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Allocable fixed inputs as a cause of joint production: an empirical investigation

Howard D. Leathers
Department of Agricultural and Resource Economics, University of Maryland, College Park, MD, USA

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ABSTRACT

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Leathers (1991) shows that while the existence of allocable fixed inputs *can* cause joint production (as in Shumway, Pope and Nash, 1984), it will not necessarily lead to joint production. The extent to which allocable fixed inputs cause joint production in agriculture is an empirical question. This paper offers an empirical answer. By estimating a short-run joint cost function, it is possible to identify levels of outputs for which joint production may be optimal in the short run but not in the long run. Only in these output regions will there be jointness caused by allocable fixed inputs. For the data in this paper (160 Wisconsin farms), these output regions are very small; thus allocable fixed inputs do not appear to be an important cause of jointness for these farms. Technical causes of jointness appear to be a significant cause of joint production.

INTRODUCTION

In a widely discussed article, Shumway, Pope and Nash (SPN, 1984) argued that many instances of multi-output production in agriculture are caused by allocable (sometimes written 'allocatable') fixed inputs. A recent paper by Leathers (1991) demonstrates that whereas allocable fixed inputs *may* cause, it is not true that they *must* cause, joint production. The issue of whether and to what extent allocable fixed inputs cause joint production is an empirical question. This paper offers an empirical answer. Utilizing the cost function approach outlined in Leathers, this paper identifies

Correspondence to: H.D. Leathers, Department of Agricultural and Resource Economics, University of Maryland, College Park, MD 20742, USA.

output regions for which joint production may be optimal in the short run, but not in the long run. Only in this region will there be joint production caused by allocable fixed inputs. For the data in this paper, these regions are very small; thus allocable fixed inputs do not appear to be an important cause of jointness for these farms. Technical causes of jointness appear to be a significant cause of joint production.

MULTI-PRODUCT COST FUNCTION APPROACH TO JOINTNESS

This paper uses notation and terminology introduced by Baumol, Panzar and Willig (BPW, 1981). A multi-product (long run) cost function $C(\mathbf{y}) \equiv C(y_1, \ldots, y_J)$ shows the minimum cost of producing, in a single firm, quantity y_1 of commodity 1, quantity y_2 of commodity two, etc. [1] The choice of the commodity set S is arbitrary: the commodities can include things (sheep, home computers) which are not produced together in reality. Estimation of a multi-product cost function does not require preidentification of products which are jointly produced. The commodity set, S, can be partitioned into mutually exclusive subsets $\mathrm{P} = \{\mathrm{T}_1, \ldots, \mathrm{T}_K\}$ and we can identify the cost function of producing the partitioned subset T_k by setting $y_j = 0$ for any commodity j not included in T_k. Obviously, one of these partitions is the 'complete partition' in which every subset contains exactly one of the J commodities; thus, the multi-product cost function contains as a special case the single-product cost function $C(0, 0, \ldots, y_j, 0, \ldots)$ for the firm producing only commodity j. For notational simplicity, the cost of producing partition T_k is written $C(y_{\mathrm{T}_k})$ and the single-product cost function is written $C(y_j)$.

BPW define the condition which leads to joint production in the long run as "economies of scope".

Definition. For any output vector, y, and any partition P, there are *economies of scope* at y with respect to P, if:

$$\sum_{k=1}^{K} C(y_{\mathrm{T}_k}) > C(y)$$

If the inequality is reversed, there are diseconomies of scope. If the inequality is weak, then there are weak economies of scope.

By concentrating on the long run, the BPW approach focuses on technological causes of jointness. BPW identify two types of technical interactions that can cause economies of scope. The first type of interaction occurs

[1] For notational simplicity, input prices are suppressed in writing cost functions.

when increased production of one good causes a decline in marginal cost of other goods:

$$\partial^2 C/\partial y_i \partial y_i < 0$$

This condition is called "cost complementarity" by BPW, and defines "technically complementary products" for Carlson (1956). The second type of interaction occurs when two products share some fixed costs. To define this condition formally, BPW divide the cost function into fixed costs (F) and variable costs (V) as $C(y) = F(T) + V(y)$. Fixed costs depend on which products are produced: $\mathrm{T} = \{i \in \mathrm{S} \mid y_i > 0\}$. Two disjoint product sets T_i and T_j share fixed costs when the F functions are subadditive:

$$F(T_i) + F(T_j) > F(T_i \cup T_j)$$

Shumway, Pope and Nash (1984, 1988), on the other hand, emphasize the fact that allocable fixed inputs can cause joint production. Obviously, inputs are fixed only in the short run; thus SPN's focus is more on short-run causes of jointness. To explore the causes of jointness in the short and long run, the following notation is used. The vector $\boldsymbol{y}$ consists of the j outputs; $\boldsymbol{x}$ consists of the i variable inputs; z consists of the k inputs which are fixed in the short run at levels $\bar{z}$; $\boldsymbol{r}$ and $\boldsymbol{w}$ are the prices of $\boldsymbol{x}$ and z respectively; Θ is the feasible production set.

