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Estimation of stochastic frontier production functions with time-varying parameters and technical efficiencies using panel data from Indian villages

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ABSTRACT

A stochastic frontier production function with time-varying technical efficiencies is estimated using panel data from ICRISAT's Village Level Studies in three Indian villages. Given the specifications of a linearized version of a Cobb–Douglas stochastic frontier production function with coefficients which are a linear function of time, the hypothesis that the traditional response function is an adequate representation of the data is accepted for only one of the three villages. The hypothesis of time-invariant technical inefficiencies is not rejected for one of the two villages for which significant technical inefficiencies exist. The hypothesis of time-invariant elasticities of the input variables is rejected for two of the three villages. Further, the hypothesis that hired and family labour are equally productive is accepted in only one of the three villages.

The technical efficiencies of individual farms exhibited considerable variation in the two villages with either time-invariant or time-varying technical efficiencies.

1. INTRODUCTION

Frontier production functions and technical efficiency of individual firms have been considered in a large number of papers in economic, statistical

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and econometric journals. Battese (1992) presents a review of the concepts and models which have been suggested and surveys applications which have appeared in agricultural economics journals.

Frontier production functions assume the existence of technical inefficiency of the different firms involved in production, such that, for specific values of factor inputs, the levels of production are less than what would be the case if the firms were fully technically efficient. The majority of the earlier applications of frontier production functions involved cross-sectional data. However, more recently attempts have been made to apply frontier production functions in the analysis of time-series data on firms involved in production. Initially the firm effects associated with the existence of technical inefficiency were assumed to be time-invariant random variables or independent and identically distributed over time. More recently, models for frontier production functions have been proposed in which the firm effects associated with technical inefficiency are assumed to be time varying [see Kumbhakar (1990), Cornwell, Schmidt and Sickles (1990) and Battese and Coelli (1992)].

In this paper, we apply the model proposed in Battese and Coelli (1992) in the analysis of panel data provided by the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT) at Patancheru (near Hyderabad) in the Indian State of Andhra Pradesh.

2. ICRISAT's VILLAGE LEVEL STUDIES

The data used in this study are from the village level studies (VLS) in which ICRISAT personnel collected a range of data from households engaged in agricultural production in different villages in India. As a part of its mandate, ICRISAT initiated its village level studies in 1975 to obtain reliable data on traditional agricultural methods in the Semi-Arid Tropics (SAT) of India so that improved technological methods could be introduced [see Jodha, Asokan and Ryan (1977), Binswanger and Jodha (1978) and Walker and Ryan (1990)].

The three villages involved in the VLS studies of ICRISAT were selected from districts which represented the broad agro-climatic subregions in the SAT of India. The main factors considered in the selection of the districts included soil types, rainfall and cropping pattern. Accessibility to agricultural universities or research stations and development programs and proximity to the ICRISAT's headquarters at Patancheru near Hyderabad were also given important consideration. Within the selected districts, talukas (subdivisions of a district) were selected which represented the typical characteristics in terms of land-use pattern, cropping, irrigation, livestock, infra-structural development, population, etc. Villages which were located near large towns or have special government or other programs were not considered in the sample. The data used in this study were collected from three villages, Aurepalle, Shirapur and Kanzara, during the years 1975–76 to 1984–85. Data on factor inputs and total production were obtained for a random sample of households in each village.

All households in each village were divided into two main groups. The agricultural labour group consisted of households operating less than 0.2 ha of land and the cultivator group consisted of households operating at least 0.2 ha of land. The cultivator group was further classified into three equal groups and ranked as small, medium and large farmers depending on the size of their holdings.

A random sample of ten households was selected from each group of farmers including the agricultural labour group so that 40 sample farmers were selected from each village. However, this study does not include the agricultural labour group for the purpose of the analysis of the frontier production functions. During the ten-year period involved, some households which were originally classified as labour farmers became small farmers in the later years and hence were included in the sample. Farmers who refused to provide information or ceased to be members of the sample were replaced by other farmers. Hence the numbers of sample households in each village, as well as the number of time-series observations for each household, were not necessarily equal.

The villages of Aurepalle, Shirapur and Kanzara were selected from the districts of Mahbubnagar, Sholapur and Akola, respectively, and are located approximately 70 km south, 336 km west and 550 km north of Hyderabad, respectively. There were 3141 people in Aurepalle, 2017 people in Shirapur and 1380 people in Kanzara in 1985.

Considerable soil heterogeneity is a characteristic of the SAT of India. Aurepalle has medium and shallow alfisols (red soils) with low water retention capacity. Soil heterogeneity is remarkably high in Aurepalle compared with Shirapur and Kanzara. Shirapur has medium and deep vertisols (black soils) with high moisture-retention capacity. Kanzara has mainly medium-deep black soils and shallow vertisols with medium moisture retention capacity. Soils in Kanzara are more homogeneous than in Aurepalle and Shirapur.

Rainfall in the SAT of India is generally erratic in distribution and the mean annual rainfall ranges from about 400 mm to 1200 mm. In the years 1975 to 1985 the average annual rainfall was 611 mm for Aurepalle, 629 mm for Shirapur and 850 mm for Kanzara. Rainfall is very erratic and uncertain in Aurepalle and Shirapur.

Walker and Ryan (1990) report that during four years of the study period Aurepalle and Shirapur had very little rainfall. Rainfall is relatively higher and less variable in Kanzara. Agriculture is predominantly dryland with two main seasons, the rainy season (kharif) which spans the months of June to October followed by the post-rainy (rabi) season.

