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# Farm management support based on mathematical programming; an example of fertilization planning

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## ABSTRACT

This paper presents electronic tool to support farmers in fertilization planning. It is developed as a spreadsheet model, utilizing optimization potential of mathematical programming techniques. Described problem of fertilization planning is rather simple from technological-expert viewpoint, but methodological application of multi-criteria paradigm makes it more complicated. In the paper focus is put on economic efficiency of fertilization. Hypothetical case illustrates its application as well as strengths and weaknesses of methodology applied.

**Key words:** spreadsheet model, fertilization planning, optimization, linear programming, goal programming

## INTRODUCTION

Farmers face numerous production challenges. One of them is nutrition management in the context of crop production. Farmers in the European Union (EU) are obliged to utilise their land in line with good farming practice. This means that fertilization should be done on the basis of performed soil analysis and fertilization plan. The latter should be prepared for all tillage units where mineral fertilizers are applied and should cover five year period. Due to farmers' often poor specialised knowledge and time shortage, fertilization plans are frequently performed by advisors without necessary information about individual plots and fertilisation practices applied by individual farmers. With increasing concerns for environment and sustainable production, it is important that fertilization planning is done by farmers themselves.

Fertilization planning is an important task of production management. It influences soil's optimal nutrients procure which is one of the basic requirement to achieve expected yield. At the same time it ensures that there will be no damage to soil's health. Beside those concerns, Ghosh et al. (2005) are stressing that applications of N, P and K higher than required are likely to cause diverse affects on crops, resulting in substantial yields decrease. In such a manner fertilization has also direct and indirect effects on crops production economics.

The core issue in fertilization planning is: at what time, in which form, how much of manure and what combination of fertilizers to apply to meet estimated nutrition requirements considering soil fertility and minimise negative effect on the environment. Due to very high and volatile prices of fertilizers, it is - besides balancing requirements of inputs - also important to search for cost efficient

combination. This means that one is searching for those fertilizers available on the market that have the best ratio between nutrients (N, P and K) and the lowest price. However, this economic viewpoint is one of those that are usually overlooked. This is especially so if the fertilization plan is prepared by agricultural advisors that are specialists in soil nutrition management and pay little or no attention to economic aspect. However, economics is indirectly considered through farmers' selection of fertilizers and might also be one of the reasons why farmers sometimes act differently from what is recommended in the plan. This unbalance between fertilization plan prepared by the advisor and economically driven behaviour of the farmer might often result in non optimal N-P-K ratio, forms and time of application. That results in lower yield and quality, negative nutrition balance and decrease in soil fertility. The main objective of this paper is to find such fertilization plan that is acceptable from fertilizing regulations and is economically efficient.

Methodologically, the problem of fertilization planning is a common problem of nutrition management that can be supported by mathematical programming methods. Therefore, the planning process could be fully or at least partially automated and thus a potential help to farmers. The main approach is constrained optimisation where, within given constraints and on the basis of defined objective function, one is looking for optimal solution – fertilization plan. From mathematical viewpoint, fertilization planning is almost identical to the problem of ration or feed mix formulation.

Linear programming (LP) is the most often applied method in fertilization planning (Hansson et al., 1999). However, in the literature one can also find other methods from the field of MP that have been applied

to solve fertilization planning problem. The most appropriate and commonly used method that also partly overcomes some of the LP drawbacks is goal programming (GP). It is a pragmatic and flexible methodology for resolving multiple criteria decision making problems which fertilization planning definitely is. Its advantage is also in familiarity with LP, since simplex algorithm is utilized to find the solution (Romero and Rehman, 1993). Some examples from the literature utilising GP paradigm in fertilisation planning might be Minguez et al. (1988), who applied GP approach supported by penalty functions (PFs). Sharma et al. (2003) applied similar approach based on Euclidean distance for problem of sugarcane fertilizer mix. Gosh et al. (2005) applied priority goal programming in nutrition management for rice production. For the same type of production, Sharma and Jana (2009) applied fuzzy goal programming (FGP) and genetic algorithm (GA) based on fuzzy GP approach. Hansson et al. (1999) extended common LP with binary variables into mixed integer program (MIP) and in such a manner enabled considering additional technical constraints.

