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Labour on the family farm: a theory under uncertainty – an extension

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ABSTRACT

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In a framework developed by P.J. Dawson, the effects of output price risk on the family labour supply and its demand for hired labour are investigated. In particular, the effects of changes in autonomous income, expected output price, family composition, and farm size are studied. Comparative statics is used to sign these effects, revealing the importance of the behavior of the measures of absolute, relative and partial risk aversion in determining them. It is shown that some of the effects may be determined only via empirical research.

1. INTRODUCTION

In his paper entitled *Labour on the family farm: a theory under uncertainty*, P.J. Dawson presents some interesting theoretical results. Dawson examines family decisions concerning the supply of family labour and the demand for hired labour under output price uncertainty. In particular, he considers the effects of: a change in autonomous income; a change in the mean output price; a change in family composition; and a change in farm size. Dawson also distinguishes between two types of family farms: family-labour-only farms and labour-hiring farms. Dawson's results, however, make use of some implicit assumptions, the essence of which can be captured by saying that the covariance of any pair of partial derivatives of the utility function vanishes. As it turns out, these are fairly strong assumptions which impose severe restrictions on the individual's preferences (i.e. the functional form of the utility function) or the distribution of

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the random variable. The purpose of this paper is to investigate the consequences of relaxing those assumptions. In other words, we generalize Dawson's results to the case in which the covariance assumptions do not hold, and we show what is required for Dawson's results to remain valid. In addition, we derive new results concerning the effect of adding a family member. In particular, the effect of an additional working member is found to be different from the effect of an additional nonworking member, a finding which is supported by an intuitive explanation.

This paper deals with only one type of farm, i.e. labour-hiring. This paper should be viewed as complementary to that of Dawson's, although it is detailed enough to be read by itself. Its structure is parallel to that of Dawson's paper. For the sake of completeness we begin with a presentation of Dawson's model. The second section analyzes the effect of a change in autonomous income. The third deals with the effect of a change in mean output price. The fourth examines a change in the family composition. The fifth looks at the effect of a change in farm size. Finally, a short summary concludes the paper.

2. THE MODEL

The family's objective is to maximize the expected value of its welfare function, which takes the form:

$$E[W] = (\alpha + \beta)E[U(m)] - \alpha D(l)$$

where α is the number of family workers, β is the number of family dependents, $U(m)$ is the von Neumann–Morgenstern utility of income per family member, $D(l)$ is the disutility of labour per family worker, and E is the expectations operator. Assume that:

$$\alpha \geqslant 0 \qquad \beta \geqslant 0 \qquad \alpha + \beta > 0$$

and

$$U_m > 0 \qquad U_{mm} < 0$$
$$D_l > 0 \qquad D_{ll} > 0$$

The income per family member, m, is:

$$m = (1/(\alpha + \beta))\,(PQ(L,N) - wH + Y)$$

where Y is autonomous income, w is the wage rate, H is hired labour, P is output price, Q is the production function, N is fixed input (land) and L is total labour ($L = \alpha l + H$). It is assumed that the production function is well behaved with $Q_l > 0$ and $Q_{LL} < 0$. Uncertainty is introduced via random output price with mean $\bar{P} = E[P]$.

The necessary conditions for a maximum are:

$$\partial E[W]/\partial H = E[U_m(PQ_L - w)] \leqslant 0 \tag{1}$$

$$H \cdot E[U_m(PQ_L - w)] = 0 \tag{2}$$

$$\partial E[W]/\partial l = \alpha\, E[U_m PQ_L] - \alpha D_l \leqslant 0 \tag{3}$$

$$F \cdot E[U_m PQ_L] - FD_l = 0 \tag{4}$$

where $F \equiv \alpha l$ is total family labour. The sufficient conditions are:

$$\partial^2 E[W]/\partial H^2 = A_1 = E\left[(1/(\alpha+\beta))U_{mm}(PQ_L - w)^2 + U_m PQ_{LL}\right] < 0$$

$$\partial^2 E[W]/\partial l^2 = A_2 = E\left[(\alpha^2/(\alpha+\beta))U_{mm}(PQ_L)^2 + \alpha^2 U_m PQ_{LL}\right] - \alpha D_{ll} < 0$$

and

$$\Delta = A_1 \cdot A_2 - B^2 > 0$$

where

$$B = \partial^2 E[W]/(\partial l\, \partial H) = E[(\alpha/(\alpha+\beta))U_{mm} PQ_L(PQ_L - w) + \alpha U_m PQ_{LL}]$$

