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Derivation of Long-run Factor Demands from Short-run Responses

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Abstract

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The concept of the restricted cost function provides a dual approach to the analysis of short-run technology. It allows also, under curvature restrictions, inference of the different possible equilibria, according to constraints on the firms. Moreover, in this paper, the properties of the restricted cost function are spelled out. Substitution possibilities related to the different regimes are also derived from the restricted cost function.

This theoretical framework is applied to characterize the French cereal-producing sector by using a cross-section of farms.

Introduction

The usefulness of empirical analysis of agricultural technology has been greatly enhanced by the use of flexible functional forms, based on cost or profit relationships rather than production functions. An important assumption underlying most cost function applications is that all inputs are in static equilibrium, according to marginal productivity pricing. This maintained hypothesis is very restrictive, especially for the agricultural sector, since certain inputs cannot be freely varied within the single observation period (Brown and Christensen, 1981; Boutitie and al., 1987; Guyomard and Vermersch, 1987; Mahé and Rainelli, 1987). So, in this paper, we develop a short-run Hicksian equilibrium model; only the variable inputs fully adjust to their cost-minimizing levels, while the quasi-fixed inputs remain fixed.

We first provide a complete characterization of the theoretical short-run Hicksian model, while paying close attention to the underlying assumptions (especially the assumptions of convexity of the restricted cost function in its arguments: variable input prices, quasi-fixed input levels, production level)

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and to the description of the different theoretical possible equilibria (short-run and long-run Hicksian equilibria, short-run and long-run Marshallian equilibria), given knowledge of short-run environment.

Following Lau (1976) and reworking Sakai's decomposition (1974), we develop and analytical framework which allows inference of the long-run uncompensated demand (and supply) functions, from the short-run Hicksian equilibrium. In order to use this methodology, sufficient curvature restrictions, which are not necessarily verified at short-run Hicksian equilibrium, are imposed: strict convexity of the restricted cost function, with respect to the quasi-fixed inputs, the output, the quasi-fixed inputs and the output, respectively.

As a final objective, this conceptual framework is used in order to assess the technological characteristics of the French cereal-producing sector. The study will use the restricted translog cost function, estimated by using a sample of farm accounts ($n=208$) related to the year 1981, and will emphasize:

- substitution possibilities (own and cross-price elasticities), economies of scale in the short-run and in the long-run equilibria;
- shadow prices and optimal quantities of fixed inputs (family labor and land).

We examine especially the implications of taking into account the simultaneous quasi-fixity of two inputs: land and family labor. This application is attractive because the assumption of long-run static equilibrium with respect to these factors has been at length questioned for French agriculture in the postwar period. We illustrate the theoretical relationships presented in Section 2 for this particular case: two quasi-fixed factors, and a restricted translog cost function.

1. A characterization of the restricted cost function

We consider a firm which uses $M+N$ inputs $(z_1, ..., z_M, x_1, ..., x_N) = (z, x)$, $x \geq 0$, $z \geq 0$ at prices (p_x, p_z) to produce one output y, $y \geq 0$. The production possibilities set, Y, is supposed to have the following properties:

(a) Y is closed and non-empty.

(b) If $y \neq 0$, then $x \neq 0$.

(c) For all $(x, y, z) \in Y$, if $x < \infty$ and $z < \infty$, then $y < \infty$.

(d) There is free disposal of inputs and output; i.e., for all $(x, y, z) \in Y$, the production plant (x', y', z') such that $(x' \geq x; z' \geq z; y' \leq y)$ is possible, i.e. $(x', y', z') \in Y$.

(e) $X(z, y) = [x; (x, y, z) \in Y]$ is convex.

The restricted cost function is then defined by:

$$\text{CR}(p_x, y, z) = \underset{x}{\text{Min}} [p'_x x; x \in X(y, z)] \tag{1}$$

With a strictly positive input price vector p_x, hypothesis (a) ensures the existence of $\text{CR}(p_x, y, z)$. Furthermore, $\text{CR}(p_x, y, z)$ is non-negative, positive when y is non-zero, non-decreasing, positively linear homogeneous, concave and continuous in p_x (MacFadden, 1978, p. 11).

It is also possible to show that property (d) implies that $\mathrm{CR}(p_x, y, z)$ is non-decreasing in y and non-increasing in z (Guyomard and Vermersch, 1988). In addition, under the convexity hypothesis (e) of the section $X(y, z)$ of Y, the knowledge of the restricted cost function $\mathrm{CR}(p_x, y, z)$ is sufficient to describe, in an exhaustive manner, the short-run Hicksian technology which is employed.[1] Finally, duality results state:

$$X^*(y,z) = [x \geq 0; p'_x x \geq \mathrm{CR}(p_x, y, z), \text{for all strictly positive } p_x] \\ = X(y, z) \qquad (2)$$

Furthermore, assuming the restricted cost function $\mathrm{CR}(p_x, y, z)$ is twice differentiable with respect to input prices, the derivative properties are:

- Shephard's lemma relates $\mathrm{CR}(p_x, y, z)$ and the cost-minimizing input demand functions through its partial derivatives:

$$\partial\, \mathrm{CR}/\partial p_{x_n} = \bar{x}_n(p_x, y, z) \qquad n=1, \ldots, N \qquad (3)$$