The total long-run cost function is:

$$C(\boldsymbol{y}) = \min_{x,z} \boldsymbol{r}'\boldsymbol{x} + \boldsymbol{w}'z \quad \text{s.t.} \quad (\boldsymbol{x}, z, \boldsymbol{y}) \in \Theta$$

where Θ is the feasible production set. The short-run cost function is:

$$c^{\mathrm{s}}(\boldsymbol{y} \mid \bar{z}) = \min_{x,z_i} \boldsymbol{r}'\boldsymbol{x} \quad \text{s.t.} \quad (\boldsymbol{x}, z_1, z_2, \ldots, z_J, \boldsymbol{y}) \in \Theta \quad \text{and} \quad \sum z_i = \bar{z}$$

The short-run cost function holds the total amount of fixed inputs constant. The cost functions are related to each other as follows:

$$C(\boldsymbol{y}) = \boldsymbol{w}'z^* + c^{\mathrm{s}}(y \mid z^*)$$

where z^* is the optimal choice of fixed input in the C minimization problem.

The relationship between the BPW and SPN approaches can be understood by referring to the following result of Leathers.

Proposition. If joint production occurs at y, then one of the following conditions must hold for the short run cost function:

(1) There are economies of scope at y with respect to the 'complete partition' in the short-run cost function.
(2) There are diseconomies of size in nonjoint production at y in the short-run cost function.

The intuition behind the proposition is the following. One of two conditions must be met in order to have joint production of a vector $\boldsymbol{y}$. The first condition is that it is cheaper in the short run to produce $\boldsymbol{y}$ jointly than to produce the y_i's in specialized firms; in a sense, the farmer can 'reduce' the cost of producing y_1 by adding production of a second good. As shown in the previous section, the existence of a fixed factor may create economies of scope in the short run for output levels at which economies of scope do not exist in the long run. In addition, unlike in the long run when economies of scope is the only reason for joint production, in the short run there may be joint production even when economies of scope do not exist. The second condition states that there may be joint production if there are decreasing returns to size for each good produced separately. In this case, the farmer chooses to produce a second good rather than to move up the average cost curve by increasing output of good 1.

The above conditions for joint production give some insight into allocable fixed inputs as a cause of jointness. Allocable fixed inputs can cause jointness insofar as they create economies of scope or decreasing returns to size. [2] Allocable fixed inputs can cause cost interdependence, but this interdependence can take the form of cost complementarity ($\partial^2 c^s/\partial y_1 \partial y_2 < 0$), creating economies of scope, or of cost anti-complementarity ($\partial c^s/\partial y_1 \partial y_2 > 0$), creating diseconomies of scope. If the fixed input creates diseconomies of scope, it may actually work against joint production. If there are diseconomies of scope, then in order to have joint production, the decreasing returns to size must be large enough to outweigh the diseconomies of scope.

USING THE MULTI-PRODUCT COST FUNCTION TO TEST FOR CAUSES OF JOINTNESS

The last section showed that the existence of allocable fixed inputs could, would not necessarily, cause joint production. The issue of whether allocable fixed inputs do cause joint production in practice is an empirical question. In this section we explore a method of getting an answer to that question.

In the long run, joint production will occur only if there is a technological reason which makes joint production cheaper than nonjoint production

[2] Note that the existence of cost interdependencies ($\partial^2 c^s/\partial y_1 \partial y_2 \neq 0$) is not sufficient to imply that joint production will take place. There are two reasons for this: (i) As mentioned above, the economies of scope condition requires that $\partial^2 c^s/\partial y_1 \partial y_2 < 0$ (if $\partial^2 c^s/\partial y_2 \partial y_2 > 0$ the condition is not met); (ii) economies of scope is necessary, but not sufficient for joint production.

(causing economies of scope). To identify regions of possible jointness caused by fixed inputs, we ask the following question: Are there output levels at which joint production would not be economical in the long run, but which might be produced jointly in the short run by firms with fixed inputs? We identify outputs which would not be produced jointly in the long run by the set of output vectors at which there are no economies of scope with respect to the complete partition in the long-run cost function. We identify outputs which might be produced jointly in the short run by the set of output vectors at which the necessary conditions for joint production in the short-run cost function are met. Note that this approach will overestimate the range of outputs on which jointness is caused by allocable fixed inputs. This overestimation is due to the fact that we are identifying regions where *necessary* conditions hold.