In Aurepalle, dryland crops include sorghum, pearl millet, pigeonpea, castor and high-yielding variety (HYV) paddy. Sorghum, pearl millet and pigeonpea are intercropped, usually with one row of pigeonpea to four rows of cereal crops. The high-yielding variety paddy is mostly grown under irrigated conditions. Of the total cropped land, about 21% is irrigated in Aurepalle, compared with 9% and 7% in the villages of Shirapur and Kanzara, respectively.

The rabi season has more reliable rainfall in the village of Shirapur. During the rabi season farmers grow mainly sorghum and chickpea. Local wheat and safflower are also grown. Irrigation is used for onions, chillies and other vegetables. However, the use of high-yielding varieties is very limited in Shirapur.

The village of Kanzara has relatively favourable rainfall in the kharif season and the crops grown include cotton, pigeonpea, hybrid sorghum, local sorghum, groundnut, green gram and black gram. Wheat and chickpea are mainly planted in the rabi season. Intercropping is more prevalent in Kanzara than in the other two villages. The use of improved technology, such as high-yielding varieties of sorghum and cotton, fertilizers and pesticides, is also high in Kanzara compared with the villages of Aurepalle and Shirapur.

There exists a large variation in the cropping patterns among the three villages. This variation is associated with differences in soil heterogeneity, rainfall pattern and other factors among the villages. Shirapur has the highest proportion of area cropped under cereals, of which local sorghum contributes about 62% of the total cultivated land in the village. The area under cereals in Aurepalle and Kanzara is about 50% and 30%, respectively. Oil crops play an important role in Aurepalle, where castor contributes about 35% of the total cropped land, followed by sorghum and paddy which contribute about 20% each. Cotton is a sole crop in Kanzara. It occupies about 40% of the cultivated land in the village.

A similar variation exists in the marketed output of crops in the three villages. The crops which have the largest proportion of marketed output are castor in Aurepalle, cotton in Kanzara and sunflower in Shirapur. The cereal crops, sorghum, pearlmillet, paddy and wheat, are mainly subsistence crops.

The labour market includes cultivators and agricultural labourers who comprise about two-thirds of the active workers in SAT India. The labour market is active in the three villages. However, the use of labour (family and hired) varies from village to village, as well as from year to year, depending on rainfall, soil type, the particular crop(s) involved, extent of irrigation, etc. Farm households depend heavily on hired labour to cultivate their land. In Aurepalle and Kanzara, hired labour provides the majority (60–80%) of the total labour used in crop production. The high demand for hired labour is due to the activities of paddy transplanting in Aurepalle and cotton picking in Kanzara. The labour force comprises men, women and children, but the latter only make a very small contribution. The contribution of men to the total family labour in crop production is substantially higher than women, while women dominate the hired labour market.

In all the villages, cultivation such as plowing, harrowing and interculturing is carried out using animal draft power, usually involving bullocks. However, many households which own small areas of land do not have bullocks. Seasonal hiring is common, especially by small farmers. It is most common in Shirapur where bullock-to-land ratios are significantly lower than in the other two villages. Single bullock owners often pool their bullocks and cultivate on an exchange basis.

Fertilizer is used almost entirely for irrigated agriculture in the study villages. However, the use of fertilizer in dryland agriculture is increasing in the rainfall-assured village of Kanzara and, to some extent, in Aurepalle. For example, the use of fertilizer in dryland farming has increased from 3% in 1975–76 to 50% by 1985–86. However, application rates per ha remained very low.

Manure plays an important role in the study villages. Many farmers apply manure to their land every year. However, the supply of manure is constrained by limited availability of fodder which restricts livestock production as well as its use for fuel.

Pesticides are applied mainly in irrigated agriculture, although the expenditure on fertilizers is much higher (about nine times) than the expenditure on pesticides. Pesticides are widely applied in the villages of Aurepalle and Kanzara.

3. FRONTIER PRODUCTION FUNCTION MODEL

The stochastic frontier production function considered in this paper is assumed to be defined by:

$$\log Y_{it} = \beta_{0t} + \beta_{1t} \log(\text{LAND}_{it}) + \beta_{2t} \log(\text{LABOUR}_{it}) + \beta_{3t} \log(\text{BULLOCK}_{it}) + \beta_{4t} D_{it} \log(\text{COST}_{it}) + \beta_{5t}(I_{it}/\text{LAND}_{it}) + \beta_{6t}(H_{it}/\text{LABOUR}_{it}) + E_{it} \quad (1)$$

where Y_{it} is the total value of output (in Rupees, expressed in terms of

1975–76 value terms) [1] for the ith farmer, $i = 1, 2, \ldots, N$, in the tth year of observation, $t = 1, 2, \ldots, T$, where $T = 10$; and:

LAND_{it} is the total hectares of unirrigated land and irrigated land (I_{it}) under production, respectively, for the ith farmer in the tth year of observation;

LABOUR_{it} is the hours of family labour and hired labour (H_{it}) for the ith farmer in the tth year of observation (in male equivalent units); [2]

BULLOCK_{it} is the total hours of owned bullock and hired bullock labour (in pairs), respectively, for the ith farmer in the tth year of observation;

COST_{it} is the total cost of inputs (involving inorganic fertilizer, organic matter applied to land, pesticides and machinery costs) for the ith farmer in the tth year of observation;

D_{it} is a dummy variable which has values zero or one if the total cost of inputs was zero or positive, respectively;

β_{jt}, $j = 0, 1, \ldots, 6$, are parameters to be estimated whose values may vary with year of observation, t; and

E_{it} is a random variable whose distributional properties are defined by equation (2) below.

