is determined mainly by the spatial variability of water application and hydrologic soil properties (e.g., Dagan and Bresler, 1988). Under these conditions there will be underirrigated and overirrigated areas and as a result, spatially variable soil salinity (Feinerman et al., 1984). Inclusion of the influence of the spatial variability in the analysis of the value of information will be another direction of significant and advantageous extension.

Appendix

Approximations of the partial derivatives of Eq. (15)

This appendix presents the numerical calculation technique of the second order partial derivatives of equation (15), with regard to two parameters: $\hat{\beta}_1$ and $\hat{\beta}_2$. For the sake of simplicity, let $Z(\beta/\beta) \equiv Z(\beta)$ and $Z(\hat{\beta}/\beta) \equiv Z(\hat{\beta})$. Hence, the loss function is (see Eq. 3): $h(\hat{\beta}-\beta) = Z(\beta) - Z(\hat{\beta})$. Since $Z(\beta)$ is constant,

$$\left.\frac{\partial^2 h(\hat{\beta}-\beta)}{\partial \hat{\beta}_i^2}\right|_{\hat{\beta}=\beta} = -\left.\frac{\partial^2 Z(\hat{\beta})}{\partial \hat{\beta}_i^2}\right|_{\hat{\beta}=\beta}$$

Utilizing a third-order Taylor expansion (all the following derivatives are computed at the point $\hat{\beta}=\beta$) and a few algebric manipulations we get:

(a)

$$-\frac{\partial^2 Z(\hat{\beta})}{\partial \hat{\beta}_1^2} = \frac{\partial^2 h(\hat{\beta}-\beta)}{\partial \hat{\beta}_1^2} \approx \frac{2Z(\hat{\beta}) - \left[Z\left(\hat{\beta}_1+\Delta, \hat{\beta}_2\right) + Z\left(\hat{\beta}_1-\Delta, \hat{\beta}_2\right)\right]}{\Delta^2}$$

(b)

$$\frac{\partial^2 h(\hat{\beta}-\beta)}{\partial \hat{\beta}_2^2} \approx \frac{2Z(\hat{\beta}) - \left[Z\left(\hat{\beta}_1, \hat{\beta}_2+\delta\right) + Z\left(\hat{\beta}_1, \hat{\beta}_2-\delta\right)\right]}{\delta^2}$$

and

(c)

$$-\frac{\partial^2 Z(\hat{\beta})}{\partial \hat{\beta}_1 \partial \hat{\beta}_2} = \frac{\partial^2 h(\hat{\beta}-\beta)}{\partial \hat{\beta}_1 \partial \hat{\beta}_2} \approx \frac{Z(\hat{\beta})}{\Delta\delta} + \frac{\Delta}{2\delta}\frac{\partial^2 Z(\beta)}{\partial \hat{\beta}_1^2} + \frac{\delta}{2\Delta}\frac{\partial^2 Z(\hat{\beta})}{\partial \hat{\beta}_2^2} - \left[\frac{Z\left(\hat{\beta}_1+\Delta, \hat{\beta}_2+\delta\right) + Z\left(\hat{\beta}_1-\Delta, \hat{\beta}_2-\delta\right)}{2\Delta\delta}\right]$$

where Δ and δ represent the marginal increments of $\hat{\beta}_1$ and $\hat{\beta}_2$, respectively. Let $X^*(\hat{\beta})$ be the optimal solution vector of the linear SLRO-model, and $\boldsymbol{C}(\hat{\beta})$ be the vector of the activities' expected net returns (in present value) in $/ha. Thus:

$$Z(\hat{\beta}) = \boldsymbol{C}(\hat{\beta}) \cdot \boldsymbol{X}^*(\hat{\beta})$$

$$Z\left(\hat{\beta}_1 \pm \Delta, \hat{\beta}_2\right) = \boldsymbol{C}(\hat{\beta}) \cdot \boldsymbol{X}^*\left(\hat{\beta}_1 \pm \Delta, \hat{\beta}_2\right)$$

$$Z\left(\hat{\beta}_1, \hat{\beta}_2 \pm \delta\right) = C(\hat{\beta}) \cdot X^*\left(\hat{\beta}_1, '\hat{\beta}_2 \pm \delta\right)$$

and

$$Z(\beta_1 \pm \Delta, \beta_2 \pm \delta) = C(\hat{\beta}) \cdot X^*\left(\hat{\beta}_1 \pm \Delta, \hat{\beta}_2 \pm \delta\right)$$