SPN (1984) argue that because "not all causes of jointness have the same implications for modeling", it is necessary to know the cause of jointness prior to estimating a joint production relation. While this is undoubtedly true when estimating a joint production function, it is not the case when estimating the dual, multi-product cost function. [3] Here we specify a cost function which is sufficiently flexible to permit economies or diseconomies of scope and increasing or decreasing returns to size.

We estimate a short run cost function by regressing short run (annual) cost on levels of output (y) and size of the capital stock (z): [4]

$$c^{s} = c^{s}(y \mid z)$$

As pointed out above, the long run cost function can be derived from the short-run function:

$$C(y) = c^{s}(y \mid z^{*}(y)) + wz^{*} \quad \text{where} \quad z^{*}(y) = \operatorname{argmin}\ c^{s}(z) + wz \text{ [5]}$$

[3] The cost function approach to joint production is also attractive for other reasons. When allocations of the fixed inputs are observed, econometric efficiency is achieved by estimating a short-short run cost function (see Leathers, 1991) subject to allocations being optimal: $(\partial c^{ss}(y, z_1, \ldots, z_n)/\partial z_i = \partial c^{ss}(y, z_1, \ldots, z_n)/\partial z_j$ for all i and j. Furthermore, optimal allocations of allocable *variable* input k is achieved by calculating $\partial c^{s}(y_i)/\partial r_k$, where $c^{s}(y_i)$ is the short run cost function of producing y_i of commodity i and zero of all other commodities, and r_k is the price of input k. These issues have received considerable attention (Just, Zilberman and Hochman, 1983; Paris, 1988; Chambers, 1989; Chambers and Just, 1989), but are neatly resolved in the cost function approach.

[4] As described below, the data used here are cross sectional data from a six-county area in Wisconsin. Input prices were not available in this data set. In any event, we would not expect much variation in input prices across observations.

[5] So z^{*} satisfies $\partial c^{s}(z^{*})/\partial z = -w$ and $\partial^2 c(z^{*})/\partial z^2$ is positive semi-definite.

BPW discuss desirable attributes for a functional form used in estimating a multiproduct cost function, and suggest that a quadratic cost function is a good choice. They suggest that, to allow for the possibility of product specific fixed costs, the intercept should be augmented with dummy variables for production of each good. In estimating a short run cost function we have the additional need to use a functional form from which optimal fixed input z^* can be calculated, and which will allow z^* to depend on y in a general and flexible way. Therefore we estimate the following functional form:

$$c^s(y, z) = \delta' m + y'Ay - \ln(z) \cdot y'By \tag{1}$$

where m is a vector of dummies, $m_i = 1$ if the farm produces product i, and is zero otherwise, and where A and B are symmetric matrices of coefficients of quadratic forms in y (the vector y includes the element 1). [6] Optimal capital stock is determined by setting $z^* = [y'By] \cdot w$ (as long as z^* is positive, or if B is positive definite). Expressing z is dollar terms, so that $w = 1$, the long run cost function, $C(y)$ is:

$$C(y, z^*(y)) = [y'By] + \delta' m + y'Ay - \ln(y'By) \cdot y'By \tag{2}$$

In the long run, joint production will occur when joint production is cheaper than non-joint production. When there are three outputs (the case in this paper), there are five possible configurations of production: (1) all three produced in a firm; (2–4) one product produced separately and the other two jointly; (5) all three produced separately. If there is joint production of all three products at levels y, then configuration (1) is the least cost configuration or producing y. If there is joint production of two of the three products at y, then one of the configurations (2)–(4) is the least cost configuration of producing y. Nonjoint production occurs when (5) is the least cost configuration.