The deterministic component of this production function is formulated from the work of Bardhan (1973), Deolalikar and Vijverberg (1983, 1987) and Battese, Coelli and Colby (1989). Bardhan (1973) considered a production function of Cobb–Douglas type in which the variables, total labour (family plus hired labour hours) and the proportion of hired labour to total labour, were separately included as explanatory variables. Bardhan (1973) used Indian farm-level data and concluded that hired and family labour were heterogeneous in some cases.

Deolalikar and Vijverberg (1983) defined a more general model of CES type in the analysis of district-level data for Indian farms. Several special cases of the CES model were considered. They concluded that the model in which hired and family labour were included as separate explanatory variables was the best one. Deolalikar and Vijverberg (1983) also considered unirrigated and irrigated land in their production function. They concluded that the best model had a weighted average of the unirrigated and irrigated areas operated as the land variable.

[1] The values of output (and input costs) were deflated using price indices which were constructed from data on prices and quantities of commodities grown in the three villages.

[2] Labour hours were converted to male equivalent units based on the conversion rule that female and child labour hours are equivalent to 0.75 and 0.50 male hours, respectively. These factors are used by ICRISAT in empirical analyses involving labour of different family members.

Battese, Coelli and Colby (1989) considered a Cobb–Douglas-type production function in which the labour and land variables were weighted averages of their respective hired and family labour hours and unirrigated and irrigated hectares. Battese, Coelli and Colby (1989) included cost of inputs in the Cobb–Douglas production function, provided input costs were positive. However, if input costs were zero (as was the case for a large proportion of the farms involved), then input costs were not included in the function.

The production function, defined by equation (1), implies that the total cost of inputs enters when it has positive values. Hence, this model implies that farms which use fertilizer, manure, pesticides and machinery have different intercept values than those farms which do not have positive input cost, but the latter farms do not have zero levels of production.

The land- and labour-ratio variables, I_{it}/LAND_{it} and $H_{it}/\text{LABOUR}_{it}$, are included to account for the possible differences in the contributions to production from unirrigated versus irrigated land and family versus hired labour, respectively. For example, if the productivities of family and hired labour were equal, then the coefficient, β_{6t}, of the labour-ratio variable would be zero. Hence a statistical test of the hypothesis that family and hired labour were equally productive can be obtained by testing that the coefficient of the labour-ratio variable was zero.

The residual random variable, E_{it}, in the production function, defined by equation (1), is assumed to be defined by:

$$E_{it} = V_{it} - U_{it} \tag{2}$$

where

$$U_{it} = \eta_{it} U_i = \{\exp[-\eta(t-T)]\} U_i \tag{3}$$

and the V_{it}'s are assumed to be independently and identically distributed as the normal random variable with mean zero and variance, σ_V^2, independently of the U_i's which are assumed to be non-negative truncations of the normal distribution with mean, μ, and variance, σ^2, where μ, σ^2 and σ_V^2 are unknown parameters to be estimated.

The decomposition of the residual random variable, E_{it}, in the production function (1), as specified in equation (2), is that which defines the stochastic frontier production function, first proposed by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977). The first term, V_{it}, is a random error, having zero mean, which is assumed to be involved in the traditional linear regression model. The second term, U_{it}, is a non-negative random variable, which is assumed to account for the existence of technical inefficiency of production of the ith firm at the tth period of observation. The subtraction of the non-negative random variable, U_{it}, from the random error, V_{it}, implies that the logarithm of production is smaller than it would otherwise be if technical inefficiency did not exist [see Battese (1992) for a more extensive review of concepts and models for frontier production functions].

The time-varying behaviour of the non-negative firm effects, U_{it}, defined by equation (3), is that defined by Battese and Coelli (1992). This model implies that if the parameter, η, is positive then the non-negative firm effects for the ith firm, U_{it}, decline exponentially to its minimum value, U_i, at the last period, T, of the panel. In this case the firms would be increasing in their technical efficiency of production over time. If, however, η was zero, then the firm effects associated with technical inefficiency of production would be constant over time (i.e., firms never improve in their technical efficiency). The estimation of the parameter, η, and testing the hypothesis that its true value is zero, is obviously of basic interest in the study of technical inefficiency of production. [3]

Parameter estimates for this stochastic frontier production function are obtained by use of the computer program, FRONTIER, Version 2.0, developed by Coelli (1991, 1992). [4] The parametric model is estimated in terms of the variance parameters, $\sigma_S^2 \equiv \sigma_V^2 + \sigma^2$ and $\gamma = \sigma^2/(\sigma_V^2 + \sigma^2)$. The parameter, γ, has a value between zero and one, such that the value of zero is associated with the traditional response function, for which the non-negative random variable, U_{it}, is absent from the model.

