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## Estimating feed utilisation matrices using a cost function approach

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### ABSTRACT

In this paper a multiple-output cost function framework is proposed to construct national feed balances or feed utilisation matrices (FUMs). The framework is applied to the Belgian compound feed industry. For estimation purposes a Symmetric Generalised McFadden (SGM) cost function is selected. The cost function is estimated using readily available time-series data for the period 1962–88. Unlike previous studies based on duality theory, this study exploits the properties of nonjointness in animal feed production to establish a complete FUM. The allocation of feed ingredients among different livestock categories as well as the composition of various compound feeds are identified. Also own- and cross-price elasticities of demand for feed ingredients by type of livestock are reported.

### INTRODUCTION

The Food and Agriculture Organization (FAO) recently concluded that “the feed-livestock sector will remain a key sector in assessing future demand for and trade in grains and [that] it would be important to monitor developments in this area” (FAO, 1989, p. 36). For this purpose, the FAO suggested, among other things, the need for (a) improved information on feed availability and utilisation, if possible by type of livestock; (b) improved methodologies for projecting future demand for all types of feeds, taking into account different types of livestock, different livestock production systems and technical and economic possibilities for substitution between feeds; and (c) assessment of the role and potential of other feeds

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such as grain substitutes, under various technical and economic conditions.

The most common procedure to construct national feed balances or feed utilisation matrices (FUMs) is with the use of technical information such as feeding requirements of different livestock categories, nutritional contributions of concentrate feeds, and various feed conversion rates (see, e.g., Parris and Tisserand, 1988; Leuck, 1985). This procedure, however, does not explicitly consider the fact that similar underlying feeding technologies may generate quite different feed utilisation patterns due to differences in relative feed *prices*. Hence, the results emerging from this procedure do not allow to quantify the impact of changing (relative) prices on the feed-livestock economy and/or to assess the distortive effects of agricultural pricing policies. Only few attempts have been made in the past to overcome this weakness by adopting an econometric approach based on duality theory (Surry and Moschini, 1984; Mahé, 1987; Mergos and Yotopoulos, 1988; Surry, 1990). Others have used a linear programming or 'pseudo-data' approach to estimate feed inclusion rates as well as animal-specific elasticity matrices of feed input demand (Paarlberg, 1979; Longmire, 1980; McKinzie et al., 1986; Peterson, 1986; Peeters, 1990).

The purpose of this paper is to propose a multiple-output cost function framework to construct a FUM. Unlike previous studies using duality theory, this study exploits the properties of nonjointness in production to establish a complete and theoretically-consistent feed balance sheet.<sup>1</sup> The conceptual framework is applied to a time series of data for the compound feed industry in Belgium. The allocation of feed ingredients to different livestock categories as well as the composition of various compound feeds will be identified. Also own- and cross-price elasticities of derived demand for feed ingredients by type of livestock will be reported. As indicated by Hall (1973), nonjointness implies that the analysis does not require data on the intermediate flows of feed resources (which are generally not available). Moreover, for econometric estimation purposes the recently developed Symmetric Generalised McFadden (SGM) cost function will be selected (Diewert and Wales, 1987).<sup>2</sup>

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<sup>1</sup> An allocation of feed resources to different livestock categories is not yet available in the EC, due to the lack of regular publications of data (see, for instance, Eurostat, 1990).

<sup>2</sup> Diewert and Wales (1987) as well as Lawrence (1989) limited their discussion of the SGM cost function to the single-output case. Hence, the multiple-output SGM cost function presented here forms a 'natural' extension of their results. In a recent paper, Kohli (1992) used a similar functional form in a multiple-output profit function context designated as a Symmetric Normalised Quadratic function. Lawrence (1989) and Kohli (1992) adopted the SGM functional form in the context of estimating price elasticities of imports and exports.

It is important to note that this study is limited to the production decisions of the Belgian compound feed industry, without explicitly modelling the links between feed manufacturing and livestock production. This approach is justified, however, by the fact that in Belgium nearly all concentrate feeds used by livestock producers are supplied by industrial feed compounders.<sup>3</sup>

The paper is structured as follows. The next section provides the conceptual framework of the analysis and specifies the basic assumptions of the model. This is followed by a description of the empirical model, the data, and the estimation method. Then the main empirical findings are presented. In the final section a number of conclusions are drawn.

### CONCEPTUAL FRAMEWORK

We assume that all feed compounders operate in perfectly competitive input and output markets. All feed compounders are cost minimisers which determine simultaneously their requirements of feed inputs subject to the level of output quantities and the prices of feed inputs.

