Nearly optimal linear programming as a guide to agricultural planning

Scott R. Jeffrey, Ron R. Gibson and Merle D. Faminow

*Department of Agricultural Economics and Farm Management, University of Manitoba,
Winnipeg, Man. R3T 2N2, Canada*

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ABSTRACT

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Linear programming has long been used as a tool in agricultural planning. This paper
presents and discusses a technique that can be used in conjunction with linear programming
to evaluate 'nearly optimal' solutions. This technique is referred to as Nearly Optimal
Linear Programming or Modelling to Generate Alternatives (MGA). MGA allows planners
to incorporate important objectives that are difficult to include in a mathematical model by
identifying and evaluating alternative 'nearly optimal' solutions. Some of these alternative
solutions may be consistent with the goals or objectives of decision makers.
    To date, MGA has received little use in addressing agricultural planning problems. A
micro-level application of MGA, concerning a dairy ration formulation problem, is pre-
sented to demonstrate the relevance of MGA to agricultural planning by decision makers.
Within this application, the use of MGA to complement and enhance normal linear
programming analysis is also discussed.

INTRODUCTION

In many countries, agricultural policy development and planning is a
difficult task, particularly because social objectives often force the adoption
of policies that are not consistent with narrow private profitability criteria.
In addition, capital constraints and limited availability of agricultural inputs
may severely restrict the options available to policy makers. Analytic tools
are required that allow policy choices to be made on the basis of objective

Correspondence to (present address): Scott R. Jeffrey, Dep. Rural Economy, University of
Alberta, 515 General Services Building, Edmonton, Alta. T6G 2H1, Canada.

economic criteria but also permit sensitivity to the often delicate process of pursuing normative social goals. Furthermore, given that data limitations can be quite serious (i.e., consistent time series of agricultural and economic variables are frequently not available), analytic models must be parsimonious in terms of data needs.

Linear programming has long been recognized as an important and useful tool in agricultural planning (Heady, 1954; King, 1953). Micro-level applications have included identification of optimal livestock rations (e.g., Klein et al., 1988) or optimal crop rotations (e.g., Henderson and Stonehouse, 1988). Macro-level applications have also been developed, including optimal facilities location studies (e.g., Faminow and Sarhan, 1983; Hilger, McCarl and Uhrig, 1977) and policy impact analyses (e.g., Graham, Webber and MacGregor, 1988). Specific benefits to using linear programming in the context of developing country economic planning are well-known. These benefits include: long time series of data are not needed; specific guidelines for achieving the 'optimal' solution are provided; and opportunity costs for alternative uses of resources can be deduced through analysis of the dual solution.

Also well known are the limitations of linear programming. The use of linear programming involves a very structured model, requiring some restrictive assumptions (e.g., linear relationships, certainty of parameters, etc.). Some of these restrictions have been overcome, to a certain extent, through the development of techniques such as separable programming, stochastic programming, goal programming, MOTAD, etc. [1] However, there are other limitations. For example, linear programming requires the assumption of complete knowledge of all parameters. As well, there is a tendency, in reporting results, to concentrate on a single optimal solution and/or arbitrary variations around the baseline. This can create inaccuracies because the reliability of optimal solutions is reduced if the system is not completely and/or accurately modelled.

The robustness of 'optimal' linear programming solutions is limited by the evidence of decision-maker behaviour that deviates from what economic modelling would consider to be optimal. This behaviour is sometimes referred to as 'quasi-rational' (Russell and Thaler, 1985). One suggested precaution in applied research to account for the possibility of quasi-rational behaviour "is to do the estimates in an unconstrained fashion whenever it is possible" (Russell and Thaler, 1985, p. 1081). While Russell and Thaler are primarily concerned with constraining behaviour in the statistical estimation of economic relationships (e.g., constraints in

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[1] Hazell and Norton (1986) provide a discussion of these techniques.

demand systems), their conclusions are also pertinent to mathematical programming models. Standard sensitivity analysis is an appropriate linear programming technique to address these concerns. However, the objective function and constraints are often an imperfect representation of the relevant decision process. Therefore, it seems reasonable to identify and analyze nearly optimal solutions in order to provide insights into quasi-rational behaviour.

This paper demonstrates a technique that allows researchers to examine some of these 'nearly optimal' feasible solutions. The technique, referred to as nearly optimal linear programming or Modelling to Generate Alternatives (MGA), may be implemented using commercial computer software. The philosophy of MGA is based on the fact that mathematical models are usually an imperfect representation of the real problem and many policy issues involve public sector planning with conflicting economic, social and political goals.

