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ELSEVIER

Agricultural Economics 10 (1994) 165–177

AGRICULTURAL ECONOMICS

Separability in farm-household economics: an experiment with linear programming

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(Accepted 12 July 1993)

Abstract

The assumption of separability between farm-household production and consumption facilitates analysis, but entails several important restrictions. The implications of assuming separability are discussed here in relation to the modelling of a representative Tongan farm-household. Econometric estimation of household demand is coupled with a linear programming (LP) model of farm-household production. When analysing consumer demand, separable farm-household economics is undoubtedly preferable to ignoring the production/consumption linkages entirely. However, the restrictions which must be imposed on the production side of the separable model are such that a realistic LP solution is unlikely to be obtained. This is likely to be a major deterrent to adopting the separable approach for studies in which the main focus is on production rather than consumption.

1. Separable farm-household model: structure and assumptions

Since the early theoretical contributions by Becker (1965) and Nakajima (1969), the farm-household has become a popular focus of attention for economists. Empirical application of farm-household economics theory is now common-place, following the lead of Barnum and Squire (1979) and the comprehensive volume edited by Singh et al. (1986a). Most applications are based on a simple conceptual model such as that illustrated in Fig. 1.

Two linked stages are identified. First, the model farm-household acts as if it seeks to maximise profits from its production activities, subject to production function constraints. The resulting revenue then forms part of its full income [1] constraint, subject to which the household is assumed to maximise its utility from consumption.

Although the two decision stages are simultaneous in time, they are commonly analysed as separate components of a sequence. This approach can be justified algebraically provided certain assumptions are made (Singh et al., 1986b,c,d). The most important assumption is the existence of markets for labour and products in which the household is a price-taker. To maximise farm profits from a fixed land area, the

[1] In this context, Becker's (1965) concept of full income is generally defined as the value of household time plus exogenous (non-wage) income plus short-run production profits.

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SSDI 0169-5150(93)E0037-N

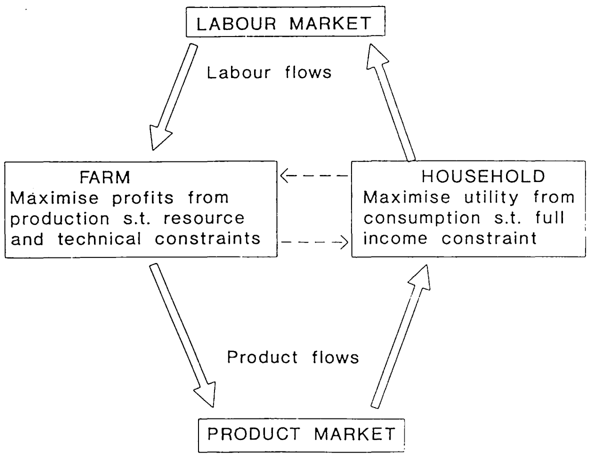

Fig. 1. Structure of the farm-household.

household should use family and hired labour until the marginal revenue product of labour equals the market wage (Singh et al., 1986b, p. 6). Assuming fixed-price markets, decisions about total labour input, and hence total production, are not affected by household preferences concerning labour and consumption goods. Both can be freely bought or sold to enable the household to achieve the subjective equilibrium position corresponding to the predetermined level of production. "In other words, the household can make its production decisions independently of its consumption and labor-supply decisions" (Singh et al., 1986b, p. 7).

The reverse is not true, however, since household consumption and labour supply are partly determined by the level of farm production. For instance, an increase in the price of a crop which is both produced and consumed will affect household consumption both directly, because of the increased purchase price, and indirectly, because the increase in the level of farm profits shifts the household's full-income constraint.

The indirect effect, whereby farm production influences household consumption, is termed the profit or production effect (Singh et al., 1986b, p. 7), and a model based on the above assumptions is 'separable' or 'recursive'. It has been demonstrated in empirical studies that the magnitudes, and often the signs, of elasticity estimates derived from household demand functions may change significantly when the profit effect is taken into account (e.g., Lau et al., 1978; Kuroda and Yotopoulos, 1978, 1980; Barnum and Squire, 1979; Ahn et al., 1981; Adulavidhaya et al., 1984; Singh and Janakiram, 1986; Strauss, 1986).