Assuming cost-minimising behaviour and using duality theory, the compound feed production technology can be represented by a strongly separable total cost function (Surry and Moschini, 1984) defined as:

$$C(w, r, y) = \min_{x,z} \{w^T x + r^T z : (x, z, y) \in T\} = G(w, y) + H(r, y) \quad (1)$$

where  $T$  is the production possibility set;  $w \equiv (w_1, \dots, w_n)$ ,  $r \equiv (r_1, \dots, r_q)$  are the price vectors associated with the vectors  $x \equiv (x_1, \dots, x_n)$  of feed inputs and  $z \equiv (z_1, \dots, z_q)$  of nonfeed inputs, respectively;  $y \equiv (y_1, \dots, y_m)$  is the vector of compound feed outputs;  $G(\cdot)$  and  $H(\cdot)$  are the feed and nonfeed cost functions, respectively, which satisfy the usual regularity conditions (McFadden, 1978). The assumption of strong separability is a realistic assumption in the context of animal feed mixing, because it means that the marginal rate of substitution between any pair of feed ingredients is not affected by the usage of nonfeed inputs, such as labour or capital (Chambers, 1988, p. 46). Hence, the demand for feed ingredients depends only on the vector of the feed input prices,  $w$ , and the vector of output

<sup>3</sup> Although exact figures are missing for Belgium, it is generally believed that the proportion of concentrate feeds fed as home-mixed rations is of minor importance (estimated to be less than 10%). Therefore, the amount of concentrate feeds processed by the compound feed industry can be taken to represent the overall demand for concentrate feeds in Belgium. On-farm mixing is principally limited to the dairy production sector, which is less vertically integrated than the other sectors of livestock production (especially pigmeat and poultrymeat/eggs production).

quantities,  $y$ . Differentiating the feed cost function in (1) with respect to input prices and output quantities, respectively, yields:

$$x = \nabla_w G(w, y) \quad (2)$$

$$g = \nabla_y G(w, y) \quad (3)$$

where  $\nabla_w G \equiv (\partial G/\partial w_1, \dots, \partial G/\partial w_n)$  and  $\nabla_y G \equiv (\partial G/\partial y_1, \dots, \partial G/\partial y_m)$ ;  $x$  gives the optimal (i.e., cost-minimising) feed input quantities, and  $g$  represents the 'marginal feed costs' of producing the compound feeds for given levels of output and given feed input prices. In order to link the marginal feed costs (which are not observable) and the unit values or prices of output (which are observable), we assume the following relationship:

$$p_k - \lambda_k p_k = g_k(w, y) \quad (k = 1, \dots, m) \quad (4)$$

where  $p_k$  is the price of output  $k$ ;  $\lambda_k$  is the proportion of price representing the nonfeed cost per unit of output  $k$ . Due to the lack of information, we treat  $\lambda_k$  as common to all types of compound feed and assume that  $\lambda_k = \lambda = H(\cdot)/C(\cdot)$ . This procedure is adopted for the sake of convenience, and is justified on pragmatic and empirical grounds. As a result, the marginal feed cost-pricing relationships in (3) can be rewritten as:

$$g \equiv p(1 - \lambda) = \nabla_y G(w, y) \quad (5)$$

Expressions (2) and (5) constitute the behavioural model of the Belgian compound feed industry. After selecting a functional form for  $G(w, y)$ , the system of feed input demand and marginal feed cost equations can be derived and estimated simultaneously.

The above model is too general, however, to identify feed utilisation by type of livestock. Therefore, we adopt (quite realistically) the following assumptions regarding the compound feed production technology: (a) constant returns to scale (CRS) in feed inputs; (b) nonjointness in feed input quantities. Nonjointness in feed input quantities means that the marginal feed cost of one output is independent of the level of production of the other outputs. Under the hypothesis of CRS, nonjointness implies that (a) the sub-Hessian of the feed cost function,  $\nabla_{yy} G \equiv [\partial^2 G/(\partial y_k \partial y_h)]$  for all  $k$  and  $h$ , is a null matrix, and (b) there is a one-to-one relationship between the marginal (unit) feed cost of output and feed input prices. As a result, the total feed cost,  $G(w, y)$ , can be written as the sum of the feed costs associated with each compound feed output separately (Hall, 1973; Kohli, 1981):

$$G(w, y) = \sum_{k=1}^m y_k g_k(w) \quad (6)$$

where  $g_k(w)$  is the marginal or unit feed cost function associated with output  $k$ ;  $y_k$  is the quantity of output  $k$ . The assumption of nonjointness allows for the determination of the quantity of feed ingredients used in the production of each compound feed ration. Thus, applying Shephard's lemma to (6) yields:

$$x_i = \frac{\partial G}{\partial w_i} = \sum_{k=1}^m y_k \left( \frac{\partial g_k}{\partial w_i} \right) = \sum_{k=1}^m x_i^k \quad (i = 1, \dots, n) \quad (7)$$

where  $x_i$  is the quantity of feed ingredient  $i$  used in the production of all compound feeds;  $x_i^k$  is the quantity of feed ingredient  $i$  used in the production of compound feed  $k$ , which is equal to  $y_k(\partial g_k / \partial w_i)$ .