- The following Hessian matrix is symmetric, negative semi-definite, and of rank $N-1$:

$$\Sigma_{p_x p_x} = [\partial^2\, \mathrm{CR}/\partial p_{x_n}\, \partial p_{x_{n'}}] \qquad \begin{matrix} n=1, \ldots, N \\ n'=1, \ldots, N \end{matrix}$$

Finally, under the assumption of differentiability of $\mathrm{CR}(p_x, y, z)$ with respect to the quasi-fixed inputs and to the output, it is convenient to define the total Hessian matrix:

$$\Sigma = \begin{bmatrix} \Sigma_{p_x p_x} & \Sigma_{p_x z} & \Sigma_{p_x y} \\ \Sigma_{z p_x} & \Sigma_{zz} & \Sigma_{zy} \\ \Sigma_{y p_x} & \Sigma_{yz} & \Sigma_{yy} \end{bmatrix}$$

The properties of the production possibilities set and the twice differentiability of $\mathrm{CR}(\cdot)$ with respect to variable input prices imply that the matrix $\Sigma_{p_x p_x}$ is negative semi-definite. Furthermore, if the restricted cost function is twice differentiable with respect to quasi-fixed factors and with respect to output, the function $\mathrm{CR}(\cdot)$ is locally strictly convex with respect to z and y (Jorgenson and Lau, 1974) and the submatrix:

$$\begin{bmatrix} \Sigma_{zz} & \Sigma_{yz} \\ \Sigma_{yz} & \Sigma_{yy} \end{bmatrix}$$

is then positive definite.

[1]The short-run Hicksian technology, at level z_0, can be defined by: $Y^0 = [(x, y);\ (x, y, z_0) \in Y]$.

2. Inferring different theoretical equilibria from the restricted cost function

The restricted (or variable) cost function, corresponding to the programm of minimizing the cost of some subset of the inputs subject to the choice of the remaining inputs, provides a functional characterization of the technology at short-run Hicksian equilibrium. In particular, it is possible to calculate the short-run Hicksian price elasticities of demand, for a variable factor:

$$\bar{\epsilon}^{SR}_{nn'} = \partial \log \bar{x}_n / \partial \log p_{x_{n'}} |_{p_{xi}, z, y} \qquad i \neq n'$$

In long-run Hicksian equilibrium, the total cost function may be written as:

$$\begin{aligned} \mathrm{CT}(p_x, p_z, y) &= \underset{x,\, z}{\mathrm{Min}} \left(p'_x x + p'_z z;\, y = \mathrm{f}(x, z) \right) \\ &= \underset{z}{\mathrm{Min}} \left(\mathrm{CR}(p_x, z, y) + p'_z z;\, y = \mathrm{f}(x, z) \right) \\ &= \mathrm{CR}(p_x, \bar{z}^{\mathrm{h}}(p_x, p_z, y), y) + p'_z \bar{z}^{\mathrm{h}}(\cdot) \end{aligned}$$

The optimal long-run Hicksian level of the input z_m, $m = 1, ..., M$, is defined as $\bar{z}^h_m(\mathrm{p}_x, p_z, y)$. The total cost function is obtained by first minimizing the restricted cost function conditional upon the level of z; and then minimizing total costs with respect to z, holding the variable inputs at their short-run Hicksian levels $\bar{x}_n(p_x, z, y)$. A sufficient condition to use this two-step decision rule is that the restricted cost function CR is strictly convex in z (in a domain which includes the observed and optimal long-run Hicksian levels of the inputs z_m); that is, the matrix Σ_{zz} is positive definite in this domain. So, it is sufficient to add to previous properties of the restricted cost function the assumption of strict convexity of CR$(\cdot)$ in z_m, $m = 1, ..., M$, if we want to infer the long-run Hicksian characteristics of the technology from the knowledge of the only restricted cost function CR.[2] For example, the long-run output-constant price elasticities of demand, or long-run Hicksian price elasticities, can be derived

[2]By direct application of the relationships between the Hessian matrices of CR and CT at the long-run Hicksian equilibrium point, it follows (Guyomard and Vermersch, 1987):

$$\partial^2 \mathrm{CR}(\cdot)/\partial \bar{z}^{\mathrm{h}2} = -[\partial \bar{z}^{\mathrm{h}}/\partial p_z]^{-1} = -[\partial^2 \mathrm{CT}/\partial p_z^2]^{-1}$$

The concavity of the total cost function in all the inputs prices implies that the matrix $[\partial^2 \mathrm{CT}(\cdot)/\partial p_z^2]$ is negative semi-definite. So a sufficient condition to infer the long-run Hicksian equilibrium from the restricted cost function is that the matrix $[\partial^2 \mathrm{CR}(\cdot)/\partial \bar{z}^{\mathrm{h}2}]$ is positive definite, and so that the restricted cost function is strictly convex in $\bar{z}^{\mathrm{h}}$.