The MGA approach deals explicitly with the fact that there are important objectives and/or constraints that cannot be fully reflected mathematically. MGA recognizes that the goals and constraints faced by different decision makers using any given model may vary considerably. In many cases, there are a myriad of sub-optimal solutions within a tolerable distance of the optimal solution so that policy makers could choose an alternative that is 'nearly optimal' in a narrow economic sense in order to achieve social objectives. The MGA technique discussed below provides a methodology whereby analysts can systematically search out these nearly optimal solutions and provide decision makers with a range of alternatives. Some of these alternatives may be consistent with their goals.

MODELLING TO GENERATE ALTERNATIVES

Given the restrictive assumptions required in mathematical programming, alternative modelling techniques generating solutions that deviate from the global optimum within an acceptable range have attracted considerable interest in recent years. The MGA approach has been used by civil engineers in water resource planning problems (Chang, Brill and Hopkins, 1982; Harrington and Gidley, 1985). More recently, MGA has been used in a farm management application to select a marketing strategy for calf producers (Burton et al., 1987).

MGA is based on the premise that modelling should be a tool of the decision maker and allow a range of possible solutions rather than replacing decision makers with a single 'answer' to the problem (Gidley and Bari, 1986). Traditional optimization techniques reject all non-optimal solutions in search of the optimal. An alternative is to generate a set of solutions that

are significantly different from each other but which are optimal or nearly optimal with respect to the modelled objective(s). The MGA concept is extended in this paper by introducing a method for 'targeting' results that are of direct interest to the decision maker. Unlike goal programming, this approach does not require that quantitative weights or priorities be assigned to individual objectives. MGA also provides information to the decision maker that is not available from sensitivity analysis. MGA can therefore be used to augment traditional post-optimality analyses common to mathematical programming applications.

In the MGA approach, solutions are generated that fall within an acceptable range of the optimal solution. A number of enumeration algorithms have been developed to identify the extreme points on a convex polytope (Dyer and Proll, 1977; Dyer, 1983). Burton et al. (1987) also provide a method of computing the number of possible extreme points for a given problem. An enormous number of alternative solutions can be generated for most empirical problems using these types of procedures. As a result, a secondary selection procedure for the alternative solutions is sometimes required to provide meaningful information for decision makers.

MGA can be implemented with a one-phase or two-phase technique. One-phase techniques generate a small number of solutions that differ significantly from each other, or target the key activities of interest to the decision maker. Two-phase techniques generate a large number of alternatives in the first phase and then select a sub-set or presentation set in the second phase. The selection of the presentation set can be accomplished with various techniques, including cluster analysis or simple ad hoc inspection. [2] Because of the computational burden of this approach, two-phase techniques are typically appropriate only for relatively small problems. A survey of one-phase and two-phase techniques is provided by Gidley and Bari (1986).

The technique employed in this paper is the HSJ (Hop-Skip-Jump) method (Brill, 1979; Chang, Brill and Hopkins, 1982). The three main advantages of this technique are: (1) unlike several of the other MGA techniques, HSJ can be performed with any commercially available mathematical programming software; (2) HSJ is a one-phase technique and therefore avoids the task of determining the relevant presentation set from a large number of alternative solutions; and (3) the concept is straightfor-

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[2] The basic procedure involved in cluster analysis is the partitioning of a set of vectors into homogeneous subsets.

ward. Gidley and Bari (1986) have concluded that the HSJ method is the most practical MGA technique.

The first step in using the HSJ method is to solve the original problem to determine the optimal solution and objective function value. For example, suppose that the original linear programming model formulation is as follows:

Maximize $\quad z = c'x$

subject to

$$Ax \leq b$$
$$x \geq 0 \tag{1}$$

where z is the objective function value, c' is the vector of objective function coefficients, x is the activity vector, A is the constraint coefficient matrix, and b is the resource vector. Once the optimal solution has been generated, the objective function in (1) is converted into a constraint. This yields the following opportunity set:

$$c'x \geq (1 - j)z^*$$
$$Ax \leq b$$
$$x \geq 0 \tag{2}$$

where z^* is the optimal objective function value from (1) and j is the tolerable deviation, or tolerance level, from the optimal objective function value. For example, if $j = 0.01$, the value of the original objective function is constrained to deviate from the optimal value by no more than one percent. [3]