After having analysed some relevant properties of a nonjoint feed technology, we are now able to identify various feed utilisation relationships by type of livestock. Based on the feed input demand and marginal feed cost equations, the (output-constant) aggregate and animal-specific price elasticities of demand for feed inputs ( $\epsilon_{ij}$  and  $\epsilon_{ij}^k$ , respectively) can be derived as:

$$\epsilon_{ij} = \left( \frac{\partial x_i}{\partial w_j} \right) \left( \frac{w_j}{x_i} \right) \quad \epsilon_{ij}^k = y_k \left( \frac{\partial^2 g_k(w)}{\partial w_i \partial w_j} \right) \left( \frac{w_j}{x_i^k} \right) \quad (8)$$

where  $\sum_j \epsilon_{ij} = \sum_j \epsilon_{ij}^k = 0$  for all  $i$  and  $k$  (Euler's theorem). In addition, the output elasticity of demand for feed inputs ( $\eta_{ik}$ ) can be derived as:

$$\eta_{ik} = \left( \frac{\partial x_i}{\partial y_k} \right) \left( \frac{y_k}{x_i} \right) \quad (9)$$

where  $\sum_k \eta_{ik} = 1$ . Furthermore, it can be shown that:

$$\eta_{ik} = \frac{x_i^k}{x_i} \quad (10)$$

and

$$\epsilon_{ij} = \sum_{k=1}^m \eta_{ik} \epsilon_{ij}^k \quad (11)$$

From expression (10) it follows that the output elasticities of feed input demand can be used to identify the allocation of feed resources to the various types of livestock. Expression (11), on the other hand, reflects the intuitively appealing result that the aggregate elasticity is a weighted average of the animal-specific elasticities, where the weights are equal to the corresponding input allocation shares.

Finally, a reasonable measure of the composition of the various compound feeds is as follows:

$$s_i^k = \frac{\frac{x_i^k w_i}{w_i^*}}{\left( \sum_{j=1}^n \frac{x_j^k w_j}{w_j^*} \right)} \quad (12)$$

where  $s_i^k$  is the quantity share of feed input  $i$  in output  $k$  (in terms of product weight);  $w_i^*$  is the implicit 'mean price' per metric tonne of feed input  $i$ , expressed in Belgian francs (not as an index!), which is defined as nominal or current value divided by quantity or product weight in the base period (1980) and adjusted for the other sample years by using appropriate price indexes (see below).

#### EMPIRICAL IMPLEMENTATION

##### *Functional form*

For the empirical implementation of the model we use an adapted version of the Symmetric Generalised McFadden (SGM) cost function recently proposed by Diewert and Wales (1987). The multiple-output SGM feed cost function for a CRS technology with 'no technological change' can be written as:

$$G(w, y) = \frac{1}{2} \left( \frac{w^T A w}{\theta^T w} \right) \phi^T y + \frac{1}{2} \left( \frac{y^T B y}{\phi^T y} \right) \theta^T w + w^T C y \quad (13)$$

where time subscripts are omitted for convenience;  $w$  and  $y$  are defined as before;  $\theta \equiv (\theta_1, \dots, \theta_n)$  and  $\phi \equiv (\phi_1, \dots, \phi_m)$  are predetermined vectors of nonnegative constants;  $A \equiv [a_{ij}] = A^T$  and  $B \equiv [b_{kh}] = B^T$  are symmetric matrices of parameters of size  $n \times n$  and  $m \times m$ , respectively;  $C \equiv [c_{ik}]$  is a nonsymmetric matrix of parameters of size  $n \times m$ . In order for  $G(w, y)$  to be a concave function of  $w$  for each  $y$ , it is necessary and sufficient that the  $A$  matrix is negative semi-definite; in order for  $G(w, y)$  to be a convex function of  $y$  for each  $w$ , it is necessary and sufficient that the  $B$  matrix is positive semi-definite. In addition, linear homogeneity means that the following adding-up restrictions on the  $a_{ij}$  and  $b_{kh}$  parameters must be imposed:  $\sum_j a_{ij} = 0$  for all  $i$ ;  $\sum_h b_{kh} = 0$  for all  $k$ . No restrictions are imposed on the  $c_{ik}$  parameters, except that they must be nonnegative. The SGM feed cost function described in (13) is linearly homogeneous in input prices  $w$  and output quantities  $y$  separately. The exogenous parameters  $\theta_i$

and  $\phi_k$  are set equal to the average cost share of input  $i$  and the average revenue share of output  $k$ , respectively.<sup>4</sup>

