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Modelling stochastic crop response to fertilisation when carry-over matters

Patrick A. Jomini [a], Robert R. Deuson [a,b], J. Lowenberg-DeBoer [a,c] and André Bationo [d]

[a] *Department of Agricultural Economics, Purdue University, West Lafayette, IN 47907, USA*
[b] *Institut d'Etudes et de Recherches Agronomiques (INERA), Ouagadougou, Burkina Faso*
[c] *Institut National de Recherches Agronomiques du Niger (INRAN), B.P. 429, Niamey, Niger*
[d] *International Fertilizer Development Center, ICRISAT Sahelian Center, International Crop Research Institute for the Semi-Arid Tropics, B.P. 12404, Niamey, Niger*

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ABSTRACT

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Soils in a large part of Niger's agricultural area are sandy and very low in nitrogen (N), phosphorus (P) and organic matter. This low soil fertility combined with low and erratic rainfall constitutes a severe constraint on food cropping in the area. Although agronomists have advised chemical fertilization as a means of improving soil fertility, little fertilizer has been used in this area of the world.

The economic management of soil fertility in the agricultural area of Niger is analyzed using a dynamic model of farmer decision-making under uncertainty. The model is based on agronomic principles of plant growth and accounts for the carry over of P, an immobile nutrient.

At current input prices, a soil P content of at least 14 ppm is found to be desirable. This target is above the natural soil fertility level of about 3 ppm. It can be maintained with a moderate annual application (12 kg P_2O_5 ha^{-1}) of simple superphosphate. Results also suggests that returns to N fertilization are too low and variable to warrant the use of this input.

1. INTRODUCTION

The low phosphorus (P) and nitrogen (N) content of many sandy soils of the West African Semi-Arid Tropics (WASAT) constitutes a major constraint to increasing food production in the region (Jones and Wild, 1975). Many soils in Niger are sandy (> 90% sand), low in organic matter (< 1%)

and plant-available P (Bray P1 test of 3 ppm), and have a low moisture holding capacity with a 3% moisture content measured at field capacity (Mahaman, 1988). This low soil fertility is complemented by low and erratic rainfall which occurs in a single cropping season from June to October (Sivakumar, 1988). Chemical fertilization is believed to be a viable way of raising soil fertility, and thus food production and farm income. However, recent estimates (Min. Agric., 1985; FAO, 1988), put average fertilizer use at less than 1 kg per hectare of cultivated land. Pearl millet, *Pennisetum glaucum* (L.) R. Br., is the main food crop in Niger. Its response to P and N fertilization is significant and well documented (Jones and Wild, 1975; Roesch and Pichot, 1958; Bationo et al., 1986; Mughogho et al., 1986) but little work has been done to determine optimal application rates. There has also been little effort to link fertilizer response and application rates to specific environmental conditions such as natural soil fertility and rainfall (Bationo et al., 1986). Carryover effects especially of P, though widely recognized by researchers (e.g. Jones and Wild, 1975) and farmers have not been taken into consideration when making fertilization recommendations. The main objective of this paper is to determine economically viable levels of P and N fertilization for millet grown on sandy soils in Niger while taking environmental conditions and carry-over effects into account.

Economists have recently recognized the value of using Linear Response and Plateau (LRP) functions in crop response analysis (e.g. Perrin, 1976; Lanzer and Paris, 1981; Grimm et al., 1987). This functional form is based on agronomic principles first advocated by the German chemist Justus von Liebig (1863) which he based on plant analyses and nutrient experiment. The LRP function is based on the Law of the Minimum according to which crop growth is proportional to the availability of the most limiting nutrient until another factor becomes limiting. Abstracting from the possibility of negative response due to an oversupply of nutrients, any increase in an input other than the limiting input does not result in any response and the function displays a horizontal plateau over a large range of the inputs considered (Redman and Allen, 1954). Recent advances in computing capacity and the development of derivative-free solution algorithms for biological compartment analysis problems and engineering systems analysis (Ralston and Jennrich, 1979) have made the estimation of LRP surfaces less costly. In this paper, an LRP surface for millet response to N, P and a measure of available moisture is estimated and used to illustrate an important property of these functions in modeling uncertain crop response. A stochastic formulation of the LRP is specified and used in a dynamic optimization model to provide viable fertilization strategies.

In the following section, previous uses of the LRP concept are reviewed. A model of fertilizer decision-making under uncertainty, taking carry over into account is presented in section three. Conditions for optimality are derived in Section 4, while data from experiments conducted in western Niger are described in Section 5. Results and recommendations are discussed in Section 6 before conclusions are offered in Section 7.