Solutions that fall within this tolerable deviation from the optimal solution are then examined and evaluated through the appropriate definition and construction of a new objective function. As described by Gidley and Bari (1986), the HSJ technique involves minimization of the sum of the decision variables that are non-zero in the original optimal solution, subject to the constraint set defined by (2). The new objective function forces variables into the basis that were non-basic in the previous solution, thereby producing a solution that differs significantly from the original. In the extreme case this approach would force all of the non-zero variables in the previous solution to zero, thus producing a solution that is completely different from the original optimal solution while still satisfying the objective function target value. The above procedure continues in this manner, each time minimizing the sum of decision variables that were non-zero in

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[3] If the original problem involves minimization of $z = c'x$, the first constraint in (2) would take the form $c'x \leq (1 + j)z^*$.
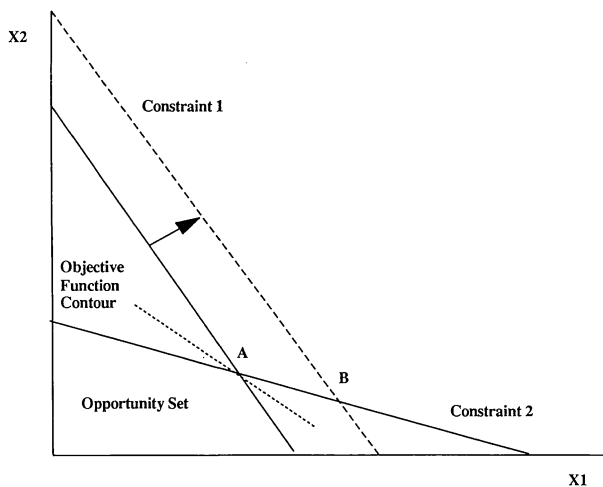
Fig. 1. Graphical representation of resource sensitivity analysis.

the previous solutions. The procedure is stopped when the MGA solutions stabilize (i.e., the set of non-zero decision variables does not change), or when a sufficient number of alternative solutions have been generated.

The basic HSJ technique may be extended in one of two ways. Additional solutions can be generated by varying the tolerance level while maintaining the same objective function. For example, if solutions within 2% of the optimum were considered acceptable by decision makers, j in model (2) could be 0.02. A further extension of the HSJ technique is to structure the objective function in an attempt to force certain results that are of direct interest to the decision maker. In general, the objective function becomes one of minimizing or maximizing the sum of target decision variables. The HSJ method does not produce the complete set of nearly optimal solutions within the tolerance level. However, it does provide a wide range of alternative solutions that may be of interest to the decision maker.

Figures 1 and 2 illustrate the basic concept of MGA relative to traditional sensitivity analysis. The original linear programming problem is a maximization problem, subject to two constraints on maximum resource availability (i.e., ≤ constraints). Sensitivity analysis is illustrated in Fig. 1. The original linear programming solution, given the objective function contour, is point A. The sensitivity of the optimal solution to adjustments in the availability of resource 1 may be examined by shifting constraint 1. In this case, the solution changes from A to B.

The basic concept underlying MGA is considerably different, as illustrated in Fig. 2. Specifically, the inferior region of the opportunity set is
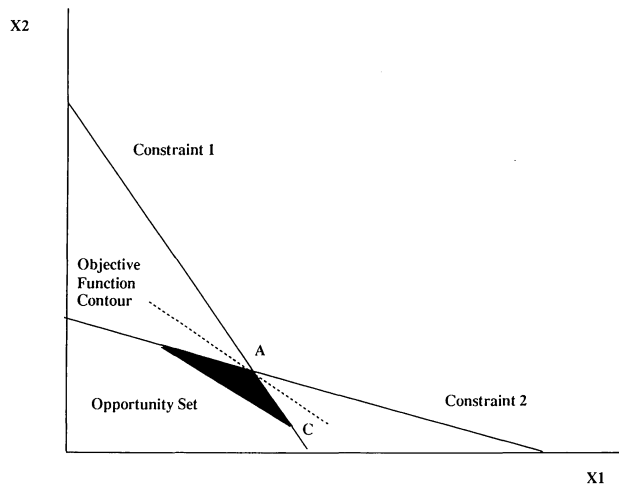
Fig. 2. Graphical representation of MGA.

examined within some particular tolerance from the optimal solution. Rather than analyzing the hypothetical issue of what happens when resource availability is altered (i.e., sensitivity analysis), the MGA approach considers interior solutions, given current resource availability. In many cases this may be a more realistic portrayal of the circumstances that are dictating the choices available to decision makers. These interior solutions are shown in Fig. 2 as the shaded area within the opportunity set. Note that point C is revealed as an alternative solution when the set of tolerable deviations is examined. It is unlikely that this point would be revealed by sensitivity analysis of either the resource vectors or the objective function coefficients.