The purpose of this paper is to demonstrate a possible approach utilising methods of mathematical programming to support fertilization planning in the frame of partial or full automation. It illustrates how such methods could be utilised in quite simple spreadsheet tool, in MS Excel framework. Due to availability of MS Excel, such tool could be accessible to a large number of potential users. After a brief methodological presentation of applied approach, a description of analysed practical example follows. In results the output of the tool is presented and discussed. Paper concludes with short concluding remarks.

## METHODOLOGY

### Mathematical programming methods

Problem of fertilization planning will be addressed in the context of single and multi-criteria decision making. As single objective programming, LP will be applied to minimize total fertilization cost. For searching compromise solution, multiple-criteria approach supported by weighted goal programming (WGP) will be applied.

### Linear programming and fertilization planning

Common to all LP problems is single objective function as its basic concept. It means that one tries to get the optimal solution in minimizing or maximizing desired objective within set of constraints imposed. In the context of fertilization planning this means that we minimise total fertilizing cost satisfying fixed constraints. However, LP's basic assumptions in some cases might prove too rigid. In the frame of diet formulation, that is significantly familiar with the problem of fertilization planning, Rehman and Romero (1987) are mentioning the two core issues. First is rigidity of constraints (no deviations are allowed) and the second is that the problem must be simplified by focusing on only one objective function. In the case of fertilization planning particularly, the philosophy of fully constraints satisfaction is not always justified and might bring us to two crucial problems. The first is mathematical one, since constrained equation system might not have a feasible solution, which could also happen due to minor deviations that are not allowed. In more open system (with relaxed constraints) there might be a solution, but it also might happen that it is not applicable due to unacceptable large deviations.

Undoubtedly minor deviations wouldn't significantly influence soil fertility and nutri-

ents procure. Consequently, expected yield wouldn't be much affected; however, such relaxation would result in feasible solution. This is especially evident if we consider robustness of nutrition requirements estimation. Nevertheless, Minguez et al. (1988) are stressing that even if the level of yield would be lower, the deficit could be compensated with lower input costs.

### Weighted goal programming and the system of penalty functions

WGP is a technique that has become a widely used approach in management science (Sharma et al., 2003). It is also often applied for nutrition management problems. WGP's formulation is expressed as mathematical model with a single objective (achievement) function (weighted sum of the deviations variables). Hence, the objective function in WGP model minimizes the undesirable deviations from the target goals levels and does not minimize or maximize goals themselves (Ferguson et al., 2006). In most cases, the obtained solution is a compromise between contradictory goals, enabled with positive and negative deviation variables. Negative deviation variables are included in the objective function for goals that are of type "more is better" and positive deviations variables are included in the objective function for goals of type "less is better". Since any deviation is undesired, the relative importance of each deviation variable is determined by belonging weights.

One of the main drawbacks of WGP is connected with the marginal changes. The method does not distinct between marginal changes within one observed goal; all changes (deviations) are of equal importance. To keep deviations within desired limits and to distinguish between different levels of deviations, system of penalty functions (PF) might be introduced into the WGP model (Jones and

Tamiz, 2010). It enables model to be more controllable. Sensitiveness of the model is defined through the number and size of defined penalty intervals. In fertilization planning PFs were applied for the first time by Minguez et al. (1988).

