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# A Linear Programming Algorithm for Determining Mean-Gini Efficient Farm Plans

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## Abstract

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A linear programming algorithm is presented for determination of farm plans that are efficient under the mean-Gini criterion. Use of the algorithm is illustrated by applying it to a standard problem from the literature. The set of efficient plans is compared with those based on mean-variance and MOTAD analysis of the same problem.

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## Introduction

Given full knowledge of a decision maker's utility function and pertinent information on the alternative choices he faces, his utility-maximizing choice can be specified unequivocally. Generally, however, full knowledge of the decision maker's utility function is lacking. The best that can then be done is to determine the efficient (i.e., undominated) set of alternatives for some assumed decision rule reflecting characteristics – such as risk aversion – thought to be held by the decision maker's unknown utility function. The decision maker can then be told that, under the assumptions made, his utility-maximizing choice must be one of the alternatives in the efficient set and that choice of an alternative not in the efficient set would be irrational. To this end, a number of approaches have been proposed to determine the efficient set of alternatives among those facing a risk-averse utility-maximizing decision maker. Best established among these approaches are second-degree stochastic dominance (SSD), mean-variance (MV) analysis, and, as a linear approximation to MV analysis, the MOTAD (minimization of total absolute deviations) approach in which mean absolute deviation is used instead of variance as a measure of

risk (Hazell, 1971; Anderson et al., 1977; Hazell and Norton, 1986). Not so well accepted (and therefore not considered further here) have been other approaches analogous to MV analysis but using, for example, semi-variance (Markowitz, 1959), confidence limits (Baumol, 1963; Hazell and Norton, 1986) and entropy (Philippatos and Gressis, 1975) instead of variance as a measure of risk.

The MV approach (and likewise MOTAD as an approximation to it) may prove misleading if its requirement that either the distribution of returns be normal or the decision maker's utility function be quadratic is not met (Hanooh and Levy, 1969). In contrast, from a theoretical perspective, SSD is far less restrictive. It places no restrictions on the probability distribution of returns and takes account of all aspects of this distribution while assuming only that the decision maker's utility function exhibits risk aversion over the relevant range of returns. SSD is thus theoretically ideal for risk-averse decision makers. However, in the context of portfolio-type problems, SSD has significant disadvantages compared to MV or MOTAD. For such problems – as exemplified by the choice of resource allocation between alternative farm enterprises – no generally applicable algorithm is available for determination of the SSD-efficient set. A process of pairwise comparison of sampled portfolios is necessary. In consequence, it is generally unfeasible to specify fully the set of SSD portfolios and there is no surety as to how many SSD-efficient alternatives may not have been located (Anderson, 1975; Bey and Howe, 1984; Shalit and Yitzhaki, 1984). Also, unless particular assumptions are made about the decision maker's degree of risk aversion as suggested, for example, by Meyer (1977) and Cochran et al. (1985), SSD is not very discriminatory; it tends to imply large efficient sets from which the decision maker has to choose (Porter and Gaumnitz, 1972; Anderson, 1974).

More recently, Yitzhaki (1982) has suggested using the mean and Gini's mean difference as an alternative approach to specification of the efficient set of alternatives for risk-averse decision makers. This mean-Gini (MG) approach possesses most of the attractive features of the MV, MOTAD and SSD approaches without their disadvantages. Thus, like MV and MOTAD, the MG approach uses a two-parameter statistic to describe the probability distribution of returns, is computationally simple, gives full specification of the MG-efficient set, and has a simple geometric representation. However, unlike MV and MOTAD, the MG approach is not restricted to normal distributions or quadratic utility; like SSD, it is applicable for any probability distribution of returns and for any utility function exhibiting risk aversion over the relevant range of returns. These attractive features are complemented further by the fact that MG efficiency implies SSD efficiency, though the reverse is only true for families of cumulative probability distributions that intersect at most once, for example the normal, lognormal, exponential and uniform distributions (Yitzhaki, 1982; Shalit and Yitzhaki, 1984). Thus, while the MG approach

delimits only a subset of the SSD-efficient set, these alternatives are fully specified in contrast to the incomplete specification of the overall SSD set achievable via the sampling-based direct approach to SSD efficiency in portfolio-type problems. Choice from the MG-efficient set, however, is at the risk that the decision maker's truly optimal choice lies not in the MG-efficient set but among the unspecified (and perhaps unascertainable) SSD-efficient alternatives lying outside the MG set. The class of decision makers (as specified in terms of utility function characteristics) for whom the risk would be real has not as yet been precisely determined. However, algebraic analysis by Buccola and Subaei (1984, p. 81) suggests that the MG approach is appropriate for decision makers who are weakly risk averse. In their particular empirical application, Buccola and Subaei (1984, pp. 83-84) found the MG and SSD sets to be identical for decision makers with an absolute risk aversion coefficient of 0.0015 or less.

