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# Regional Cooperation in the Use of Irrigation Water: Efficiency and Income Distribution

D. Yaron<sup>1</sup> and A. Ratner<sup>2</sup>

<sup>1</sup>Levi Eshkol School of Agriculture, Faculty of Agriculture, The Hebrew University of Jerusalem and the Center for Agricultural Economic Research, P.O. Box 12, Rehovot 76-100 (Israel)

<sup>2</sup>The Tel-Aviv College for Business Administration, P.O. Box 48068, Tel-Aviv 61480 (Israel)

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## Abstract

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The paper presents an analysis of the economic potential of regional cooperation in water use in irrigation under conditions characterized by a general trend of increasing salinity. Income maximizing solutions for the region are derived and the related income distribution schemes are solved for, with the aid of cooperative game theory algorithms and shadow cost pricing. Distinction is made between distribution policies with and without side payments. The reasonableness and the acceptability of these schemes is later critically evaluated. The Nash–Harsanyi approach seems to be the most appropriate for the conditions studied.

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## Introduction

In most parts of the world with irrigated agriculture, the allocation of irrigation water to farms is dictated by water rights (quotas) which have been institutionally determined many years previously. Generally, these rights have not been changed since their determination, nor have they been adjusted to the significant technological changes in agriculture and in the farming systems which have occurred. The inevitable result is inefficiency in the interregional and interfarm allocation of water.

The inefficiency of the institutional water allocation system has been exacerbated recently by the increasing use in irrigation of low quality water (e.g. drainage water or brackish water from marginal sources) in regions which suffer from water scarcity (e.g., Western United States, Israel). In those regions where a dual supply is being developed with differentiation according to water quality (in terms of salinity or other quality parameters), the institutional system can hardly cope with water allocation problems. In most situations

until now, with one water quality being supplied to a region, the problem of quota allotment to farms involves two parameters – the annual and the peak-season quantities.<sup>1</sup> With a dual water supply there are four allocation parameters to be considered.

An interesting problem arises when the ‘National Water Authority’ (or any other authority which allocates water to regions and farms) and a particular region are faced with the option of increasing the salinity of the water supply to the region and compensating the region by increased quantity. If the water supplied to the region has to be of just one salinity level (contrary to the dual supply situation) and the region’s farms have different preferences with respect to the desired quantity–salinity (Q–S) mix, the determination of the ‘optimal’ mix is a difficult problem. This problem is addressed in the present paper.

The growing complexity of water-allocation issues in the situations described above and in others will, necessarily, increase the inefficiency of the institutional allocation system and emphasize the need for interfarm and interregional water mobility in correspondence with economic considerations.

One way to increase the efficiency of water allocation among farms within regions is through the establishment of farmers’ regional water associations or cooperatives. The cooperatives must be established voluntarily: the historical institutional water rights of its farm members will be retained (at least during the first few years; changing them seems to be an extremely difficult or even impossible task), but the members of the association will be able to exchange water quotas (a) among themselves, and (b) with other entities (e.g., the National Water Authority). Such associations already exist and operate in certain regions with irrigated agriculture.

The objective of this paper is to analyze the economic potential of such cooperative associations from the point of view of both *efficiency* and *equity* under conditions characterized by a general trend of increasing salinity of the water supplied to agriculture. The approach can be easily extended to other parameters of water quality.

## **A model for the determination of the optimal water quantity–salinity mix for a regional water users’ cooperative**

### **Assumptions and framework for the analysis**

Consider a region with  $I$  farms and a given allotment of water ( $\overline{GW}$ ) of a relatively low salinity level (high quality),  $R_0$ ;  $\overline{GW}$  is the sum of the individual farm quotas ( $GW_i$ ):

<sup>1</sup>Sometimes the number of parameters is larger.

$$\overline{GW} = \sum_{i=1}^I GW_i$$

Assume that the National Water Authority can supply the region with a higher quantity of water at the cost of increasing its salinity. For a given  $\overline{GW}$  the substitution between water quantity  $\overline{BW}$  ( $\overline{BW} \geq \overline{GW}$ ) and its salinity  $R$  ( $R \geq R_0$ ) is subject to a transformation curve determined by the National Water Authority (for the same quantity of water lower salinity is preferred by farmers):

$$F(\overline{BW}, R | \overline{GW}) = 0 \quad (1)$$

It is assumed that the salinity of water ( $R$ ) must be the same for the whole region and all of its farms. Any decision regarding  $R$  and the receipt of a larger quantity of water but of higher salinity must be mutually agreed upon by all the farms in the region. This provides the essential motivation to encourage the region's farms to cooperate within the framework of a water users' association.