### Tool for fertilization planning

Our tool for fertilization planning was developed in MS Excel. It was constructed as two phase model (it merges two sub-models) utilising mathematical programming techniques (LP and WGP supported by PFs). The tool is relatively open and enables potential end-user to change or adjust different parameters in the process of solution searching. One can change goals priority levels, set of goals, constraints for different technologies and practices. End-user can also decide which sub-model to apply in the process of solution searching. He/she also has an option to decide whether to consider estimated price for organic manure or not. Even though manure is usually not bought, it has some value. The tool calculates it on the basis of weighted average of pure nutrients (N, P and K) it contains, while pure nutrients' prices are calculated from market prices of available and most often used fertilizers. The main problem in such approach is that estimated price could be very high and consequently domestic manure does not enter into the optimal fertilization plan. Therefore, current version of the tool has option to ignore prices of manure or to reduce estimated cost.

In further work it would be necessary to estimate also the economic value of organic matter. Particularly on arable land in crop rotation it has an important role that could be expressed in monetary terms as reduced cost of manure. In the current version of the tool this reduction is possible only through subjective judgement.

First sub-model of the tool is an example of a common least-cost manure-fertilizers combination, based on LP paradigm. Obtained optimal fertilization plan should cover all requirements of crops included in rotation and enable targeted five-year nutrients (N, P and K) balance in the soil, which is the level 'C'. If obtained solution is acceptable (deviations of N, P and K are in reasonable values), the planning process could stop at this level. However, if this sub-model does not find appropriate solution (N, P and K deviations are too big), the achieved result should be applied only to estimate expected fertilizing cost. In this manner, the tool calculates the target economic goal, which is one of the goals in the second sub-model.

Main benefit of such approach is that the model autonomously estimates fertilizing costs (goal's value) that significantly change with volatility of fertilizers' market prices. Such approach substitutes expert estimate of expected costs that are necessary in the second step, based on a compromise solution searching. However, only in such a manner economics can be considered. Therefore, WGP supported by a system of PFs yields technologically appropriate fertilizing plan, which is also acceptable from the economic perspective.

Second sub-model includes four goals. They capture desired nutrients levels (N, P and K) and total fertilizing costs. The problem of fertilizing planning is organised in such a manner that each nutrient needs should be balanced as much as possible in five year period. As target fertilizing cost enters estimated costs from the first sub-model. From pre-definition of the problem it is dependent only if bought fertilizers' prices are considered or the ones of the estimated manure values. To enable better accuracy of the second sub-model and to avoid problems mentioned above, PFs were also applied.

Due to the fact that farmers have to prepare fertilization plan once in five year time, the tool is organised in such a manner that it calculates fertilization plan for each year separately. However, the nutrients are balanced in five year period in line with crop requirements and soil analyses result. The latter is focusing mainly on P and K. Possible situations for each nutrient in the soil is that it is in deficit, optimally provided or in surplus. Regarding the soil fertility (P and K availability), correction coefficients defined by Leskošek (1993) are included to add or reduce crop requirements.

Current version of the tool is from technological viewpoint relatively simple and includes only the most important constraints that should be met by farmers. This version of the tool does not consider additional 'time' constraints that would bind manure application within one year. However, both sub-models consider basic constraints related to maximal nutrients input from organic manure (170 kg/ha of nitrogen, 120 kg/ha of phosphorus and 300 kg/ha of potassium). If the farmer has not got enough manure on disposal, or it was already applied on other plots, he can also further reduce application of organic manure with additional constraints. Model also ensures that application of manure is not allowed on particular plots (due to transportation costs, other logistics problems etc.). User also has a possibility to restrict application of organic fertilizers on individual crops.

Crop rotation has fundamental role in fertilization planning. The tool enables one to select crops from a long list of crops that enter into five year production. Regarding the expected yield, the tool calculates N-P-K requirements (Leskošek, 1993). New crops can also be added simply to this list. Crops can enter into five year rotation as main or subsequent crops.

### Description of analysed example

Application of the tool is illustrated on an example of a hypothetical parcel (1.28 ha) of arable land. The plan refers to the period of 2011-2016, where the basic assumption is that the prices of fertilizers will remain the same as in year 2011. It is supposed that soil analysis has point on poor supply with K (14 mg/100g) and P (6 mg/100g), which means that in five year period it is necessary to apply more nutrients as required by expected yields (shift from level B to level C).