This note presents a linear programming procedure for determination of the MG-efficient set and, by way of application to an empirical example already well established in the literature, illustrates its use in a farm-planning context. Resultant plans are compared with those obtained using the MV and MOTAD approaches.

## MG efficiency

Following Yitzhaki (1982), alternative  $F$  is MG-efficient over alternative  $G$  if:

$$\mu_f \geq \mu_g \quad (1)$$

and

$$\mu_f - \Gamma_f \geq \mu_g - \Gamma_g \quad (2)$$

where, for the relevant distribution of returns,  $\mu$  denotes the mean and  $\Gamma$  denotes half of Gini's mean difference. The latter is the expected absolute difference between all possible pairs of observations of a random variable. Thus, if returns follow a continuous distribution over the range  $[a, b]$ :

$$\Gamma = \frac{1}{2} \int_a^b \int_a^b |x-y| f(x) f(y) dx dy \quad (3)$$

If returns are discrete with  $n$  possible values:

$$\Gamma = \sum_{i=1}^n \sum_{j>i}^n |x_i - x_j| p(x_i) p(x_j) \quad (4)$$

which, if the returns are equi-probable, reduces to:

$$\Gamma = \sum_{i=1}^n \sum_{j>i}^n |x_i - x_j| / n^2 \quad (5)$$

Condition (2) above for MG efficiency is analogous to comparison between alternatives on the basis of their certainty equivalents, the latter being defined for each alternative as  $\mu - \pi$  where  $\pi (> 0)$  is the (risk-averse) decision maker's subjectively determined risk premium. However, comparison on the basis of certainty equivalence does not require condition (1). Also, as pointed out by Yitzhaki (1982), the MG approach is somewhat analogous to Baumol's (1963) expected-gain confidence-limit (EL) criterion where  $E = \mu$  and  $L = \mu - \phi\sigma$  with  $\phi$  a positive constant and  $\sigma$  the standard deviation of returns. For  $\phi\sigma = \Gamma$ , the EL and MG criteria are the same. More generally, just as with the EL approach, the MG approach allows a prospect with greater risk (as measured by  $\Gamma$ ) to be preferred over another as long as its mean is sufficiently large.

### Generation of the MG-efficient set

The typical farm-planning problem consists of deciding on the allocation of available resources between possible activities (or, equivalently, of choosing the best feasible combination or portfolio of activity levels) for the coming production period. With information on the probability distribution of returns (usually specified as gross margins) for each activity, an efficient set of plans for consideration by the (assumed risk-averse) decision maker may be generated using SSD, MV, MOTAD or MG analysis. As outlined by Anderson et al. (1977) and Hazell and Norton (1986), procedures for SSD, MV and MOTAD in the context of farm planning are well established.

Construction of the MG-efficient set is facilitated by first determining the  $M\Gamma$ -efficient set defined by conditions (6) and (7):

$$\mu_f \geq \mu_g \quad (6)$$

$$\Gamma_f \leq \Gamma_g \quad (7)$$

Once the  $M\Gamma$ -efficient set has been determined, the MG-efficient set is easily determined by using the fact that the  $M\Gamma$ -efficiency conditions are sufficient for MG efficiency (Yitzhaki, 1982, p. 181). The procedure is to delete from the  $M\Gamma$ -efficient set all plans having the same  $\mu - \Gamma$  value but a lower  $\mu$  value than any other plan in the  $M\Gamma$  set. The plans remaining constitute the MG-efficient set. This can be shown schematically as per Fig. 1 where the  $M\Gamma$ -efficient set is represented by the curve ABC. By virtue of the geometry implied by the  $45^\circ$ -degree line tangential to ABC at B, plans such as those represented by points D and E lying the same horizontal distance ( $D'D = E'E$ ) from the  $45^\circ$ -degree line have the same  $\mu - \Gamma$  value. However, E has a higher  $\mu$  value than D. There-

fore, by condition (1), E dominates D in terms of MG efficiency. Thus the MG-efficient set consists of the segment BC of the  $M\Gamma$ -efficient set ABC.

