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Evaluation of risk in farm planning: a case study

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Many studies suggest that farmers frequently show risk averse attitudes, and choose the “risk minimizing” and “safety first” survival strategy rather than pursuing the profit maximization. This article reports on a study of the impact of risk caused by different events: climate, stock levels, price volatility and other causes affecting the yield and price variability of agricultural crops. This study will simulate the risk in farm decisions using a sumex utility function that allows to parameterize the risk for specific traits of the function, and MOTAD (minimization of total absolute deviations) to simulate an efficient combination of crops in the whole farm planning (WFP). The empirical analysis is represented by a case study consisting in the risk simulation of a farm of 100 Ha, growing vegetable crops located in the Northern region of Italy. The risk is modelled using 15 years historical observations with discrete probability distribution of some of the most diffused cereal and oilseed crops (source: Eurostat). The objective is to evaluate the risk aversion by designing a utility frontier of crop combinations using a LP approximation model. The results indicate the trade off between expected returns and risk: if the value of gross income is expected to increase, the farmers tend to specialize in the most profitable portfolio enterprise while it is not so evident that the diversification will contribute to curb the risk.

Key word: whole farm planning, risk, sumex utility function, LP-MOTAD, portfolio analysis

1 - Introduction

Farmers are frequently facing their decisions in condition of uncertainty due to their limited capacity to anticipate future events about climate, market changes and biological responses to different farming practices. (Chen et al., 1999a). Then their belief and preferences are used to planning processes instead of a rational evaluation using available information to convert the uncertainty into risk and decide consequently the best action to be undertaken (Knight, 1971; Pannell et al., 1988; Hardaker, 1991; Rosa, 1987; Bakus et al., 1997; Anderson et al., 2002; Anderson et al., 2004; Bazeman, 2006). The literature is plenty of studies describing risk and uncertainty and their consequences for decision making, yet risk and uncertainty, by their nature, are very difficult to deal with. Because uncertainty is widespread in its origins and pervasive in its impacts, it cannot be fully accommodated in any planning model, then the analyst must simplify his modelling depending on the perceptions of reality and ability to convert those perceptions into an 'appropriate' planning model (Matlon, 1991; 1997; Knight, 1971; Chavas et al., 1994; Martin et al., 2000; Anderson et al., 2002). Risk is widely recognized as an issue of critical importance to farmers' decision and policies must be addressed to reduce the impact of risk in their decisions (Anderson et al., 1977)

A realistic farm planning needs to take into account the farmer's subjective probabilities assigned to the occurrence of events affecting their risky prospects; i.e use of a technology to protect crop yield with irrigation, hail or frost protection or prevent market risk with storage or insurance. The subjective expected utility (SEU) involves the disaggregation of risk decision problem into separable assessments of the decision's maker beliefs about the uncertainty captured subjectively with a Utility function and translated into decisions. Then SEU is a suitable tool for structuring the uncertainty into a feasible model of risk driving to an optimal decision which yield the highest expected utility. (Hardaker et al., 1991, 1997). In a questionnaire reporting different risky prospects about yield and price submitted to a number of farmers the answers suggested that the farmers

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tended to exhibit a risk aversion behaviour when asked to select higher risky prospects associated to higher income level. (Meuissen, 2010). This study is based on a case study drawn from experiences of arable farm having different options to combine cereals and oilseed enterprises. The risky prospects are simulated using “ad hoc” utility functions that account of the risk in problems of WFP.² A typical applications of this approach is the combination of Bernoulli utility function and Bayesian analysis to elaborate probabilities defined in two steps: i) “ex ante” by using an historical series of data; ii) “ex post” with experiments. These sources of information are combined to find the optimal solution with risky prospects and ordered preferences (Rosa, 2002). In fig. 1 is presented a schematic overview of these various approaches of risk analysis in farm management. There are three main input modules (see first column): 1) farm accounting data, 2) normative optimisation rules; 3) risk-management instruments. The empirical analysis is presented in the second column dealing with the premises of the first column, showing different methods for capturing information about farm results: expected gross margins, mean-variance approach or variability of state of natures. These data suggest the choice of the normative analysis based on three types of optimisation methods (third column): linear programming (LP), quadratic risk programming and (QRP) and Utility-efficient programming (UEP) technique. The last column reports the results obtained from the three approaches..

The object of this paper is to present an approach to the whole farm planning under risk caused by changes in prices and yield due to climate events, price volatility or government policies using a case study referred to representative farm situation with 100 Ha size located in the Northern Italy. Four major cereal crops: maize, barley, wheat, sorghum, and three oilseeds crops: soybean, rapeseed, sunflower are included in the portfolio simulation; the data to compute the gross margin (GM) of the crop enterprises are provided by the Eurostat. The optimal combination of farm enterprises with risk is found with linear programming MOTAD model; using the the sumex utility function as the objective function allowing to parameterise the risk in the stochastic efficiency frontier (UEP). The basic LP model is briefly introduced and main results are presented and possibilities to reduce costs in the short and medium term is discussed. Subsequently, the limits of LP optimisation and the effectiveness of this methodology in estimating activity levels and related costs as well as the welfare impact of public policy is questioned.. Formal aspects of the "Interval Linear Programming (ILP)" approach are summarized and the use of the min-max regret criterion within the ILP framework is then presented. The implementation procedure and results thereof are provided and the decision support methodology is proposed that integrates multiple criteria, and the decision making process is simulated through illustrative examples, followed by some conclusions.