The recursive property does not hold if any prices in the model are affected by production decisions. This is the case when markets do not exist or are not competitive. If there is no labour market, for instance, the household must equate its labour demand and supply according to an 'implicit' or 'virtual' wage determined by "all the variables that influence household decisionmaking" (Singh et al., 1986b, p. 8). Levels of production, consumption and family labour use will then all be simultaneously determined by the virtual price. Similarly, imperfections in the product market, such as restrictions on the volume of trade or marked differences between buying and selling prices, would invalidate the assumption that the household is indifferent between producing a commodity and purchasing it. The separability assumption also generally breaks down if risk and risk aversion are recognised to be significant factors. This is because:

> risk aversion in consumption induces risk aversion where profits are concerned. Thus, the expected utility of profit must be maximized. The form of this function depends on the form of the consumption utility function and consumption decisions (Roe and Graham-Tomasi, 1986, p. 257).

In terms of Nakajima's (1986) categorisation of farm-household types, the separable model can only be strictly relevant to the farm firm/labourer's household/consumer's household in which some output is sold and a labour market exists. Even in this case, the market conditions may not be adequately satisfied, and risk cannot be accommodated. The separable model, in other words, appears to have very limited validity. The question may therefore be raised whether there can be any justification for pursuing such a restrictive approach.

The response of Singh et al. (1986d) is that separability should be assumed unless there is compelling evidence to the contrary. The main reason for this is that separable models are relatively simple to estimate using standard econometric procedures. Abandoning the separability assumption results in a much less tractable model (Singh et al., 1986d, p. 52).

At least some aspects of separability can now be tested empirically (for tests of the labour market assumptions, see e.g. Benjamin, 1992). Even without formal tests, intuitive assessments may be made of the bias likely to result from wrongly assuming separability. These are based on the extent to which violations of separability are likely to affect the specific variables of policy interest (Singh et al., 1986d, p. 50).

In many cases, it can be argued that the assumption of separability will result in only minor distortions, and should therefore be adopted in the interests of model tractability. Most farm-household modelling to date has been undertaken on this basis (e.g., Kuroda and Yotopoulos, 1978, 1980; Barnum and Squire, 1979; Ahn et al., 1981; Adulavidhaya et al., 1984; Van Kooten and Arthur, 1985), although there have been some exceptions (e.g., Lopez, 1986; Roe and Graham-Tomasi, 1986).

2. Empirical evidence

In the study described in this paper, the first stage was to assess the acceptability of the separability assumptions in the light of available evidence from the study area, the Kingdom of Tonga in the South Pacific. [2] Crucial considerations included the nature of product and labour markets in Tonga, the risks faced by producers, and the extent to which Tongan farmers' objectives could be adequately represented within the separable framework. [3]

Labour market

An active labour market existed in three of the four survey villages. [4] Labour shortages brought about by migration plus a reservation price of labour pushed up by access to overseas labour markets and rising domestic prices, have apparently been largely responsible for a high level of wage inflation in recent years. While the assumption that all households face a single wage rate is clearly an oversimplification, the rate for casual agricultural labour is fairly uniform within each village, and could be regarded as a reasonable proxy for the marginal wage rate available to all village households.

Product markets

Although produce markets exist in the main towns and most farm-households participate as both buyers and sellers, there are several complicating factors. The virtual absence of village-level markets for staple commodities imposes considerable costs on sellers, who must travel to a central marketplace and wait for sales. Gift exchange is an important part of the village economy which may well affect production decisions as well as consumption patterns. Moreover, there appear to be prestige factors associated with home production of staple food requirements, which would invalidate the assumption of indifference between production and purchase. In the interests of model simplicity, it may be argued that such factors should be overlooked, but the implications of doing so must be fully considered.

Goals of smallholder farm-households

As part of an attitudinal survey, an attempt was made to elicit information about the factors which contribute to farm-household utility in

[2] Research was undertaken as part of the South Pacific Smallholder Project, which was funded by the Australian Centre for International Agricultural Research (ACIAR) and organised by the Department of Agricultural Economics and Business Management, University of New England, Armidale, N.S.W., Australia, in conjunction with the Ministry of Agriculture, Forests and Fisheries, Tonga. The main focus of the Project was on identifying constraints and opportunities in smallholder agriculture through the collection and analysis of detailed socio-economic data from a sample of about 30 farm-households in each of four villages. Data were collected over a 12-month period commencing in November 1984. For further details, see Hardaker et al. (1988).

[3] See Delforce (1993) for a detailed analysis.

[4] The exception is in a relatively remote and poorly developed area.