It can be shown, in a way similar to that of Batra and Ullah (1974, p. 541), that the expected marginal product of labour is strictly greater than the wage rate, w. That is:

$$E[P]\, Q_L > w$$

The economic implication of this relationship is that less labour is employed and less output is produced under uncertainty conditions than under certainty conditions. Two remarks conclude this section:

(1) The analysis continues under the assumption that no family member works off the farm. Such a situation may result from: transaction costs (as noted by one of the referees) such as those involved in commuting from isolated farms, high returns on the farm relative to the market wage, or ideological reasons. Nevertheless, the model does not explicitly exclude the possibility of work off the farm (H can be negative).

(2) The model treats family composition as an exogenous variable. An interesting extension would be to endogenize it. In the long run, the family may choose the optimal number of working and nonworking members. This decision, of course, may not be possible in the short run, and is beyond the scope of this paper.

3. CHANGE IN AUTONOMOUS INCOME

In this section, we derive comparative static results concerning a change in autonomous income. Formally, let $Y = Y + \delta$; totally differentiating the necessary condition with respect to l, H, and δ, and evaluating the results at $\delta = 0$ yields:

$$\partial H/\partial \delta = \frac{-Z_1 A_2 + Z_2 B}{\Delta}$$

and

$$\partial l/\partial \delta = \frac{-Z_2 A_1 + Z_1 B}{\Delta}$$

where

$Z_1 = \partial^2 E[W]/(\partial H \partial\delta) = (1/(\alpha+\beta))E[U_{mm}(PQ_L - w)]$

and

$Z_2 = \partial^2 E[W]/(\partial l \partial\delta) = (\alpha/(\alpha+\beta))E[U_{mm}PQ_L]$

Substituting A_1, B, Z_1, and Z_2 into the equation for $\partial l/\partial\delta$ and rearranging yields:

$$\partial l/\partial\delta = -\frac{\alpha\, E[U_m PQ_{LL}]\cdot E[U_{mm}]w}{\Delta}$$
$$+\left(\alpha/(\alpha+\beta)^2\right)\frac{E\left[U_{mm}(PQ_L - w)^2\right]\cdot E[U_{mm}](-w)}{\Delta}$$
$$+\left(\alpha/(\alpha+\beta)^2\right)\frac{E[U_{mm}(PQ_L - w)(w)]\cdot E[U_{mm}(PQ_L - w)]}{\Delta}$$

This expression can only be reduced to that of Dawson's, i.e. the first term,[1] in the special case when $\mathrm{VAR}(U_{mm}(PQ_L - w)) = \mathrm{COV}(U_{mm}(PQ_L - w)^2, U_{mm})$. In general, these covariances are not equal. In fact, their equality is guaranteed in only two cases. The first is constant P, i.e. no uncertainty, and the second is a linear utility function, i.e. risk neutrality.

Without any additional assumptions, it is possible to sign the three terms on the right-hand side. The first and second terms are clearly negative. The last term is nonnegative because it is a product of a positive number and a square. For nonconstant absolute risk the third term is strictly positive (Sandmo, 1971). Hence Dawson's qualitative result, that the sign of $\partial l/\partial\delta$ is unambiguously negative, holds under constant absolute risk aversion. Assuming only increasing or decreasing absolute risk aversion causes the effect of a change in autonomous income on individual and total family labour supply to be indeterminate.