This paper is organized as follows: in section 2 is presented an overview of the methods to manage the risk, in session 3 is analyzed the theoretical model of the utility efficient frontier, in session 4 is discussed the estimation of the utility efficient frontier and farmers' preferences for the risk, session 5 reports the data used for the analysis, session 6 discuss the results of the utility efficient frontier, and session 7 reports the conclusions and further implementation of the risk analysis.

² This is a problem of decision making under risk. There is a rich literature on this subject that constitutes a subject matter of decision making on its own (see [Hardaker et al., 1988, 1991, 1997](#)). One could mention the E-V model non-linear or quadratic as well as its linearized versions such as MOTAD and target-MOTAD but also models based on game theory reasoning such as maximin, minmax, safety-first etc. models. For all these models, availability of covariance matrices – that require gross margins of individual crops-related to different states of nature or years- are fundamental for efficient diversification among farm activities as a means of hedging against risk. Non-interactive methodologies attempting to assess multi-criteria utility functions [1] are including at least one risk criterion, always requiring detailed information at the farm level. As experimental applications of this method state the risk criterion ranks second after the gross margin maximisation one in the multi-objective function having weights around 30%. This is probably the reason that all of the above method modelling implementation contain at best a few dozen of farms.

2 – The risk management

The major concern for the farm planning is the consequences of price and yield variability affecting income volatility. (Manfredo and Leuthold, 1998). Some methods to curb the risk have been proposed: risk pooling and risk spreading technique. The risk pooling techniques are based on price smoothing mechanisms; for example, the average pricing methods proposed by the co-operative groups for crop delivered by their members is an example of risk hedging. Farmers, when delivering their crops receive an advanced payment with a bonus to compensate the difference with the market prices of the past cropping year (Pannell et al., 1988). Other co-operatives calculate the average price in a defined period of time to smooth the extreme values; for livestock (a continuous production) the price is calculated with moving average technique to smooth the cyclical pattern of the prices. In all cases the risk pooling method needs information from reference markets: centralised and organised markets, spot markets and/or futures markets to hedge the risk according with a selected price simulator.

The whole farm planning is a systemic approach to farm management that uses the portfolio analysis to combine risky prospects generated by farm enterprises. (Kobzar, 2006) A typical strategy to face the risk is the crop diversification; the positive side is the risk spreading on a number of activities. Planning the whole farm with risk implies to face the following problems:

- 1) select the enterprises to be included in the farm plan;
- 2) select “ad hoc” technologies (combination of land, labour and machinery factors);
- 3) select the resources to be allocated in each enterprise;
- 4) evaluate the risky prospects of the farm enterprises;

Risk spreading techniques have solid theoretical background on the optimum design of sets of portfolios investments for distributing the risk in a set of feasible activities with variable expected return. Correlation coefficients among the risky assets, suggest to combine the risky prospects: farmers are used to face the risk by diversifying their enterprises, but this will imply higher cost for capital investments, diseconomies of scale, and labour inefficiency. (Cordier, 2000). Formally the optimal portfolio is found with stated utility function to emulate the risk behaviour and the set of optimal combination of activities allowing to track the efficient frontier.³ The economists have progressed in developing a suite of contingency plans for the state-events of farm plans to minimise the consequences of negative risky prospects (Kaine et al, 1994 ; AA.VV., 2002)..

3 – Utility frontier and efficient programming

The stochastic programming has been applied to the whole farm planning, with the risk considered a stochastic component of the O.F. Many programming models have been tested by using alternative forms of utility functions to elaborate the stochastic production frontier (Hardaker et al., 1991, Ghodake and Hardaker, 1981; Meyer, 1977; Lee et al., 1985; Kaiser and Aplan, 1989; Hossain et al., 2002). The simpler linear programming method has been adapted to the risk with a matrix of activity net revenues by states (rows) and activity (column), using historical data corrected for inflation and trends or even subjectively and assigning probabilities to states to give stochastic nature to the risky prospects for revenues. (Hardaker et al., 1991; Hossein et al., 2002)). The quadratic risk programming (QRP) approach is based on a matrix deviation of net revenues from the mean value (E-V efficient set of solutions) with variance minimized subjected to a parametric constraint of expected income. This method requires that the distribution of total net revenue are normal and the farmer's utility function is quadratic, the two assumption are related in that the farmers could change their behaviour according with the distribution of revenues. The Motad programming is an approximation of the QRP (in that the E-M frontier adapt to E-V frontier) improved by the Target Motad which generate a set of efficient E-D solution for a target T that is the level of total revenues from which are evaluated their deviations. (Mc Carl et al., 1989). This

³ In livestock statistics show that only dairy farms with more than one thousand heads generate positive returns

approach generate a set of stochastically efficient solutions. The problem is that T is a discretionary value to be specified and from this level the values of deviations could change consistently. Among the utility maximization method (Lambert and Mc Carl, 1986) the utility efficient programming proposed by Patten *et al.* consists in a definition of separable utility function $U = G(z) + \beta(H(z))$ where parameter β is the variation of risk preferences and the optimizing model will be the following: $\text{Max } E(U) = p' G(z) + \beta(p' H(z))$ with β parametric subject to $Ax \leq b$; $Cx - Iz = u_f$ and $x \geq 0$. This programming approach assumes:

- i) a quadratic utility function with positive marginal utility defined in a bounded range to emulate the increasing risk aversion (Hanoch and Levy, 1970);
- ii) the assumption of normality distribution of GM (gross margins) prospects.