Tonga. Five goals were defined, covering the general areas of 'religion', 'cash', 'home production', 'family' and 'leisure'. In spite of inconsistencies, it did prove possible to obtain goal rankings from 49 of the 53 interviewees. The 'religion' goal had the highest mean rank score when results from all four survey villages were taken into account. This was followed by 'cash', 'home production', 'family' and finally 'leisure'. However, the high overall rank of the 'religion' goal was due to a very high score in one of the villages. In the other three villages, the 'cash' goal was placed first.

The goal rankings are indicative of the relative importance of five factors in Tongan smallholder decision making. Within the separable framework, it must be assumed that these preferences are captured in the estimated expenditure model.

Risk

Tongan farmers face several types of production risk. Droughts and cyclones are regular occurrences, while pests and diseases have had devastating effects on a range of crops in recent years. Market price fluctuations also affect some crops. In particular, seasonal products such as tomato and capsicum have highly variable prices from month to month, whereas the prices of most root crops are fairly stable.

Relatively little is known about the risk attitudes of Tongan smallholders, [5] although there are some grounds for believing that they are averse to risk. Case study material from 16 farmers, for instance, revealed that the resistance or susceptibility of a crop to natural hazards was an important consideration in farmers' choice of crops (Delforce, 1988). The large number of different crops grown on most agricultural allotments also suggests a desire to reduce total risk by diversification.

As noted above, it is generally not possible to accommodate risk in a separable model. The degree to which a risk-neutral model would misrepresent farmers' objectives is difficult to gauge a priori.

Discussion

Evidence from Tongan villages suggests that there must be some doubts about the validity of the separability assumptions at farm-household level. Nonetheless, it may be argued that the assumptions are plausible enough to justify adoption of the separable approach. For instance, although labour and product markets may not be perfectly competitive, the assumption that farm-households are price takers does seem reasonable. Similarly, such evidence as is available concerning risk and risk attitudes may not, in itself, warrant the abandonment of the separable framework. Therefore, the separable approach was retained in the first instance, pending an evaluation of the usefulness and plausibility of the model developed.

3. A separable model of the Tongan farm-household

3.1. Production model

Choice of technique

Tongan farm-households typically cultivate several different crops, so either multi-crop production functions or mathematical programming (MP) could have been used to model farm production. The programming approach appeared to have greater potential for the types of policy analysis of interest in the current study. [6] In contrast, econometric models have limited predictive capacities if conditions are varied beyond the range represented in the data used for estimation. Results based on extrapolation may be inconsistent with what is feasible as determined by resource and other constraints on the real system. Yet it is just such evaluations outside the range of previous experience that are of most

[5] Difficulties experienced in obtaining information about smallholders' goals suggested that it would not be worthwhile to attempt direct elicitation of risk attitudes.

[6] Specific policy options were assessed in the areas of land, labour, capital and credit, technology and the terms of trade (see Delforce, 1993).

Table 1
Structure of profit-maximising LP model

Row	Grow and sell crops (41)	Borrow land (1)	Use labour (6)	Transfer working capital (5)	Use credit (6)	Grow coconuts (1)	Sell coconuts (6)	Make and sell copra (6)	Grow paper mulberry (1)	Make handicrafts (12)	Reef collection (6)	Line fishing (6)	Sell fish (6)	Livestock (2)	Fallow (50)	Dir	RHS
Objective function (1)	+	−	−		−	−	+	+	−	+	+	−	+	+			
Land (6)	+	−				+			+					+	<	2.2	
Secure land (1)	(+) [a]					+			+						<	2.2	
Labour (6)	+	+	−			+	+	+	+	+	+	+	+	+	+	<	0
Working capital (6)	+/−	+	+	+/−	+/−	+	−	−	+	−	−	+	−	−		<	[b]
Cassava (6)	(−)													(+)		<	0
Coconuts (6)						−	+	+						(+)		<	0
Copra (6)																<	0
Paper mulberry (6)									−	+						<	0
Fish (6)												−	+			<	0
Livestock (24)														−		<	0
Min. coconuts (1)						+										>	[b]
Rotations (118)	+/−					+			+/−						+/−	</=	0
Bounds (2)		+												(+)		<	[b]

[a] Brackets around the sign indicate that not all activities in the group are involved.
[b] Non-zero right-hand sides or bounds.

interest for policy purposes. The highly constrained nature of typically resource-poor smallholder systems makes constrained optimisation models, such as those provided by MP, particularly appropriate.