Obviously,  $\sum_i BW_i = \overline{BW}$ , with  $BW_i$  denoting the quantity of water of quality  $R$  allocated to farm  $i$ .

Farm  $i$ 's income  $y_i$  is a function of  $BW_i$  and  $R$ :

$$y_i = y_i(BW_i, R) \quad i = 1, 2, \dots, I \quad (2)$$

To encourage cooperation farm  $i$ 's income must be higher than or equal to  $y_i^0$ :

$$y_i \geq y_i^0 \text{ for all } i \quad (3)$$

with  $y_i^0$  being the income before the establishment of the regional association.

In view of the above, referring to the region and its water supply-demand relationships as a competitive market seems inappropriate: (a) the number of farms in the region is small<sup>2</sup>, the farms are not anonymous and partial cooperation agreements are possible, and (b) it is assumed that the deliberate increase of water salinity in the region necessitates an agreement by all the farms in the region and by the National Water Authority. Accordingly, the cooperative's problem is to determine simultaneously: (a) the optimal quantity-salinity ( $\overline{BW}-R$ ) mix for the regional cooperative, and (b) the quantity of water ( $BW_i$ ) of salinity  $R \geq R_0$  allotted to each of the individual farms. The objective is to maximize the region's welfare, that is to *increase* the region's income, subject to an acceptable income distribution among its farms. The achievement of the region's *maximal* income ( $\max \sum_i y_i$ ) could be in conflict with the distributional goal.

It is further assumed that good drainage conditions prevail in the region and

<sup>2</sup>The authors have in mind a viable region with 20-25 villages, a number considerably larger than in our expository discussion.

that rainfall is sufficient to prevent long-term salt accumulation in the soil. Accordingly, the analysis is restricted to a 1-year planning horizon.

### Efficiency frontier

The efficiency frontier in the sense of Pareto-optimal points with the property that any move from such a point aimed at improving the income of one farm must necessarily reduce the income of some other farm (s), can be derived by solving the following problem:

$$\text{Maximize } W = \sum_{i=1}^I \lambda_i y_i \quad (4)$$

subject to

$$\begin{aligned} F(\overline{BW}, R | \overline{GW}) &= 0 \\ \sum_i GW_i - \overline{GW} &= 0 \\ \sum_i BW_i - \overline{BW} &= 0 \end{aligned} \quad (5)$$

$$y_i = y_i(GW_i, BW_i, R)$$

$$y_i \geq y_i^0 \quad \text{all } i$$

with  $W$  being the cooperative's welfare function,  $\lambda_i$  relative weight assigned to the  $i$ th farm income ( $\lambda_i > 0$ ,  $\sum \lambda_i = 1$ ), with all other symbols as previously defined. By parametrically varying the  $\lambda_i$  weights, the 'efficiency frontier' in the  $I$  farms' income ( $y_i$ ) space can be derived.

A linear programming (LP) model designed for the derivation of the efficiency frontier is presented in Appendix A. More details on the LP model can be found in Yaron and Ratner (1985) or Ratner (1983, in Hebrew). The derivation of the efficiency frontier involves cumbersome computations and it is not recommended as an approach towards solving the regional problem. It is discussed here for expository purposes and as a stage towards the presentation of our model.