While the HSJ technique is straightforward in its use, both conceptually and empirically, it has received little usage in empirical agricultural research. [4] The application presented below provides an example of how the HSJ method may be used in addressing an empirical farm-level agricultural problem; that is, livestock ration formulation. This example demonstrates the potential applicability of MGA to agricultural planning in terms of both farm management and policy issues.

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[4] Burton et al. (1987) utilize a form of MGA in their analysis. However, the technique used in that study is more complex than HSJ and cannot be carried out using only commercially available mathematical programming software.

A FARM MANAGEMENT APPLICATION OF MGA – LEAST-COST RATION
FORMULATION

The issue addressed in this example is the influence of policy change on
farm-level decisions, and the potential role of MGA in the analysis of this
problem. Current transportation policy in western Canada involves subsi-
dization of rail freight rates for grains and oilseeds, in support of export
activities. Proposals have been made that would eliminate this subsidy,
raising the cost of transporting grain and making it more expensive to
export. It has been suggested that one effect of this adjustment would be to
reduce grain prices in the region. This would, in turn, affect demand for
grains and forages in livestock rations in western Canada.

The potential effects of this policy change on dairy rations are addressed
through the use of a linear programming model. Ration formulation is one
of the more common farm management applications of linear programming
(e.g., Klein et al., 1988). This type of problem is typically well defined, with
the objective being to minimize the total ration cost. The minimization
problem is subject to a set of constraints, defined by factors relating to
nutrient requirements and nutrient balance. While nutrient relationships
are not strictly linear, they may be approximated as linear with little loss of
accuracy. As a result, linear programming provides a useful method of
determining economically optimal rations.

While linear programming is a useful tool for this application, there are
shortcomings arising from the required assumptions. For example, this type
of ration model implicitly assumes that all feeds are of uniform quality; that
is, the nutrient content is constant. In reality, while purchased commercial
ration mixes may have a guaranteed nutrient analysis, home-grown feeds
(e.g., forage) will not. Another potentially restrictive assumption is that of
cost minimization being the single objective of the decision maker. Typi-
cally, decision makers may have other goals or objectives in formulating a
ration. For example, a farmer may wish to minimize cost, but also maximize
the use of home-grown feeds, or minimize the use of a particular feed
because of unreliability in supply. A traditional least-cost ration model
cannot incorporate these alternative objectives. [5] However, MGA may be a
useful tool in addressing these concerns.

In this example, nearly optimal rations for a western Canadian dairy/
cash crop operation are examined. The farm has a 50-cow dairy herd, and

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[5] There are several other ways to address these shortcomings. Techniques such as stochastic
programming or goal programming may be used to incorporate stochastic feedstuff quality
and multiple objectives, respectively (e.g., D'Alfonso and Roush, 1990).

the herd average annual milk production is 8160 kg of 3.5% butterfat milk. The herd has a calving interval of 12 months, with an average lactation period of 300 days. The crop mix for the farm operation consists of home-grown feeds for the dairy ration, as well as wheat, oats and canola, which are sold as cash crops. In formulating the ration for the dairy herd, the farmer has several specific objectives: minimization of ration cost, minimization of the variability of nutrient quality (i.e., feed composition), and maximization of convenience. Convenience in ration formulation would involve using feeds that are familiar and/or easily accessible.

The objective of this analysis is to assess the impact of the proposed policy change on the dairy ration for this farm operation. This problem is formulated using a least-cost linear programming model. Separate daily rations are formulated for each 100-day trimester of lactation, along with a daily ration for the dry period. The objective function for the model is the total annual ration cost for an individual cow, which is minimized.