from the restricted cost function and are conditional upon the optimal quantity $\bar{z}^{\mathrm{h}}(p_x, p_z, y)$. These elasticities include both pure substitution and expansion effects, involved by quasi-fixed factor variations:

$$\bar{\epsilon}^{\mathrm{LR}}_{nn'} = \partial \log \bar{x}^{\mathrm{h}}_n / \partial \log p_{xn'} \,|_{p_{xi}, p_z, y} \quad i \neq n'$$

$$= \partial \log \bar{x}_n(\bar{z}^{\mathrm{h}}, y) / \partial \log p_{xn'} \,|_{p_{xi}, z, y} \quad \Big\} \text{(A)} \qquad \text{(7a)}$$

$$+ \sum_{m=1}^{M} \partial \log \bar{x}_n(\bar{z}^{\mathrm{h}}, y) / \partial \log z_m \times \partial \log \bar{z}^{\mathrm{h}}_m / \partial \log p_{xn'} \,|_{p_{xi}, p_z, y} \Big\} \text{(B)}$$

The generalization of this decomposition work is possible for all theoretical possible equilibria: short-run and long-run Hicksian equilibria, short-run and long-run Marshallian equilibria, and is summarized in Table 1. The long-run Marshallian situation, not necessarily achieved by the observed technology, can be approached by three equivalent ways: either by cost minimization with respect to z given y and then by profit maximization with respect to y; or by profit maximization with respect to y given z and then by cost minimization with respect to z, or by profit maximization with respect to z and to y. In order to infer the different possible equilibria from the short-run Hicksian equilibrium, the restricted cost function must verify the following sufficient curvature restrictions, derived from economic theory[3]:

- $\Sigma_{zz}(\bar{z}^{\mathrm{h}})$ must be positive definite to infer the long-run Hicksian demand equations from the short-run Hicksian demand equations.
- $\Sigma_{yy}(y^{\circ})$ must be positive definite to infer the short-run Marshallian demand equations from the short-run Hicksian demand equations.
- The submatrix:

$$\begin{bmatrix} \Sigma_{zz}(z^{\mathrm{m}}, y^{\mathrm{m}}) & \Sigma_{zy}(z^{\mathrm{m}}, y^{\mathrm{m}}) \\ \Sigma_{yz}(z^{\mathrm{m}}, y^{\mathrm{m}}) & \Sigma_{yy}(y^{\mathrm{m}}, y^{\mathrm{m}}) \end{bmatrix}$$

must be positive definite to infer the long-run Marshallian demand equations from the short-run Hicksian demand equations.

For example, under the assumption of strict convexity of the restricted cost function $\text{CR}(p_x, y, z)$ with respect to z and y, the long-run Marshallian demand function of factor x_n may be written as:

$x^{\mathrm{m}}_n(p_x, p_z, p_y)$

[3]These sufficient conditions can be clearly proved by using the relationships which exist between the Hessian matrices of CR, CT, πR and πT at the respective optimum levels and by using a analogous method as noted in footnote 2.

TABLE 1

Characterization of the different theoretical equilibria from the knowledge of the only restricted cost function $\text{CR}(p_x, z, y)$

Minimization of the restricted cost function
$\text{CR}(p_x, z, y)$ (1)
SHORT-RUN HICKSIAN EQUILIBRIUM (SRHE)

Sufficient conditions:
strict convexity of CR in z
$\text{Min}_z\, [\text{CR}(\cdot) + p_z, z]$ (2)
Minimization of the total cost function:

$\text{CT}(p_x, p_z, y)$

LONG-RUN HICKSIAN EQUILIBRIUM (LRHE)

Sufficient condition:
strict convexity of CR in y
(strict convexity of CT in y)
$\text{Max}_y\, [py - \text{CR}(\bar{z}^h) - p_z, \bar{z}^h]$ (4)
$= \text{Max}_y\, [py - \text{CT}]$ (4′)

Sufficient condition of strict convexity of CR in z and in y:
$\text{Max}_{z,y}\, [py - \text{CR}(\cdot) - p_z, z]$

Sufficient condition:
strict convexity of CR in y
$\text{Max}_y\, [py - \text{CR}(\cdot)]$ (3)
Maximization of the restricted profit function:

$\pi R(p_x, z, p_y)$

SHORT-RUN MARSHALLIAN EQUILIBRIUM (SRME)

Sufficient condition:
strict convexity of CR in z
(strict concavity of πR in z)
$\text{Min}_z\, [\text{CR}(y^0) + p_z, z]$ (5)
$= \text{Max}_z\, [py^0(\cdot) - \text{CR}(y^0(\cdot)) - p_z, z]$ (5′)

Maximization of the total profit function
$\pi T(p_x, p_z, p_y)$
LONG-RUN MARSHALLIAN EQUILIBRIUM (LRME)

$$=\bar{x}_n(p_x, \bar{z}^{\mathrm{h}}(p_x, p_z, y^{\mathrm{m}}(p_x, p_z, p_y)), y^{\mathrm{m}}(p_x, p_z, p_y)) \quad (4)$$

$$=\bar{x}_n(p_z, z^{\mathrm{m}}(p_x, p_z, p_y), y^{0}(p_x, z^{\mathrm{m}}(p_x, p_z, p_y), p_y)) \quad (5)$$

$$=\bar{x}_n(p_x, z^{\mathrm{m}}(p_x, p_z, p_y), y^{\mathrm{m}}(p_x, p_z, p_y)) \quad (6)$$