As mentioned, the partial derivatives were calculated three times, according to the following marginal increments of the parameters:

- Alternative I
  $\Delta\hat{S}_0 = 0.5$ $\Delta\hat{a} = 0.2$ $\Delta\hat{b}_1 = 4$
- Alternative II
  $\Delta\hat{S}_0 = 0.25$ $\Delta\hat{a} = 0.1$ $\Delta\hat{b}_1 = 2$
- Alternative III
  $\Delta\hat{S}_0 = 0.25$ $\Delta\hat{a} = 0.5$ $\Delta\hat{b}_1 = 2$

References

Antonovitz, F. and Roe, T. (1984) The value of rational expectations forecast in a risky market: a theoretical and empirical approach. Am. J. Agric. Econ., 66: 717–723.

Bickel, P.I. and Yahav, J.A. (1969) Some contributions to the asymptotic theory of Bayes solution. Z. Wahrscheinlichkeitstheorie Verw., II: 257–276.

Bie, S.W. and Ulph, A. (1972) The economic value of soil survey information. J. Argon. Econ., 23: 285–295.

Dagan, G. and Bresler, E. (1988) Variability of yield of an integrated crop and its causes. III. Numerical simulation and field results. Water Resour. Res., 24: 395–401.

Dantzig, G.B. and Madansky, A. (1961) On the solution of two-stage linear programs under uncertainty. In: Proc. 4th Berkeley Symp. Mathematical Statistics and Probability, Berkeley, Univ. California, Vol. 1.

Davis, D.R. and Dvoranchik, W.M. (1971) Evaluation of the worth of additional data. Water Res. Bull., 7: 700–707.

DeGroot, M.H. (1970) Optimal Statistical Decisions. Carnegie Mellon University, McGraw-Hill, New York.

Duckstein, L., Szidarorszky, F. and Yakowitz, S. (1977) Bayes design of a reservoir under random sediment yield. Water Resour. Res., 13: 713–718.

El Agizy, M. (1967) Two-stage programming under uncertainty with discrete distribution function, Oper. Res., 15: 55–70.

Feinerman, E. and Yaron, D. (1983a) The value of information on the response function of crops to soil salinity. J. Environ. Econ. Manage., 10: 72–85.

Feinerman, E. and Yaron, D. (1983b) Economics of irrigation water mixing within a farm framework. Water Resour. Res., 19: 337–345.

Feinerman, E., Knapp, K.C. and Letey, J. (1984) Salinity and uniformity of water infiltration as factors in yield and economically optimal water application. Soil Sci. Soc. Am. J., 48: 477–481.

Klemes, V. (1977) Value of information in reservoir optimization. Water Resour. Res., 13: 837–850.

Maas, E.V. and Hoffman G.J. (1977) Crop salt tolerance – current assessment, ASCE J. Irriq. Drain. Div. IR2, 103: 115–134.

Maddock, T. (1973) Management model as a tool for studying the worth of data. Water Resour. Res., 9: 270–280.

Mjelde, J.W., Sonka, S.T., Dixon, B.L. and Lamb, P.J. (1988) Valuing forecast characteristics in a dynamic agricultural production system. Am. J. Agric. Econ., 70: 674–684.

Pratt, J.W., Raiffa, H. and Schlaifer, R. (1965) Introduction to Statistical Decision Theory, McGraw-Hill, New York.

Preckel, P.V., Loehman, E.T. and Kaylen, M.S. (1987) The value of public information for microeconomic production decisions. West. J. Agric. Econ., 12: 193–197.

Quandt, R.E. (1972) A new approach to estimating switching regressions. J. Am. Stat. Assoc., 67: 306–310.

Ryan, J.G. and Perrin, R.K. (1974) Fertilizer response information and income gains: the case of potatoes in Peru. Am. J. Agric. Econ., 56: 337–43.

Sadan, D. and Berglas, M. (1980) Unpublished data set. Extension Service, Ministry of Agriculture, Israel.

Zacks, S. (1971) The theory of Statistical Inference. Wiley, New York.