These two conditions may be inconsistent with the expected nature of the true preferences of the decision makers because of the asymmetric perception of risk generated by the skewed function of the expected incomes (Collender and Chalfant, 1986). However, this function has many desirable properties: i) decrease of risk aversion with the increase of z and possibility to module the absolute risk aversion with β parameter ranging in the interval between g for $\beta = 0$ and h for β very large. Then the optimal farm plans is consistent with the stochastic dominance when the risk attitude of the decision maker is embedded into the utility function defined in a given interval of risk aversion. Further, with linear segmentation of the Utility function it is possible to use a parametric linear programming. This introduces the way to afford the problem with a discrete stochastic programming; a method is to solve the problem in a two stage approach in which state of nature and activities are modified passing from the first to the second stage. Alternative approaches have been followed to the solution of the problem with embedded risk and modulation: Monte Carlo, Game theory, Maximum admissible loss and others. Assuming a non-risk neutrality of the entrepreneurs and knowledge about the relevant form of utility function embedding the risk preference, Hardaker has recommended to use a UEP (Utility efficient programming) when the advice to a group of decision-makers is given to obtain an efficient set of farm plans using methods somewhat similar to the stochastic dominance. (Patten *et al.*, 1988). With the UEP any form of concave utility function can be conveniently used to investigate whether utility functions generate different results about the optimal solutions. For our purposes, it has been chosen the following utility with constant relative risk aversion (CRRA)

$$1 - \quad U(z) = \frac{1}{1-a} \left(Z^{1-a} \right)$$

Z is the vector of wealth in this case given by the net income per year, a is the coefficient of relative risk aversion, $U(z)$ is the CRRA function with positive 1st order condition, $U'(z) = z^{-a} > 0$ and negative 2nd order condition: $U''(z) = -a \cdot z^{-a-1} < 0$. A common measure of risk aversion function showing decreasing absolute risk aversion is given by: $r_a(z) = -U''(z)/U'(z) = a/z$; this ratio shows that that $r_a(z)$ will decrease with increasing z while the CRRA $r_r(z) = z r_a(z) = a$ suggests that $z r_a(z)$ is the measure of risk aversion and is a pure number that can be used in the international contest for different currencies. It is important to notice that $r_r(z)$ and $r_a(z)$ are subordinated to the choice of the utility function. In addition to the CRRA power function a negative exponential function is:

$$2 - \quad U(z) = 1 - \exp(-cz),$$

$r_a(z)$ is equal to the constant relative risk aversion meaning that the preferences for risky prospects are unchanged if a constant amount is added or subtracted from the all payoff.

Then it is possible to define these properties:

if $a = 1$, the CRRA power function reduces to the logarithmic function, $U(z) = \ln(z)$.

if $a = 0$, $U(z) = z$ that is the solution for a risk neutral farmer. (Arrow, 1970; Lien and Hardaker, 2001). The sumex is a class of separable utility function proposed by Lambert and Mc Carl (1985)

$$3 - \quad \text{Max } E(U) = \sum_{k=1}^s p_k (G(z) + \lambda H(z))$$

G and H are appropriate functions of the variable net income z satisfying the desired properties of separability and concavity; the risk aversion is simulated by varying parametrically λ in the range

between a and b. This function tracks the linear approximation of the frontier function (Patten and others, 1988).

$$4 - \quad U(z) = -\exp(-az) - \lambda \exp(-bz)$$

The sumex function satisfies the separability and concavity conditions that can be demonstrated with the first and second derivatives:

$$5 - \quad U'(z) = -a \exp(-az) + \lambda b \exp(-bz) > 0$$

$$6 - \quad U''(z) = -a^2 \exp(-az) - \lambda b^2 \exp(-bz) < 0$$

The absolute risk aversion coefficient $r_a(z)$ is obtained from the ratio $-U''/U'$:

$$7 - \quad r_a(z) = (a^2 \exp(-az) + \lambda b^2 \exp(-bz)) / (a \exp(-az) + \lambda b \exp(-bz))$$