The SGM functional form has several desirable properties: (a) The symmetry of the cost function means that we are not forced to single out one particular input or output to play an asymmetric role<sup>5</sup>; (b) The sub-Hessians of the feed cost function,  $A$  and  $B$ , are matrices of constants. As a result, global curvature conditions can easily be checked and/or imposed if needed without destroying the (local) flexibility of the cost function. This feature gives the SGM an edge over, for example, the translog (Diewert and Wales, 1987, p. 44); (c) Under the combined hypothesis of CRS and nonjointness in feed input quantities the  $B$  matrix is a null matrix. Hence, the number of free parameters is reduced substantially, which makes the estimation process less cumbersome.

The  $n$  feed input demand equations can be derived by applying Shephard's lemma to the SGM feed cost function in (13):

$$x_i = \frac{\partial G}{\partial w_i} = \left( \frac{a_i w}{\theta^T w} - \frac{1}{2} \theta_i \frac{w^T A w}{(\theta^T w)^2} \right) \phi^T y + c_i y \quad (i = 1, \dots, n) \quad (14)$$

where  $a_i$  and  $c_i$  are the  $i$ th row of matrix  $A$  and matrix  $C$ , respectively (recall that the  $B$  matrix is a null matrix). The feed input demand equations are linearly homogeneous in output quantities, and homogeneous of degree zero in input prices.

The  $m$  marginal feed cost equations derived from the SGM feed cost function in (13) are as follows:

$$g_k = \frac{\partial G}{\partial y_k} = \frac{1}{2} \phi_k \left( \frac{w^T A w}{\theta^T w} \right) + w^T c_k \quad (k = 1, \dots, m) \quad (15)$$

where  $c_k$  is the  $k$ th column of matrix  $C$ . The marginal feed cost equations are linearly homogeneous in input prices; the marginal feed costs are independent of the output mix.

<sup>4</sup> As suggested in the literature (for example, Diewert and Wales, 1988 and 1991) this ensures that  $\theta^T 1_n = \phi^T 1_m = 1$ , where  $1_n$  and  $1_m$  are the  $n$ - and  $m$ -dimensional unit vectors, respectively.

<sup>5</sup> Diewert and Wales (1987) show that the SGM form (single-output case) is flexible for a price vector  $w$  satisfying  $A \cdot w = 0$ . While the *nonsymmetric* Generalised McFadden functional form (analogous to the Normalised Quadratic functional form) has superior flexibility properties in that it is not restricted to being flexible at just one point, the results obtained are sensitive to the choice of the *numéraire* good which plays an asymmetric role. This sensitivity is eliminated by the use of the SGM form.



The  $n + m$  equations in (14) and (15) constitute the model to be estimated. If concavity is rejected by the data, it can easily be imposed by a reparameterisation of the  $A$  matrix using a Cholesky decomposition (Lau, 1978; Jorgenson and Fraumeni, 1981; Morey, 1986).

### *Data construction*

The parameters of the system of feed input demand and marginal feed cost equations in (14) and (15) are estimated using annual data for the period 1962–88. The data required to estimate the model are prices and quantities of feed ingredients and compound feeds. Data are available on (a) the *total* use of feed ingredients by *all* livestock (without revealing the feed allocation among livestock categories), and (b) compound feed prices and quantities by livestock category. The data were obtained from publications of the Federation of Belgian Feed Compounders (BEMEF). It is estimated that the sample represents 85% of the total amount of compound feeds produced in Belgium.

In analysing the technology of the Belgian compound feed industry a certain level of data aggregation is inevitable, due to the fact that there are simply too many separate feed inputs and outputs to obtain meaningful statistical results. For this reason, four input aggregates and three output aggregates are distinguished. The input aggregates included in the model are: (a) cereals (maize, barley, sorghum, wheat, other cereals); (b) cereal substitutes<sup>6</sup> (molasses, manioc, by-products, fats and oils); (c) high-protein feeds (animal meals, pulses and dehydrated products, corn gluten feed and 'other feed items with less than 25% protein content', soymeal and 'other feed items with more than 25% protein content'); (d) additives (minerals, vitamins, synthetic lysine and methionine, drugs, preservatives, flavourings). The output aggregates included in the model are: (a) poultry feeds (pullets, layers, broilers, turkeys, other poultry feeds); (b) pig feeds (piglets, fattening pigs, gestating sows); (c) cattle feeds (breeding cattle, beef cattle, dairy cows). Dairy by-products (mainly used in the production of 'milk replacers') are not included in the analysis. Accordingly, compound feeds for veal calves are excluded from the analysis as well.