2. PREVIOUS LRP FUNCTION APPLICATIONS

The LRP specification arises from von Liebig's 'Law of the Minimum' according to which plant growth is proportional to an increase in the supply of the most limiting factor called the "minimum factor" (Redman and Allen, 1954). The two-input LRP function shown in Fig. 1 illustrates this principle in the case of two inputs: an initial response is obtained as the supply of P is increased. Increasing the supply of N does not improve yield before a minimum soil P content has been achieved. As the maximum plateau is reached, yield is not affected by added nutrients in this area of the response surface. Anderson and Nelson (1975) have shown that the concept of the LRP function provides a useful framework from the design of experiments to the analysis of agronomic data and the formulation of

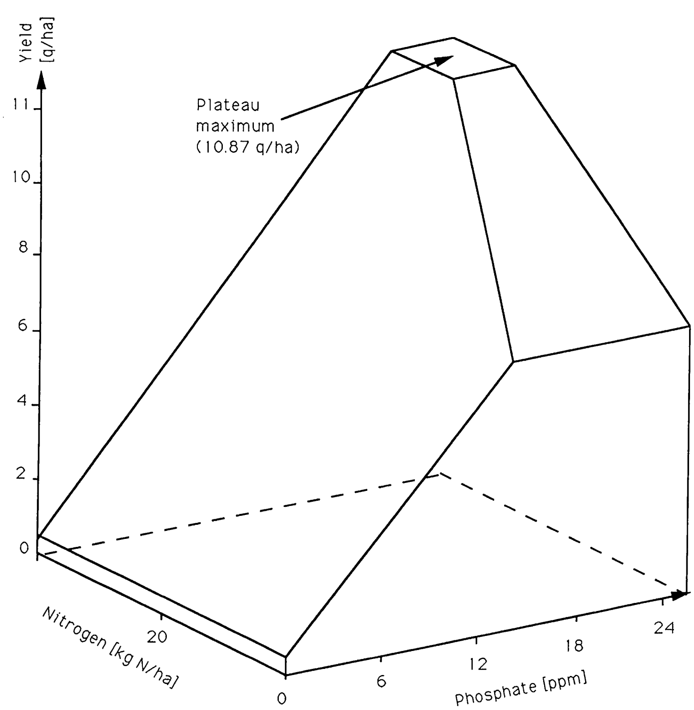

Fig. 1. Two-input linear response and plateau function for modeling millet response to nitrogen and phosphorus. q, metric quintal = 100 kg.

recommendations. The LRP framework is particularly useful in modeling biological processes in which some factor plays a limiting role in determining the process (Ralston and Jennrich, 1979).

While the concept of proportional response was criticized by various authors (e.g. Mitscherlich-Königsberg, 1909), the plateau concept is generally well accepted among biological researchers. Waggoner and Norvell (1979) found the LRP formulation to be an adequate approximation for corn (*Zea mays* L.) and red clover (*Trifolium pratense* L.) grown in Iowa. Cate and Nelson (1971), Anderson and Nelson (1975), Ackello-Ogutu et al. (1985) and Grimm et al. (1987) found the LRP specification to be a good yield-predicting model by various statistical and goodness-of-fit criteria for a wide variety of crops grown in temperate areas. Lanzer and Paris (1981) showed that maintenance fertilization recommendations for a wheat-soybean cropping system in southern Brazil could be significantly improved from a cost reduction standpoint by using a LRP framework. While most of these authors' discussion centers on the superiority of the LRP specification over continuously differentiable functions, this will not be done here. The reader is referred to Jomini et al. (1988) and Jomini (1990) for detailed procedures and criteria for discriminating between response models.

3. AN ECONOMIC MODEL OF THE FERTILIZATION PROBLEM

The model developed in this section presents three important characteristics:

(1) it is developed in a stochastic framework to account for the risk surrounding response to fertilization in Niger;
(2) it is developed in a dynamic framework to account for the carry-over effects of some nutrients;
(3) it is based on the LRP specification for modeling crop response.

As a first approximation, the farmer is assumed to maximize in each period t the expected income from cultivating millet on 1 hectare, with input and output prices assumed known:

$$\max \pi_t^e = p_y Y_t^e - \sum_{i \in I} c_i X_{it} \tag{1}$$

where c_i and p_y are input and output prices, respectively, Y_t^e is expected yield in period t, X_{it} is the amount of input i purchased in period t, and π_t^e is expected profit in period t.