Several alternative feeds are included in the model. Wheat and oats are included, as these crops are presently grown on the farm. Alfalfa hay, barley, corn for silage and grass hay (brome/timothy mix) are also included, as these represent crops that the farmer would consider growing if economical and appropriate for use in the dairy rations. Finally, commercial dairy ration, corn, canola meal and vitamin/mineral supplement are included as potential purchased feeds. [6] Nutrient content for most feeds is based on National Research Council estimates (NRC, 1978). [7]

The mathematical model is defined so as to minimize total ration costs, subject to nutritional requirements. Nutrient requirements provide the structure for the constraints of this problem, and are determined by NRC guidelines, based on body weight and milk production. [8] Formally, the model may be stated as follows:

$$\text{Minimize} \quad z = \sum_i \sum_j X_{ij} \, \text{CST}_j \, \text{DAYS}_i \tag{1}$$

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[6] While corn for silage may be grown in parts of western Canada, the climatic restrictions (e.g., growing season length) in most of the region prevent corn for grain from being a economically viable crop.

[7] The only exception to this is the commercial dairy ration. The guaranteed analysis for a typical 16% crude protein ration is used to represent the nutrient content for this feedstuff.

[8] Average daily production levels in the first, second and third-lactation trimesters are 34.5 kg, 28.0 kg and 18.6 kg, respectively.

subject to:

$$\sum_j X_{ij} \, \text{NUT}_{jk} \geq \text{NUTREQ}_{ik} \qquad \forall i \quad \forall k \tag{2}$$

$$\sum_j X_{ij} \leq \text{MAX}_i \qquad \forall i \tag{3}$$

$$\sum_j X_{ij} \, \text{FIB}_j - \text{FIBREQ}_i \sum_j X_{ij} \geq 0 \qquad \forall i \tag{4}$$

$$\sum_j X_{ij} \, \text{CA}_j - \sum_j X_{ij} \, \text{PHOS}_j \geq 0 \qquad \forall i \tag{5}$$

$$\sum_j X_{ij} \, \text{CA}_j - 3.0 \sum_j X_{ij} \, \text{PHOS}_j \leq 0 \qquad \forall i \tag{6}$$

$$X_{ij} \geq 0 \qquad \forall i \quad \forall j \tag{7}$$

where subscripts are used to indicate the individual rations for each stage of lactation (i), type of feed (j) and type of nutrient (k). The activities, z and X_{ij}, represent total annual cost and kilograms of feed j in ration i, respectively. The following constants refer to the fixed coefficients that are used in the model. They represent nutrient content and requirements, feed costs, and maximum feed intake for each ration.

CST_j	cost per kg for feed j
DAYS_i	number of days over which ration i is fed
NUT_{jk}	kilograms of nutrient k per kg of feed j
NUTREQ_{ik}	daily requirement (kg) for nutrient k in ration i
MAX_i	maximum dry matter intake for ration i
FIB_j	fibre content of feed j
FIBREQ_i	fibre requirement for ration i
PHOS_j	kilograms of phosphorus per kg of feed j
CA_j	kilograms of calcium per kg of feed j

All coefficients, constants and activities are expressed on a 100% dry matter (DM) basis.

Equation (1) is the objective function, which represents the total annual cost of the dairy ration. Equation (2) ensures that nutrient requirements are met by each ration. Included in this set of constraints are minimum requirements for net energy (NE_l), crude protein, calcium, phosphorus and Vitamin A.[9] Equation (3) ensures that the total ration in each period will not exceed the maximum dry matter intake for the dairy cow. Equation (4)

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[9] Within this set of constraints is an additional constraint limiting wheat to be no more than 40% of the non-forage portion of each ration. This constraint is required because of the digestive problems for ruminants associated with high concentrations of wheat in the diet.

requires each ration to meet the minimum fibre content, defined in percentage terms. The next two constraints require the ratio of calcium to phosphorus in each ration to be within a lower (equation 5) and upper (equation 6) limit, respectively. Finally, equation (7) ensures that all activities are non-negative.

Initially, prices of alternative feeds are based on typical 1990 market prices, with the exception of the forage crops, for which variable production costs are used. [10] All values are determined on a 100% DM basis. Feed quantities are expressed in kg of dry matter.

The solution for this model represents the base ration for the dairy herd. The four least-cost rations are provided in Table 1. For each ration, the first column represents the daily amount fed to the cows. The second column is the reduced gradient for each feed. This represents the amount by which the per-unit cost, on a 100% DM basis, would have to decrease before the feed would enter the solution. All feeds in the solution have a zero reduced gradient, by definition. Several non-basic feeds also have zero-reduced gradients. This indicates the presence of degeneracy in the linear programming model, likely resulting from redundancy in the constraint set. The final column in Table 1 represents the total annual ration for an individual cow. The feeds utilized in the rations are alfalfa hay, grass hay, barley and vitamin/mineral supplement. The total annual cost of this ration is $437.87.