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<sup>6</sup> The term 'cereal substitutes' refers to the products (carbohydrates) mentioned in Annex D of the basic cereals Regulation (EEC) 2727/75, including corn gluten feed (CGF). It should be noted, however, that in the present analysis CGF is included in the category of high-protein feeds. This classification may be questionable (Boyd and Brorsen, 1986; McKinzie et al., 1986; among others). Conducting tests of this classification is hampered by the lack of specific data on the use of CGF.

The data used as explanatory variables in the estimation process are feed ingredient prices ( $w_i$ ,  $i = 1, \dots, 4$ ) and compound feed production quantities ( $y_k$ ,  $k = 1, 2, 3$ ), normalised to equal 1.0 in 1980 (which is taken as the base period). The input price indexes of cereals, cereal substitutes, and high-protein feeds were computed as approximative Divisia indexes. For additives only one representative price figure was available. The output quantity indexes were computed as approximative Divisia indexes. The data used as dependent variables are feed input quantities ( $x_i$ ,  $i = 1, \dots, 4$ ) and marginal feed costs of output ( $g_k$ ,  $k = 1, 2, 3$ ), expressed in billions of 1980 Belgian francs. The feed input quantities were computed by dividing current feed input expenditures by the corresponding input price index. The marginal or unit feed costs were computed by dividing current feed costs of output (that is, total output values or revenues times 1 minus  $\lambda$ ) by the corresponding output quantity index.

### *Estimation procedure*

To facilitate econometric estimation, the four input demand equations in (14) and three marginal feed cost equations in (15) are transformed into nominal or current value terms by multiplying their elements by the corresponding feed input price index and output quantity index (see also Kohli, 1981; Livernois and Ryan, 1989). Hence, the dependent variables are defined as  $R_i = w_i x_i$  ( $i = 1, \dots, 4$ ) and  $R_k = g_k y_k$  ( $k = 1, 2, 3$ ). Prior to estimation, additive error terms are appended to each of the equations. The error terms are assumed to be jointly normally distributed with zero means and with constant but unknown variances and covariances. The model is then estimated using the iterative Zellner technique (IZEF), which is asymptotically equivalent to FIML (Judge et al., 1985). The estimation is conducted using the 4.2A version of the TSP computer program (Hall, 1991).

Since the sum of the first four equations ( $\sum_i w_i x_i$ ) is equal to the sum of the last three equations ( $\sum_k g_k y_k$ ), as required by the linear homogeneity of the feed cost function, the seven equations are linearly dependent. As a result, the covariance matrix of the error terms is singular, and one of the seven equations must be dropped for estimation purposes. Since the IZEF/FIML estimates are invariant to the equation deleted, we arbitrarily drop the feed input demand equation associated with additives.

Preliminary estimation of the linear system of input demand and marginal feed cost equations strongly suggested the presence of autocorrelation. Therefore, the stochastic specification of our model considers both contemporaneous and intertemporal correlation of the error terms. Specifically,

let  $x_{i,t} = \beta[w_t, y_t]$  and  $g_{k,t} = \gamma[w_t]$ , then the autoregressive model specification is as follows:

$$x_{i,t} = \beta[w_t, y_t] - \rho\beta[w_{t-1}, y_{t-1}] + \rho x_{i,t-1} + \mu_{i,t} \quad (16)$$

$$g_{k,t} = \gamma[w_t] - \rho\gamma[w_{t-1}] + \rho g_{k,t-1} + \epsilon_{k,t} \quad (17)$$

where  $\mu_{i,t}$  and  $\epsilon_{k,t}$  are assumed to be independently and normally distributed with mean zero and constant covariance matrix;  $\rho$  is the coefficient of autocorrelation, which is common to all equations due to the linear dependency in the system (Berndt and Savin, 1975, p. 939). The coefficient of autocorrelation and the structural parameters of the model are then estimated jointly using the nonlinear IZEF technique.

## EMPIRICAL RESULTS

Since our primary objective is to derive a set of feed demand elasticities as well as a complete FUM, the adopted structural properties of the compound feed production process (i.e., separability, nonjointness, CRS, and absence of technological change) are treated as maintained hypotheses in the estimation process.<sup>7</sup>

### *Parameter estimates*

Table 1 reports the estimates of the SGM parameters and their (asymptotic) standard errors. Overall, the estimation results are satisfactory. All the coefficients have theoretically correct signs and most parameter estimates (18 out of 23) are significantly different from zero at the conventional 5% level. The model fits the data fairly well; the  $R^2$  values range from 0.969 to 0.992. The parameter estimates were checked for acceptance of curvature conditions. Since all the Cholesky values associated with the  $A$  matrix are nonpositive, the data are consistent with global concavity. Hence, the SGM cost function is ‘well-behaved’ over the entire sample period.<sup>8</sup>

<sup>7</sup> Testing for the various structural hypotheses is beyond the scope of this paper. Further research on these issues is currently undertaken by the authors.