From economic point of view three different analyses were conducted. In the first example only the prices from mineral fertilizers were considered and economic value of manure was neglected. In the second run full estimated manure costs were also considered. To consider also the positive effect of organic matter in manure, the third analysis assumes 50 % reduction of manure costs. For all three cases fertilization plans were prepared with both sub-models. In our analysis we were able to include in the plan only 4 different fertilizers and cattle manure. Beside the balance of P in K, we were mostly interested in total fertilization cost in five year period.

On the analysed arable land we considered crop rotation that is frequently applied on Slovene cattle farms. First year the main crop is maize for silage with expected yield of 45 t/ha, followed by triticale. After its harvest green cover during the winter is composed of grass-legume mixture. In the third year main crop is again maize for silage, followed by winter wheat. Five year rotation concludes with grass-legume mixture in the fifth year.

It is supposed that manure application is possible on analysed plot. However, from technological point of view it is reasonable to apply it only in the year of maize production.

## RESULTS AND DISCUSSION

Two examples of fertilization plans are presented in table 1. Both assume zero manure cost. First fertilizing plan was prepared by LP sub-model, minimising total cost. It could be seen that the model couldn't totally balance K and P in five year period. There is a minor surplus of K (11.6). However, this is a solution that would undoubtedly be acceptable in practice. If the farmer would use the tool for preparing fertilization plan, he/she could stop at this point. From the second example it is obvious that the application of more advanced sub-model (WGP+PF) would yield totally balanced solution. From economic perspective it is obvious that the first plan is cheaper (for 148.5 € resp. in five year period). Both results are obtained under assumption that manure price equals zero. However, considering estimated cattle manure cost, the result changes<sup>4</sup>. In both examples more mineral fertilizers are used instead. In simulation of reduced manure costs for 50 % there was no change and plan equals presented ones (table 1). The difference is only in higher total costs. In such an example the difference between estimated total costs is also reduced. The first sub-model solution costs increase for 442 € and the second sub-model for 317 €.

From obtained results no general conclusions could be drawn regarding appropriateness of the applied methods. In analysed case one gets acceptable solution already with the first sub-model and it is not necessary to proceed further with the more complex second model.

Due to automation process through the system of macros, we can conclude that the tool enables one easy formulation of fertilization plan with the first or the second sub-model. In practice, complete fertilization recommendation is certainly much

more complex than presented in this paper. However, presented approach also enables additional considerations, such as mineralisation dynamics, application time, organic fertilizer quality, volatility of fertilizers prices over time etc. that would make fertilization plan more realistic. From technical point of view this means adding additional inequality constraints.

## CONCLUSIONS

From the results obtained it is apparent that combination of deterministic LP technique and WGP supported by PFs is a useful approach for formulating five year fertilization plans, particularly if this is the 'engine' from user-friendly optimization tool. Such approach enables end-user to formulate technologically acceptable and economically efficient fertilization plans in relatively simple and prompt way.

In the analysed case it proves that with the second sub-model, based on MCDM, one could find more balanced plan. However, there are also examples where already with the first sub-model one could find convenient solutions that are completely identical to those obtained with the second sub-model. In testing the tool there were also examples where the first sub-model yield solution with deviations of K or P greater than 150 kg. In such an example the second sub-model should be strictly applied.

Presented tool could be further upgraded considering more goals and constraints. It is deficient, particularly from time horizon perspective within one year, since it gives no information when to apply particular fertilizer (application time). Another challenge is to include, beside already mentioned organic matter, further constraints to control soil acidity and mineralisation dynamic.

<sup>4</sup> We don't present those results due to space limit.