Before proceeding to the empirical section, it is useful to express (4), (5) and (6) in terms of price elasticities of demand. For example, using the decomposition (4), the long-run Marshallian elasticity matrix may be written as:

$$\begin{aligned}[\bar{\epsilon}^{\mathrm{LR}}_{nn'}] = \; & [\bar{\epsilon}^{\mathrm{SR}}_{nn'}] \;\} \text{ (A)} \\ & +[x_{nm}]\,[z_{mn'}] \;\} \text{ (B)} \\ & \left.\begin{aligned} & +[x_{nm}]\,[z_{m1}]\,[y_{1n'}] \\ & +[x_{n1'}]\,[y_{1n'}] \end{aligned}\right\} \text{ (C)}\end{aligned} \quad (7b)$$

where

$$[x_{nn}] = [\partial \log \bar{x}_n(\bar{z}^{\mathrm{h}}(y^{\mathrm{m}}), y^{\mathrm{m}})/\partial \log z_m] \quad (N \times M)$$

$$[z_{mn'}] = [\partial \log \bar{z}^{\mathrm{h}}_m(y^{\mathrm{m}})/\partial \log p_{x_{n'}}] \quad (M \times N)$$

$$[z_{m1}] = [\partial \log \bar{z}^{\mathrm{h}}_m(y^{\mathrm{m}})/\partial \log y] \quad (M \times 1)$$

$$[y_{1n}] = [\partial \log y^{\mathrm{m}}/\partial \log p_{x_{n'}}] \quad (1 \times N)$$

$$[x_{n1}] = [\partial \log \bar{x}_n(\bar{z}^{\mathrm{h}}(y^{\mathrm{m}}), y^{\mathrm{m}})/\partial \log y] \quad (N \times 1)$$

Therefore, the total change in x^{m}_n, in response to a change in $p_{x_{n'}}$, may be decomposed into three effects:

- a pure substitution effect along the old isoquant: (A)
- an expansion effect due to the quasi-fixed factor variation until the Hicksian optimal level: (B)
- an expansion effect due to the change in the output level along the new expansion path associated with inputs prices: (C).

3. Econometric model

In this section, we describe the econometric model used to characterize the structure of the French cereal sector. For econometric estimation, a translog specification is chosen; this function, which imposes the least restrictions on the technology, must be viewed as a local second-order approximation to the true restricted cost function. One exogenous output, four variable inputs (fuel and oil, fertilizers, capital, hired labor) and two quasi-fixed factors (family labor, land) are considered; the data concern a cross-section of farms and are detailed in the following section[4]:

[4]A full quadratic expansion of the restricted cost function around the approximation point would include quadratic terms for y and z_h, and y and p_i. However, the likelihood ratio test for the hypothesis saying that the corresponding coefficients equal zero is not rejected at conventional level (5%).

$$\ln \text{CR}\,(y, p_i, z_h) = a_0 + a_1(\ln y) + \tfrac{1}{2}a_2(\ln y)^2 + \sum_{i=1}^{4} c_i(\ln p_i) + \tfrac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4} d_{ij}(\ln p_i)(\ln p_j) + \sum_{h=1}^{2} f_h(\ln z_h) + \tfrac{1}{2}\sum_{h=1}^{2}\sum_{k=1}^{2} g_{hk}(\ln z_h)(\ln z_k) + \sum_{i=1}^{4}\sum_{k=1}^{2} k_{ih}(\ln p_i)(\ln z_h) + \epsilon_{\text{CR}} \quad (8)$$

where CR is restricted cost; y is output; p_i are variable input prices, as for fuel and oil $(i=1)$, fertilizer $(i=2)$, capital $(i=3)$, and hired labor $(i=4)$; z_h are the quasi-fixed inputs: family labor $(h=1)$, and land $(h=2)$.

Without loss of generality, symmetry is imposed on the parameters d_{ij} and g_{hk}. Shephard's lemma gives the cost-share equations, on which we add a disturbance term, ϵ_i, to reflect errors in optimization:

$$M_i = p_i x_i / \text{CR} = c_i + \sum_{j=1}^{4} d_{ij}(\ln p_j) + \sum_{h=1}^{2} k_{ih}(\ln z_h) + \epsilon_i \qquad i=1,2,3,4 \quad (9)$$

By construction:

$$\sum_{i=1}^{4} M_i = 1$$

This additivity constraint implies:

$$\sum_{i=1}^{4} c_i = 1, \qquad \sum_{i=1}^{4} d_{ij} = 0 \quad \forall\, j; \qquad \sum_{i=1}^{4} k_{ih} = 0 \quad \forall\, h$$

Symmetry and additivity constraints ensure the theoretical restriction of homogeneity of degree one in input prices.

The set of equations (8) and (9) will be used to estimate the parameters of CR, from which the short-run Hicksian price elasticities of demand will be derived:

$$\bar{\epsilon}_{ij}^{\text{SR}} = (d_{ij} + M_i M_j)/M_i \qquad \forall\, i,\ \forall\, j,\ j \neq i$$
$$\bar{\epsilon}_{ii}^{\text{SR}} = (d_{ii} + M_i^2 - M_i)/M_i$$

By solving equations (10) with respect to the quasi-fixed factors, we can derive the optimal long-run Hicksian levels $\bar{z}_m^{\text{h}}$ of these quasi-fixed inputs.

$$\partial\, \text{CR}(p_x, \bar{z}^{\text{h}}, y)/\partial z_m + p_{z_m} = 0 \quad m=1, \ldots, M \quad (10)$$