<sup>8</sup> The Cholesky decomposition replaces the matrix  $A$  by the product  $LDL^T$ , where  $L \equiv [\ell_{ij}]$  is a unit lower triangular matrix with  $\ell_{ij} = 0$ , for  $i < j$ ,  $\ell_{ii} = 1$ ; and  $D \equiv [d_{ii}]$  is a diagonal matrix of Cholesky values of size  $4 \times 4$ . Global concavity requires that the elements of the matrix  $D$  are nonpositive. The estimated Cholesky values from applying nonlinear IZEF are as follows (asymptotic standard errors are between parentheses):  $d_{11} = -6.9583$  (1.3275);  $d_{22} = -2.2716$  (1.0429);  $d_{33} = -0.3671$  (0.4508);  $d_{44} = 0$  due to the linear homogeneity property (see also Jorgenson and Fraumeni, 1981, p. 26).

TABLE 1

Parameter estimates of the SGM feed cost function

Parameter	Value	Asymptotic standard error	Parameter	Value	Asymptotic standard error
$a_{11}$	-6.9583	1.3275 *	$c_{11}$	2.4227	0.3374 *
$a_{12}$	3.6359	1.2125 *	$c_{12}$	7.0047	0.6891 *
$a_{13}$	3.3843	0.9005 *	$c_{13}$	1.0442	0.3189 *
$a_{14}$	-0.0619	0.5373	$c_{21}$	0.7406	0.3179 *
$a_{22}$	-4.1714	1.6067 *	$c_{22}$	5.3876	0.5945 *
$a_{23}$	0.7035	0.8191	$c_{23}$	2.2030	0.3303 *
$a_{24}$	-0.1680	0.5231	$c_{31}$	3.1813	0.2113 *
$a_{33}$	-4.7030	0.8581 *	$c_{32}$	6.3646	0.4487 *
$a_{34}$	0.6152	0.3555	$c_{33}$	2.8960	0.2384 *
$a_{44}$	-0.3853	0.3877	$c_{41}$	1.2263	0.1580 *
$\rho$	0.9237	0.0382 *	$c_{42}$	0.6636	0.3109 *
			$c_{43}$	0.7763	0.1615 *

\* Denotes significance at the 5% level.

The subscripts  $i, j$  of the  $a_{ij}$  parameters take the following values: 1 for cereals; 2 for cereal substitutes; 3 for high-protein feeds; 4 for additives. The subscript  $k$  of the  $c_{ik}$  parameters takes the following values: 1 for poultry feeds; 2 for pig feeds; 3 for cattle feeds.

### *Allocation of feed ingredients to livestock categories*

The characteristic of nonjointness imposed on the compound feed production technology allows us to break down the total use of feed ingredients by category of livestock (equation 10). Estimated allocation shares of the feed ingredients are presented in Table 2, for four selected years. For instance, in 1988, 21.4% of total cereals usage enters the production of poultry feeds, 68.6% of total cereals usage goes to pig feeds, and so on. An inspection of the results in Table 2 reveals that (since 1970) 50 to 70% of all cereals, cereal substitutes, and high-protein feeds is used in producing compound feeds for pigs. These figures clearly reflect the growing importance of the pork sector in Belgium. This development has been accompanied by a steadily declining share of feed resources entering the production of poultry feeds. The results in Table 2 also show that the share of feed ingredients going to the beef/dairy sector remains fairly stable over time.

### *Composition of compound feeds*

Using the total amounts for each feed ingredient combined with the allocation shares estimated in the previous section, we are able to calculate the composition of the various compound feeds (equation 12). The esti-

TABLE 2

Estimated feed input allocation to livestock categories, selected years (percentage shares)

	Poultry	Pigs	Cattle	Total
Cereals				
1963	41.5	46.5	12.0	100.0
1970	31.7	61.1	7.1	100.0
1980	23.1	66.9	10.0	100.0
1988	21.4	68.6	10.0	100.0
Substitutes				
1963	16.6	49.1	34.3	100.0
1970	13.4	65.5	21.1	100.0
1980	8.9	64.7	26.4	100.0
1988	8.7	64.0	27.3	100.0
Proteins				
1963	42.0	32.7	25.3	100.0
1970	35.6	47.4	17.0	100.0
1980	25.6	51.2	23.3	100.0
1988	23.8	51.7	24.4	100.0
Additives				
1963	61.4	12.9	25.8	100.0
1970	58.8	21.7	19.5	100.0
1980	46.0	24.9	29.1	100.0
1988	44.3	24.1	31.6	100.0