### Income distribution considerations

For the sake of simplicity and the benefit of graphical exposition we refer in this section to cooperation between two farms only. Figure 1 presents the efficiency frontier of incomes (AB curve) which can be generated by the two farms under conditions of a possible change in the 'regional' water quota (that is, the total quota of the two farms in this simplified case), accompanied by a change in water salinity and exchange of water quotas among the farms. The

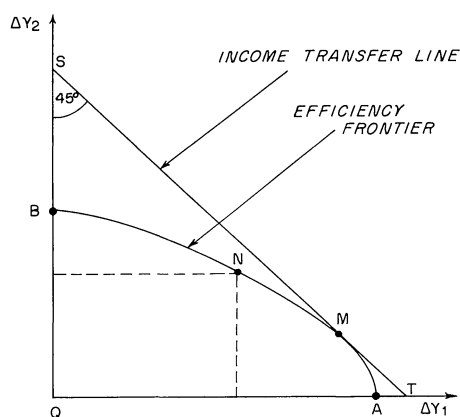


Fig. 1. Income efficiency frontier and income transfer line.

incomes of the two farms with no cooperation are shown by the point Q (coordinates  $y_1^0$  and  $y_2^0$ ), with the horizontal and the vertical axes showing the incremental incomes due to cooperation ( $\Delta y_i = y_i - y_i^0$ ). At point A,  $\Delta y_1$  is maximized, with  $y_2$  remaining at  $y_2^0$ ; the reverse holds at B. If the incomes of the two farms are assigned equal weights in (4), i.e.,  $\lambda_1 = \lambda_2$ , the total income of the cooperative is maximized. This situation is represented by the point M where the gains from the cooperation are  $\Delta y_1 + \Delta y_2$ . In the example of Fig. 1,  $\Delta y_1$  is significantly higher than  $\Delta y_2$ , implying that Farm 1 is more likely to benefit from exchange of water quotas than Farm 2. Accordingly, Farm 1 will be referred to in the following as being more efficient.

As we move from M to the 'south east' along the AB curve the income generated by Farm 1 rises and that of Farm 2 falls; the opposite holds for a move in the 'north west' direction along AB. In both cases the total income of the cooperative decreases for any move away from M.

Any point on the AB curve corresponds to a certain reallocation of water between the region and the National Water Authority, and between the farms within the region. The *decision variables* are  $\overline{BW}$ ,  $BW_i$ ,  $R$  and the areas of the various crops on each farm (see Appendix A for details).<sup>3</sup>

### Income distribution policies – with or without side payments

Theoretically, two major groups of policies are open to the cooperative:

(1) The cooperation is restricted to the exchange of water quotas only; the distribution of income is determined solely by water transfers (for example Farm 1 transfers water to Farm 2 in June and a reverse transfer takes place, say, in July).

<sup>3</sup> $\overline{GW}$ ,  $GW_i$  and  $R_0$  are considered as a special case of  $\overline{BW}$ ,  $BW_i$  and  $R$ .

(2) The cooperation involves both water quota exchange and side payments, i.e. direct income transfers. If a policy with side payments is chosen the cooperative will maximize its income, at M in our example, and then redistribute it according to the income transfer line ST (with slope  $-1$ ).<sup>4</sup> In such a case income maximization and income distribution can be analyzed and modelled independently; whereas with no side payments these two issues are interdependent.

There is, in general, resentment among farmers to redistribute income via the side payments mechanism. Such policy implies, in our example, that a share of the income generated by the more efficient farm (Farm 1) will be transferred to the less efficient one (Farm 2).

Over and above the resentment to such policy per se the question of what should be the magnitude of the side payments is not easy to answer. An attempt to apply cooperative game theory models to determine it (Yaron and Ratner, 1985) and specifically the Core, the Shapley Value (Shapley, 1953), and the Nucleolus solution (Schmeidler, 1969), with reference to a quasi-empirical case, lead to questionable results, which could not be used as a starting point for designing a sound policy. These results are presented in the following section.

Redistribution of income through payments for water transfers according to its shadow price was also considered. However, this approach, while a priori very sound from the theoretical point of view, poses a major difficulty under numerous situations prevailing in Israel (and, possibly in other countries). The reason is that water is rationed according to institutionally determined quotas, or historical water rights, and its price to farmers is subsidized, and in many situations is lower than the shadow price. In such situations water quotas allocated to farmers bear a considerable component of rent as is the case in the study here reported (see the following section). With water considered to be a national resource owned and administered by the nation, it is claimed that farmers have the right to benefit from their water quotas if and only if they use water for production. If water had been *private* property of the farmers, the objection to rent on water quotas would have been removed.