Another reason for opting for MP was the perceived inadequacy of some of the available data. Although records were kept of the inputs and outputs of specific productive activities, there were few complete 'observations' suitable for use in an econometric model. Certainly, it would not have been possible to estimate separate production functions for each activity. At best, an aggregate production function similar to that estimated by Strauss (1986) could perhaps have been specified, but considerable difficulties were anticipated. On the other hand, the information collected did prove useful for the development of activity budgets, as required in programming models (e.g., see Delforce, 1988, 1993).

Structure of the model

The assumption of separability imposes several important restrictions on the production model. If separability is valid, then farm-household utility is maximised by applying the principle of profit maximisation to production activities, subject only to technical constraints and some (short-run) resource constraints such as land and working capital. Family labour is not considered to be constraining, since it is assumed that hired labour is available at the equilibrium market wage, and that the family and the market are equally acceptable sources of labour. Similarly, household consumption requirements are irrelevant since food can be purchased if enough is not produced. There are assumed to be no differences between buying and selling prices, no preferences for either home-produced or purchased foods, and risk is assumed to be unimportant.

With these considerations in mind, the LP matrix of a representative smallholder farm-household in Tonga comprised 155 activities and 188 constraints, with maximisation of net revenue from production as the objective (Table 1). Production activities in the model included the growing of staple and non-staple crops, crop processing (for vanilla and coconuts), handicraft production, fishing and livestock husbandry. Six periods were identified, corresponding to peaks and slacks in the production cycles of the main crops.

The input–output coefficients used in the model were derived from farm-household data collected by the South Pacific Smallholder Project (see Delforce, 1988, 1993; Gyles et al., 1989). In accordance with the separability assumptions, all crops were valued at their market prices. Similarly, although handicraft, fishing and livestock husbandry tend to be largely subsistence activities, they too were valued in the objective function at market prices.

Land constraints were set at the average area of land held under secure tenure by the sample households. Land could also be borrowed, under conditions similar to those frequently encountered in the survey villages.

Rotational constraints were included in the model, to ensure that the optimal cropping pattern did not violate the rotational sequences normally followed by Tongan smallholders. In particular, fallow land was constrained to be twice the area of the cropped land.

The use of labour for farm-household production was monitored through six labour activities. [7] There was assumed to be a single source of labour, valued at the market wage.

It is not appropriate to include cash constraints in a separable model, since cash, labour and kind are assumed to be freely convertible. The cash/labour conversion has been described in the previous paragraph. The value of cash and kind was assumed to be covered by the term 'working capital'. There were working capital constraints for each period; their right-hand sides were the value of gifts received by the representative household. Inter-period transfers were allowed. There was also the opportunity to borrow money at the prevailing bank interest rate.

[7] Labour costs could simply have been included as variable costs in the activity budgets. However, the inclusion of labour use activities removed the need to calculate labour use ex post from the solution.

Results

The optimal solution to the profit-maximising model involved using the available land (own and borrowed) for the cultivation of yams, sweet potato (*kumala*) and vanilla, in addition to the coconuts and fallow areas which were required to be present. Sensitivity analysis showed that yams, *kumala*, vanilla and watermelon remained dominant as returns to particular crop activities were varied. No fishing or handicraft activities were undertaken, but the keeping of poultry was at the maximum level allowed. [8] A total of 1830 hours of labour was used, and T$67 was borrowed in period 3 and repaid in period 4. [9] The objective function value was T$6838.

The assumed cost of labour was an important determinant of the optimal solution. In the basic model, labour was valued at T$1 per hour. With labour at T$2 per hour, its usage was reduced to 1236 hours by growing only watermelon and vanilla. At T$3 per hour, 1038 hours were used for the production of vanilla, kava, *kumala* and pumpkin. At this wage rate, it would no longer be economical to keep poultry. However, the full amount of land available (own and borrowed) continued to be used, subject to the rotational restrictions.

Discussion

The optimal solution to the profit-maximising LP is not entirely unrealistic in that yams and vanilla are crops of considerable importance in practice, and the cultivation of *kumala* has also expanded greatly in recent years. Rotational constraints are satisfied and total labour use, although somewhat lower than the norm, is quite plausible given that only a subset of all the possible uses of time are included in the model.

However, the extent to which the solution can be said to be representative of actual production patterns among farmers in the study area is limited. It would certainly be unusual for farmers to confine their cropping activities to just three types. Cassava, the most widely grown staple, does not appear in the optimal solution, nor do any of the fishing or handicraft activities which are commonly practised in the survey villages. Moreover, the objective function value of over T$6800 is considerably higher than average household incomes in the study villages (less than T$3000 in three of the villages; T$4500 in the fourth).