mated quantity shares of the various feed ingredients (in terms of product weight) for each type of compound feed are reported in Table 3, for four selected years.<sup>9</sup> Obviously, this table is comparable to a conventional input-output matrix containing technical coefficients. Not surprisingly, the results in Table 3 indicate that throughout the sample period the share of cereals in compound feed production has been steadily declining compared with the other feed inputs. The quantity share of cereals dropped from 52% in 1963 to 26% in 1988 in poultry feeds; from 52% to 29% in pig feeds; from 27% to 11% in cattle feeds. By contrast, the proportions of cereal substitutes, and particularly high-protein feeds, have expanded significantly over the sample period. Moreover, it is interesting to note that the share of cereal substitutes, as well as the share of additives, have fallen slightly since 1980, whereas the share of high-protein feeds has continued to grow sharply. This result probably reflects the observation that the EC

<sup>9</sup> The estimated cost shares of the various feed inputs for each compound livestock feed are not reported in this paper, due to space limitations. They are available from the authors upon request.

TABLE 3

Estimated composition of compound feeds, selected years (percentage shares of rations)

	Cereals	Substitutes	Proteins	Additives	Total
Poultry					
1963	52.0	9.5	31.1	7.4	100.0
1970	46.0	11.5	34.1	8.4	100.0
1980	35.8	14.4	37.8	12.1	100.0
1988	26.4	13.7	48.1	11.8	100.0
Pigs					
1963	52.0	25.0	21.6	1.4	100.0
1970	45.7	29.2	23.5	1.6	100.0
1980	35.7	36.0	26.0	2.3	100.0
1988	28.7	34.0	35.2	2.2	100.0
Cattle					
1963	26.6	34.7	33.2	5.5	100.0
1970	21.7	38.2	34.2	5.8	100.0
1980	15.4	42.6	34.3	7.6	100.0
1988	11.0	37.9	43.6	7.5	100.0

import of, for example, CGF has doubled since the early 1980s, at the expense of cereal substitutes (Agra Europe, 1991).

From Table 4 we may compare our findings with those of two other studies of the Belgian compound feed industry. The first study was conducted by one of the authors (Peeters, 1990), who adopted a 'pseudo-data' approach to analyse the EC compound feed sector. The second study was undertaken by Schlitz (1987), who estimated a FUM for Belgium using technical-nutritional information. The corresponding feed inclusion rates for 1984/85 are shown in Table 4. Even though the results of the three studies are consistent in a qualitative sense, there are several differences in the magnitudes of the various feed inclusion rates. For example, the present approach predicts a poultry feed mix which is characterised by a lower share of energy-rich cereal substitutes and a considerably higher proportion of protein feeds and additives. Also, noticeable differences can be observed in the composition of cattle feeds, while the three studies seem to provide broadly similar results for the composition of pig feeds. However, since this study differs from the two other studies in many respects (e.g., data sources, product definitions, methodology), inferences drawn from this comparison should be qualified carefully. Yet the present approach has a particularly important advantage in that the estimation is based on the feed compounders' *actual* behaviour.

TABLE 4

Comparison of the composition of compound feeds with other studies, 1984/85 (percentage shares of rations)

	This study <sup>a</sup>	Peeters (1990) <sup>b</sup>	Schlitz (1987) <sup>c</sup>
<b>Poultry</b>			
Cereals	32.7	50.2 <sup>d</sup>	61.4
Substitutes	12.5	13.7	18.8
Proteins	42.4	29.3	17.3
Additives	12.3	6.8	2.2
<b>Pigs</b>			
Cereals	33.7	28.8	30.3
Substitutes	33.6	37.7	40.8
Proteins	30.4	31.6	24.9
Additives	2.3	1.9	4.1
<b>Cattle</b>			
Cereals	14.4	7.9 <sup>e</sup>	6.7
Substitutes	38.8	44.8	23.0
Proteins	38.9	46.8	63.4
Additives	7.9	0.5	7.0

<sup>a</sup> Estimated mean values for 1984 and 1985.

<sup>b</sup> Estimated values using pseudo data, evaluated at the prices of the 1984/85 season (point of translog approximation).

<sup>c</sup> Estimated values for the 1984/85 season using technical information (fixed input-output coefficients or Leontief structure).

<sup>d</sup> Weighted average for layers and broilers, where the weights are equal to the shares in total compound feed production for poultry (0.65 and 0.35, respectively).

<sup>e</sup> Standard (complete) dairy feed.

### *Price elasticities of demand for feed ingredients*

To complete our empirical results the estimated own- and cross-price elasticities of feed input demand are shown in Table 5, both at the aggregate and livestock-specific level (equations in 8). Space limitations, however, preclude reporting all the elasticity estimates. Therefore, the elasticity estimates are presented for 1988 only.<sup>10</sup> Feed inputs are substitutes when the cross-price elasticity is positive and complements when it is negative.