Further investigation of the expression for $\partial l/\partial\delta$ reveals that the effect of a change in autonomous income on individual labour supply can be signed under milder conditions as well. A_1 is negative by the second order conditions and $-Z_2$ is positive by assumption, hence $-Z_2A_1$ is negative. Z_1 is positive under decreasing absolute risk aversion (Sandmo, 1971); the B term has two components, of which the second is negative by assumption and the first is negative under the assumption of increasing partial risk aversion with respect to the random variable P. Note that the latter assumption is compatible with the commonly accepted assumptions of decreasing absolute risk aversion and increasing relative risk aversion.

[1] With the exception that the expectation operator was taken out of the product.

Increasing partial risk aversion with respect to P [i.e. $(\partial/\partial P)(-U_{mm}/U_m)PQ > 0$] is sufficient for $E[U_{mm}(PQ_L - w)PQ_L]$ to be negative. This can be shown formally: let superscript * denote a variable at the point where $PQ_L - w = 0$. Then for $P > P^*$, $-(U_{mm}/U_m)PQ > (-(U_{mm}/U_m)PQ)^*$. By multiplying both sides by $-U_m(PQ_L - w)$ and taking the expectation we get: $E[U_{mm}(PQ_L - w)P]Q < 0$. Dividing by Q and multiplying by PQ_L gives the desired result. The same reasoning can be used for $P < P^*$. Hence, under the assumption of increasing partial risk aversion, the effect of an increase in autonomous income on the individual's labour supply is unambiguously negative.

As it was shown by Dawson:

$$\partial H/\partial \delta = -\partial F/\partial Y + \frac{(\alpha/(\alpha+\beta))D_{ll}\, E[U_{mm}(PQ_L - w)]}{\Delta}$$

That is, the effect of an increase in autonomous income on hired labour equals the negative of the effect of the same increase on total family labour plus an additional term. Thus, this additional term, which is positive under the assumption of decreasing absolute risk aversion (Sandmo, 1971), is the effect of an increase in the autonomous income on total labour. Under Dawson's assumptions, $\partial F/\partial Y$ is ambiguous, leading to an ambiguity of the derivative of hired labour with respect to a change in autonomous income. However, under the assumptions allowing negative signing of $\partial F/\partial Y$, the effect of an increase in autonomous income on hired labour is positive.

The mechanism underlying these results appears to be as follows: Assuming decreasing absolute risk aversion, a lump-sum income makes the family wealthier and less risk averse. As a result, total labour input and hence output are increased. Ceteris paribus this would be the effect of hired labor as well. Assuming that leisure is one of the normal goods, the direct effect of an increase in autonomous income is to decrease the family labor supply. However, the total effect of such an increase either on owner or hired labour must include an interaction effect between the two. Unlike under conditions of certainty, the decisions regarding hired and owner labour are not independent. They are made simultaneously rather than sequentially, leading to an interaction between them, represented by the B term. We will elaborate with regard to owner labour; the intuition with regard to hired labor is similar.

As aforesaid, an increase in autonomous income causes an increase in hired labour. The increase in hired labour has two effects on family labour. The first effect is a decrease in marginal product of labour on the farm and hence is negative. The second is related to the family risk attitudes. The increase in hired labour increases the risky component of the family income. Assuming the plausible assumption that the measure of partial risk aversion is increasing in the risky component of income, the certainty equivalent of the random variable – price – decreases, leading to further reduction in the value marginal product of labor on the farm. Thus both components of the interaction effect are negative. To summarize, under the maintained assumptions, the direct effect of the wealth increase and the two components of the interaction effect are all negative, resulting in an overall negative effect on the family labour supply from an increase in autonomous income.