We can then use the analytical framework briefly presented in the previous section in order to estimate the long-run Hicksian price elasticities of demand, not only for a variable factor x_n, but also for a quasi-fixed input z_m. Lau (1976) first established the theoretical relationships between the Hessian of the restricted cost function CR(·) and the total cost function CT(·). Brown and Christensen (1981) proposed a practical procedure for deriving the matrix of long-run compensated elasticities using knowledge of only the restricted cost function. Nevertheless, in the most recent studies using a translog specification, only one quasi-fixed factor is considered (Brown and Christensen, 1981; Halvorsen et Smith, 1986; Squires, 1987; Guyomard, 1989). For the specific case with two quasi-fixed inputs, we obtain the following expressions for the long-run Hicksian price elasticities[5]:

$$\bar{\epsilon}_{ij}^{\mathrm{LR}} = \bar{\epsilon}_{ij}^{\mathrm{SR}} - (1/M_i)\,(A(i,j)/B(h,k)) \qquad \text{(a)}$$

$$\bar{\epsilon}_{ih}^{\mathrm{LR}} = (M_h/M_i)\,(C(i,h,k)/B(h,k)) \qquad \text{(b)}$$

$$\bar{\epsilon}_{hi}^{\mathrm{LR}} = -C(i,h,k)/B(h,k) \qquad \text{(c)}$$

$$\bar{\epsilon}_{hk}^{\mathrm{LR}} = -M_k(g_{kh} + M_h M_k)/B(h,k) \qquad \text{(d)}$$

$$\bar{\epsilon}_{hh}^{\mathrm{LR}} = M_h(g_{kk} + M_k^2 - M_k)/B(h,k) \qquad \text{(e)}$$

with

$$M_h = -p_h \bar{z}_h^{\mathrm{h}} / \mathrm{CR}(y, p_{\mathrm{x}}, \bar{z}^{\mathrm{h}})$$

$$M_k = -p_k \bar{z}_k^{\mathrm{h}} / \mathrm{CR}(y, p_{\mathrm{x}}, \bar{z}^{\mathrm{h}})$$

$$\begin{aligned} A(i,j) = (M_i M_k + k_{ik})\,[&(g_{hh} + M_h^2 - M_h)(M_j M_k + k_{jk}) \\ &- (g_{hk} + M_k M_h)(M_j M_h + k_{jh})] \\ + (M_i M_h + k_{ih})\,[&(g_{kk} + M_k^2 - M_k)(M_j M_h + k_{jh}) \\ &- (g_{hk} + M_k M_h)(M_j M_k + k_{jk})] \end{aligned}$$

[5]For instance, using the approach in terms of Hessian identities developed by Lau (1976), one finds:

$$\begin{aligned} [\partial^2 \mathrm{CT}(\cdot)/\partial p_{x_i}\,\partial p_{x_i}] = {} & [\partial^2\, \mathrm{CR}(\cdot)/\partial p_{x_i}\,\partial p_{x_i}] \\ & - [\partial^2\, \mathrm{CR}(\cdot)/\partial p_{x_i}\,\partial z]\,[\partial^2 \mathrm{CR}(\cdot)/\partial z\,\partial z]^{-1}[\partial^2\, \mathrm{CR}(\cdot)/\partial z\,\partial p_{x_i}] \end{aligned}$$

The above expression, calculated at the optimal levels of the quasi-fixed factors, can be written in terms of elasticities matrices. In the particular case of a translog specification with two quasi-fixed factors we obtain the following equation:

$$\bar{\epsilon}_{ii}^{\mathrm{LR}} = \bar{\epsilon}_{ii}^{\mathrm{SR}} - \left(\frac{1}{M_i}\right)\left[\frac{A(i,i)}{B(h,k)}\right]$$

Note that the generalization of the previous derivation to the case of M ($M \geq 3$) quasi-fixed factors is not straightforward. It is also theoretically possible to compute the short-run and long-run Marshallian price elasticities of demand (and of supply) using the parameters of the only restricted cost function (Guyomard, 1988). Unfortunately the function CR(·) is not convex in y and so this calculation is not correct.

$$B(h,k) \;= (g_{kk}+M_k^2-M_k)(g_{hh}+M_h^2-M_h)-(g_{hk}+M_hM_k)^2$$

$$C(i,h,k) = (g_{kk}+M_k^2-M_k)(M_iM_h+k_{ih})-(g_{hk}+M_hM_k)(M_iM_k+k_{ik})$$

where i and j refer to a variable factor, and h and k to a quasi-fixed factor.

Equations (a) to (e) provide expressions for calculating long-run Hicksian price elasticities in the case of a translog approximation of the restricted cost function with two quasi-fixed inputs. For instance, $\bar{\epsilon}_{ih}^{\mathrm{LR}}$ is the long-run Hicksian price elasticity of the variable factor x_i with respect to the price of the quasi-fixed factor z_h. $\bar{\epsilon}_{ii}^{\mathrm{LR}}$ is the own compensated long-run price elasticity of demand for the factor x_i; in this case, the second term of equation (a) is negative by convexity of the restricted cost function in z. Furthermore, if the firm operates at long-run Hicksian equilibrium, the observed and optimal levels of the quasi-fixed factor are the same; the short-run Hicksian own-price elasticity $\bar{\epsilon}_{ii}^{\mathrm{SR}}$ is then smaller, in absolute value, than the long-run Hicksian price elasticity $\bar{\epsilon}_{ii}^{\mathrm{LR}}$, consistent with the Le Chatelier's principle. Outside the long-run Hicksian equilibrium, the optimal level of the quasi-fixed factor is not equal to the actual level; as a consequence, the two price elasticities $\bar{\epsilon}_{ii}^{\mathrm{SR}}$ and $\bar{\epsilon}_{ii}^{\mathrm{LR}}$ can not be theoretically compared in the view of the Le Chatelier's principle because the points of derivation are different.