Assuming that the change in transportation policy (i.e., elimination of the subsidy) occurs, relative prices for alternative feeds will change. The degree of change is unknown. However, for the purposes of this analysis, a 10% decrease in non-forage feed prices is assumed to occur as a result of the change in policy. [11] The coefficients in the least-cost ration model are adjusted and the model is re-solved. The resulting least-cost ration is summarized in the first column of Table 2.

As may be seen from a comparison of Tables 1 and 2, a 10% decrease in non-forage feed prices has no effect on the least-cost ration. While the ration cost decreases ($437.87 to $412.99) by 5.7%, the least-cost ration still consists of alfalfa and grass hay, and barley. These results would seem to indicate that the policy change will have no effect on feed demand in dairy rations.

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[10] Market prices are used to represent opportunity costs for the various feeds. Costs of production are used for the forage crops because stable markets for these feeds do not exist in western Canada.

[11] Forages are not presently eligible for the subsidized rates.

TABLE 1

Least-cost dairy ration problem – base linear programming solution

| Feed | Dairy rations (kg) [a] | | | | | | | | Total annual ration [b] |
| | First trimester | | Second trimester | | Third trimester | | Dry period | | |
	Level	Reduced gradient	Level	Reduced gradient	Level	Reduced gradient	Level	Reduced gradient	Level
Commercial ration	–	–	–	0.005	–	0.005	–	0.012	–
Alfalfa hay	8.83	–	6.73	–	3.63	–	1.15	–	1 994.23
Grass hay	1.60	–	0.62	–	1.74	–	1.74	–	509.25
Barley	10.16	–	10.10	–	8.11	–	4.82	–	3 149.65
Oats	–	0.003	–	–	–	–	–	0.038	–
Wheat	–	0.021	–	0.026	–	0.026	–	–	–
Corn silage	–	0.096	–	0.092	–	0.092	–	–	–
Corn	–	0.237	–	0.235	–	0.235	–	–	–
Canola meal	–	–	–	0.022	–	0.022	–	0.934	–
Vitamins/minerals	0.03	–	0.03	–	0.01	–	0.003	–	7.83

[a] All quantities are expressed on a 100% dry matter basis. Dashes (–) represent zero values.

[b] Total annual rations are calculated by summing the daily rations, with each being multiplied by the number of days over which it is fed.

TABLE 2

Alternative solutions for the least-cost dairy ration problem – basic MGA approach

Feed	Annual dairy rations (kg) [a,b]					
	Least cost	MGA1	MGA2	MGA3	MGA4	MGA5
Commercial ration	–	–	–	–	–	169.98
Alfalfa hay	1 994.23	1 065.10	2 024.88	2 030.77	1 999.48	1 924.60
Grass hay	509.25	–	468.94	527.26	539.90	552.72
Barley	3 149.65	–	1 537.10	2 719.90	2 753.56	3 002.60
Oats	–	4 248.92	293.10	–	–	–
Wheat	–	–	1 220.19	–	–	–
Corn silage	–	267.50	74.30	–	–	–
Corn	–	–	–	351.10	312.30	–
Canola meal	–	177.90	–	19.37	44.50	–
Vitamins/minerals	7.83	3.59	5.50	8.29	7.60	7.60
Ration cost	$412.99	$433.64	$433.64	$433.64	$433.64	$433.64

[a] All quantities are expressed on a 100% dry matter basis. Dashes (−) represent zero values.
[b] The nearly optimal rations (MGA1 to MGA5) are obtained by minimizing the sum of activities that were basic in any previous solution, assuming a 5% tolerance from the minimum ration cost ($412.99). For example, MGA1 is obtained by minimizing the sum of alfalfa hay, grass hay and barley in each time period.