The frontier production function (1)–(3) is equivalent to the traditional response function if the parameters, γ, η and μ, are simultaneously equal to zero. Hence a test of the null hypothesis, H_0: $\gamma = \eta = \mu = 0$, is desirable to test whether the traditional response function is an adequate representation, given the specifications of the stochastic frontier production function involved. Also, if the parameter, μ, was zero, then the farm effects associated with the last period of observation in the panel would have half-normal distribution. Hence testing the null hypothesis that η is zero is

[3] As stated in Battese and Coelli (1992), the exponential specification of the behaviour of the firm effects, U_{it}, over time, given by equation (3), is a rigid parameterization. Alternative models are being investigated, but no programming algorithms for them have been completed.

[4] The program, FRONTIER, Version 2.0, can be obtained without cost from Tim Coeli, Department of Econometrics, University of New England, Armidale 2351, Australia (e-mail address: tcoelli@metz.une.edu.au). The program assumes that the stochastic frontier production function is of Cobb–Douglas type and so the logarithmic transformation of the variables of the model is performed. Hence the input data for the land- and labour-ratio variables, in the model of equation (1), must be in their exponential (or anti-logarithmic) form so that, when logarithms are taken, the variables involving the proportions of irrigated land to total land and hired labour to total human labour are correctly obtained.

of interest to test if the stochastic frontier production function has time-invariant inefficiencies of production. The generalized likelihood-ratio test statistic can be easily calculated from the logarithms of the likelihood function associated with the unrestricted and restricted maximum-likelihood estimates for the special case in which the appropriate parameters are zero by using the program, FRONTIER, Version 2.0.

The coefficients of the explanatory variables in the production function (1), β_{0t}, $\beta_{1t}, \ldots, \beta_{6t}$, are permitted to vary with year of observation, t. However, in order to obtain a model which is more parsimonious so far as the number of independent parameters is concerned, we consider a simplified frontier production function in which the coefficients of the explanatory variables are assumed to be a linear function of time, as defined by equation (4):

$$\beta_{jt} = \beta_j + \delta_j(\text{year}_{it}) \equiv \beta_j + \delta_j \times t, \quad j = 0, 1, \ldots, 6 \tag{4}$$

where β_j and δ_j, $j = 0, 1, \ldots, 6$, are unknown parameters to be estimated.

Thus the frontier production function (1)–(4) has time-varying coefficients of the explanatory variables and time-varying technical inefficiencies. The coefficients of the explanatory variables in the production frontier would be time invariant if the time coefficients, δ_0, $\delta_1, \ldots, \delta_6$, had zero values. The hypothesis of time-invariant parameters could be tested by performing the likelihood-ratio test of the null hypothesis, H_0: $\delta_0 = \delta_1 = \ldots = \delta_6 = 0$. Further, the hypothesis that Hicksian-neutral technical change applies could be tested by performing the likelihood-ratio test of the null hypothesis that all coefficients of the explanatory variables, other than the intercept, are time invariant, i.e., H_1: $\delta_1 = \delta_2 = \ldots = \delta_6 = 0$.

The following section deals with the empirical analyses of the data obtained from the three villages, Aurepalle, Shirapur and Kanzara. It is expected that different parameter estimates and technical efficiencies are likely because of the substantial differences in the agro-climatic environments among the three villages.

4. EMPIRICAL RESULTS

A summary of the data on the different variables in the frontier production function (1) is given in Table 1. It is evident from these statistics that Aurepalle farmers tend to be smaller in terms of value of output and total land operated. Kanzara farmers had the highest mean value of output, human labour and bullock labour. Kanzara farmers have the least amount of irrigation because of the relatively assured rainfall, whereas Aurepalle farmers have the greatest amount of irrigation because of the prevalence of growing paddy.

TABLE 1

Summary statistics for variables in the stochastic frontier production function for farmers in Aurepalle, Shirapur and Kanzara

Variable	Sample mean	Sample standard deviation	Minimum value	Maximum value
Value of output (Rs. in 1975–76 values)				
Aurepalle	3559.9	4482.7	7.2	18094
Shirapur	3689.1	3437.2	22.0	26423
Kanzara	5206.7	7207.7	121.6	39168
Land (ha) ≡ Unirrigated + Irrigated land (I)				
Aurepalle	4.23	3.80	0.16	20.97
Shirapur	6.63	5.45	0.61	24.19
Kanzara	5.99	7.38	0.40	36.34
Proportion of Irrigated land ≡ $I/$Land				
Aurepalle	0.14	0.21	0	1.0
Shirapur	0.13	0.24	0	1.0
Kanzara	0.06	0.13	0	1.0
Labour (h) ≡ Family + Hired labour (H)				
Aurepalle	2133.5	2697.4	18	12916
Shirapur	1658.9	1558.6	40	11146
Kanzara	2565.7	3138.7	58	15814
Proportion of Hired labour ≡ $H/$labour				
Aurepalle	0.42	0.29	0	0.98
Shirapur	0.45	0.20	0.06	0.98
Kanzara	0.56	0.27	0.016	0.996
Bullock labour (h of bullock pairs)				
Aurepalle	518.9	592.8	8	4316
Shirapur	340.6	280.5	14	1240
Kanzara	567.3	763.5	12	3913
Cost of other inputs (Rs.)				
Aurepalle	626.4	963.3	0	6205
Shirapur	458.8	1023.8	0	6746
Kanzara	626.0	975.8	0	5344

Bullock labour is used considerably more in Kanzara and Aurepalle than in Shirapur. Cost of inputs had a high proportion of zero observations in all three villages and so the sample means were not very large in all three cases.