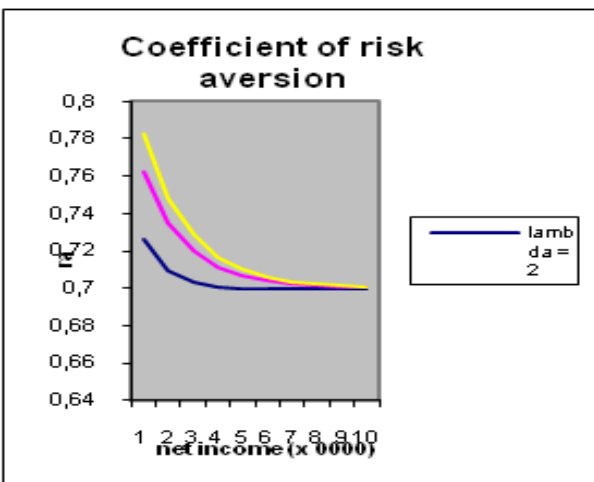
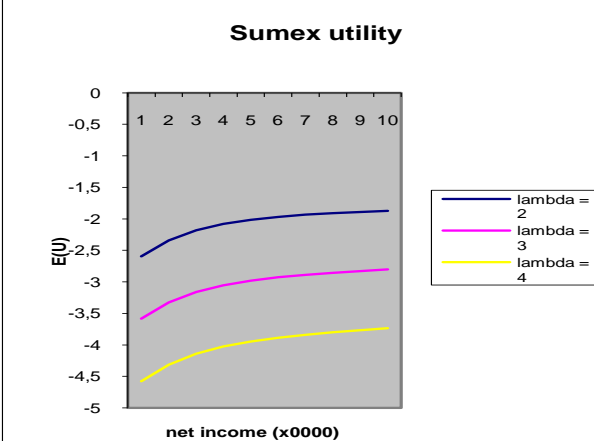
This ratio allows the function to manifest an absolute decreasing risk aversion with growing income z in the range between a and b . The sumex utility function allows to emulate the absolute risk aversion that tends to decrease by the increase of the income (z) in the lower (a) and upper (b) bounds. Anderson has proposed a classification of the risk preference based on relative risk aversion with respect to wealth $r_r = f(w)$ varying in the range between 0,5 (hardly risk aversion at all) and 4 (extreme risk aversion) with the coefficient of absolute risk aversion respect to wealth $r_a(w)$ estimated by: $r_a = r_r / w$. (Anderson et al, 1977, 2002). The variable w is an appropriate measure of the wealth and r_r , the absolute risk aversion, is the elasticity of the marginal utility of wealth with an approximate value of 2, varying in the range between 1 and 3 (Little and Mirrless, 1974). The sumex utility function allows to emulate the absolute risk aversion that decreases with the increase of income (z) and lower (a) and upper (b) bounds.

In our analysis the risk aversion is referred to the gross margin and for this purpose it is needed to find the relations between $r_r(w)$, $r_a(w)$, $r_r(z)$, $r_a(z)$. (Lien and Hardaker, 2001).

The relationship between wealth and revenue is defined with the following equation: $w = w_0 + z$; Where w_0 is the initial wealth value and z is stochastically independent from w . The value that gives the amount of w is obtained with the capitalization of the future expected gross margin risky prospects varying between $2 \cdot 10^{-6} \leq r_a \leq 6 \cdot 10^{-6}$ suggesting the interval to find the optimal plan with risk aversion. In tab. 2 and fig. 1 are reported the analytic and graphic development of the Sumex utility function with evidence of their properties and level of risk aversion. decreasing for higher values of Z and increasing values of λ . At the maximum value of z the three level of risk tend to converge to the same value.

z	Sumex Utility		
	$\lambda = 2$	$\lambda = 3$	$\lambda = 4$
10000	-2,59237916	-3,58539921	-4,57841927
20000	-2,3398247	-3,32591352	-4,31200234
30000	-2,18133071	-3,16053669	-4,13974266
40000	-2,07990669	-3,05227785	-4,02464901
50000	-2,01312355	-2,97870762	-3,94429168
60000	-1,96738145	-2,92622578	-3,88507011
70000	-1,93443397	-2,88658562	-3,83873727
80000	-1,90928076	-2,85478644	-3,80029212
90000	-1,88888965	-2,82779574	-3,76670184
100000	-1,87142185	-2,80377443	-3,73612701

z	ra - risk aversion coefficient		
	$\lambda = 2$	$\lambda = 3$	$\lambda = 4$
10000	0,72626353	0,76273501	0,78276361
20000	0,7096561	0,73483916	0,74821667
30000	0,70309591	0,71973843	0,72838475
40000	0,70068348	0,7113658	0,71683633
50000	0,69989402	0,70662298	0,71003769
60000	0,69970222	0,70389086	0,70600439
70000	0,69970987	0,70229797	0,70359929
80000	0,69976987	0,7013617	0,70216036
90000	0,69983218	0,7008085	0,70129768
100000	0,69988279	0,70048056	0,70077983



4 – Estimation of Utility efficient frontier with MOTAD approach

The problem is formulated as it follows:

$$\text{Max } E(U) = \sum p_k (G(z) + \lambda H(z))$$

$$\text{subject to } \bar{R}'x = e$$

$$Ax \leq b; \quad x \geq 0$$

where p_k is the probability of state k , λ is a non negative parameter varied parametrically to simulate the different levels of risk aversion, G and H are the two components of the sumex utility

function U , $\bar{R} = E(GI)$ is the vector of expected gross margin (GM) and $\bar{R}'x$ is the vector of expected GI of activities x to be included in the production plan; x is the vector of positive values referred to activity level, e is the maximum limit of risk acceptance by farmers, A is the matrix of technical coefficients of resources used by activity x and constrained to b , that is the maximum quantity of available resources. (Patten et al. 1982, Pope, 1982). The solution generates a mean-variance portfolio frontier conditional on e with λ varied parametrically between the lower limit $\lambda = 0$ when $r_a = a$ and the upper limit with $\lambda \rightarrow \infty$ when $r_a = b$. By varying e , an efficient set of portfolio solution is generated with $e = g(v)$; the optimal farm plan can be estimated with the O.F. $\text{Max } E(U) = h(e, p)$, subject to $e = g(v)$. Among alternative forms of the model it has been chosen the MOTAD