The farmer's decisions are constrained by the technology he is using and the natural conditions (in this case soil fertility and moisture) under which he operates. The following constraints define the feasible set:

$$Y_t^e = \sum_{s \in S} p^s Y_t^s \tag{2}$$

$$Y_t^s = \min_{i \in I} [\alpha_{i0}^s + \alpha_{i1}^s W_{it}, M^s] \tag{3}$$

$$b_{it} = a_{i0} + a_{i1} W_{it} + a_{i2} Y_t^e \tag{4}$$

$$W_{it} = b_{it} + \lambda_i X_{it} \tag{5}$$

$$b_{i0} = \boldsymbol{b}_i \tag{6}$$

where new variables are: Y_t^s is the expected yield in state of nature s and time period t, W_{it} the amount of input i available for plant uptake during cropping season g, X_{it} the amount of input i applied during cropping season t, b_{it} the plant-available stock of input i at the beginning of cropping season t. New parameters are: p^s the probability associated with state of nature s, α_{i0}^s and α_{i1}^s, the linear response parameters to input i which may depend on state of nature s, M^s the maximum yield plateau associated with state of nature s, a_{i0}, $a_i 1$, and a_{i2}, the carry over parameter for input i, and λ_i the rate at which applied input i is transformed into plant-available form during the cropping season.

The relations between soil and inputs are described in the carry over equations (3). In the problem at hand, the inputs considered in set I are N, P and useful rainfall (R). Potentially each of these inputs can carry over from one season to the next, but in the sandy soils of Niger, there is little carry over of soil moisture. The soil dries up almost completely between cropping seasons. This means $b_{\mathrm{R}t} = b_{\mathrm{R}t+1} = 0$. Different measures of available moisture are discussed later.

In the case of N, losses through volatilization are very high due to high temperatures (Jones and Wild, 1975). This means a large part of any applied N is lost before the following cropping season. In addition, the accurate measurements of plant-available N is difficult. For these reasons, the carry over function for N is not estimated, but is assumed to be $W_{\mathrm{N}t} = X_{\mathrm{N}t}$.

The only carry-over function left in the specific model therefore describes the behavior of plant-available P. This function is based on a very simplified nutrient balance approach (Frissel, 1978), accounting for additions and decreases to the nutrient stock. The interpretation of the carry-over coefficients in (4) follows:

$a_{\mathrm{P}0}$ is the net rate by which the stock of plant-available P is increased from one year to the next. This coefficient results from the sum of the rate at which plant-available P is released by the natural weathering of soil particles, and the amount of P brought with wind-borne dust, less the losses due to erosion and leaching.

$a_{\mathrm{P}1} = \partial b_{\mathrm{P}t+1} / \partial b_{\mathrm{P}t}$ is the rate at which plant-available P carries over into the following year.

$a_{\mathrm{P}2} = \partial b_{\mathrm{P}t+1} / \partial Y_{\mathrm{P}t}^{\mathrm{e}}$ is the average rate at which P is taken up by harvested plant matter.

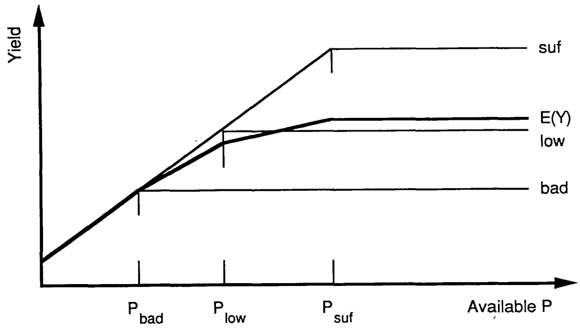

Fig. 2. Single-input linear response and plateau functions for three states of nature and corresponding expected yield function.

Assuming initial soil fertility is known to be b_P (6), the amount of P available for plant growth during season t is obtained by (5), and the carry over equation (4) supplies the residual soil P after cropping in season t has occurred. The LRP concept is applied in defining millet grain yield response (Y_t^s) to the inputs considered (3). For N, the response function is expressed in terms of nutrients applied, while for P, the soil level of P is the argument of the response function. This function is defined for sandy soils similar to those on which the experiments were conducted and for meteorological conditions prevailing in state of nature s. States of nature are defined in terms of moisture availability.

While it is possible that the slope parameters α_{i0}^s and α_{i1}^s may change with the state of nature, the largest effect would probably shift the plateau levels because the limiting factor would often depend on the state of nature. Thus the simplifying assumption is made that only the plateau (M^s) varies with the state of nature. The response to a given nutrient is therefore assumed not be affected by the moisture supply if this supply is sufficient ot promote adequate plant growth.