The stochastic frontier production function (1)–(4) consists of 18 parameters, fourteen being associated with the explanatory variables of the function and four being parameters which specify the distributions of the random variables, V_{it} and U_{it}. The maximum-likelihood estimates for the parameters of the frontier production functions with time-invarying parameters and technical inefficiencies for the three villages are presented in

TABLE 2

Maximum-likelihood estimates for parameters of the stochastic frontier production functions with time-varying coefficients for farmers in Aurepalle, Shirapur and Kanzara

Variable	Parameter	M.L. estimates for production frontiers in		
		Aurepalle	Shirapur	Kanzara
Constant	β_0	1.66	4.27	2.40
		(0.96)	(0.64)	(0.49)
Year	δ_0	−0.16	−0.32	−0.09
		(0.17)	(0.10)	(0.12)
Log(LAND)	β_1	0.42	0.38	0.45
		(0.22)	(0.11)	(0.20)
Year × Log(LAND)	δ_1	−0.031	−0.047	−0.062
		(0.042)	(0.021)	(0.026)
Log(LABOUR)	β_2	1.12	0.48	0.915
		(0.32)	(0.10)	(0.092)
Year × Log(LABOUR)	δ_2	0.035	0.068	−0.034
		(0.055)	(0.017)	(0.014)
Log(BULLOCK)	β_3	−0.54	−0.10	−0.24
		(0.36)	(0.12)	(0.14)
Year × Log(BULLOCK)	δ_3	−0.007	−0.001	0.055
		(0.068)	(0.020)	(0.023)
Log(COST)	β_4	−0.056	−0.058	−0.003
		(0.039)	(0.027)	(0.026)
Year × Log(COST)	δ_4	0.0094	−0.0098	0.0196
		(0.0065)	(0.0049)	(0.0039)
I/LAND	β_5	0.84	0.43	0.80
		(0.88)	(0.30)	(0.66)
Year × (I/LAND)	δ_5	−0.15	−0.113	−0.068
		(0.14)	(0.046)	(0.052)
H/LABOUR	β_6	−0.58	0.35	−0.18
		(0.57)	(0.29)	(0.22)
Year × (H/LABOUR)	δ_6	0.043	−0.049	0.002
		(0.091)	(0.043)	(0.032)
	$\sigma_S^2 = \sigma_V^2 + \sigma^2$	0.230	0.179	0.111
		(0.084)	(0.033)	(0.014)
	$\gamma = \sigma^2/\sigma_S^2$	0.26	0.403	0.16
		(0.29)	(0.083)	(0.20)
	η	−0.066	0.236	0.02
		(0.081)	(0.035)	(0.15)
	μ	−0.43	−0.56	0.30
		(0.78)	(0.31)	(0.19)
	Log-likelihood	−157.67	−120.20	−85.87

Estimated standard errors produced by FRONTIER, Version 2.0, are given below the estimates to two significant digits. The parameter estimates are given correct to the corresponding number of digits behind the decimal places.

Table 2. Before discussing individual estimates for the parameters in the models, we consider various tests of hypotheses about some of the parameters to determine the preferred frontier production functions for the different villages.

Tests of hypotheses about the distribution of the random variables associated with the existence of technical inefficiency and residual error are given in Table 3. The last column of Table 3 gives the probability of exceeding the calculated χ^2-value if the respective null hypothesis is true. This value is called the 'prob-value' and the null hypothesis is rejected if the prob-value is smaller than the desired value for the probability of a Type I error.

The statistics in Table 3 suggest that, given the specifications of the stochastic frontier production function with time-varying coefficients and time-varying technical inefficiencies (1)–(4):

(1) For Aurepalle farmers, the hypothesis that the traditional response function is an adequate representation of the data would not be rejected, given that the desired probability of a Type I error was not greater than 0.10. Hence it could be concluded that technical inefficiency is not evident for Aurepalle farmers, given their level of technology.

TABLE 3

Statistics for tests of hypotheses for parameters of the distribution of the farm effects, U_{it}, associated with the stochastic frontier production function with time-varying coefficients for the farmers in Aurepalle, Shirapur and Kanzara

Null hypotheses	Log-likelihood	χ^2-value $\equiv c$	Prob-value $\equiv p$ $P(\chi^2 > c \mid H_0$ true)
H_0: $\gamma = \eta = \mu = 0$			
Aurepalle	−158.21	1.08	$0.75 < p < 0.90$
Shirapur	−173.94	107.48	$p < 0.005$
Kanzara	−94.09	15.64	$p < 0.005$
H_0: $\eta = \mu = 0$			
Aurepalle	−158.12	0.90	$0.50 < p < 0.75$
Shirapur	−150.82	61.24	$p < 0.005$
Kanzara	−87.59	2.64	$0.25 < p < 0.50$
H_0: $\eta = 0$			
Aurepalle	−157.79	0.24	$0.50 < p < 0.75$
Shirapur	−149.52	58.64	$p < 0.005$
Kanzara	−85.97	0.60	$0.50 < p < 0.75$
H_0: $\mu = 0$			
Aurepalle	−158.09	0.84	$0.25 < p < 0.50$
Shirapur	−146.38	52.36	$p < 0.005$
Kanzara	−87.83	3.12	$0.05 < p < 0.10$

(2) For Shirapur farmers, the null hypotheses, that the traditional response function is an adequate representation, or that the farm effects associated with technical inefficiency are time invariant and/or have half-normal distribution, would be rejected, given that the probability of a Type I error was 0.10. Thus none of the sub-models considered would be an adequate representation of the frontier production function for Shirapur farmers. However, the technical inefficiency of production of Shirapur farmers decreases over time because of the estimate of the time-varying inefficiency parameter being positive, i.e., $\hat{\eta} = 0.236$.