Several conclusions emerge from the reported elasticities. Firstly, all the elasticity estimates are plausible, both in sign and magnitude. Secondly,

<sup>10</sup> The elasticity estimates for the other sample years are available from the authors upon request.

TABLE 5

Estimated own- and cross-price elasticities of feed input demand, 1988

	Elasticity with respect to price of:			
	Cereals	Substitutes	Proteins	Additives
<b>Poultry</b>				
Cereals	-0.956	0.486	0.465	0.004
Substitutes	1.336	-1.440	0.178	-0.073
Proteins	0.277	0.039	-0.381	0.065
Additives	0.006	-0.038	0.154	-0.122
<b>Pigs</b>				
Cereals	-0.618	0.314	0.301	0.003
Substitutes	0.379	-0.408	0.050	-0.021
Proteins	0.266	0.037	-0.365	0.062
Additives	0.022	-0.144	0.588	-0.466
<b>Cattle</b>				
Cereals	-1.652	0.840	0.805	0.007
Substitutes	0.346	-0.373	0.046	-0.019
Proteins	0.219	0.030	-0.301	0.051
Additives	0.007	-0.043	0.174	-0.138
<b>Aggregate</b>				
Cereals	-0.794	0.404	0.387	0.003
Substitutes	0.454	-0.489	0.060	-0.025
Proteins	0.257	0.036	-0.353	0.060
Additives	0.010	-0.065	0.265	-0.210

most of the own-price elasticities of feed input demand are less than 1.0 in absolute value (with the exception of cereal substitutes in poultry feeds, and cereals in cattle feeds), indicating an overall inelastic response to changes in own prices. Obviously, cereals have the highest price elasticity of demand. The relatively small own-price elasticity of high-protein feeds reflects the fact that these ingredients are essential inputs, given their role as major suppliers of digestible protein in livestock feeding. The magnitudes of the aggregate elasticities found in this study are broadly consistent with the findings of Surry and Moschini (1984). Using a translog approximation (based on data from 1962 to 1978), they found the following elasticity values for Belgium (evaluated at the mean point): cereals -0.632; cereal substitutes -0.225; high-protein feeds -0.160. Thirdly, the signs of the cross-price elasticities between cereals, cereal substitutes, and high-protein feeds are all positive, implying substitution relationships. An interesting implication of this finding is that cereal substitutes and high-protein feeds are both substitutable for cereals. Hence, our results support the hypothesis that high-protein feeds may also serve as energy suppliers. This finding accords with observations by other analysts (Longmire, 1980; Surry



and Moschini, 1984; Peeters, 1990). The relationship of substitutability between cereals and high-protein feeds probably arises because at low prices of high-protein feeds, the sizeable energy content of these feeds enables cereals to be displaced. On the other hand, the cross-price elasticities between cereal substitutes and high-protein feeds are very small and not significantly different from zero. Fourthly, additives (which include synthetic proteins) substitute for high-protein feeds and (to a lesser extent) cereals, while additives are complementary to cereal substitutes. As expected, the elasticities associated with additives are extremely small and not significantly different from zero.

#### CONCLUDING REMARKS

The lack of data on the use of compound feed ingredients by livestock category is generally viewed as a major impediment to the construction of feed utilisation matrices. However, this study presented a relatively simple econometric approach which enables the analyst to largely overcome this problem by using data which are readily available. It is shown that under the maintained hypothesis of nonjointness in production, a complete feed utilisation matrix can be derived from a multiple-output cost function model. At the same time, price elasticities of feed input demand by type of livestock can be obtained from the model. The study also illustrated the use and importance of the recently developed Symmetric Generalised McFadden functional form. It turned out that global curvature properties were met by the sample. Although the results are conditional upon the separability assumptions implied by the data aggregation, we argue that the analysis reported in this paper provides more reliable estimates than other studies which are solely based on pseudo data and/or technical-nutritional information.

The empirical results manifestly showed that feed compounders in Belgium have persistently substituted cheap imported feed resources (cereal substitutes and particularly high-protein feeds) for domestic cereals in the production of compound feeds. In fact, the cereals content of compound feeds has been halved during the last three decades! Clearly, a reversal of this dramatic development can only be accomplished by drastically lowering EC support prices of cereals and/or raising the import prices of nongrain feed resources ('rebalancing').

Although the empirical evidence presented in this paper is specific to one country, we think that the results may serve as an indication of existing trends in feed utilisation by other EC member states where the compound feed industry plays an important role as feed supplier. Moreover, the methodology outlined in this paper is general enough to have a broader

applicability. Of course, further research at a more disaggregated level remains to be done in order to enhance our understanding of the feed–livestock interrelationships.

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