However, while this ration analysis incorporates one producer objective (i.e., cost minimization), the alternative objectives are not considered. For example, if feed grain prices are reduced relative to forage prices, farmers may choose to substitute grain for forage to reduce nutrient content variability. Convenience may become a more significant consideration, in relative terms. The basic linear programming analysis does not consider this possibility. [12]

Analysis of reduced gradients, or the use of sensitivity analysis, may provide an indication of how prices must change for a different dairy ration to be optimal, in terms of cost. However, the presence of degeneracy reduces the reliability of dual values and reduced gradients (McCarl, 1977). Also, sensitivity analysis is not very useful in assessing the objectives related to convenience or uniformity of nutrient quality. As a result of these factors, the original linear programming model does not furnish complete information for providing recommendations to policy makers or farmers.

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[12] Goal programming could be used to address this issue, but would require identification of specific weights for each objective.

Because of the limitations associated with the least-cost model and solution, the MGA technique is used to generate 'nearly optimal' solutions. This is done to (a) identify alternative rations that achieve, to a certain extent, the alternative objectives, and (b) provide an indication of the stability of the least-cost solution. The framework discussed earlier (i.e., HSJ technique) is used to construct the MGA model. The original linear programming problem is further constrained by requiring the annual ration cost to be no more than 5% greater than the minimum cost; in other words, no greater than $433.64 per cow. [13] This 5% is the allowable tolerance from the original solution, and represents $20.65 per cow per year or $1032.50 annually for the herd. This is a substantial deviation from the minimum cost. However, the additional cost may be justified if the resulting ration is optimal with respect to alternative decision-maker objectives (i.e., variability of nutrient quality or convenience). Also, the allowable ration cost ($433.64) is still lower than the total cost associated with the original base linear programming solution ($437.87).

Table 2 provides five alternative rations (MGA1 to MGA5) that are generated using the basic HSJ technique. These rations are obtained by minimizing the sum of activities that are basic in any previous solution. For example, MGA1 is obtained by minimizing the sum of basic activities from the original least-cost solution. MGA2 is obtained by minimizing the sum of basic variables from the original least-cost solution and the MGA1 solution. All five of these rations meet the nutritional requirements specified in the original model and are within the allowable tolerance for cost. As may be noted from Table 2, the five rations utilize a variety of alternative feeds in various combinations.

The MGA results in Table 2 indicate that there is some degree of potential substitutability between forages, and a significant degree of substitutability between non-forage feeds. For example, the primary forages in the least-cost ration are alfalfa hay and grass hay. In MGA1, however, corn silage substitutes for grass hay. Also, oats and wheat are substitutes for barley in some MGA rations. However, with the exception of MGA1, there is a degree of stability in all of the rations with respect to the overall forage content (approximately 45% of total dry matter). In MGA1, the forage content drops to below 25% of total dry matter.

The results in Table 2 provide useful information to policy makers who are interested in the potential impacts on feed demand of the change in transportation policy. The original least-cost analysis would suggest that

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[13] Initially, tolerance values less than 5% were used but the MGA procedure was unable to generate rations that were significantly different from the original solution.

TABLE 3

Alternative solutions for the least-cost dairy ration problem – MGA approach with specific objectives

Feed	Annual dairy rations (kg) [a,b]		
	MGA6	MGA7	MGA8
Commercial ration	–	–	189.20
Alfalfa hay	935.96	1 358.48	1 908.88
Grass hay	361.22	103.96	560.90
Barley	–	–	2 989.16
Oats	4 258.46	4 590.82	–
Wheat	–	–	–
Corn silage	–	–	–
Corn	–	–	–
Canola meal	194.30	–	–
Vitamins/minerals	3.51	0.40	4.30
Ration cost	$433.64	$433.64	$433.64

[a] All quantities are expressed on a 100% dry matter basis. Dashes (−) represent zero values.
[b] The nearly optimal rations (MGA6 to MGA8) are obtained by minimizing or maximizing total use of specific feeds in the annual ration, assuming a 5% tolerance from the minimum ration cost ($412.99). MGA6 is obtained by minimizing total use of all forages, MGA7 is obtained by maximizing total use of all cereal grains, and MGA8 is obtained by maximizing total use of commercial dairy ration.

there may be little or no effect on dairy rations in western Canada. The MGA analysis indicates that there may be some effects on livestock feed demand in the region. However, the policy change will probably not have a significant detrimental effect on overall forage demand, at least with respect to the dairy sector.

The MGA analysis may also be of value in providing extension information to farmers. The results in Table 2 provide a least-cost ration, along with several nearly optimal alternative rations. Each ration achieves, to a certain extent, the objectives outlined earlier. MGA1 utilizes primarily alfalfa and oats (minimum number of feeds), MGA2 utilizes some wheat in the ration (a crop already grown on the farm), and MGA5 utilizes commercial dairy ration (least variable in terms of nutrient content). All six rations (including the least-cost ration) achieve cost efficiency, to a certain degree.