As above, our notation here is that $C(y)$ denotes the cost of producing the vector y containing all three outputs; $C(y_i)$ denotes the cost of producing amount y_i of good y and zero amounts of the other outputs. In addition $C(y_{j \neq i})$ denotes the cost of producing zero amount of good i and

[6] For notational simplicity identify the elements of A as:

$$A = \begin{bmatrix} \alpha_0 & \frac{1}{2}\alpha_1 & \frac{1}{2}\alpha_2 & \frac{1}{2}\alpha_3 \\ \frac{1}{2}\alpha_1 & \alpha_{11} & \frac{1}{2}\alpha_{21} & \frac{1}{2}\alpha_{31} \\ \frac{1}{2}\alpha_2 & \frac{1}{2}\alpha_{12} & \alpha_{22} & \frac{1}{2}\alpha_{32} \\ \frac{1}{2}\alpha_3 & \frac{1}{2}\alpha_{13} & \frac{1}{2}\alpha_{23} & \alpha_{33} \end{bmatrix}$$

This means that for $y = (1, y_1, y_2, y_3)$, $y'Ay$ is $\alpha_0 + \Sigma_i \alpha_i y_i + \Sigma_i \Sigma_j \alpha_{ij} y_i y_j$. Similarly, for B.

amount y_j of other goods. Therefore, the cost of configuration (1) is $C(y)$; the cost of configurations (2)–(4) is $C(y_i)+C(y_{j\neq i})$ for $i=1, 2, 3$; and the cost of configuration (5) is $\Sigma_k C(y_k)$.

In terms of the quadratic long-run cost function (2), the conditions for non-joint production in the long run are:

$$\sum_k C(y_k) < C(y)$$

or

$$2(\beta_0+\alpha_0)-(\beta_{12}+\alpha_{12})y_1y_2-(\beta_{23}+\alpha_{23})y_2y_3-(\beta_{13}+\alpha_{13})y_1y_3 \\ +y'By\ \ln(y'By)-\sum_k\left[\beta_0+\beta_k y_k+\beta_{kk}y_k^2\right]\ln\left[\beta_0+\beta_k y_k+\beta_{kk}y_k^2\right]<0 \tag{3}$$

and

$$\sum_k C(y_k) < C(y_i)+C(y_{j\neq i}) \quad \text{for} \quad i=1, 2, 3$$

or

$$(\beta_0+\alpha_0)-(\beta_{jk}+\alpha_{jk})y_jy_k-\sum_{j\neq i}\left[\beta_0+\beta_j y_j+\beta_{jj}y_j^2\right]\ln\left[\beta_0+\beta_j y_j+\beta_{jj}y_j^2\right] \\ +\left[\beta_0+\beta_j y_j+\beta_k y_k+\beta_{jj}y_j^2+\beta_{kk}y_k^2+\beta_{jk}y_jy_k\right]\ln\left[\beta_0+\beta_j y_j+\beta_k y_k \right. \\ \left. +\beta_{jj}y_j^2+\beta_{kk}y_k^2+\beta_{jk}y_jy_k\right]<0 \quad \text{for all } i \text{ and } j,\ k\neq i \tag{4}$$

By identifying output levels y for which these conditions hold, we identify the set of output points over which non-joint production will occur in the long run.

Next we need to identify a subset of this set which contains output points at which non-joint production will occur in the long run, but at which joint production may occur in the short run. The necessary conditions for joint production at y in the short run are that short-run costs of joint production are less than short run costs of nonjoint production or that nonjoint production exhibits decreasing economies of size at output level y_i.

In terms of the quadratic short-run cost function, nonjoint production is cheaper than joint production in the short run if:

$$\sum_k c^s(y_k, z) < c^s(y, z)$$

or

$$2(\alpha_0-\ln(z)\beta_0)-(\alpha_{12}-\ln(z)\beta_{12})y_1y_2-(\alpha_{13}-\ln(z)\beta_{13})y_1y_3 \\ -(\alpha_{23}-\ln(z)\beta_{23})y_2y_3]<0 \tag{5}$$

and

$$\sum_k c^s(y_k, z) < c^s(y_i, z) + c^s(y_{j \neq i}, z) \quad \text{for } i = 1, 2, 3$$

or

$$(\alpha_0 - \ln(z)\beta_0) - (\alpha_{jk} - \ln(z)\beta_{jk})y_j y_k < 0 \quad \text{for} \quad i = 1, 2, 3; \quad j, k \neq i \quad (6)$$

Decreasing returns to size of nonjoint production at output y_i exist when the marginal cost of nonjoint production exceeds the average cost of nonjoint production at y_i. In terms of the quadratic cost function (1):

$$\partial c^s(y_i, z)/\partial y_i > c^s(y_i, z)/y_i$$

or

$$\alpha_i + 2\alpha_{ii} y_i - \ln(z) \cdot [\beta_i + 2\beta_{ii} y_i]$$
$$> [\alpha_0 + \alpha_i y_i + \alpha_{ii} y_i^2 - \ln(z) \cdot (\beta_0 + \beta_i y_i + \beta_{ii} y_i^2)]/y_i$$

or

$$\alpha_{ii} y_i^2 - \alpha_0 + \ln(z) \cdot [\beta_0 - \beta_{ii} y_i^2] > 0 \quad (7)$$