Outside the long-run Hicksian optimum, the shadow price (or dual price) for each quasi-fixed factor can be evaluated as:

$$p_{z_m}^* = -[\partial\,\mathrm{CR}(\cdot)/\partial z_m] \tag{11}$$

If the ex-post shadow price and the ex-ante observed price of a quasi-fixed factor are equal, then the actual level z_m corresponds to the long-run Hicksian levels $\bar{z}_m^{\mathrm{h}}$. Moreover, the concept of shadow price allows us to characterize, in a dual and simple way, the disequilibrium induced by fixity (Kulatilaka, 1985). Indeed, assuming that the restricted cost function is strictly convex in z_m, $m=1, ..., M$; it is easy to show (in the case of one quasi-fixed factor):

$$\begin{bmatrix} p_{z_m}^* \le p_{z_m} \Leftrightarrow \bar{z}_m^{\mathrm{h}} \le z_m \\ p_{z_m}^* \ge p_{z_m} \Leftrightarrow \bar{z}_m^{\mathrm{h}} \ge z_m \end{bmatrix.$$

Finally, a measure of short-run returns to scale can be defined as:

$$\mathrm{ECH}^{\mathrm{SR}} = [\partial \ln \mathrm{CR}/\partial \ln y]^{-1}\Big|_{p_x, z} \tag{12}$$

Caves et al. (1981), followed by Halvorsen and Smith (1986), suggest a different measure which takes into account not only changes of the variable inputs but also changes of the quasi-fixed inputs:

$$\text{ECH}^0 = [1 - \sum_{m=1}^{M} (\partial \ln \text{CR}/\partial \ln z_m)]/[\partial \ln \text{CR}/\partial \ln y]\Big|_{px} \tag{13}$$

This measure does not represent returns to scale along the expansion path where total cost is minimized at every level of output and generally differs from a long-run measure as:

$$\text{ECH}^{\text{LR}} = [\partial \ln \text{CT}/\partial \ln y]^{-1}\Big|_{px,pz} \tag{14}$$

However, ECH^0 et ECH^{LR} give equal measures of returns to scale at the long-run Hicksian equilibrium (Guyomard and Vermersch, 1987). Consequently, considering the restricted cost function CR and the optimal long-run Hicksian levels $\bar{z}^{\text{h}}$, it is possible to infer:

$$\text{ECH}^{\text{LR}} = [1 - \sum_{m=1}^{M} (\partial \ln \text{CR}(\bar{z}^{\text{h}})/\partial \ln z_m)]/[\partial \ln \text{CR}(\bar{z}^{\text{h}})/\partial \ln y]\Big|_{px} \tag{15}$$

4. Data and empirical results

The model is estimated using data from a sample of farm accounts ($n=208$) related to the year 1981; for each farm, cereal production represents at least 80% of the total production. Four variable inputs and two quasi-fixed factors are taken into account: the model is specified with prices for fuel and oil, fertilizers, capital and hired labor; acreage under cultivation and family labor are included as fixed inputs. For the capital input, it is assumed that the service flow from the stock of capital is proportional to this stock and, as in Dormont and Sevestre (1986), the user cost of capital is the only the apparent interest rate. The level of output is measured by cash sales and variable shares are the values of these inputs divided by the restricted cost.

The restricted cost function is jointly estimated with the cost-share equations which sum to unity; so, one of them is dropped. The system of equations is estimated by the maximum-likelihood method whose corresponding estimates are invariant to choice of deleted equation (Barten, 1969).

The parameter estimates for the final form of the model are shown in Table 2 together with their estimated standard errors. For each point, the estimated-cost shares are positive and the concavity of the restricted cost function in input prices is verified at the sample average. If z_h and z_k represent, respec-

TABLE 2

Parameter estimates

Parameter	Estimate	Standard error
a_0	12.146	0.035
a_1	0.379	0.121
a_2	0.055	0.117
c_1	0.109	0.004
c_2	0.306	0.011
c_3	0.139	0.013
d_{11}	0.027	0.015
d_{12}	− 0.026	0.007
d_{13}	− 0.022	0.011
d_{22}	0.107	0.016
d_{23}	0.002	0.018
d_{33}	0.101	0.033
f_1	− 0.137	0.086
f_2	0.730	0.144
g_{11}	− 0.080	0.136
g_{12}	0.011	0.204
g_{22}	− 0.228	0.311
k_{11}	0.004	0.100
k_{12}	− 0.002	0.009
k_{21}	0.035	0.029
k_{22}	0.034	0.025
k_{31}	− 0.095	0.033
k_{32}	0.009	0.030

tively, the levels of family labor and land, at the sample average, the fitted cost function is convex in z_h and concave in z_k; the wrong sign of parameter f_k is probably derived from multicollinearity between z_k and y, the level of output. However, f_k is reestimated by a production function model relating y (the level of output) to z_k (the level of the quasi-fixed factor), in order to solve the optimal level $\bar{z}_k^h$; in this case, the multicollinearity problems are replaced by simultaneity problems.