Note that the magnitude of the rent derived from water depends on the profitability of staple crops which are the marginal water users. If their long-term profitability would fall or if the water price paid by farmers would be raised, the issue of rent on water quotas would be resolved and shadow cost pricing of water would become acceptable.<sup>5</sup>

To sum up this section, exchange of water quotas and payment for water transfers according to the shadow price is not acceptable under numerous sit-

<sup>4</sup>Note that ST is tangent to AB.

<sup>5</sup>The reference by the authors to the inefficient system of quota allocation and water pricing in Israel as given, does not imply its approval by them. On the contrary, suggestions for its revision have been repeatedly set forth (e.g. Yaron, 1971, 1979). This issue, however, falls beyond the scope of the present paper.

uations prevailing in Israel, with water quotas bearing a considerable rent component.

Accordingly, we turn, in the following section, to the discussion of policy with no side payments.

### Selection of the 'best' point on the income efficiency frontier

Given the income efficiency frontier, the next question is which point on it should be chosen and how? The question amounts to assigning relative weights ( $\lambda_i$  in expression 4) to the incomes of the members of the cooperative, which is equivalent to formulation of the cooperative's welfare function. Several approaches to this problem have been discussed in the literature (e.g. among others, the conceptual issues by Sen, 1970, the empirical approach by Keeney and Raiffa, 1976, and a historical overview by Cohon, 1978); they are all based on arbitrary judgements and may involve very difficult negotiations to reach an agreement.

If we assume, as an approximation, that utility is linear in money, the approach originally proposed by Nash (1950) for two 'players' (in game theory semantics) and extended by Harsanyi (1959) to  $n$  players with  $n > 2$ , leads to an objectively and uniquely determined point on the income efficiency frontier. It is only necessary that the members of the cooperative agree upon four basic and a priori logical assumptions.<sup>6</sup>

In terms of our problem and with reference to  $n$  farms the Nash-Harsanyi solution can be derived with the aid of the following model:

Maximize  $Z$ :

$$Z = \prod_{i=1}^n (y_i - y_i^0) \quad (6)$$

subject to restrictions (5) as previously specified. By logarithmic transformation of (6) and maximizing  $\log Z$  (a monotonic function of  $Z$ ) a separable objective function is obtained and the problem can be solved by a separable programming routine available for most computers. The optimal water salinity  $R$  is determined by solving the problem parametrically, for the discrete levels of  $R$  chosen (see above). A detailed mathematical formulation of the above model is presented in Appendix B.

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<sup>6</sup>The assumptions are: (a) joint efficiency; (b) invariance with respect to linear utility transformations; (c) symmetry; and (d) independence of irrelevant alternatives.



## A quasi-empirical application

The models discussed in the previous section have been applied to the analysis of potential cooperation among three farms residing in a small 'region' in the Negev area of Israel. The farms and their characteristics are real and only the boundaries of the 'region' are artificial. The reason for such problem definition is to decrease the computational burden in our expository analysis and at the same time to keep reality represented as truthfully as possible.

Farms 1 and 2 are kibbutzim<sup>7</sup> and farm 3 is a moshav.<sup>8</sup> Each kibbutz has two water sources: (a) the regional water project, operated by Mekorot Co. (with current salinity of 220 ppm Cl), and (b) self-owned wells of saline water (1000–1200 ppm Cl). Farm 3 has only one water source, which is the regional water project. Each of the three farms has at its disposal annual and high season – monthly – quotas – from each water source. The quotas are determined by the National Water Authority.

The major differences between the farms are:

(1) Farm 3 is much more sensitive to salinity than the two kibbutzim. This is due to a large share of salinity-sensitive (perennial) fruit crops in its crop mix (40% of the irrigated area as compared with none and 5% respectively on Farms 1 and 2).

(2) The land area of Farms 1 and 2 is practically unlimited while on Farm 3 it is a limiting factor.

(3) The share of the high-season water quotas supplied by the regional project out of the seasonal total is 20%, 12% and 15%, respectively, for Farms 1, 2 and 3. The high-season pumping quota from the self-owned wells is 10% and 30%, respectively, for Farms 1 and 2.

(4) A large share of the land of Farm 1 is sandy soil with the rest being loess. All the land of Farms 2 and 3 is loess. The differences in soil types lead to differences in the cropping patterns.

(5) Farm 1 uses its well water in mix with the Regional Project water on some of its lands; Farm 2 irrigates some of its lands by the self-owned wells, while the rest is irrigated with the Regional Project water.