4. CHANGE IN EXPECTED OUTPUT PRICE

Consider the effect of a change in expected output price. As Dawson shows, by a method similar to the one used in Section 3, the effect of a change in mean output price on family labour for labour-hiring farms is ambiguous. That is

$$\partial l/\partial \bar{P} = (\partial l/\partial Y)Q + \frac{(\alpha/(\alpha+\beta))E[U_{mm}(PQ_L - w)]\, wE[U_m Q_L]}{\Delta}$$

is indeterminate. The first term is an autonomous-income effect, which is negative under either the assumption of constant absolute risk aversion or that of increasing partial risk aversion with respect to the random output price. The second term is positive under decreasing absolute risk aversion, but its interpretation as a substitution effect should not be confused with the familiar substitution effect existing under certainty conditions. A substitution effect under certainty conditions exists only in family-labour-only farms where an increase in expected output price increases the shadow price of leisure and hence affects the consumption of leisure in a direction opposite to that of the income effect, provided leisure is one of the normal goods. In labour-hiring farms, however, the shadow price of leisure is w. The hired labour in this case is used as a buffer, i.e. the farm owner can always get one more hour of leisure for the price of w because hired labour and owner labour are perfect substitutes.

The effect of a change in the expected output price on hired labour is given by: [2]

$$\partial H/\partial \bar{P} = (\partial F/\partial Y)Q + \frac{(\alpha/(\alpha+\beta))D_{ll}\, E[U_{mm}(PQ_L - w)]\, Q}{\Delta} - \frac{(\alpha^2/(\alpha+\beta))E[U_{mm}PQ_l]\cdot E[U_m Q_L]w}{\Delta} + \frac{\alpha D_{ll}\, E[U_m Q_l]}{\Delta}$$

[2] Note that the difference between this expression and the original expression is that the expectation operator has not been taken out of the product.

The last three terms on the right-hand side of this expression are obviously positive. The sign of the whole expression however, is positive under an additional assumption, either that of constant absolute risk aversion or that of increasing partial risk aversion with respect to the random price. The effect of a change in the expected output price on total labour, $\partial F/\partial \bar{P} + \partial H/\partial \bar{P}$, is unambiguously positive under the assumption of decreasing absolute risk aversion alone.

5. CHANGES IN FAMILY COMPOSITION

We turn now to investigate the effect of changes in family composition on individual labour supply. In a similar way, the effect of additional family working members on individual labour supply is found to be:

$$\begin{aligned}
\partial l/\partial \alpha = {} & (\alpha/(\alpha+\beta)) \frac{E[U_{mm}(PQ_L - w)]\, w \cdot E[U_m PQ_{LL} l]}{\Delta} \\
& - (\alpha/(\alpha+\beta)) \frac{E[U_m PQ_{LL}] \cdot E[U_{mm}(PQ_L l - m)]\, w}{\Delta} \\
& + \left(\alpha/(\alpha+\beta)^2\right) \\
& \qquad \times \frac{E[U_{mm}(PQ_L - w)] w \cdot E[U_{mm}(PQ_L l - m)(PQ_L - w)]}{\Delta} \\
& - \left(\alpha/(\alpha+\beta)^2\right) \frac{E\left[U_{mm}(PQ_L - w)^2\right] \cdot E[U_{mm}(PQ_L l - m)]\, w}{\Delta}
\end{aligned}$$

Similarly, the effect of additional family nonworking members is:

$$\begin{aligned}
\partial l/\partial \beta = {} & (\alpha/(\alpha+\beta)) \frac{E[U_m PQ_{LL}] \cdot E[U_{mm} m]\, w}{\Delta} \\
& + \left(\alpha/(\alpha+\beta)^2\right) \frac{E[U_{mm}(PQ_L - w)]\, w \cdot E[U_{mm}(-m)(PQ_L - w)]}{\Delta} \\
& - \left(\alpha/(\alpha+\beta)^2\right) \frac{E\left[U_{mm}(PQ_L - w)^2\right] \cdot E[U_{mm}(-m)]\, w}{\Delta}
\end{aligned}$$

The finding that the effect of an additional nonworking member on individual labour supply is different from that of an additional working member can be intuitively understood as follows. The additional nonworking member is getting an income of m dollars and is therefore reducing the income to be shared among the rest of the family members by m. An additional working member reduces the family income by the same amount but he/she also increases the family income by $PQ_L l$.