Table 1: Obtained fertilization plans from the first and second sub-model

Fertilizing plan for farm with ID

xxxxxxxxxx

Analyses conducted by:

Zgajnar J.

Parcel data:

Home name

GERK:

Code of arable land

Soil analyses

Date

P2O5 mg/100g

K2O mg/100g

Test

1.28 ha

ababab

N 1

2011

6

B

14

B

Average nutrients per year

Average year correction to achieve 'C' level

Deviation on annual basis

Nutrients in fertilizer (kg/ha)

Crop requirements (kg/ha)

Year	Main crop	Subsequent crop	Code	Organic / mineral fertilizer	Price	Quantity	Nutrients in fertilizer (kg/ha)			Crop requirements (kg/ha)		
							N	P	K	N	P	K
2013	Maize for silage	Wheat	G1	Cattle manure	0.0	34.0	132.7	85.0	204.0	146.3	90.0	180.0
			M2	NPK 15 15 15	44.2	2.0	30.0	30.0	30.0			
						Year 3						
2014	Wheat	GLM	M1	NPK 5-15-30	56.3	3.3	16.7	50.0	100.0	122.5	50.0	100.0
			M2	NPK 15 15 15	44.2	1.7	25.0	25.0	25.0			
			M14	UREA N 46	33.2	1.8	80.8	0.0	0.0			
2015	GLM	GLM	M1	NPK 5-15-30	56.3	4.0	20.2	60.5	121.0	121.0	49.5	110.0
			M2	NPK 15 15 15	44.2	0.9	14.0	14.0	14.0			
			M14	UREA N 46	33.2	1.9	86.8	0.0	0.0			

Fertilizing plan for farm with ID

xxxxxxxxxxxxxx

Analyses conducted by:

Zgajnar J.

Parcel data:

Home name

GERK:

Code of arable land

Soil analyses

Date

P2O5 mg/100g

K2O mg/100g

Test

1.28 ha

ababab

N 1

2011

6

B

14

B

Average nutrients per year

Average year correction to achieve 'C' level

Deviation on annual basis

Nutrients in fertilizer (kg/ha)

Crop requirements (kg/ha)

Year	Main crop	Subsequent crop	Code	Organic / mineral fertilizer	Price	Quantity	Nutrients in fertilizer (kg/ha)			Crop requirements (kg/ha)		
							N	P	K	N	P	K
2011	Maize for silage	Triticale	G1	Cattle manure	0.0	24.4	95.3	61.0	146.5	146.3	90.0	180.0
			M1	NPK 5-15-30	56.3	0.3	1.5	4.5	9.1			
			M2	NPK 15 15 15	44.2	3.3	49.4	49.4	49.4			
2012	Triticale	GLM	M1	NPK 5-15-30	56.3	4.6	22.9	68.8	137.5	137.5	55.0	123.8
			M2	NPK 15 15 15	44.2	0.8	11.3	11.3	11.3			
			M14	UREA N 46	33.2	2.2	103.3	0.0	0.0			
2013	Maize for silage	Wheat	G1	Cattle manure	0.0	24.4	95.3	61.0	146.5	146.3	90.0	180.0
			M1	NPK 5-15-30	56.3	0.3	1.5	4.5	9.1			
			M2	NPK 15 15 15	44.2	3.3	49.4	49.4	49.4			
2014	Wheat	GLM	M1	NPK 5-15-30	56.3	3.3	16.7	50.0	100.0	122.5	50.0	100.0
			M2	NPK 15 15 15	44.2	1.7	25.0	25.0	25.0			
			M14	UREA N 46	33.2	1.8	80.8	0.0	0.0			
2015	GLM	GLM	M1	NPK 5-15-30	56.3	4.0	20.2	60.5	121.0	121.0	49.5	110.0
			M2	NPK 15 15 15	44.2	0.9	14.0	14.0	14.0			
			M14	UREA N 46	33.2	1.9	86.8	0.0	0.0			

GLM - grass-legume mixture

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