The expected yield response (Y_t^e) is defined in (2) as a linear combination of the response functions characterizing each state of nature (Y_t^s). This linear combination is itself a minimum function as illustrated in Fig. 2 with a single-input, three states of nature (*bad*, *low*, sufficient, or *suf*) combination (assuming parameters α_0^s and α_1^s do not depend on the states of nature).

In the single input case, the expected response function is composed of one linear segment for each state of nature and a plateau. With an increasingly detailed description of the environment in which the decision-maker operates, the number of states of nature becomes large, and the expected response function approaches a smooth curve. Superficially, this almost smooth curve composed of many segments resembles commonly used polynomial curves. However, in the expected response function resulting from the LRP framework, yield variability is modeled in some detail while variability in the polynomial curves is only reflected in an undifferentiated error term.

4. OPTIMALITY CONDITIONS

Optimality conditions for the fertilizer problem when carry-over matters have been reviewed by Dillon (1977) and Kennedy et al. (1973). When nutrients carry over into following production periods, the problem is to choose application rates and timing that maximize the net present value of the stream of income generated by the cropping activity. In general, the cost of fertilization must be covered by the value of the marginal yield increases over the entire planning period that can be attributed to the fertilizer application.

The problem of determining optimal fertilization rates can be divided into two sub-problems once parameters are known. In a first step, the optimization process, in combination with the biological response function and the probabilistic relation (2), determines jointly the yield (Y_t^{e}) and fertility level ($W_{\mathrm{P}t}$ and $W_{\mathrm{N}t}$) to achieve. These results serve in turn as input to find the optimal fertilizer application levels ($X_{\mathrm{P}t}$ and $X_{\mathrm{N}t}$) given an initial soil fertility ($b_{\mathrm{P}t}$) level.

Assuming the farmer maximizes current and future benefits due to the current fertilizer application, the problem can be stated in the recurrance relation:

$$\pi_t(b_{\mathrm{P}t}) = \max_{W_{\mathrm{P}t}, W_{\mathrm{N}t}} \left[p_y Y_t^{\mathrm{e}} - \sum_{i \in I} c_i X_{it} + (1+r)^{-1} \pi_{t+1}(b_{\mathrm{P}t+1}) \right] \tag{9}$$

where π_t is the maximum expected profit that results from the optimal application choice given the pre-season soil test level of $b_{\mathrm{P}t}$ and assuming all future decisions are taken optimally, and r is the private rate of discount reflecting the decision-maker's rate of time preference.

Since time enters the problem only through the discount term, this is an autonomous problem, which tends towards a steady state equilibrium in the long run (Kamien and Schwartz, 1981). The steady-state equilibrium is attained when the state variable $b_{\mathrm{P}t}$ is maintained constant at b_{P}^*. The corresponding control variables X_{P}^* and X_{N}^* (the fertilization rates) are held constant in order to replenish the nutrient stock and maintain the long-term equilibrium expected yield (Y^{e}).

The steady-state P fertilizer application necessary to maintain soil fertility constant at b_{P}^* is found by solving carry over equation (4) for X_{P}^*, replacing W_P^* by its expression in terms of native and applied P:

$$X_P^* = (a_{P1}\lambda_P)^{-1}[b_P^*(1 - a_{P1}) - a_{P0} - a_{P3}Y^e] \tag{10}$$

The steady-state fertilization X_P^* corresponds to the net losses incurred by the soil P stock, adjusted for the fertilizer's efficiency in raising soil P (λ_P). For simplification, expected yield (Y^e) is included instead of the actual yield.

All variables in (10) are dependent on the optimal long-term soil fertility b_P^*. This optimal level is obtained by solving the maximization problem stated in (9). Noting that in the steady state $\pi_t(b_{Pt}) = \pi_{t+1}(b_{Pt+1}) = \pi^*(b_P^*)$, the recursive equation in (9) can be rewritten as:

$$\pi^*(b_P^*) = \max_{W_P^*, W_N^*} (r^{-1} + 1)\left[p_y Y^e - \sum_{i \in I} c_i X_i^*\right] \tag{11}$$

i.e. the discounted sum of all current and future contributions to steady state profits due to current decisions. Assuming differentiable functions, marginal conditions for the problem stated in (11) are:

$$\partial\pi^*/\partial W_P^* = \partial\pi^*/\partial b_P^* = (r^{-1} + 1)[PY(\partial Y^e/\partial b_P^*) - c_P] \geq 0 \tag{12}$$

$$\partial\pi^*/\partial W_N^* = \partial\pi^*/\partial X_N^* = (r^{-1} + 1)[pY(\partial Y^e/\partial X_N^*) - c_N] \geq 0 \tag{13}$$