The sign of $\partial l/\partial\alpha$ is ambiguous because no matter what the sign of $E[U_{mm}(PQ_L l - w)]$, the second and fourth terms have opposite signs. To sign $\partial l/\partial\beta$ we need to make two additional assumptions, that of increasing relative risk aversion and that of positive income. While the first assumption is reasonable (Arrow, 1970) the second one is controversial, because under uncertainty a positive income is not guaranteed. Nevertheless, under these two assumptions, the effect of an additional nonworking member on family labour is unambiguously positive. Note that the expression for $\partial l/\partial\beta$ is reduced to Dawson's finding when $\text{cov}(U_{mm}(PQ_L - w), U_{mm}(PQ_L - w)) = \text{cov}(U_{mm}(PQ_L - w)^2, U_{mm})$. However, equality between these two covariances is only achieved under constant P (i.e. no uncertainty) or a linear utility function (i.e. risk does not affect the decision-maker).

The effect of an additional family member on total family labour supply is considered next. The change in total family labour supply caused by an additional working member is given by:

$$(\partial F/\partial\alpha) = \alpha(\partial l/\partial\alpha) + l$$

The analysis above clearly indicates that $\partial F/\partial\alpha$ is ambiguous. The effect of an additional nonworking family member on total family labour supply is given by:

$$\partial F/\partial\beta = \alpha(\partial l/\partial\beta)$$

Under the assumptions used to sign $\partial l/\partial\beta$, it is unambiguously positive.

Next we consider the effect of an additional family member on hired labour. An additional working member would affect hired labour in the following way:

$$\begin{aligned}\partial H/\partial\alpha = & -\partial F/\partial\alpha + (1/(\alpha+\beta))\frac{D_{ll}\,E[U_{mm}(wl-m)(PQ_L-w)]}{\Delta} \\ & -\left(\alpha/(\alpha+\beta)^2\right)\frac{E[U_{mm}(wl-m)]\cdot E[U_{mm}(PQ_L-w)]\,w^2}{\Delta} \\ & -\left(\alpha/(\alpha+\beta)^2\right)\frac{E[U_{mm}(wl-m)(PQ_L-w)]\cdot E[U_{mm}]\,w^2}{\Delta}\end{aligned}$$

It appears that the term $E[U_{mm}(wl - m)]$ is ambiguous, hence the whole expression is ambiguous and the effect of an additional working family member on hired labour is indeterminate. Note that the last term of this equation vanishes when there is no risk or when risk does not matter. In each of these cases, the result coincides with Dawson's.

The change in hired labour resulting from an additional nonworking member is as follows:

$$\partial H/\partial\beta = -\partial F/\partial\beta + (1/(\alpha+\beta))\frac{D_{ll}\,E[U_{mm}(-m)(PQ_L - w)]}{\Delta}$$
$$-\left(\alpha/(\alpha+\beta)^2\right)\frac{E[U_{mm}(-m)]\cdot E[U_{mm}(PQ_L - w)]\,w^2}{\Delta}$$
$$-\left(\alpha/(\alpha+\beta)^2\right)\frac{E[U_{mm}(-m)(PQ_L - w)]\cdot E[U_{mm}]\,w^2}{\Delta}$$

In this expression, none of the terms is ambiguous, and hired labour is increased when the family takes on an additional dependent.

The change in total labour resulting from the addition of a family member can be found by summing up the change in total family labour and the change in hired labour. The above results show that in the case of an additional working member the effect is ambiguous, while in that of an additional nonworking member the effect is unambiguously positive.