(Hardaker et al, 1997; Ignizio, 1982; Mc Carl et al., 1989). The values of \bar{R} requires to be evaluated with unbiased estimator while the historical data may be subjected to sampling error (Pope, 1982). However, many authors have suggested that there isn't any need to assume a standard distribution of states and observations from recent years can be assumed as a representative random sample of equally likely outcomes or states with subjectively assessed probabilities. (Patten et al., 1982). The MOTAD approach has been proposed to generate a mean-variance frontier built on an efficient set of portfolio plans to incorporate the assessed probability of the occurrence of states of nature (of risky prospects) inherent yield, prices or costs while the payoff are the farm gross margin (GM) Hazell 1971, Anderson et al., 2002). These are assessed with probability distribution, having a solid empirical ground when there is an exhaustive knowledge of the production coefficients; in this case the objective function will capture the utility dimension embedded into the expected profit function.

The MOTAD requires a representative sample of observations about crop activities (enterprises) included in the farm plan, giving a reliable measure of the minimum absolute deviation of the gross margin (GM) from the mean value. At these conditions it is possible to compute the density function inherent the probability of occurring alternative states of nature (yield or prices) of the enterprises included in the whole farm plan. The Sumex utility function $U = -\exp(-az) - \lambda \exp(-bz)$ is specified with restrictions on parameters, for risk aversion and relative ranges a and b.

The sumex utility function is the objective function ⁴ formulated as follows:

$$8 - \quad U(z) = -\exp(-0,0001z_k); \quad H(z) = \exp(-0,000001z_k)$$

Since the functions G and H are concave the Duloy and Norton procedure is used for the linear approximation. In table 1 are reported the data for the period 95-08 (Eurostat) used for the estimation of the states (yield and prices) generating risky prospects about of major commodities for the selected region. The standard deviation of yields (s.d) fluctuate in the range between 0,18 for sunflower and 0,59 for Maize; these values are much lower compared with the s.d of prices that fluctuate in the range between 1,84 of Barley and 10,22 of rapeseed. The covariance value shows the risk generated by the combination of price and yield variations, affecting the farmer's GM: the lowest absolute value is for Barley (0,01), the highest is for soybean (0,71); the price variability compared to yield variability is ranging from moderately high (less than 10 times for rapeseed, sorghum, maize and wheat) to very high (for barley, soybean and sunflower).

Tab 2 – Yield and price variability in Italy, period: 1995-2008

Crop	Yield			Price			cov	cov ratio
	mean	s.d	cv	mean	s.d	cv	Y-P	Y/P
Wheat	3,34	0,37	0,11	14,62	2,43	0,17	0,14	8,43
Maize	9,27	0,59	0,06	10,34	1,97	0,19	0,34	8,48
Barley	3,63	0,20	0,06	12,88	1,84	0,14	-0,01	33,59
Sorghum	5,95	0,37	0,06	10,36	1,71	0,17	-0,03	7,05
Rapeseed	1,48	0,46	0,31	26,55	10,22	0,38	3,31	3,49
Soybean	3,46	0,35	0,10	22,49	6,76	0,30	-0,71	50,66
Sunflower	2,11	0,18	0,09	24,00	6,95	0,29	0,34	199,52

Source: our elaborations from Eurostat data

The sample mean activity GM includes 14 years observation (period 1995-2008) about the states of nature assumed to have the same probability; p_r , for $r = 1..14$ with $\sum_{r=1}^{14} Pr = 1$ and $p_1 = p_2 =$

⁴ M is a measure of risk because is an unbiased measure of the population variance assumed to be normal

...p14 = 1/14; the unbiased estimator of the mean absolute deviation (MAD) of the expected farm GM is:

$$9 - \quad M = s^{-1} \sum_{r=1}^s | \sum_{j=1}^n (c_{rj} - c_j^*) x_j$$

s, is the sample size equal to 14, c_{rj} is the GM per unit of the activity j^{th} in the year r^{th} (for $r = 1..14$), c_j^* is the sample mean of the GM per unit of activity j^{th} ⁵ and X_j is the dimensional variables of crop production. With the solution proposed by Hazell the risk is computed using the LP-MOTAD approach that minimize the MAD for a given level of expected GM indicated by $E(z)$. The utility efficient programming model assumes the linearity condition, and the all a_{hj} , b_h and c_j are assumed to be known and constant. The objective function is formulated as it follows:

$$10 - \quad \text{Max } E(U) = \sum_{k=1}^s p_k (G(z) + \lambda H(z)) \text{ with } \lambda \text{ varied parametrically}$$