Estimates of short-run Hicksian price elasticities of demand, evaluated at sample average, are shown in Table 3 with their estimated t ratios. Except for hired labor, all the estimated own-price elasticities are significant at the 5% level. Demands for energy and capital are the most price-responsive; significant short-run net substitution possibilities between capital and hired labor, capital and fertilizers are found. At the sample average, $\mathrm{ECH}^{\mathrm{SR}}=2.63$; this figure may be viewed as too high but multicollinearity problems or omitted factors can distort this result. Nevertheless, the result is consistent with other measures of returns to scale based on a total cost function where all inputs are

TABLE 3

Short-run Hicksian price elasticities of demand, evaluated at sample average, z_h and z_k; h, family labor; k, land

Quantity	Price: Fuel and oil	Capital	Hired labor	Fertilizers
Fuel and oil	− 0.638	0.075	− 0.065	0.628
	(4.63)	(1.24)	(0.62)	(4.53)
Capital	0.026	− 0.344	0.145	0.173
	(1.24)	(6.68)	(3.91)	(2.98)
Hired labor	− 0.051	0.321	− 0.136	− 0.135
	(0.62)	(3.91)	(0.56)	(0.6)
Fertilizers	0.153	0.118	− 0.042	− 0.23
	(4.53)	(2.98)	(0.6)	(2.65)

TABLE 4

Long-run Hicksian price elasticities of demand, evaluated at sample average, z_h and z_k; h, family labor; k, land; assuming that the quasi-fixed factors are optimal

Quantity	Price: Fuel and oil	Capital	Hired labor	Fertilizers	Family labor	Land
Fuel and oil	−0.706	−0.047	−0.16	0.307	0.119	0.492
Capital	−0.0168	−0.425	0.065	−0.022	0.130	0.268
Hired labor	−0.129	0.144	−0.496	−0.337	0.961	−0.143
Fertilizers	0.075	−0.015	−0.105	−0.634	−0.039	0.718
Family labor	0.034	0.106	0.355	−0.047	−1.09	0.64
Land	0.170	0.262	−0.063	1.025	0.763	−2.16

variable (Boutitie and al., 1987); both studies show that there are scale economies.

Assuming that observed levels of quasi-fixed factors are long-run Hicksian levels, estimates of long-run price elasticities of demand are shown in Table 4; own-price elasticities are greater, in absolute value, than own-price elasticities calculated in Table 3 and so are consistent with the Le Chatelier's principle. All long-run Hicksian own price elasticities have the expected algebraic sign; all factors have inelastic long-run compensated demand, except for family labor and land. Long-run net substitution possibilities appear between family labor and hired labor, family labor and capital, land and energy, capital or fertilizers. Nevertheless, the substitution relationship between the two types of labor may be interpreted with caution because the departures between z and $\bar{z}^{h}$ are important; the assumption of a full static equilibrium in which all inputs

TABLE 5

Long-run Hicksian price elasticities of demand, evaluated at sample average, z_h and $\bar{z}_k^h$; h, family labor; k, land

Quantity	Price				
	Fuel and oil	Capital	Hired labor	Fertilizers	Land
Fuel and oil	−0.734	−0.192	−0.205	0.228	0.89
Capital	−0.07	−0.606	0.012	−0.22	0.87
Hired labor	−0.156	0.030	−0.283	−0.575	0.966
Fertilizers	0.06	−0.15	−0.18	−0.63	0.895
land	0.013	0.036	0.018	0.054	−0.119

TABLE 6

Long-run Hicksian price elasticities of demand, evaluated at sample average, $\bar{z}_h^h$ et $\bar{z}_k^h$; h, family labor; k, land

Quantity	Price					
	Fuel and oil	Capital	Hired labor	Fertilizers	Family labor	Land
Fuel and oil	−0.734	−0.175	−0.247	0.256	0.013	0.856
Capital	−0.06	−0.545	−0.16	−0.119	0.04	0.859
Hired labor	−0.193	−0.35	0.845	−1.18	−0.244	1.10
Fertilizers	0.06	−0.082	−0.368	−0.521	0.044	0.86
Family labor	0.066	0.587	−1.61	0.94	0.36	−0.34
Land	0.011	0.031	0.018	0.045	-8.4×10^{-4}	−0.105

fully adjust to their optimal level seems restrictive and inappropriate for the French cereal technology.

Table 5 shows the long-run Hicksian price elasticities in the case where only fixity of land is relaxed; the previous relations of substitutability with land also appear. The long-run Hicksian price elasticities of demand in the case where the two constraints upon family labor and land can be relaxed are shown in Table 6. These two last long-run Hicksian price elasticities (Tables 5 and 6) cannot be compared, in the view of the Le Chatelier's principle, with the previous short-run Hicksian price elasticities, because, in each case, the point of approximation is different:

$$(z_h, \bar{z}_k^h(z_h)), \quad (\bar{z}_h^h, \bar{z}_k^h), \quad (z_h, z_k), \quad \text{respectively}$$

The results presented in Table 6 show that own long-run Hicksian price elasticities for hired labor and family labor are positive; these wrong signs indicate that the long-run technology defined by minimizing total costs is not well behaved, although the restricted cost function is well behaved not only with respect to variable inputs but also with respect to quasi-fixed factors. Note that the optimal level of family labor is smaller than the observed level indicating a surplus of self-employed labor. On contrary, the optimal level of land is greater than the actual level. These results are consistent with our previous findings based on macrodata for French agriculture from 1960 to 1984 (Guyomard, 1988, 1989).