(3) For Kanzara farmers, the null hypothesis, that the technical inefficiencies of farmers are time invariant and the inefficiency effects have half-normal distribution, would not be rejected by the data, given that the probability of a Type I error did not exceed 0.10. This implies that, although Kanzara farmers exhibit significant technical inefficiency, they do not improve their level of efficiency over the period of the panel of observations.

Given these conclusions about the time-varying nature of the technical inefficiencies of farmers (when present) in the stochastic frontier production functions, various null hypotheses about the coefficients of the frontiers for the three villages are considered. The relevant test statistics are presented in Table 4 for three particular null hypotheses.

TABLE 4

Statistics for tests of hypotheses for coefficients of the explanatory variables of the appropriate stochastic frontier production functions for farmers in Aurepalle, Shirapur and Kanzara

Null hypotheses	Log-likelihood	χ-value $\equiv c$	Prob-value $\equiv p$ $P(\chi^2 > c \mid H_0 \text{ true})$
H_0: $\delta_0 = \delta_1 = \ldots = \delta_6 = 0$			
Aurepalle	−180.74	45.06	$p < 0.005$
Shirapur	−131.13	21.86	$p < 0.005$
Kanzara	−100.54	25.90	$p < 0.005$
H_0: $\delta_1 = \delta_2 = \ldots = \delta_6 = 0$			
Aurepalle	−165.95	15.48	$0.010 < p < 0.025$
Shirapur	−131.01	21.62	$p < 0.005$
Kanzara	−100.53	25.88	$p < 0.005$
H_0: $\beta_6 = \delta_6 = 0$			
Aurepalle	−163.82	11.22	$p < 0.005$
Shirapur	−121.80	3.20	$0.10 < p < 0.25$
Kanzara	−89.04	2.90	$0.10 < p < 0.25$

The loglikelihood values are calculated assuming that the stochastic frontier production functions for the three villages have $\gamma = \eta = \mu = 0$ for Aurepalle; γ, η and μ are free parameters for Shirapur; and $\eta = \mu = 0$ for Kanzara.

The first null hypothesis, H_0: $\delta_0 = \delta_1 = \ldots = \delta_6 = 0$, which implies that the stochastic frontier production function (1)–(4) has time-invariant coefficients, would be rejected for all three villages, given that the appropriate probability of a Type I error was 0.01. The second null hypothesis, H_0: $\delta_1 = \delta_2 = \ldots = \delta_6 = 0$, which specifies that the elasticities of the input variables in the production frontier are time invariant, would be accepted for Aurepalle, but rejected for Shirapur and Kanzara, if the desired probability of a Type I error was 0.01. The third null hypothesis, H_0: $\beta_6 = \delta_6 = 0$, which implies that hired and family labour are equally productive, would be rejected for Aurepalle, but accepted for Shirapur and Kanzara, if the probability of a Type I error was 0.01.

On the basis of the above tests of hypotheses, we conclude that the preferred models for the frontier production functions for the three villages are:

(1) For Aurepalle: the traditional response function, involving no technical inefficiency, in which the elasticities of the input variables are time invariant, but year of observation is included to account for technological change.

(2) For Shirapur: the stochastic frontier production function with time-varying technical inefficiency and time-varying elasticities for the explanatory variables, for which the labour-ratio variable is absent.

(3) For Kanzara: the stochastic frontier production function with time-invariant technical inefficiencies, which have half-normal distribution, but time-varying coefficients of the explanatory variables, such that family and hired labour are equally productive.

Given the specifications of the stochastic frontier production functions with time-varying parameters and technical efficiencies which are considered in this paper, the estimated parameters for the preferred frontier models are presented in Table 5. For Aurepalle, the rate of technical change of production was estimated to be about 5.5% per year for the sample period; the elasticities of land and labour were about 0.29 and 1.44, respectively, whereas the elasticities of bullock labour and costs were negative with values about -0.62 and -0.01, respectively. The coefficient for bullock labour is significantly different from zero. Negative elasticities of bullock labour have been found in other studies [e.g., Saini (1979), Battese, Coelli and Colby (1989) and Battese and Coelli (1992)]. Various explanations have been suggested for this phenomenon.

For Shirapur, the rate of technical change declined by about 38% per year, but technical inefficiency of production also declined during the sample period, as indicated by the positive estimate for η of about 0.26. The elasticities of production with respect to the factors of production were

TABLE 5

Maximum-likelihood estimates for parameters of the preferred stochastic frontier production functions with time-varying coefficients for farmers in Aurepalle, Shirapur and Kanzara