ESTIMATION OF A MULTI-PRODUCT COST FUNCTION FOR WISCONSIN FARMS

In the next part of this paper, we estimate a cost function of the type suggested above for a sample of wisconsin farms surveyed in 1960. The survey includes data from 160 farms in three eastern Wisconsin counties (Calumet, Manitowoc, and Sheboygan) and three western Wisconsin counties (Barron, Pierce, and St. Croix). The data set contains information on annual operating cost, the value of the farm's capital stock, and seven output measures: pounds of milk sold, m; head of beef cattle sold; number of hogs sold; number of laying hens on the farm; number of broilers sold; number of lambs sold; and total value of crops sold, C. Only four farms sold broilers, only six sold lambs, and only 14 sold beef cattle during 1960. These categories were combined with hogs and laying hens to make an output category called "animal products other than milk", A.[7] By reducing the number of outputs to three, we greatly reduce the number of coefficients to be estimated. The value of crops sold is a perfect measure of quantity of crops sold only if the price received for crops is the same across

[7] The prices used in aggregating these quantities into a value of animal products variable were derived from data in *1965 Wisconsin Agricultural Statistics,* Wisconsin State Department of Agriculture, Madison, WI, and *Conversion Factors Weights and Measures,* ESCS Stat. Bull. 616, USDA, Washington, DC.

farms; this condition is almost surely violated since different farms sell different types of crops and sell them at different times during the year. Similarly, the "other animal products" variable is in dollar value terms and is not a precise measure of quantity. The fixed input, z, is measured as services derived from the capital stock of the farm, or the value, in dollars, of total farm assets. [8]

The coefficients of expression (1) were estimated by OLS regression of operating cost on the three output measures and the measure of value of capital services. The resulting estimate of the matrix B was not positive definite; thus it was impossible to calculate optimal capital stocks for most farms. [9] Positive definiteness can be imposed a priori, by estimating the elements of the Cholesky decomposition of the matrix using non-linear least squares (see Lau, 1972, 1978). The results of this are:

$$\begin{aligned} c^s(y, z) = {} & 13.6654 + 2.115 * DM - 5.5713 * DA - 0.7084 * DC \\ & + 1.3223\mathrm{E}(-2) * M + 3.9879\mathrm{E}(-3) * A + 6.1830 * C \\ & + 1.3751\mathrm{E}(-6) * M^2 + 2.2178\mathrm{E}(-7) * A^2 - 1.4798 * C^2 \\ & - 5.9917\mathrm{E}(-7) * M * A + 6.9516\mathrm{E}(-5) * M * C \\ & + 1.0071\mathrm{E}(-3) * A * C \\ & - \ln(z) * [1.5257\mathrm{E}(-4) - 4.4344\mathrm{E}(-7) * M \\ & \qquad + 8.0937\mathrm{E}(-7) * A - 6.6071\mathrm{E}(-3) * C \\ & \qquad + 1.2889\mathrm{E}(-9) * M^2 + 4.2938\mathrm{E}(-9) A^2 + 0.2861 C^2 \\ & \qquad - 2.3525\mathrm{E}(-9) * M * A + 1.9204\mathrm{E}(-5) * M * C \\ & \qquad - 3.5051\mathrm{E}(-5) * A \cdot C] \end{aligned}$$

Econometric details are contained in the Appendix.

In the long-run cost function, $C = y'By + \delta'm + y'Ay - \ln(y'By)y'By$, fixed costs are $C(0, 0, 0) = \alpha_0 + \beta_0 - \beta_0 \ln(\beta_0) = \$1366.69$. Cost comple-

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[8] The value (z) of services from the capital stock is derived from the total value of farm assets (S) by using the formula $z = S(1 - e^{-rt})/r$, where r is the interest rate, and t is the future life of the asset. Thus V is a fixed percentage of S. For interest rates in the 4–6% range and t in the 20–30 year range, this percentage is in the 6–8% range. Therefore, the value of services derived from the capital stock is calculated as 7% of the value of the capital stock.

[9] If $y'\hat{B}y$ is negative for a given y, then $\partial c^s(y, z)/\partial z$ is negative and the optimal capital stock is zero for this level of output. This was the case for 154 of the 160 farms using the OLS estimate of B.

mentarities are functions of all three outputs, the complementarity between good i and good j being:

$$\partial^2 C/\partial y_i \, \partial y_j = \alpha_{ij} - \ln(y'By)\beta_{ij} - \left[\beta_i + 2\sum_k \beta_{ik} y_k\right]\left[\beta_j + 2\sum_k \beta_{jk} y_k\right]/y'By$$

Over the range of outputs in the sample, this is always positive between animal products and crops and is always negative between milk and animal products. Between milk and crops, cost complementarity is negative for high levels of output of milk or crops and is positive for low levels of output.