The concentration of resources on a small number of highly remunerative activities is, of course, a common problem in LP models in which the full range of objectives and constraints have not been incorporated. In the specific instance being considered here, there appear to be four main explanations for the discrepancies between the optimal solution and actual production patterns:

(1) The desire to grow staple foods and produce handicrafts for home use and to satisfy social obligations seems to be almost universal among the rural inhabitants of Tonga. The relative market values of such products may be of little importance in determining cropping patterns and time allocation. Thus, it seems that household consumption requirements *do* influence farmers' production decisions.

(2) Pests, diseases and climatic hazards can reduce crop yields substantially. Such risks are no doubt well known to farmers, but risk aversion cannot be accounted for in a separable model.

(3) The prices assumed to be received for productive output are fixed in the model before the optimal production pattern is determined. Producers, of course, do not face such price certainty. Aversion to price risk may be an important factor influencing production decisions.

(4) There may be limits to market demand for some products, which restrict the amount that the *representative* farm-household would consider producing. Such restrictions cannot be included in the model because they would violate the market assumptions embodied in the separable approach. There appears to be a fundamental inconsistency between the concept of the repre-

[8] Initially, the model was unbounded because the poultry activity used no farming land and labour supply was not restricted. An upper bound of 15 broilers was therefore placed on the keeping of poultry (since few village households have more than this).

[9] At the time of the Project, the Tongan dollar was at par with the Australian dollar (T$1 = A$1).

Table 2
Elasticity estimates from AIDS model

	Expenditure category [a]						
	1 LOCSTAP	2 IMPSTAP	3 LOCPROT	4 IMPROT	5 OFBT	6 NONFD	7 LEIS
Mean budget share (w_i)	0.081	0.015	0.072	0.042	0.057	0.121	0.612
Elasticities at means							
Expenditure	0.791	0.755	1.122	0.650 **	0.782	1.053	1.053
Own-price	−1.844 *	−0.218	1.718	1.785	1.598	−2.658	−0.078 **
Cross-price							
$j=1$	–	0.045	−0.174	−0.676	0.278	−1.382 **	0.424 **
$j=2$	−5.958 **	–	1.408	−4.703 **	−6.471 **	10.082 **	−0.466
$j=3$	9.081 *	0.283	–	5.982	4.683	−5.811	−1.219
$j=4$	6.399 **	−1.143	0.177	–	−0.475	−12.732 **	1.534
$j=5$	−6.229	3.167	−1.312	1.462	–	15.298 **	−3.209 **
$j=6$	−7.251	−2.436	−5.136	−3.603	2.474	–	1.961
$j=7$	−0.595	−0.453	2.196	−0.898	−2.869 **	−3.847 **	–

* Based on a corresponding expenditure or price coefficient with a $T-$ ratio greater than 1.645.
** Based on a corresponding expenditure or price coefficient with a $T-$ ratio greater than 1.960.

sentative farm and the requirements of separability. What may be an optimal production pattern for an individual profit-maximiser may be quite implausible for the representative farmer.

The implications of these factors are considered further below. First, however, the other component of the separable model – the household demand system – is outlined, and procedures for integrating production and consumption models are described.

3.2. Consumption model

Choice of technique

The Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980) was selected as the preferred model for use here. The AIDS has several advantages over 'competitors' such as the Linear Expenditure System (LES). For instance, as it is not derived from an additive utility function, price and income elasticities vary across observations, so Engel curves may be nonlinear. Luxuries and necessities can be distinguished. Homogeneity and symmetry are not automatically satisfied, so this model can be used to test their empirical validity. Although the system has limitations when used for time-series estimation (Wohlgenant, 1984), these were not seen as crucial in the context of the present study.

Model specification

Six categories of cash and subsistence expenditure were identified: local staples (LOCSTAP), imported staples (IMPSTAP), local protein (LOCPROT), imported protein (IMPROT), other food, beverages and tobacco (OFBT) and non-food (NONFD). In addition, leisure (LEIS) was identified as another component of the expenditure of 'full income'.

The model was estimated using 150 quarterly observations from three villages.[10] Expenditure and price elasticities calculated from the results are presented in Table 2. LOCSTAP, IMPROT and OFBT are classified as necessities, since their expenditure elasticities are less than 1.0. All of the expenditure elasticities are positive, indicating that expenditure on all categories increases (as expected) as total expenditure increases.