These conditions provide optimal soil fertility targets and are analogous to the first-order conditions for the single-period profit maximization problem. Since the response function in this problem is formed of splines, these conditions are expressed as:

$$\Delta Y^e/\Delta W_i^* = \sum_{s \in S} p^s(\Delta Y^s/\Delta W_i^*) \geq c_i/p_y \quad \forall\, i \text{ simultaneously} \tag{14}$$

This is illustrated graphically with the two-input expected minimum yield function shown in Fig. 3. The optimal plant-available nutrient combination to maintain in the steady state is found at the tangency the isoprofit plane (gray in Fig. 3) with slopes c_N/p_y and c_P/p_y and the planes defining the response function. Given our earlier assumptions about nutrient behavior, the optimal application of N is $X_N^* = W_N^*$. The optimal P fertilization level is obtained by solving equations (10) and (5) simultaneously for X_P^*, the application level, and b_P^*, the carry-over level of P.

5. FERTILIZER RESPONSE DATA FROM NIGER

Since 1982, the International Fertilizer Development Center (IFDC) has collaborated with the International Crop Research Institute for the Semi-Arid Tropics (ICRISAT) on an extensive fertilizer research program. The data used in this study were collected by these institutes at ICRISAT's

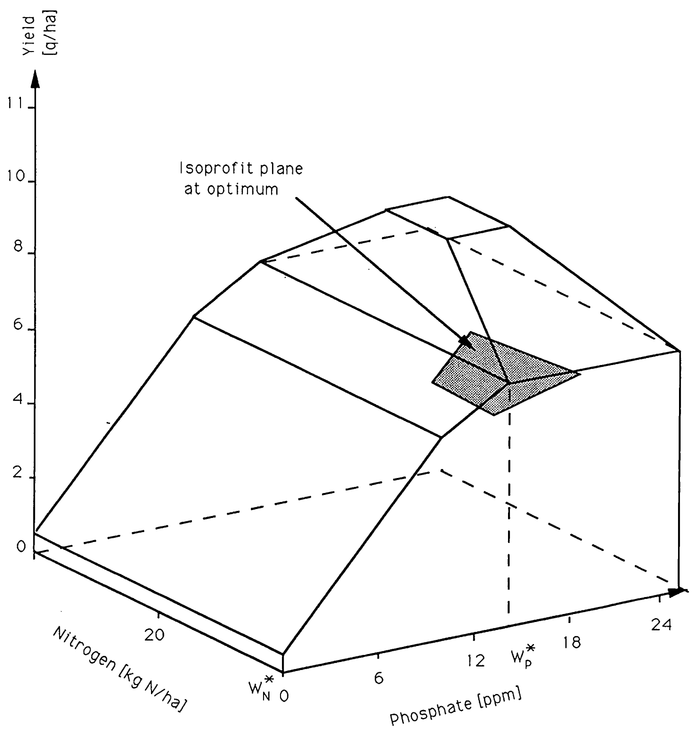

Fig. 3. Expected linear response and plateau function for millet response to nitrogen and phosphorus under three states of nature.

Sahelian Center located in Sadoré, near Say, 40 km southeast of Niamey, Niger. Soils at the experiment station are well characterized by the data presented in the introduction. Data collected at this site during 1982, 1984, 1985 and 1986 were used. Nineteen different combinations of N and P were used in the ranges 0–45 kg P_2O_5 ha^{-1} in the form of simple superphosphate (SSP) and 0–120 kg N ha^{-1} in the form of urea. A detailed presentation of the data and experimental conditions are found in Jomini (1990).

6. RESULTS AND DISCUSSION

The expression for W_{Pt} (5) was substituted into (4) to estimate the annual P carry-over equation. This equation was corrected for heteroskedasticity after a Breush-Pagan test (Judge et al., 1988) showed variance increasing with higher soil test levels. The estimated carry over equation for P is:

$$b_{Pt+1} = 1.95 + 0.45\, b_{Pt} + 0.14\, X_{Pt} + 0.14\, Y_t \qquad (15)$$

(standard errors) (0.25) (0.07) (0.02) (0.03) where b_{Pt} is the Bray P1 soil test level, expressed in ppm, X_{Pt} is the level of P fertilizer application, expressed in kg P_2O_5 ha^{-1}, and Y_t is millet grain yield, expressed in 100 kg millet grain ha^{-1}.