The above differences among the region's farms open options for cooperation in the use of their water sources.

The results of the analysis of the potential for income increase due to cooperation, and of the alternative policies for the allocation of the cooperation gains, applying the approaches discussed in the previous section, are presented below.

<sup>7</sup>Kibbutz (plural Kibbutzim) is a collective settlement with voluntary membership and democratic management.

<sup>8</sup>Moshav – a cooperative village of 60–150 small family farms. The village cooperative acts as a credit association and provides production and marketing services. For simplicity the moshav village is referred to as an aggregate in our quasi-empirical analysis.

TABLE 1

Income with and without cooperation and comparison of alternative schemes of allocation of the cooperation gains with reference to policies with side payments

	Farm		1	2	3	Total
(1)	Income with no cooperation	(US\$1000) <sup>a</sup>	946	769	605	2320
(2)		(%) <sup>b</sup>	41	33	26	100
(3)	Income generated under cooperation	(US\$1000)	1579	568	558	2705
(4)		(%)	58	21	21	100
Allocation of cooperation gain						
(5)	Nucleolus	(US\$1000)	112	179	94	385
(6)		(%)	29	47	24	100
(7)	Shapley Value	(US\$1000)	159	194	32	385
(8)		(%)	41	51	8	100
(9)	Shadow cost pricing	(US\$1000)	52	333	0	385
(10)		(%)	14	86	0	100

<sup>a</sup>Rounded numbers.

<sup>b</sup>Percentages computed with reference to unrounded numbers.

Table 1 presents the results with reference to policies which assume that side payments are acceptable.

The upper part of the table presents the farms income and the region's total with no cooperation and the income generated on the farms under cooperation and efficient interfarm allocation of the region's water. As shown, the total income is increased by 385 thousand dollars or 17%. This increase in the total regional income originates in a 633-thousand dollar increase in the income generated on Farm 1 ( $1579 - 946 = 633$ ) which is offset by a decrease in the incomes generated on Farms 2 and 3 ( $-201$  and  $-47$  thousand dollars, respectively). The increased income generated on Farm 1 is due to transfers of water as follows: Farm 1 receives 1 719 000 m<sup>3</sup> from Farm 2 (1 247 000 m<sup>3</sup> from the Regional Project and 472 000 m<sup>3</sup> from wells) and 60 000 m<sup>3</sup> from Farm 3 (Regional Project water).

The lower part of Table 1 presents the allocation of the cooperation gains among the region's farms according to the Nucleolus<sup>9</sup>, Shapley Value and Shadow Cost Pricing schemes. Scrutiny of the allocations of the cooperation gains according to the cooperative game theory approaches – Nucleolus and Shapley Value – suggests that Farm 1, which contributes the major part to the region's cooperative income, receives a smaller share of the cooperation gains than Farm 2, which contributes considerably less to the cooperative income. This rather unacceptable result follows from the fact that the above game so-

<sup>9</sup>The Core, the extreme points of which were computed, was quite large and therefore not useful.

lutions reflect the bargaining power of the participants rather than an allocation based on some notion of justice. Shadow cost pricing leads to even more unacceptable results; the reasons for this were discussed in the previous section.

To support the conclusion that the above results are unacceptable, we compute from Table 1 the side payments given away or received (minus sign) by the three farms.

Denote:

- INC, income with no cooperation (row 1 in Table 1)
- IC, income generated on farm  $i$  under cooperation (row 3)
- SP, side payments given away (negative sign if received)
- CG, cooperation gains (rows 5, 7, 9, respectively).

Using the above definitions we get:

$$CG = IC - INC - SP$$

and

$$SP = IC - INC - CG \quad (7)$$

Applying (7) we get for Farm 1 side payments of 521, 474 and 581 thousand dollars respectively for the Nucleolus, Shapley Value and Shadow Cost Pricing allocations.

The side payments of Farm 2 (a net donor of water and payments receiver) are  $-380$ ,  $-395$ , and  $-534$  thousand dollars, respectively, for the same allocation schemes. The side payments received by Farm 3 are less dramatic, but still large. Recall that the net transfers of water quotas given away by Farms 2 and 3 are respectively 1719 and 60 thousand  $m^3$ .