For present purposes, the most interesting of the significant price elasticities relate to the own prices of LOCSTAP and LEIS. Both LOCSTAP and LEIS demand are inversely related to own price, but demand for LOCSTAP is own-price elastic, while demand for LEIS is own-price inelastic.

[10] One of the four survey villages was excluded because of poor data quality.

3.3. Integration of production and consumption models

Integration of the LP model of farm-household production with the AIDS model of consumption involves adding the 'profit effect' to elasticity estimates derived from the consumption model (see Appendix). It was assumed that only three of the AIDS expenditure categories – LOCSTAP, LOCPROT and LEIS – were affected by the production side of the model. These categories were all present in some form in the optimal LP solution. Thus, yams and *kumala* were taken to represent staple crop production and the keeping of poultry was taken to represent local protein food production. The profit effect could therefore be calculated for these categories. However, the LOCPROT price elasticity obtained from the AIDS model was unfortunately not statistically significant. Discussion here is therefore confined to LOCSTAP and LEIS.

The AIDS and integrated elasticities estimated with the separable household model are presented in Table 3. The first point to note is that the sign of the LOCSTAP elasticity remains unchanged when the profit effect is incorporated, but the sign of the LEIS own-price elasticity changes from negative to positive. For LOCSTAP, the absolute value of the integrated elasticity is lower than that of the AIDS elasticity. This is to be expected, since a rise in the price of staples brings about an increase in production profits, and hence full income. The income elasticity of LOCSTAP is positive, so the income effect of the price rise cancels out the price effect to some extent. However, as the income response is relatively inelastic, it is not strong enough to cause a change in direction of the overall response.

Table 3
Comparison between AIDS elasticities (ϵ) and integrated elasticities (η)

	Expenditure category	
	LOCSTAP	LEIS
Own-price		
ϵ	-1.844	-0.078
η	-1.677	0.363
Effect of wage rate		
ϵ	-0.595	–
η	-0.322	–

Similar logic can be applied to the case of the effect of a wage rate increase on demand for leisure. On the consumption side, demand for leisure decreases as its (opportunity) cost rises. However, the fall in profits which results from an increase in the wage rate is offset by the increased value of household time, so full income rises. The income elasticity for leisure is positive and elastic, so the rise in income brought about by the wage rate rise serves to change the price response from negative to positive.

The effect of a change in the wage rate on demand for LOCSTAP is also shown in Table 3. A wage rate rise increases full income and causes demand to rise, so the integrated elasticity is less negative than the AIDS elasticity.

4. Discussion and Conclusion

The above comparison between the AIDS and integrated elasticities confirms the finding of other practitioners of farm-household modelling, that the effect of price changes on farm-household production, and hence on profits, can have an important influence on price elasticities of demand. The results show that the own-price elasticity of demand for a good is likely to be higher (in absolute value) for a non-producing household than for one which does produce that good. Moreover, the fact that the value of household time is a component of full income means that the negative relationship often found in consumption studies between demand for leisure and the wage rate may become positive (as in the current study) when the profit effect is taken into account.

For a researcher primarily interested in the consumption or expenditure behaviour of farm-households, the separable approach is undoubtedly superior to an analysis which takes no account of the links between production and consumption. In this situation, the main purpose in modelling production activities is to determine the effect on production (and hence on full income) of changes in the prices of some of the expenditure categories modelled in the consumption analysis. As long as at least some of the categories modelled on the consumption side are represented in the production-side solution, the profit effect can be derived and integrated elasticities can be calculated. Details of how that profit effect is obtained may be of minor concern.

On the other hand, if the production activities are of interest in themselves, and the programming approach is the preferred method of modelling these, then the separable approach may well prove inadequate. [11] This is likely to be the case even when, as in Tonga, the a priori evidence indicates that the market and risk assumptions of separability are not unreasonable. Had it been practical (or preferred) to use econometric estimation on the production side, a standard separable model would probably have been acceptable, and the validity of specific restrictions could have been tested. However, it has been argued in this paper that the econometric approach is neither practical nor optimal in certain situations. If LP is used, the required assumption that production decisions are made solely on the basis of profit maximisation, and are not influenced by consumption requirements or other non-profit considerations, is far too restrictive. There seems little doubt that Tongan smallholders (in common with semi-subsistence farmers elsewhere) prefer to provide a large part of household consumption requirements from own production, and that price, climatic and other risks are important. Moreover, the assumption required in a separable model that producers are price takers in a competitive market (i.e. they face perfectly elastic demand curves for their output) seems incompatible with the notion of the representative farm-household, whereby 'average' results can be aggregated up to a regional or even national level.