Variable	Parameter	M.L. estimates for production frontiers in		
		Aurepalle	Shirapur	Kanzara
Constant	β_0	0.46 (0.43)	4.6 (3.7)	2.38 (0.91)
Year	δ_0	0.0547 (0.0099)	−0.38 (0.18)	−0.09 (0.15)
Log(LAND)	β_1	0.287 (0.086)	0.40 (0.58)	0.48 (0.15)
Year × Log(LAND)	δ_1	0	−0.051 (0.057)	−0.064 (0.024)
Log(LABOUR)	β_2	1.441 (0.096)	0.47 (0.65)	0.87 (0.20)
Year × Log(LABOUR)	δ_2	0	0.075 (0.047)	−0.032 (0.033)
Log(BULLOCK)	β_3	−0.618 (0.074)	−0.12 (0.15)	−0.24 (0.15)
Year × Log(BULLOCK)	δ_3	0	−0.001 (0.020)	0.055 (0.025)
Log(COST)	β_4	−0.010 (0.017)	−0.055 (0.036)	−0.002 (0.025)
Year × Log(COST)	δ_4	0	−0.0100 (0.0068)	0.0196 (0.0046)
I/LAND	β_5	0.04 (0.23)	0.49 (0.50)	0.91 (0.49)
Year × (I/LAND)	δ_5	0	−0.121 (0.050)	−0.078 (0.060)
H/LABOUR	β_6	−0.35 (0.11)	0	0
Year × (H/LABOUR)	δ_6	0	0	0
	$\sigma_S^2 \equiv \sigma_V^2 + \sigma^2$	0.191	0.23 (0.30)	0.151 (0.026)
	$\gamma \equiv \sigma^2/\sigma_S^2$	0	0.49 (0.69)	0.36 (0.12)
	η	0	0.26 (0.89)	0
	μ	0	−1.5 (10.1)	0
	Log-likelihood	−165.95	−121.80	−89.04

time varying, such that the elasticity of labour increased over time, whereas the elasticities of land, bullock labour and input costs decreased over time.

For Kanzara, the rate of technical change was estimated to decline by about 9%; the elasticities of land and labour declined over time, whereas the elasticities of bullock labour and input costs increased over time.

From the elasticity estimates from the preferred stochastic frontier production functions in Table 5, it can be shown that the returns-to-scale parameter for the tth year, λ_t, for the three villages is estimated by:

– Aurepalle

$$\hat{gl}_t = \underset{(0.033)}{1.1100}$$

– Shirapur

$$\hat{\lambda}_t = \underset{(0.066)}{0.807} + \underset{(0.020)}{0.012t}$$

– Kanzara

$$\hat{\lambda}_t = \underset{(0.056)}{1.106} - \underset{(0.0084)}{0.0211t}$$

The estimates for the standard errors of the estimators, obtained by using the estimated covariance matrix for the maximum-likelihood estimators calculated by FRONTIER, Version 2.0, are presented below the corresponding estimates. The returns-to-scale parameter for Aurepalle is time invariant but for Shirapur and Kanzara it is time varying, given the nature of the preferred stochastic frontier production functions for those villages. The returns-to-scale parameter is estimated to increase over time for Shirapur but the rate of increase per year does not appear to be significant, whereas the decline in the returns-to-scale parameter per year appears to be significant for Kanzara. Using asymptotic t-tests, the hypothesis of constant returns to scale would be rejected in each case. Generalized likelihood-ratio tests for the null hypothesis of constant returns to scale yielded chi-square statistics of 16.28, 23.56 and 5.60 for Aurepalle, Shirapur and Kanzara. These statistics are highly significant for Aurepalle and Shirapur (even at the 0.5% level). However, the hypothesis of constant returns to scale would be accepted for Kanzara, unless the level of significance was as high as 0.10. [5]

Estimated technical efficiencies of the farmers in Shirapur and Kanzara, obtained by using the predictor suggested by Battese and Coelli (1992), are presented in Tables 6 and 7, respectively. Since farmers in Shirapur have time-varying technical efficiencies, predictions for the technical efficiencies

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[5] The generalized likelihood-ratio tests are preferred to the asymptotic t-tests for testing hypotheses for the frontier functions because there is some concern about the estimated standard errors produced by FRONTIER, Version 2.0 in many cases.