In the long run, the existence of cost complementarities between milk and animal products combined with the joint fixed costs leads us to expect that the minimum cost configuration will always require that milk and animal products be produced jointly. However, cost anti-complementarities exist between crops and animal products and, in certain cases between crops and milk. Thus, we expect stand alone production of crops to be least cost, when the anti-complementarity dominates the joint fixed costs, and we expect joint production of all three commodities to be optimal when the joint fixed costs dominate.

This is in fact exactly what we find. For most levels of output, the joint production of all three commodities is least cost. At high levels of output, the joint fixed cost reason for joint production becomes less important a part of total cost, and the cost anti-complementarities dominate; thus at high levels of output, producing crops separately from milk and animal products becomes the least cost configuration.

This enables us to draw our first important conclusion: the existence of allocable fixed inputs does not cause joint production of milk and animal products. Joint production of milk and animal products is optimal in the long run at all levels of output. This jointness is caused by technological factors which make it cheaper to produce the two products jointly. Intuitively, veal and beef seem to be natural joint products with milk production and such items as veterinary knowledge or shared feed storage and grinding equipment are applicable to production of both goods.

The remaining question is whether allocable fixed inputs may cause joint production of crops with dairy and animal products. We answer this question by first by examining whether there are regions of output (and levels of fixed input z) at which total long run cost is minimized by producing crops nonjointly but short run variable cost is minimized by joint production. In BPW terminology we are concerned with whether there are economies or diseconomies of scope of the short run cost function with respect to the partition which produces crops separately and (milk, animal products) jointly. Second, we examine regions of output over which there are decreasing returns to size.

Obviously, the characteristics of the short run cost function depend on the value of the fixed input z. For the data set here, z (denominated in \$100) ranged from 1.80 to 630 averaging 26 (this corresponds to an average farm value of about \$37,000 in 1960 dollars). We examine characteristics of the operating cost for values of z in this range.

In the short run cost function, fixed costs shared jointly by all three products are $1366.54 - 0.015\,257 * \ln(z)$. This declines as z increases, but is positive for any realistic size of z. The cost complementarity between milk and animal products is $-0.00\,06 + 0.000\,000\,2 * \ln(z)$. This increases with z, but is negative for any realistic value of z. The cost complementary between milk and crops is $0.006\,95 - 0.001\,92 * \ln(z)$. This declines with z, being positive for values of z less than 37, and negative for $z > 37$. The cost complementarity between crops and animal products is $0.100\,71 + 0.003\,51 * \ln(z)$. This is positive for all realistic levels of z and increases with z. The same pattern exists here as for the long run cost function. High levels of z reduce shared fixed costs, making non-joint production more likely, but increase cost complementarities between milk and crops, making joint production more likely. Furthermore, high levels of z increase cost anti-complementarities between crops and animal products. Which factor is more important in determining economies of scope is an empirical question.

A comparison of short run vs. long run scope economies is shown in Fig. 1. Figure 1 illustrates regions where non-joint production of crops is least cost in short run and long run. The illustration is for a representative cross section of output space, holding milk production constant at 1820 cwt. [10] Other cross sections are virtually identical. As described in the last paragraph, the effect of size of fixed input z on the existence of economies of scope is an empirical question. In this case, for high levels of z, the areas in which joint production is less costly in the short run are actually smaller than the area of joint production in the long run.

It is only for very small levels of z that economies of scope in the short run cost function exist where there are not economies of scope in the long run cost function. Even then, as Fig. 1 shows, these regions are very small. For short run cost when $z = 3$, there are no output points with crop sales less than \$450 for which joint production is least cost in the short run but not the long run. For crop sales of \$600, it is only for animal products sales

[10] Average milk production is 1820 cwt, with a maximum of 5200 cwt; cwt, short or nett hundredweight = 100 lb ≈ 45.36 kg.