Some alternative to the separable approach clearly must be sought if a realistic programming model of the farm-household is to be developed. A full consideration of the possibilities is beyond the scope of this paper, but one procedure which deals with the above difficulties, while still retaining the theoretical framework of farm-household economics, is to integrate both production and consumption aspects of the farm-household within a single non-deterministic MP model. Several alternatives are available (e.g., see Anderson et al., 1977; Wills and Perlack, 1980; Hazell and Norton, 1986), although in many cases an assumption of separability between production and consumption is implicitly required. The non-separable model developed for the Tongan study had an assumed utility function which included as attributes not only cash profits, but also subsistence consumption and leisure. Target MOTAD (Tauer, 1983) and MOTAD (Hazell, 1971) techniques were used to capture the production risks associated with subsistence and cash crops, respectively. [12] This model was then used to assist in the analysis of alternative policy measures which the Tongan government might consider implementing in order to promote national economic development based on a dynamic smallholder sector.

Acknowledgements

The study described in this paper was undertaken while the author was in the Department of Agricultural Economics and Business Management of the University of New England, Armidale, N.S.W., Australia. Particular thanks are due to Professor J.B. Hardaker.

[11] Singh and Janakiram (1986), for instance, found that the results of their Nigerian LP model were not in accordance with normal patterns of farm production, apparently because production risks were ignored in the model. To lessen the divergence between the model and reality, they incorporated into the LP the minimum subsistence requirements estimated on the consumption side of the farm-household model. The implication here (although the authors were apparently unaware of it) is that the assumption of separability between production and consumption decisions was invalid.

[12] The non-separable model is described in full by Delforce (1993).

Appendix

A.1. Formulae for calculating integrated elasticities

The starting point for the derivation of formulae for integrated elasticities is the demand function for each expenditure category. Following Barnum and Squire (1979) and Singh et al. (1986c), these can be expressed in general terms as:

$$X = X(p, w, Y; a_i) \tag{A1}$$

with

$$Y = \pi(p, w, \alpha_i, A) + wT + E \tag{A2}$$

where p is commodity prices, w is the wage rate, Y is full income, a_i is household characteristics, π is profit, α_i is a set of technical parameters governing production, A_i is land area operated, T is total time available to the household, and E is exogenous income.

Given the above functions, the effect on demand for subsistence commodities (LOCSTAP and LOCPROT) of a change in own-price is given by:

$$\mathrm{d}S/\mathrm{d}p = (\partial S/\partial p) + (\partial S/\partial Y)(\partial\pi/\partial p) \tag{A3}$$

where S is expenditure on a home-produced commodity.

Expressing (A3) in terms of elasticities (i.e. multiplying both sides by p/S) gives:

$$(\mathrm{d}S/\mathrm{d}p)(p/S) = (\partial S/\partial p)(p/S) + (p/S)(\partial S/\partial Y)(\partial\pi/\partial p) \tag{A4}$$

If the second term on the right hand side is multiplied by $Y\pi/Y\pi$, the above expression can be rearranged and simplified to give:

$$\eta = \epsilon + (\partial S/\partial Y)(Y/S)(\partial\pi/\partial p)(p/\pi)(\pi/Y) \tag{A5}$$

where

$$\eta = (\mathrm{d}S/\mathrm{d}p)(p/S)$$

and

$$\epsilon = (\partial S/\partial p)(p/S)$$

The term ϵ is easily recognisable as the own-price elasticity of demand estimated from a consumption model such as the AIDS discussed in Section 3.2, while $(\partial S/\partial Y)(Y/S)$ is the expenditure elasticity. The profit effect is captured by the term $(\partial\pi/\partial p)(p/\pi)$, which can be derived from the LP production model of Section 3.1 (see below). Thus, η can be interpreted as the own-price elasticity of demand for commodity S "on the assumption that farm profits are allowed to vary according to the dictates of profit maximisation" (Barnum and Squire, 1979, p. 79).