The results of the Nash–Harsanyi solution with no side payments are presented in Table 2. Scrutiny of Table 2 suggests that the region's income increases due to cooperation by 194 thousand dollars or 8%, about one half of the increase in income when cooperation involves side payments.

The percentage-wise distribution of the total income generated on the farms is similar to that as before establishing the cooperative. From this point of view, the results seem intuitively acceptable. The aggregate income of the cooperative rises by 8% in comparison with the non-cooperative situation, while the incomes of Farms 1–3 rise by 9, 5 and 11%, respectively.

Computationally the Nash–Harsanyi model is simple in comparison with the other game theory models applied in this study. It involves finding the optimal solution for each farm in the region and its income under no cooperation conditions ( $y_i^0$ ), and then finding the regional solution according to the model here described. If there are say, 20–25 farms in the region, each represented in the model by 40–50 restrictions and 30–50 activities, this amounts altogether to 800–1250 restrictions and 600–1250 activities. A limited number of additional restrictions should describe the regional pool of water from which

TABLE 2

Nash–Harsanyi solution

Farm	1	2	3	Total
Income with no cooperation				
(US\$1000)	946	769	605	2320
(%) <sup>a</sup>	41	33	26	100
Income generated under cooperation				
(US\$1000)	1031	811	672	2514
(%)	41	32	27	100
Cooperation gains				
(US\$1000)	85	42	67	194
(%)	43	22	35	100

<sup>a</sup>Percentages computed on the basis of unrounded numbers.

‘water transfer activities’ convey water to the farms; their number depending on the complexity of the water supply system.

As mentioned previously, the objective function is separable non-linear and concave. This is certainly a manageable model which can be solved with a reasonable amount of effort.

## Summary and conclusions

In this paper the potential for economic gains from regional cooperation in the use of water resources was analyzed assuming exchange of water quotas and water mobility among farms. Several alternative policies for the allocation of cooperation gains were studied with a distinction being made between policies with and without side payments.

The application of the alternative models to a quasi-empirical case shows that the regional income can be increased by 17% by means of cooperation with side payments. However, several difficulties are involved in the adoption of policies with side payments: (a) general resentment of farmers to adopt side payments as a policy; (b) all schemes of allocation of the cooperation gains among the farms lead to unsound results.

Payments for transfers of water according to a shadow cost pricing scheme also lead to apparently unacceptable results because: (a) water is subsidized and rationed, with its shadow price being considerably higher than the price paid by the farmers (in the case studied and many other situations); (b) it is objected that farmers will benefit from the rent derived from water quotas, unless it is used for production.

On the other hand, an apparently acceptable solution was derived with the aid of the Nash–Harsanyi model with no side payments.

It should be noted that the above results were obtained with reference to a particular situation and generalizations at this stage are premature. At the same time the application of the Nash–Harsanyi model to the problem discussed in this paper (or to similar problems) seems promising:

(1) The assumptions or the axioms underlying are simple, easy to comprehend and acceptable. In effect, there is some empirical evidence that the Nash model is applied in real life situations (Bell and Zusman, 1976).

(2) It is computationally simple and manageable with a reasonable amount of effort.

The difficulty envisaged in the application of the above model could be in the explanation to the farmers of the link between the underlying assumptions (the axioms) and the solution, namely why or how the underlying axioms lead to the solution arrived at. Therefore, it seems that the use of the model implies the involvement of a qualified and authoritative arbitrator who may use the model as an aid in his arbitration task.

Another solution which might be outlined as a result of this study is a ‘modified shadow cost pricing’ for water. If the rent from water quotas is high and it is objected that the quota ‘owners’ who transfer water to others will benefit from the quota rent, a levy could be imposed on the water transferred. Shadow cost price will be paid by the buyer with the levy deducted from the amount received by the seller. The total amount of the levy collected might be redistributed among the farms, or be used for regional projects or for funding other regional activities. The details of such a policy should take into account a premium for the risk undertaken by the buyer, the mechanism of collection of the levy and its allocation among alternative uses. The details fall beyond the scope of the present paper.

## Appendix A

### Linear programming model for the derivation of the efficiency frontier

In the transition from the theoretical background to the operational model it was decided, for computational convenience, to refer to  $R$  as an exogenous parametrically varying variable<sup>10</sup>,  $R = 220, 260, 300, 350, 400$  ppm Cl, with 220 being the current salinity level ( $R_0$ ).