All parameters are significantly different from zero as seen from standard errors supplied in parentheses. A high adjusted $R^2 = 82\%$ indicates this model was able to explain a large part of the observed variation in the data. Parameters are reasonable and of expected signs. As a matter of quick validation, assuming no fertilization, the low-level equilibrium yield is about 250 kg ha^{-1} and carry over soil P is 3 ppm, which is typical of unfertilized sandy soils under cereal cultivation in Niger (Mahaman, 1988).

The estimated intercept indicates that nearly 2 ppm are added each year to the stock of plant-available P through natural causes (i.e., wind-blown dust, weathering of soil particles, breakdown of organic matter). According to the coefficients on Y_t, each 100 kg in grain yield removes 0.14 ppm P from the soil. This removal rate is consistent with rates found through plant analysis (Jomini, 1990). Removal is also reflected in the parameter on b_{Pt} which shows that nearly 45% of the P available at the beginning of any season is available in the following season.

Finally, each kg P_2O_5 ha^{-1} applied increases the soil P in the following year by 0.14 ppm. Assuming a relevant soil depth of 15 cm and a soil bulk density of 1.6 g cm^{-3}, 1 kg P_2O_5 ha^{-1} is converted to 10^6 $(2.4 \times 10^6)^{-1}$, or 0.42 ppm. Multiplying X_{Pt} in kg ha^{-1} by this factor results in an application rate expressed in ppm. The corresponding level of a_{P2} adjusted for these new units, indicates that close to 30% of applied P is found in the stock of plant-available P in the following season. An immediate rate of P availability is obtained by assuming P from the soil and from fertilizer carry over in a similar fashion. Replacing W_{Pt} in equation (4) by its expression in (5), parameter λ_P is then obtained by dividing the parameter on X_{Pt} by the parameter on b_{Pt} (i.e. a_{P1}). This immediate rate of P solubility is found to be 70%, which again is reasonable since SSP is a soluble fertilizer.

Results for a multiple input LRP function are presented in Table 1. The inputs included are useful rainfall, P and N. Useful rainfall is defined as the rainfall occurring during a period of 80 days following the first occurrence of a 3-day rainfall exceeding 20 mm. The 80-day period is assumed to cover the critical growth periods of the millet variety used (CIVT), including plant establishment and flowering. Although other factors affect crop growth, especially early sand storms which impede proper stand establishments and dry spells during the cropping season, this measure of available moisture was found to be useful in the estimation of crop response. The estimation shows that in 1984, yield was significantly limited by the low useful rainfall (196 mm). Among the 140 observations made during that

TABLE 1

Linear response and plateau function relating millet yield (q ha^{-1}) to rainfall, available P from SSP, and N from urea, Sadoré, Niger, 1982–1986

Response parameters	Units of variables	Estimates	Asymptotic error standard
Rainfall	(cm)		
α_{R0}		−2.4674	1.0387
α_{R1}		0.4014	0.0462
Phosphate	(ppm)		
α_{P0}		0.5448	0.0361
α_{P1}		0.4697	0.0347
Nitrogen	(kg ha^{-1})		
α_{N0}		7.0932	0.3443
α_{N1}		0.1261	0.2808
Plateau maximum			
M		10.8754	1.1726
Observations		440	
MSE		6.87	

q, metric quintal = 100 kg.

year, predicted yield is limited by rainfall in 109 cases; otherwise yield is limited by very low available P of the order of 2.5–5.0 ppm.

According to these results, no response to N is expected before the soil P level reaches 14 ppm. This result is consistent with observations by Bationo et al. (1986) and especially Jones and Wild who refer to the West African savana (1975, p. 150):

> Most cereals show some response to nitrogen, but this element is the first limiting nutrient only in some of the more humid parts of the region (...), where long grass fallows have just been cleared (...). Elsewhere, phosphate deficiencies must be remedied before applied nitrogen can give substantial yield increases.

Twenty-two observations for the four other years of trials were in the area where none of the inputs considered (N, P, R) was limiting. For these observations the yield is expected to reach 1087 kg ha^{-1} and it is presumed that either some other nutrient, management or the genetic potential of the variety used limit the yield. Figure 1 provides a graphical representation of the estimated LRP when rainfall is not limiting.