$E(U)$ is the expected Sumex Utility, $K = 1..s$ indicates one of the equally probable 14 state of nature of the crops, p_k . Because $E(U)$ is non linear, a procedure to approximate the Utility has been used (Mc Carl and Onal, 1989)

$$11 - \quad G(z) = -\exp(-0,0001z); \quad H(z_k) = \exp(-0,000001z)$$

z is the expected gross margin ($c_{rj} - c_j^*$) to be maximized with a parametric constraint on the sum of negative deviations.

$$12 - \quad \sum_{j=1}^n a_{hj} x_j \geq \text{or} \leq b_h \text{ for } h = 1..m$$

$$13 - \quad \sum_{j=1}^n (c_{rj} - c_j^*) x_j + y_r \geq 0 \text{ for } r = 1..s$$

$$14 - \quad \sum_{r=1}^s y_r \leq \lambda \text{ for } \lambda = 0 \rightarrow \lambda \max$$

Constraint 12 specify the restrictions of land, labour and working capital resources to b_h .

Constraint 13, is specific of the MOTAD and refers to the $r = 1..n$ deviations from the sample mean c_j^* , of each of the activity j (1..7) in year r (1..14). The sample includes the all states of nature that are assumed with the same probability. The non negative variables Y satisfies the required condition that the deviations of the gross margin in a single state r should be non negative.

Constraint 14 takes into account the sum of total negative deviations throughout the all 14 states and allows to compute the lambda critical values at each change of basis.

The all x and y are nonnegative variables and λ is a value varied parametrically from 0 to its maximum relevant value.

5 – Data for the analysis

The risk aversion behaviour of farmers, is simulated to generate a farm plan with a set of crop enterprises (Hardaker and others, 1997; Rosa et al., 2002, 2010; Ray et al., 1998; Richardson et al., 2000) with Gross margin (GM) per unit collected for the period 1995-2008 (source Eurostat for prices and yield). Each period represents a given state of nature and it is assumed that the all states are equally probable.⁶ The selected enterprises are: Wheat, Mais, Barley, Sorghum, Rapeseed, Soybean and Sunflower; it is assumed that the decision to sow these crops are not constrained by the need to recycle feed into farm for animal feeding, and rotation is simulated with the usual constraints. The evidence of the risk, is observed in table 3 where are reported the gross margins with mean, standard deviation and coefficient of variation ($CV = S.D./M$ CV). It is observed

⁵ M is a measure of risk being an unbiased measure of the population variance assumed to be normal

⁶ Our knowledge regarding possible shifts in the frequencies of extreme events with a new climate regime is limited. There also remains work to be done to incorporate the current information on changes in variability, as represented in climate models, into methods for assessing impacts on agriculture. (Chen et al, 1999)⁶

a wider fluctuation of the CV for rapeseed and barley, that are the highest risky prospects, while for the other crops the range fluctuate between the minimum 0,12 and maximum 0,22 value.

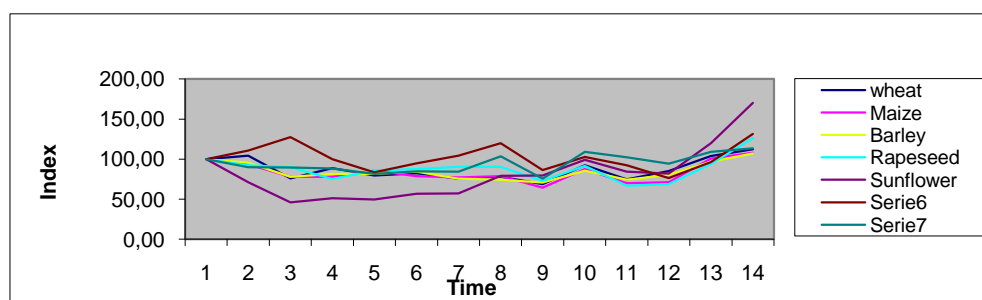
Table 3 – Vectors of GM per Ha for a set of crop products and a sample of 14 year observations

Year	Wheat	Maize	Barley	Sorghum	Rapeseed	Soybean	Sunflower
1995	577,87	1698,23	649,35	699,35	320,41	693,14	441,52
1996	603,05	1586,58	621,38	644,59	228,16	766,48	396,66
1997	439,59	1323,76	507,29	629,58	147,76	884,80	395,85
1998	513,98	1326,04	530,84	525,51	164,34	694,05	390,08
1999	460,67	1451,08	529,86	602,06	159,03	580,00	359,99
2000	475,69	1340,38	548,03	611,32	181,51	657,77	374,04
2001	437,00	1312,91	492,05	633,22	183,88	723,57	373,19
2002	450,00	1341,13	482,30	632,53	254,18	831,05	457,72
2003	396,94	1094,52	459,80	508,31	254,79	599,99	333,75
2004	534,69	1489,25	554,63	641,23	317,20	712,04	481,71
2005	431,49	1185,70	482,00	468,10	270,14	639,05	451,14
2006	491,16	1207,15	522,18	481,64	262,80	528,37	415,78
2007	600,00	1713,41	629,53	658,21	384,54	667,46	482,00
2008	650,00	1833,22	695,83	884,44	544,81	910,73	501,17
Mean state r	504,44	1421,67	550,36	615,72	262,40	706,32	418,18
dev st	111,73	217,68	279,52	104,85	106,90	111,21	51,47
CV	0,22	0,15	0,51	0,17	0,41	0,16	0,12

Source: Eurostat Agricultural data

For the risk generated by GM, using the index values for the yield of these crops (1995 = 100): there is no clear evidence of an increasing trends through time but some yearly fluctuations with evidence of increasing trend in the last period 06-08 due to price increase (see tab. 1) and closer GM correlation in the last six years.