TABLE 6

Predicted technical efficiencies of Shirapur farmers from 1975–76 to 1984–85

Farmer	Technical efficiencies									
	75–76	76–77	77–78	78–79	79–80	80–81	81–82	82–83	83–84	84–85
1	–	0.505	0.589	0.664	0.729	0.784	0.829	0.866	0.895	0.918
2	–	0.227	0.319	0.414	0.507	–	–	–	–	–
3	–	0.848	0.880	0.905	0.926	0.942	0.955	0.965	0.973	0.979
4	–	–	–	–	–	0.807	0.847	0.879	0.905	0.926
5	0.225	0.317	0.413	0.506	0.592	0.668	0.734	0.788	0.833	–
6	0.738	0.789	0.832	0.868	0.896	0.919	0.937	0.951	0.962	0.971
7	0.583	0.658	0.724	0.779	0.825	–	–	–	–	–
8	0.218	0.309	0.404	0.498	0.585	0.662	0.728	0.784	0.829	0.866
9	0.020	–	–	–	0.252	0.346	0.443	0.534	0.618	0.691
10	0.499	0.583	0.659	0.725	0.781	0.826	–	0.893	0.917	0.935
11	0.637	0.704	0.762	0.811	0.851	0.883	0.908	0.929	0.945	0.957
12	0.899	0.921	0.938	–	–	–	–	–	–	–
13	0.541	0.621	0.692	0.753	0.803	0.845	0.878	0.905	0.926	0.943
14	0.650	0.715	0.771	0.818	0.857	0.888	0.912	0.932	0.947	–
15	0.405	0.497	0.583	0.660	0.726	0.782	0.827	0.864	0.894	0.918
16	0.346	0.441	0.531	0.615	0.687	0.749	0.801	0.843	–	–
17	0.399	0.492	0.579	0.656	0.723	0.779	–	–	–	–
18	0.439	0.529	0.612	0.685	0.747	0.799	0.841	0.876	0.903	0.925
19	0.521	0.603	0.677	0.740	0.793	0.836	0.871	0.900	0.922	0.939
20	0.830	0.865	0.894	0.917	0.935	0.950	0.961	–	–	–
21	0.295	–	–	0.569	–	0.716	0.773	0.820	0.859	0.889
22	0.801	0.841	0.875	0.902	0.923	0.940	0.954	0.964	0.972	0.979
23	0.176	–	–	–	0.540	–	–	–	–	–
24	0.579	0.654	0.720	0.776	0.823	0.860	0.891	0.915	0.934	0.949
25	0.664	0.728	0.782	0.827	0.863	0.893	0.916	0.935	0.950	0.961
26	0.402	0.495	0.581	0.658	0.724	0.780	0.826	0.864	0.893	0.917
27	0.865	0.894	0.917	0.935	–	0.961	0.970	0.977	0.982	0.986
28	0.772	0.817	0.855	0.886	0.911	0.931	0.946	0.958	0.968	0.975
29	0.705	0.762	0.810	0.850	0.882	0.908	0.928	0.944	0.957	0.967
30	0.844	0.877	0.903	0.924	0.941	–	–	–	–	–
31	0.895	0.917	0.935	–	–	–	–	–	–	–
32	0.409	–	–	–	–	–	–	–	–	–
33	–	0.337	0.432	0.523	0.607	0.681	–	–	–	–
34	–	–	–	–	–	0.790	0.833	0.868	0.896	0.919
35	–	0.699	0.757	0.806	0.846	–	–	0.926	0.943	0.956
36	–	–	–	–	–	–	–	0.912	0.931	0.946
37	–	0.875	0.902	0.923	0.940	0.953	0.964	0.972	0.978	0.983
38	–	–	–	–	–	–	–	–	–	0.941
Mean	0.548	0.637	0.696	0.750	0.796	0.835	0.869	0.896	0.918	0.936

Values of technical efficiencies are not obtained in years when no observations are observed.

TABLE 7

Predicted technical efficiencies of Kanzara farmers

Farmer	Technical efficiency
1	0.724
2	0.784
3	0.871
4	0.842
5	0.837
6	0.766
7	0.841
8	0.752
9	0.949
10	0.941
11	0.898
12	0.819
13	0.889
14	0.654
15	0.912
16	0.742
17	0.888
18	0.946
19	0.702
20	0.883
21	0.757
22	0.822
23	0.742
24	0.834
25	0.947
26	0.688
27	0.795
28	0.964
29	0.857
30	0.754
31	0.907
32	0.856
33	0.906
34	0.856
35	0.860
Mean	0.839

of the sample farmers are presented for each of the years for which observations were obtained. The technical efficiencies of Shirapur farmers in the first year of observation showed great variability (from 0.020 to 0.899) but increased over the ten-year period to as high as 0.986. However, predictions for the time-invariant technical efficiencies of the Kanzara farmers, presented in Table 7, varied from 0.654 to 0.964. In both cases, the level of technical inefficiency of the farmers involved was considerable for most farmers.

5. CONCLUSION

Our application of a stochastic frontier production function with time-varying parameters and technical inefficiencies in the analysis of panel data from three Indian villages has indicated some important findings:

(1) Different conclusions about technical inefficiency were obtained in the different villages. In Aurepalle, the frontier function was not significantly different from the traditional response function, which can be estimated efficiently by ordinary least-squares regression. Hence it was concluded that technical inefficiency of production was not present among farmers in Aurepalle, given their level of technology. However, in Shirapur and Kanzara, technical inefficiency of production was highly significant. In Kanzara, technical inefficiency was not significantly different over time. However, in Shirapur, where it could not be concluded that technical inefficiency was time invariant, the farmers involved demonstrated increasing levels of technical efficiency over the years observed.

(2) The hypothesis that the coefficients of the explanatory variables (other than the intercept) were time invariant was rejected for the villages of Shirapur and Kanzara. Thus the elasticities of production with respect to the different inputs were found to be time varying in these two villages.

(3) The returns-to-scale parameters for production in the three villages were estimated to be close to one, but they were significantly different from one, given the sample sizes and the model assumptions.

The above results indicate that the inclusion of year-of-observation as an explanatory variable in the frontier model to account for neutral technological change, provided that it is appropriate, does not necessarily imply that technical inefficiency will then be absent from the model, as was found in the empirical example reported by Battese and Coelli (1992).

Given that the agricultural enterprises and environments in the three different villages are significantly different, it is perhaps not surprising that different conclusions are drawn concerning the preferred stochastic frontier production functions in the three villages. The conclusions of our analysis should stimulate further research to explain the basis for the different conclusions which have been made in our analysis. It is not claimed that our theoretical model and the empirical results obtained are the most appropriate for Indian agriculture. We hope that further work can be conducted by, or in collaboration with, researchers with more detailed knowledge of Indian agriculture to obtain better models and empirical results which can describe the agricultural operations involved and stimulate appropriate development.

The empirical application of stochastic frontier production functions for the analysis of panel data requires that the deterministic component of the functions be appropriately modelled, in addition to the stochastic elements associated with technical inefficiency and random error. This is obviously a challenging exercise.

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