For each level of  $R$  the following problem was solved:

Maximize  $f$ :

$$f = \sum_i \lambda_i \sum_j c_{ij}^R x_{ij}^R \quad (\text{A.1})$$

<sup>10</sup>Feinerman (1980) and Feinerman and Yaron (1983) have incorporated  $R$  into a linear programming model as an endogenous decision variable, along with the quantities of water to be applied to the farms’ crops. Their approach however, is computationally cumbersome and has not been applied here.

subject to

$$\begin{aligned}
\sum_j w_{ij}^R x_{ij}^R - BW_i^R &\leq 0 \quad i=1, 2, \dots, I \\
\sum_j d_{ij}^R x_{ij}^R - SW_i^R &\leq 0 \\
\sum_i BW_i^R - \overline{BW}^R &\leq 0 \\
\sum_i SW_i^R - \overline{SW}^R &\leq 0
\end{aligned} \tag{A.2}$$

$$\frac{1}{\alpha^R} \overline{BW}^R \leq \overline{GW}$$

$$\frac{1}{\alpha^R} \overline{SW}^R \leq \beta \overline{GW} \quad i=1, 2, \dots, I \quad p=1, 2, \dots, p$$

$$\sum_j a_{pij} x_{ij} \leq b_{pi}$$

$$\sum c_{ij}^R x_{ij}^R \geq y_i^0 \tag{A.3}$$

$$x_{ij}^R, BW_i^R, SW_i^R \geq 0 \tag{A.4}$$

where  $\lambda_i$  is the relative weight assigned to the income of the  $i$ th farm;  $\overline{BW}^R$ ,  $BW_i^R$ , respectively, total annual quantity of water of salinity  $R$  allocated to the region and Farm  $i$ ;  $\overline{SW}^R$ ,  $SW_i^R$  respectively, high season quantity of water of salinity  $R$  allocated to the region and Farm  $i$ ;  $c_{ij}$  income per activity unit  $j$  on farm  $i$  (US\$);  $x_{ij}$  level of activity  $j$  on farm  $i$ ;  $w_{ij}$ ,  $d_{ij}$  respectively, total and high-season water inputs per activity unit  $j$  on farm  $i$ ;  $\alpha^R$  regional coefficient of substitution between low-salinity water (GW) for water of salinity ( $R \geq R_0$ );  $\alpha^R$  varies parametrically according to  $R$ . These coefficients are computed from the quantity–salinity regional transformation function (1);  $\beta$  maximal share of high-season water quota out of the annual total ( $0 < \beta < 1$ );  $a_{pij}$  and  $b_{pi}$ , respectively, input coefficient and availability level of restriction  $p$  other than water; and  $y_i^0$  income of the  $i$ th farm before cooperation (computed by LP models for each of the  $I$  farms).

The model is solved parametrically for each set of  $\lambda_i$ 's and the five levels of  $R$ . The optimal solution for a given set of  $\lambda_i$ 's and  $R$  values is the solution which yields the highest value of the objective function (A.1); it points out the optimal  $R$ , and, it yields discrete points on the efficiency frontier as an approximation to the continuous efficiency surface. Note that in this formulation  $\overline{GW}$  is considered to be special case of  $\overline{BW}$ .

The computational procedure is feasible for a small number of farms; say  $I \leq 4$  or 5. With a larger number of farms, the computational load becomes prohibitive. Therefore, this model should be regarded as one of an expository nature and as a basis for the Nash–Harsanyi model presented in the text and in Appendix B.

## Appendix B

### A model for the derivation of the Nash–Harsanyi solution

Maximize  $Z^*$ :

$$Z^* = \prod_{i=1}^n (y_i^R - y_i^0) \quad (\text{B.1})$$

subject to restrictions (A.2) and (A.4) detailed in Appendix A and

$$y_i^R - \sum_j c_{ij}^R x_{ij}^R = 0 \quad \text{for all } i \quad (\text{B.2})$$

The optimal water salinity  $R$  is determined by solving the problem parametrically, for the discrete levels of  $R$ , and choosing  $R$  which leads to highest  $Z^*$ .

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