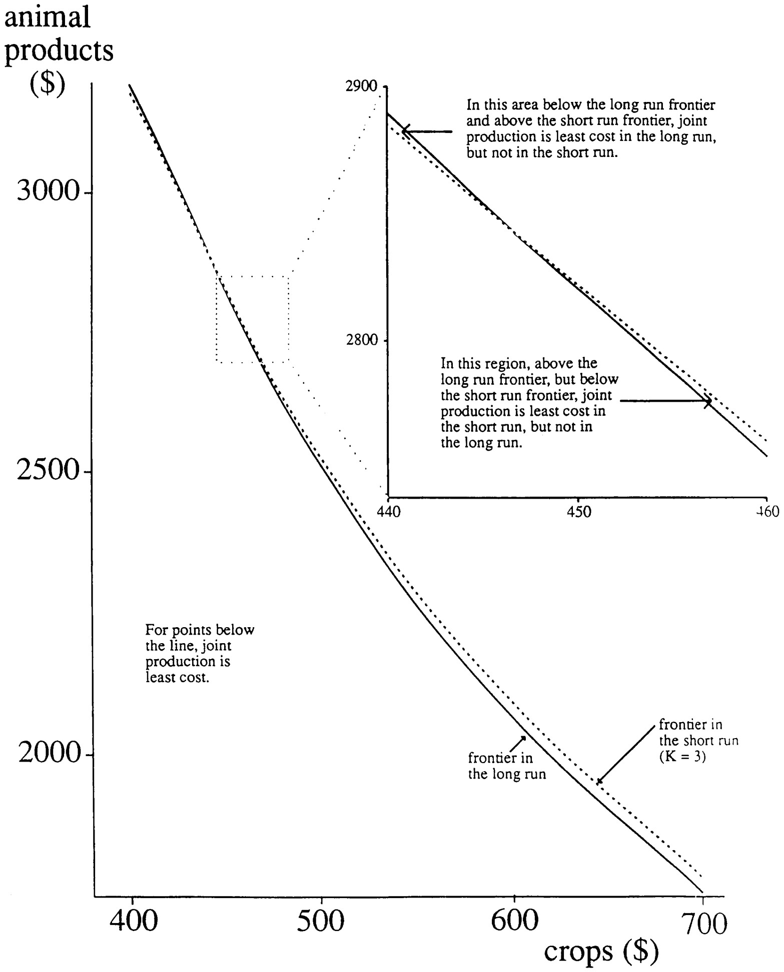

Fig. 1. Economies of scope in the short and long run.

between \$2050 and \$2065 that joint production is least cost in the short run but not the long run.

Above we have searched for regions of output and levels of fixed input at which there are economies of scope in the short run but not in the long run. These regions are non-existent for most levels of fixed input, and very small for any levels of fixed input. The next step in our search for jointness which might be caused by allocable fixed inputs is to look for regions of output at which there are decreasing returns to size. Equation (7) defines these regions.

For crops, economies of size are decreasing when:

$$[-1.4798 - 0.2861 \ln(z)]C^2 > 13.6654 - 0.7084 - 0.000\,15 \ln(z)$$

This does not occur for any realistic level of z. A possible explanation for this is that only small levels of crop production are observed in this data set (maximum sales of crops is \$700). The possibility exists that crops are produced for sale only "by mistake" – when actual yields exceed yields needed for animal feed [11]. If this is the case, then there is really no joint production of crops, and the only jointness that needs to be considered here is dairy and animal products (discussed above). The model here (as the models in SPN and BPW) does not include yield risk, and thus does not allow for the possibility of joint production by mistake.

For animal products, economies of size are decreasing when:

$$[2.2178 - 0.0429 \ln(z)]A^2 > [13.6654 - 5.5713 - 0.000\,15 \ln(z)] \times 10^7$$

For $z = 630$ (maximum value) this occurs when $A > 6457$. For $z = 1.8$ (minimum value), this occurs when $A > 6075$. Average production of other animal products is 1146, well below this cutoff, but animal production ranges as high as 13 607.

For milk, economies of size are decreasing when:

$$[1.3752 - 0.001\,29 \ln(z)]M^2 > [13.664 + 2.115 - 0.000\,15 \ln(z)] \times 10^6$$

For $z = 630$ (maximum value), this requires that $M > 3377$. For $z = 180$ (minimum value), this requires that $M > 3377$. For $z = 180$ (minimum value) this requires that $M > 3397$. Average milk production is 1820, but ranges to a high of 5200.

Therefore, the necessary condition for joint production caused by allocable fixed inputs is met only for very large farms – producing almost twice the average milk production or producing more than five times the average animal products. For such firms, decreasing returns to size exist and allocable fixed inputs may cause joint production.

The results here suggest that technological causes of jointness are important in agriculture. Joint production of milk and animal products has a technological cause, for virtually any range of output and capital stock. The data presented here provide little evidence that allocable fixed inputs are an important cause of jointness in agriculture. The necessary conditions for jointness caused by allocable fixed inputs are met only for a small region of observed outputs. Furthermore, even in these regions we have only shown that allocable fixed inputs *may* cause jointness.