Fig.3 – Index of hystorical trend of GI for different crops: period 95-08; 1995 = 100



Source – Our elaboration from Eurostat

Data in table 3 are used to compute the mean absolute deviations reported in table 4

Tab. 4 – Mean absolute deviations

State (years)	Mean absolute deviation from the mean r				(Crj - C*j)		
	Wheat	Maize	Barley	Sorghum	Rapeseed	Soybean	Sunflower
1995	73,43	276,57	98,99	83,63	58,01	13,18	23,33
1996	98,61	164,91	71,02	28,87	34,24	60,16	21,52
1997	64,85	97,91	43,07	13,86	114,63	178,48	22,34
1998	9,54	95,63	19,52	90,21	98,06	12,27	28,10
1999	43,77	29,41	20,51	13,66	103,37	126,33	58,20
2000	28,75	81,29	2,34	4,40	80,89	48,55	44,14
2001	67,43	108,76	58,31	17,50	78,52	17,25	44,99
2002	54,44	80,54	68,06	16,81	8,21	124,73	39,54
2003	107,50	327,15	90,56	107,41	7,61	106,33	84,44
2004	30,25	67,58	4,27	25,51	54,80	5,72	63,52
2005	72,94	235,97	68,37	147,62	7,75	67,27	32,96
2006	13,28	214,51	28,18	134,08	0,40	177,95	2,41
2007	95,56	291,75	79,17	42,48	122,14	38,86	63,81
2008	145,56	411,55	145,46	268,71	282,42	204,41	82,98
Mean r	64,71	177,39	56,99	71,05	75,07	84,39	43,74

Source – Our elaboration from Eurostat

6 - Results of the Utility efficient frontier

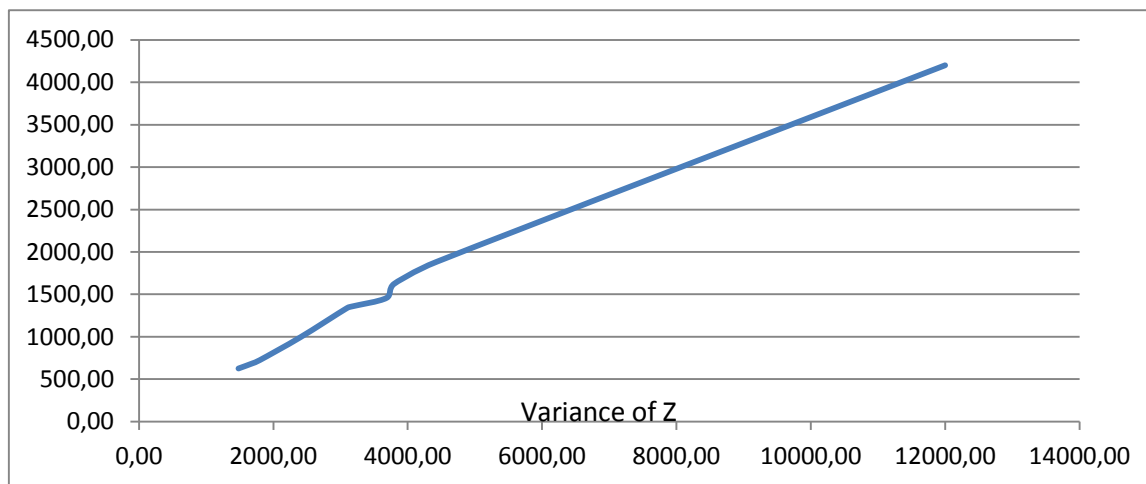
The solutions obtained with the linear approximation of the Utility function vector $U(z)$ represent the loci of the base solutions (U - M indifference curve) with the efficient set of activities, obtained from the simulation of 14 years of states which are the historical information disclosed to farmers for their planning decisions. The efficient frontier has required to find the utility values combined with the risk M by varying parametrically the lambda coefficient in the limit between a and b to find the efficient portfolio solutions given by the corner points of the Utility frontier. Starting with an initial solution the lambda is progressively increased and the OF values indicate the reaction to changes in lambda until one of the constraint is met or one of the variables is driven to zero. At this point a change of the base is needed and the lambda value can be further increased with the activity level varying now differently to find the next combination set of activities; this procedure is continued to find an exhaustive number of combinations to draw the frontier compatible with the risk aversion. In table 5 are reported the values of ten corner solutions with the values of the objective function $U(z)$ and variance M corresponding to given levels of risk aversion imposed by lambda; the points of corner solution determined by change in the base represent the efficient frontier. The solutions are targeted to find the degree of risk aversion with respect to the set of activities in the whole farm plan to indicate the degree of specialization or diversification with the implied risk.