In similar fashion, the effect on leisure expenditure of a change in the wage rate can be identified as:

$$\mathrm{d}wL/\mathrm{d}w = \partial wL/\partial w + \partial wL/\partial Y(\partial\pi/\partial w + T) \tag{A6}$$

where wL is the value of leisure expenditure, as modelled in the AIDS. Expressing this equation in elasticity terms, and rearranging and simplifying as before, gives:

$$\eta = \epsilon + (\partial wL/\partial Y)(Y/wL)(\partial\pi/\partial \mathrm{w}) \times (w/\pi)(\pi/Y) \tag{A7}$$

but this time:

$$\epsilon = (\partial wL/\partial w)(w/wL) + [(\partial wL/\partial Y)/wL](wT) \tag{A8}$$

Computation of the second term on the right-hand side of Eq. (A8) is simplified by the fact that $(\partial wL/\partial Y)(Y/wL)$ is the expenditure elasticity of leisure in the AIDS model, so $[(\partial wL/\partial Y)/wL]$ is obtained by dividing the elasticity by Y.

The effect on food and non-food commodity groups of a change in wage rate can be expressed as:

$$\mathrm{d}X/\mathrm{d}w = \partial X/\partial w + \partial X/\partial Y(\partial\pi/\partial w + T) \tag{A9}$$

and rearranged as before to give:

$$\eta = \epsilon + (\partial X/\partial Y)(Y/X)(\partial\pi/\partial w)(w/\pi)(\pi/Y) \tag{A10}$$

where

$$\epsilon = (\partial X/\partial w)(w/X) + [(\partial X/\partial Y)/X]wT$$

However, since imported foods and non-foods are not represented in the LP model, the profit effect is zero and $\eta = \epsilon$ for those commodities. Eq. (A10) is consequently only applied to the analysis of home-produced foods (LOCSTAP and LOCPROT).

A.2. Information from the LP model

Using a programming model, production-side elasticities are, of necessity, arc elasticities. For instance, the effect on profit of a change in the price of local staples would be estimated as follows:

$$\epsilon = (\Delta\pi/\Delta P)(p/\pi) \qquad \text{(A11)}$$
$$= (\pi_1 - \pi_0)/(p_1 - p_0)(p_0 + p_1)(\pi_0 + \pi_1)$$

This simple formula disguises a rather complex issue when dealing with programming models, namely, which points to choose for p_0 and p_1. The response of output to price in an LP model cannot be represented as a continuous function, since the levels of the activities change in a step-wise fashion as relative prices change. Thus, simply calculating the difference in profits resulting from a price change of given magnitude (say, 10%) may be inappropriate. For instance, the arc elasticity calculated for a 10% rise in price may be quite different from that which corresponds to a 10% price *fall*. In previous studies in which the production side of a separable household model has been modelled using LP, the precise procedure followed in determining arc elasticities is not clear. Singh and Janakiram (1986, p. 103), for instance, explain that "arc elasticities are calculated simply by carrying out the parametric changes in the LP models and calculating the changes in profits (and incomes) that result...".

Given the nature of the LP model and its solution, it would seem to be most appropriate to calculate arc elasticities across the entire range of solutions within which the relevant activity or group of activities remains in the basis. The easiest way to do this, for instance in the case of the effect on profit of a change in local staple price (Eq. A6), is to take p_0 as being the lowest price at which local staple activities enter the basis, and p_1 as being the highest price within which staple crop production remains price responsive (i.e. no further increases in supply occur when price is raised further).

Table A.1
Elasticity estimates from LP model

	Expenditure category		
	LOCSTAP	LOCPROT	LEIS
P_0	0.95	0.009	0.475
P_1	1.57	0.095	13.25
π_0	6645.1	7021.5	6827.6
π_1	9860.4	−430.1	13109.8
$(\partial\pi/\partial P)(P/\pi)$	0.7918	−1.367	0.3385

The price elasticities calculated using the above method are summarised in Table A1 for the three expenditure categories represented in both the production and consumption models: local staples, local protein foods and leisure.

The other information required from the LP model for the computation of integrated elasticities is the proportion of income which comes from production profits (π/Y). In Eq. (A2), Y was defined as the sum of production profits, the value of available time and exogenous income. In annual terms and for the representative profit-maximising household, profit is T$6838, total time is 18132 hours,[13] the wage rate is T$1 per hour and exogenous income is assumed to be T$522. This gives a value of Y of:

$$Y = 6\,838 + 18\,132 + 522 = 25\,492 \qquad \text{(A12)}$$

and

$$\pi/Y = 6\,838/25\,492 = 0.268 \qquad \text{(A13)}$$

With the above information, Eqs. (A5), (A7) and (A10) can be calculated, giving the results presented in Table 3 and discussed in the text.

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[13] Total time is estimated as 14 h/day × 13.558 adults × 7 days × 52 weeks.

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