An alternative use of the HSJ technique, discussed earlier, is to optimize with respect to specific objectives. This variation of the HSJ technique may be useful in identifying nearly optimal solutions that are of particular interest to decision makers. Table 3 provides three nearly optimal rations (MGA6 to MGA8) that are obtained in this manner, again allowing a 5% tolerance from the minimum cost. MGA6 is obtained by minimizing the

overall use of forages in the rations, MGA7 is obtained by maximizing the use of cereal grains (i.e., barley, oats, wheat and corn) and MGA8 is obtained by maximizing the use of commercial dairy ration. The resulting rations achieve a degree of cost efficiency while maximizing the use of feeds that are less variable in terms of nutrient content. As with the earlier MGA solutions, these rations utilize a variety of feeds in various combinations.

The MGA solutions provide useful information with respect to ration formulation. Information is provided to policy makers concerning the possible implications of policy changes for livestock feed demand. If the objective of this modelling procedure is to provide extension recommendations to the farmer, the analysis provides a least-cost ration, along with several nearly optimal alternatives. All of the rations may achieve one or more of the farmer's objectives. The use of the MGA technique allows farmers to choose the type of ration that may most closely relate to specific farm objectives and constraints while also ensuring that the ration meets the necessary criteria, including economic and non-economic considerations.

OTHER APPLICATIONS OF MGA

The MGA technique is not limited to use in this type of micro application. Linear programming has been used in a variety of studies related to farm-level applications. These include analyses of farm production decisions, marketing decisions and studies designed to predict producer response to policies and technologies. Examples include papers by Adesina and Sanders (1991), Kaiser and Apland (1987), Perry et al. (1989) and Trelawny and Stonehouse (1989). The empirical models in these studies involve the use of stochastic programming, MOTAD, dynamic modelling, etc. Results can be used to direct research and policy planning. MGA is a complementary technique, and can be used in conjunction with these models and procedures, rather than replacing them. [14]

Linear programming is also popular in addressing macro- or sector-level problems and issues. These include analyses related to optimal transportation patterns, aggregate demand and trade problems, etc. MGA may also have a role or use in complementing the linear programming results from these types of studies. An example of this type of analysis is provided by Gibson, Faminow and Jeffrey (1990).

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[14] Caution must be used in employing the MGA methodology in conjunction with risk programming analysis. The implications of MGA with respect to the underlying theoretical models of risk behaviour have not, as of yet, been explored.

CONCLUSION

As discussed by Monke and Pearson (1989), trade-offs between efficiency and nonefficiency objectives frequently arise in policy analysis for agricultural development. Linear programming techniques can be quite useful in the pursuit of efficiency objectives, but they tend to result in rigid conclusions and recommendations that may vary considerably from other nonefficiency policy objectives. Techniques such as goal programming have been devised to address this concern, but they also suffer from rigidity because it is necessary to predetermine the weighting of preferences.

Some analysts may suggest that economists should strive to provide the 'best' solution to decision makers. However, the inflexible recommendations arising from this process often result in economists being spectators, rather than participants, in policy development. Also, accuracy is a relative term when referring to linear programming solutions, as they are 'optimal' only in the sense of the mathematical formulation. These solutions may not accurately model the actual decision framework for economic agents. Finally, confidence in optimal solutions may not be appropriate when, in many cases, significantly different solutions exist within a small tolerance of the optimal level of the objective function.

This paper has suggested 'Modelling to Generate Alternatives' as an alternate technique. There are two primary advantages to this approach. First, analysts do not need to explicitly specify goals or preference weights. Instead, a predetermined tolerance from the optimal solution is specified and a range of solutions is generated within that tolerance. This leaves the issue of choice to decision makers, but they are provided with a clearly defined set of alternatives. Second, the solution technique proposed in this paper is extremely simple, can be solved using any commercially available mathematical programming software package and requires no special computer skills.

The use of this technique was demonstrated with a common problem in agricultural planning and decision making; livestock ration analysis. This example is presented not only to demonstrate the use of the technique but to also illustrate how the results can be used in an applied analysis. The technique would appear to be particularly relevant to the case of developing countries because social objectives often weigh heavily in the determination of public policy towards agriculture.

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