The two extreme solutions with minimum and maximum values of $U(z)$ are reported in 1.st and last line of the table 5. The last line reports the solution of the most specialized plan represented by activity mais, using 85 of the 100 Ha (85% of the available land is used); the utility is at the maximum absolute value with the absolute risk aversion given by the ratio M/U is 2,86 (slope of the Utility frontier as a measure of the risk aversion). The minimum value of the $U(Z)$ is reported in line 1 with four enterprises (annual crops) activated and three of them in rotation: wheat, maize and soybean. The land used is only the 62% of the total; by the way also the following two plans use a limited quantity of land. The second part of the table 5 reports diversified plans with better results; the best is the one in line 9 with six activities and 99% of land used with a limited coefficient of risk aversion. The most diversified plan reported in line 7 did not produce higher results in terms of U and land used but one of the lowest values in term of risk aversion. Finally, comparing these results with the ones obtained with traditional solution reported in the last column it is possible to notice great differences between the $U(z)$ values and GI values. The results suggest to select the plan reported in the line 7 that is the most diversified one.

Tab 5 – Corner solutions and combination of activities

Utility frontier with risk			Level of the activities included in in the whole farm plan								GI
M	U(Z)	M/U	Wheat	Maize	Barley	Sorghum	Rapeseed	Soybean	Sunflower	Land use	
1480,47	626,11	2,36	20	20	0	0	2	20	0	62	1002,1
1690,62	687,45	2,46	0	40	2	0	10	0	0	52	704,86
1797,42	724,49	2,48	20	20	2	0	10	20	0	72	1240,26
2401,43	993,83	2,42	0	60	2	0	10	0	0	72	911,66
3092,66	1336,80	2,31	20	40	0	0	0	4	0	64	795,96
3176,48	1358,65	2,34	20	40	0	0	2	4	0	66	849,06
3685,21	1458,01	2,53	20	30	8	10	10	10	10	98	1539,64
3799,31	1624,87	2,34	20	40	20	0	0	0	0	80	963,6
4430,48	1883,68	2,35	20	40	20	0	2	10	7	99	1409,6
12000,00	4200,00	2,86	0	85	0	0	0	0	0	85	878,9

Fig. 4 - Efficient frontier for whole farm plan for a farm of 100 Ha



7 – Conclusion

This research has been dedicated to the whole farm planning using the optimal portfolio analysis to maximize the individual utility (gross revenues) consistent with personal preferences about risk aversion. The risk aversion is modulated in a given range of the utility function and for the probability distribution of the states of nature it is assumed having the same probability to simplify the analysis without loss of generality or precision. The search for a stochastic production frontier is proposed by using the sumex utility and the MOTAD linear approach to obtain the solutions of different risky prospects. The farmer's risk aversion reflects the farmer's perception of risk for states affecting the activities: with the growing expected values of GM farmers tend to specialize in Maize enterprise that shows one of the lowest values of CV of gross income. The lower utility values of the more diversified portfolio combinations suggest that the trade off between utility and risk drives to the more specialized solution and suggests that the diversification leads to a lower risk and lower utility that is not very attractive for the larger farm. Another consideration is for the risk growth driven by volatility in the energy market more than climate change. The commodity prices fluctuate in a wider range due to higher volatility in agricultural markets due to speculation in future and financial markets. (Rosa and Vasciaveo, 2010; 2012a, 2012b, 2012c). Beside some level of speculation is desirable for the market activity, the farmers, especially the smaller ones are more cautious in their farm decision for the risk to lose their assets if something will be wrong in prices or yields. The protected environment of the farmers may have postponed the use of financial instruments, which could limit the demand for other risk management tools. Emergency measures are also blamed for undermining existing risk management systems to the extent that farmers are

relying on ad hoc government intervention in case of a crisis rather than on long term risk hedging strategies. Reduced support prices and other protection mechanism would require instruments to help farmers to cope with the resulting increase in income-related risk. We conclude that the optimal portfolio solution depends on the variety of the rural environment. Weather condition influences soil moisture, plant root uptake, water and temperature-related stress on plants; furthermore, soil characteristics and cropping practices can also affect the crop growth. An alternative to the historical crop yield data used in this work a crop modelling approach based on simulation will cope with the uncertainty caused by weather conditions, soil characteristics or cropping practices in a more dynamic contest.

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Tab 2 – Yield and price variability in Italy, period: 1995-2008

Crop	Yield			Price			cov	cov ratio
	mean	s.d	cv	mean	s.d	cv	Y-P	Y/P
Wheat	3,34	0,37	0,11	14,62	2,43	0,17	0,14	8,43
Maize	9,27	0,59	0,06	10,34	1,97	0,19	0,34	8,48
Barley	3,63	0,20	0,06	12,88	1,84	0,14	-0,01	33,59
Sorghum	5,95	0,37	0,06	10,36	1,71	0,17	-0,03	7,05
Rapeseed	1,48	0,46	0,31	26,55	10,22	0,38	3,31	3,49
Soybean	3,46	0,35	0,10	22,49	6,76	0,30	-0,71	50,66
Sunflower	2,11	0,18	0,09	24,00	6,95	0,29	0,34	199,52

Source: our elaborations from Eurostat data