The rainfall coefficients are indicative of the millet response to moisture, but they should be interpreted with caution. They are based on 4 years of data and the standard errors of the estimates are high. They vary considerably with the starting point used in the estimation process. Their interpre-

TABLE 2

Minimum input requirements for various yield levels using SSP

Target yield ($kg\ ha^{-1}$)	Minimum		
	Useful rainfall (mm)	P_2O_5 (ppm)	Nitrogen (kg/ha)
200	111	3.1	0
540	196	10.3	0
709	238	13.9	0
1087	332	22.0	30

tation may not be reliable since we do not have enough points to correctly estimate the spline corresponding with this input to the crop response function.

According to the function's parameters, the minimum level of rainfall needed to obtain the maximum plateau of 1089 kg ha^{-1} is 332 mm in the 80 days assumed to be relevant to plant growth. Similarly, if rainfall is 238 mm in the first 80 days of the season, the maximum obtainable yield is 709 kg ha^{-1}. Rainfall of 196 mm (as in 1984) further limits yield to 540 kg ha^{-1}. To each of these yield levels corresponds a minimum level of P and N. These minimum levels are found in Table 2 along with the corresponding useful rainfall. These results show that relatively low useful rainfall of less than 350 mm is sufficient to produce 1000 kg grain millet yields. Whenever this level of moisture is attained (through reasonably distributed useful rainfall), soil fertility is expected to limit yield. This confirms recent suspicions formulated by agronomists who have identified soil fertility as the major impediment to increasing food crop production in the WASAT (El-Swaifi, 1984). The rainfall limits presented in Table 2 are used next to define intervals of R for the purpose of identifying probability distributions of various moisture conditions.

Two different distributions of three moisture intervals are shown in Table 3. The distribution of useful rainfall for Sadoré (in the second column) is based on the data used in estimating the response function. The other distribution (third column) is based on daily rainfall data from Niamey Airport. For the two locations presented, the probability of sufficient moisture ($R > 332$) is seen in Table 3 to be much lower for Niamey. This may reflect a difference in the years sampled, but also illustrates the worsening moisture conditions that can be expected as latitude increased in the region. Since the response estimates are supposed to reflect physiological reactions of a given millet variety on sandy soils to varying levels of N, P, and R, they can be used along with a variety of rainfall patterns

TABLE 3

Frequency distributions of useful rainfall for two different sites in western Niger

Rainfall intervals (mm)	Sadoré 1982–1986 (%)	Niamey Airport 1968–1987 (%)
less than 238	40	35
238–332	20	45
greater than 332	40	20

stemming from different locations with similar soil conditions. The expected yield function (Y_t^e) is shown in Fig. 3. It is used next in the optimization model to determine optimal fertilizer application rates.

It is assumed that a substantial amount of the organic matter produced under improved soil fertility conditions is left on the field. This is assumed to maintain the level of N if none is applied, and to avoid some of the acidification problems that has been observed by Pichot et al. (1981) and others with continued N application on soils with low organic matter.

Fertilizer is assumed to be available at 50 Francs CFA (FCFA, 1 US$ = 300 FCFA in 1987) kg^{-1} SSP (18% P_2O_5) and 65 FCFA kg^{-1} urea (40% N), or US$ 0.17 and 0.22 kg^{-1}, respectively. These prices are representative of those observed during the period 1986–1989. While official input prices are relatively constant, millet grain prices vary widely from place to place and with seasons. Three prices are used in this analysis to represent this wide range: 100 FCFA kg^{-1} grain, a high price that may be found after a poor crop, 10 FCFA kg^{-1}, a low price that may be found in an isolated area after a large crop; and 50 FCFA kg^{-1}, a common price in rural areas at harvest time. These three output prices are used to illustrate the interaction between the biological expected yield function and expected prices.

Results using these three output prices are shown in Table 4. In the first column, output price is assumed to be 100 FCFA kg^{-1} grain. This high price results in a high optimal steady-state fertilization rate. Soil fertility is maintained at a relatively high level ($b_P^* = 7.5$ ppm). This level of soil fertility is obtained by applying at least 21 kg P_2O_5 ha^{-1} (or 117 kg SSP ha^{-1}) and 30 kg N ha^{-1} (or 63 kg urea ha^{-1}). The expected yield is 726 kg ha^{-1} with a high variability (coefficient of variation, cv = 27%). While a relatively high per-hectare income is expected (64,812 FCFA or US$216), cash outlays of 7, 788 FCFA (US$26) are relatively high for farmers facing cash constraints. At the lower output price level, the farmer's expected yield and soil fertility goals are lower at 650 kg ha^{-1} grain and 5.3 ppm, respectively. This is obtained by applying yearly 12 kg P_2O_5 ha^{-1} (67 kg

TABLE 4

Summary of optimal steady-state fertilization under various input–output price assumptions