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[11] Thus crops are 'produced' only as an intermediate input to animal production. Chambers (1989) shows a simple way of avoiding the distinction between inputs and outputs by identifying commodities as thru-puts.

CONCLUSIONS

In this paper, the multi-product cost function concepts introduced by Baumol, Panzar and Willig are used to analyze causes of joint production in U.S. agriculture. Although the existence of a fixed input may cause cost interdependency ($\partial^2 c/\partial y_i \partial y_j \neq 0$), this does not imply that joint production will be optimal. Two necessary conditions for joint production in the short run are that there be economies of scope in the short run production function or that there be diseconomies of size of nonjoint production for at least one of the goods.

Econometric estimation of the short run cost function allows empirical examination of the extent to which these necessary conditions hold in the short run when jointness is not optimal in the long run. Since jointness cannot be caused by allocable fixed inputs in the long run, this will give an indication of whether or not allocable fixed inputs is an important cause of jointness. For a sample of Wisconsin farms in 1960, we find that these necessary conditions hold only in limited areas. Thus, the data here does not find much empirical support for the assertion that allocable fixed inputs are an important cause of jointness. The failure to find an important role for allocable fixed inputs as a cause of jointness may be the result of the data used in this paper, or of the functional form [12]. Further investigation using other data sets will be necessary before any definitive statements can be made on the importance of allocable fixed inputs as a cause of jointness. This paper presents an analytical method for such investigations.

APPENDIX

Estimation of the multi-product cost function

In the short-run cost function $c^s = \delta' m + y' A_1 y - \ln(z) y' B y$, we expect the matrix B to be positive definite. To impose this restriction we use the Cholesky decomposition: $B = L'DL$, where:

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ L_{21} & 1 & 0 & 0 \\ L_{31} & L_{32} & 1 & 0 \\ L_{41} & L_{42} & L_{43} & 1 \end{bmatrix} \qquad D = \begin{bmatrix} D_1 & 0 & 0 & 0 \\ 0 & D_2 & 0 & 0 \\ 0 & 0 & D_3 & 0 \\ 0 & 0 & 0 & D_4 \end{bmatrix}$$

[12] The cost function was also estimated using a functional form quadratic in K: $O = y'Ay + Ky'By + K^2 y'Cy$. The resulting cost function was not decreasing and concave in K for most levels of output, and as in the results reported above, it was not possible to solve for an optimal K. This is some evidence that the results reported here are somewhat robust over choice of functional form.

The estimated coefficients of the A_1 matrix are:

intrcpt	13.6654	C	6.1830
	(7.3398)		(26.3443)
DM	2.1150	M^2	1.38E(−6)
	(7.8649)		(1.00E(−6))
DA	−5.5713	$M \cdot A$	−6.0E(−7)
	(3.4193)		(1.00E(−6))
DC	−0.7084	$M \cdot C$	6.95E(−5)
	(24.4401)		(8.23E(−3))
M	0.0132	A^2	2.2E(−7)
	(0.0051)		(1.00E(−6))
A	0.0040	$A \cdot C$	1.01E(−3)
	(0.0053)		(9.55E(−3))
		C^2	−1.4798
			(5.0595)

The estimated coefficients of the Cholesky decomposition are:

D_1	0.0.124	L_{21}	−2.91E(−3)
	(10.3265)		(2.2822)
D_2	0	L_{31}	5.31E(−3)
	(0)		(4.5727)
D_3	0	L_{41}	−43.3061
	(0)		(35 946)
D_4	0	L_{32}	0
	(0)		(0)
		L_{42}	0
			(0)
		L_{43}	0
			(0)

Sum of squared residuals: 30 044.
Standard errors in parentheses.

The positive definiteness constraint is imposed by requiring that the diagonal elements of D be non-negative, or that each $D_i = d_i^2$. Maximum likelihood estimates of the parameters of A_1, D (the d_i's), and L are obtained using non-linear least squares (NLS) estimation techniques. When no constraints are imposed, the estimation yields the OLS results shown in Table 1, with a sum of squared residuals equal to 24 457. As mentioned in the text, the constrained results imply that the optimal capital stock is zero for almost all farms. A capital stock of zero is imposed, and the resulting estimates of the A_1 coefficients are used as starting values in the constrained NLS. The sum of squared residuals from the regression where capital is omitted is 30 082. A number of starting values for the elements of the L and D matrices were tried; only one showed an improvement of the sum of squared residuals below 30 080. Those results are reported here.

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