6. CHANGE IN FARM SIZE

This section deals with the effect of an increase in farm size on both family and hired labour. The comparative statics of the problem gives:

$$\partial l/\partial N = -\frac{(\alpha/(\alpha+\beta))E[U_{mm}PQ_N]\cdot E[U_m PQ_{LL}]\,w}{\Delta}$$
$$+\frac{(\alpha/(\alpha+\beta))E[U_m PQ_L N]\cdot E[U_{mm}(PQ_L - w)]\,w}{\Delta}$$
$$-\frac{\left(\alpha/(\alpha+\beta)^2\right)E\left[U_{mm}(PQ_L - w)^2\right]\cdot E[U_{mm}PQ_L PQ_N]}{\Delta}$$
$$+\frac{\left(\alpha/(\alpha+\beta)^2\right)E[U_{mm}PQ_L(PQ_L - w)]\cdot E[U_{mm}(PQ_L - w)PQ_N]}{\Delta}$$

This result differs from Dawson's result in that it has two additional terms. However, its interpretation and conclusion are similar. The first and third terms on the right-hand side are negative and represent the wealth effect. The second and fourth terms are positive and represent the substitution effect. Hence, the overall effect of an increase in farm size on individual family labour and on total family labour is indeterminate. Note that under certainty conditions the substitution effect vanishes because the individual family labour decision is independent of the hired labour decision.

The effect of a change in farm size on hired labour is given by:

$$\partial H/\partial N = +\frac{(\alpha^2/(\alpha+\beta))E[U_{mm}PQ_N]\cdot E[U_mPQ_{LL}]w}{\Delta}$$
$$-\frac{(\alpha^2/(\alpha+\beta))E[U_{mm}PQ_L]\cdot E[U_mPQ_LN]\,w}{\Delta}$$
$$+\frac{(\alpha/(\alpha+\beta))D_{ll}E[U_{mm}(PQ_L-w)PQ_N]}{\Delta}$$
$$+\frac{\alpha D_{ll}E[U_mPQ_LN]}{\delta}$$

The first, second and fourth terms are all positive. The third can be signed under the additional assumption of decreasing partial risk aversion with respect to the random variable P. [3] Note that this assumption does not exclude the possibility of increasing relative risk aversion: PQ sufficiently larger than m, or alternatively wH sufficiently larger than Y will do. Hence, under the assumption of decreasing partial risk aversion, the effect of an increase in farm size on hired labour is unambiguously positive.

Calculating $\alpha\partial l/\partial N + \partial H/\partial N$ yields an ambiguous expression for the effect of an increase in farm size on total labour. This result coincides with Dawson's finding.

7. SUMMARY AND CONCLUSIONS

The effects of changes in autonomous income, expected output price, family composition, and farm size on the supply of family labour and the demand for hired labour were considered. Under Dawson's assumptions, the effects of an increase in autonomous income on hired labour and family labour are ambiguous. Dawson's finding of a decrease in family labour and an increase in hired labour hold qualitatively under either the assumption of constant absolute risk aversion or that of increasing relative risk aversion with respect to the random income. However, the effect of lump-sum income on total labour, as Dawson showed, is positive under the assumption of decreasing absolute risk aversion alone.

With regard to a change in expected output price, a similar conclusion can be derived. It appears that there does not exist a set of assumptions under which its effect on owner labour can be signed. However, the effect of an increase in expected output price on hired labour is positive under

[3] One may prove this by a method similar to that presented in Section 3.

either the assumption of constant absolute risk aversion or that of increasing relative risk aversion with respect to the random income, while decreasing absolute risk aversion alone is sufficient to sign such an effect on total labor to be positive.

With respect to a change in family composition, we show that it does matter whether the additional member is working or nonworking, in contrast to Dawson's finding. The effect of an additional working member on family labour is found to be ambiguous, while the effect of an additional dependent is positive. The latter result coincides qualitatively with that of Dawson. The effect on hired labour is ambiguous when there is an increase in the number of family workers, and positive when there is an increase in the number of dependents, the latter contrasting with Dawson's finding of ambiguity. An increase in farm size ambiguously affects the family labour supply. Hired labour is shown to be positively affected by increased farm size, under the assumption of decreasing partial risk aversion, representing a refinement of Dawson's general finding that hired labour is ambiguously affected by a change in farm size.

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