Millet price (FCFA[a] kg^{-1} grain)	100	50	10
Relative cost of P, c_P/p_y	2.78	5.56	27.80
Soil phosphorus target			
carry-over, b_P^* (ppm)	7.5	5.3	4.5
available, W_P^* (ppm)	22.0	13.9	10.3
Fertilization			
phosphorus, X_P^* (kg P_2O_5 ha^{-1})	21	12	8
nitrogen, X_N^* (kg N ha^{-1})	30	0	0
Yield			
expected, Y^e (kg ha^{-1})	726	650	540
cv (%) [b]	27	12	0[3]
Net income			
expected (π^*) (FCFA)	64,812	29,164	3,176
cv (%) [b]	31	14	0[3]
Total fertilizer cost, $\Sigma_i c_i x_i^*$ (FCFA)	7,788	3,336	2,224

[a] FCFA, Franc CFA; in 1987, 1 US$ = 300 FCFA.

[b] Coefficients of variation (cv) are calculated as $s_x/E(x)$, where s_x is the standard deviation of x over the states of nature, and $E(x)$ is the mean of variable x over the states of nature.

[c] cv = 0 because the supply of P is just sufficient to sustain the maximum yield obtainable in the worst state of nature. While this may seem unreasonable, it is reasonable to expect a lower distribution of yields if a nutrient is limiting than when it is not limiting and the yield distribution is governed by the distribution of states of nature.

SSP ha^{-1}) and no N. By giving up higher yield, the variability of yield decreases (cv = 12%). Expected income is also much lower at 29,164 FCFA (US$97) and so are fertilizer costs (3,336 FCFA, or US$11). In spite of this fall in income, it is still more than double the income expected from unfertilized yields (12,500 FCFA or US$42).

When the price of millet drops to 10 FCFA kg^{-1} grain, the steady state fertilization level is 8 kg P_2O_5 ha^{-1} and no N. This results in a drastic drop in expected income to 3,176 FCFA (US$11). These expected benefits are less than twice the additional costs linked to fertilization (2,224 FCFA, or US$8). According to the FAO rule of thumb requiring that benefits be at least twice the extra costs entailed by a new technology, this would lead to rejecting the use of fertilizer at this low price of output. This rule of thumb is commonly used in cost-benefit evaluations of new technologies (CIMMYT, 1988). In addition, in this partial analysis of the fertilizer problem, no allowance was made for procurement and application costs. It is worth noting however, that the expected yield is double the level expected from a low fertility field ($b_P^* = 3$ ppm) and 140 kg above the national average during 1980–1985. (Min. Agric., 1986). A more extensive use of moderate P fertilization would therefore significantly contribute to the government's objective of food self-sufficiency (USAID/Niger, 1981).

This latter result indicates that even when rainfall is limiting, yield increases can be expected from the improvement of fertility in the poorest soils of Niger. Although a distribution of soils by classes of P content is not available, it is suspected that P-starved soils (soil $P < 5$ ppm) prevail over large areas of the country. Moderate phosphate fertilization in these areas would lead to increased yields, food production and income.

While the optimal approach path to maintenance fertility and application levels is not investigated here, Dillon (1977), Kennedy et al. (1973) and Kennedy (1986a, b) show that when no constraints impede it and the rate of time preference is positive, the best path to the steady-state is one by which the target soil fertility is reached the quickest. In this case, the optimal short run strategy with a low fertility soil would be to apply the amounts of P and N necessary to obtain the optimal soil fertility in the current period.

7. CONCLUSIONS

A dynamic model of millet response to P, N and available moisture was developed within a linear response and plateau framework. Results from the agronomic response model indicate, first that little response to N applications is to be expected unless a soil P of at least 14 ppm is obtained. This P level can be maintained in the long run by applying 12 kg P_2O_5 ha^{-1} annually in the form of SSP. Secondly, relatively little moisture (350 mm during the first 80 days of a cropping season) is expected to be sufficient to support a yield of 1 metric tonne of millet, well above unfertilized yield levels. The results seem valid for a wide area of Niger's agricultural zone composed in large part (> 75%) of sandy soils.

From an economic standpoint, moderate P fertilization is found to be optimal to maintain soil fertility at a level that permits yield and income increases over the unfertilized situation. At current price levels, annual applications of 12 kg P_2O_5 ha^{-1} as SSP are expected to result in yields of up to 70% kg millet grain ha^{-1}, twice the yield expected if no fertilizer were used, if rainfall is not limiting. Only at relatively high output prices are the returns to N sufficient to warrant its use.

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