In order to evaluate the importance of taking into account the quasi-fixity of some inputs, we also estimated a translog total cost function CT4 with only four inputs: fuel and oil, capital, hired labor, and fertilizers. It is interesting to note the relationships between the elasticities (short-run and long-run Hicksian elasticities) derived from the restricted cost function, and those would be provided by a cost function including only the variable inputs. If the $[(x_i), (z_h)]$ production function weak separability is valid[6], the elasticities estimated with this last model would reflect net substitution elasticities, i.e. along a four-input isoquant. Therefore, they would be upward-biased estimates of net long-run elasticities calculated along a six-input isoquant. Following Berndt and Wood (1979), it is possible to evaluate this upward bias using the following equality:

$$\bar{\epsilon}^{LR}_{nn'} = \partial \log \bar{x}^h_n / \partial \log p_{xn'} \,|_y$$
$$= \partial \log \bar{x}^h_n / \partial \log p_{xn'} \,|_v$$
$$+ [(\partial \log \bar{x}^h_n / \partial \log v)(\partial \log v / \partial \log p_v)(\partial \log p_v / \partial \log p_{xn'})\,|_y]$$

where v (p_v) is the output (the price) of the weakly separable subfunction (x_i); $i=1, ..., 4$.

Since the term in brackets is negative, $\bar{\epsilon}^{LR}_{nn'}$ is always inferior to $\bar{\epsilon}^{LR(CT4)}_{nn'} = \partial \log \bar{x}^h_n / \partial \log p_{xn'} \,|_v$. Empirically, this inequality is rejected by the data for only two out of the sixteen cases (Tables 6 and 7).

TABLE 7

Long-run Hicksian price elasticites of demand, evaluated at sample average with a four-input total cost

	Price			
Quantity	Fuel and oil	Capital	Hired labor	Fertilizers
Fuel and oil	−0.65	0.18	−0.12	0.60
Capital	0.07	−0.61	0.34	0.20
Hired labor	−0.10	0.71	0.0085	−0.63
Fertilizers	0.14	0.13	−0.19	−0.08

[6]The hypothesis that the restricted cost function is logarithmically separable into a function of variable inputs and quasi-fixed inputs is not rejected at a 5% level (test based on the likehood ratio method).

It is not possible to compare, analytically and theoretically, the short-run Hicksian elasticity of demand derived from the six-input restricted cost function and the long-run Hicksian elasticity of demand calculated with the four-input total cost function. This mainly comes from the fact that, in both cases, the point of approximation is different and the second term (B) in equation (7a) may be either positive or negative. Note that some variable inputs (for example seeds) are not included in the model because no measure of unit price could be computed from the available data. As a result, even if these inputs are weakly separable from the others, all the elasticity estimates would be upward-biased for the same reason as before.

Finally, the long-run measures of returns to scale give:

$$\mathrm{ECH}^{\mathrm{LR}}(z_h, \bar{z}_k^{\mathrm{h}}(z_h)) = 3.69$$

$$\mathrm{ECH}^{\mathrm{LR}}(\bar{z}_h^{\mathrm{h}}, \bar{z}_k^{\mathrm{k}}) = 4.05$$

where h corresponds to family labor, and k to land.

So, at the sample average, there exist economies of scale, whatever measure is utilized.

Concluding remarks

The use of a restricted cost function permits the estimation of the characteristics of French cereal technology in a framework of short-run static Hicksian equilibrium. Significant short-run net substitution possibilities between capital and hired labor, capital and fertilizers are found. Moreover, at the sample average, technology exhibits positive economies of scale.

The use of a restricted cost function allows also the characterization of the other theoretical possible equilibria (long-run Hicksian equilibrium, short-run and long-run Marshallian equilibria), under the sufficient assumptions of strict convexity of the function CR in $\bar{z}^{\mathrm{h}}(y)$ (long-run Hicksian demand of the quasi-fixed inputs), in $y^0(z)$ (short-run Marshallian supply of the output), and in z^{m} and y^{m} (long-run Marshallian demand of the quasi-fixed inputs and long-run Marshallian supply of the output).

For all practical purposes, the data used in this study make it possible to infer the characteristics concerning only the long-run Hicksian equilibrium. The fixedness hypothesis of family labor and land can be relaxed and some insights on long-run Hicksian price elasticities of demand are derived; input substitutability is still valid in the long run. Nevertheless, all the results must be viewed with caution due to poor significance of certain parameters, wrong sign of some coefficients, or violation of economic theory. Especially, further research seems necessary in order to impose curvature restrictions so that theoretical properties of the short-run or long-run technologies are not violated.

The restricted cost function can also be used to evaluate other characteristics of the short-run Hicksian technology. It is especially efficient to measure the capacity utilization that has an explicit economic theoretical foundation, and to calculate the capacity output elasticities with respect to input prices. The methodology used in this paper and the theoretical works of Morrison (1985, 1986) offer a fruitful approach for further research in this area.

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