Consider now the farm-planning problem. Suppose the farmer has information (perhaps subjectively derived) on the unit gross margin  $X_{ij}$  of each activity  $j=1, 2, \dots, m$  in the prior production period  $i=1, 2, \dots, n$ . Denoting the units of the  $j$ th activity by  $x_j$ , the total gross margin ( $T_i$ ) of any feasible farm plan in period  $i$  is given by:

$$T_i = \sum_{j=1}^m x_j X_{ij} \quad (8)$$

Assuming  $X_{1j}, X_{2j}, \dots, X_{nj}$  are equ-probable, the Gini parameter  $\Gamma$  of equation (5) for the plan generating  $T_i$  of equation (8) is given by:

$$\Gamma = \sum_{i=1}^n \sum_{k>1}^n |T_i - T_k| / n^2 \quad (9)$$

The  $M\Gamma$ -efficient set is obtained by minimizing  $\Gamma$  for any given level of  $\mu$  and subject to the relevant constraints. Thus the problem may be stated as minimizing  $\Gamma$  of equation (9) subject to:

$$\sum_{j=1}^m x_j \mu_j = \mu \quad (\mu=0 \text{ to unbounded}) \quad (10)$$

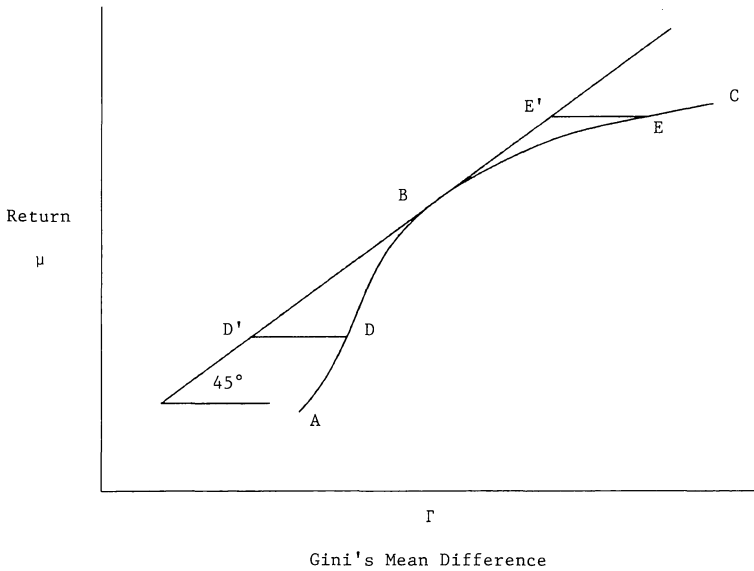


Fig. 1.  $M\Gamma$  (curve ABC) and MG (curve BC) efficient sets in  $[\Gamma, \mu]$  space (adapted from Yitzhaki, 1982, p. 183).

$$x_j \geq 0 \quad (\text{for all } j=1, 2, \dots, m) \quad (11)$$

and the resource constraint  $b_r$  for the  $r$ th resource:

$$\sum_{j=1}^m a_{rj}x_j \leq b_r \quad (\text{for all } r=1, 2, \dots, s) \quad (12)$$

Because of the absolute values in its objective function (9), the above problem cannot be solved directly by linear programming. However, using the approach applied by Hazell (1971) for MOTAD, it can be transformed to a linear programming problem as follows:

Define:

$$T_i - T_k = y_{ik}^+ - y_{ik}^- \quad (13)$$

for all  $i$  and  $k$  of equation (9) such that:

$$y_{ik}^+, y_{ik}^- \geq 0 \quad (14)$$

$T_i - T_k$  is thus unconstrained in sign. If  $y_{ik}^+$  and  $y_{ik}^-$  are then selected in some minimal way so that one of them is zero, it must result that:

$$|T_i - T_k| = y_{ik}^+ + y_{ik}^- \quad (15)$$

for all  $i$  and  $k$ . This can be done concurrently with seeking optimal  $x_j$  ( $j=1, 2, \dots, m$ ) via the following linear programming problem which is equivalent to minimizing  $n^2\Gamma$  subject to the relevant constraints:

Minimize:

$$\sum_{i=1}^n \sum_{k>i}^n (y_{ik}^+ + y_{ik}^-) \quad (16)$$

subject to

$$\sum_{j=1}^m x_j(X_{ij} - X_{kj}) - y_{ik}^+ + y_{ik}^- = 0 \quad (\text{for all } i=1, 2, \dots, n \text{ and } k>i) \quad (17)$$

$$\sum_{j=1}^m x_j\mu_j = \mu \quad (\mu=0 \text{ to unbounded}) \quad (18)$$

$$x_j \geq 0 \quad (\text{for all } j=1, 2, \dots, m) \quad (19)$$

and

$$\sum_{j=1}^m a_{rj}x_j \leq b_r \quad (\text{for all } r=1, 2, \dots, s) \quad (20)$$

Parametric solution of the above problem relative to  $\mu$  generates the  $M\Gamma$ -efficient set. Within this, the  $MG$ -efficient set can be delimited as discussed above in relation to Fig. 1.

Should  $X_{ij}$  not be equi-probable over  $i=1, 2, \dots, n$  but have probability  $p(X_{ij})$ , the constraint of equation (17) should be amended to:

$$\sum_{j=1}^m x_j (X_{ij} - X_{kj}) p(X_{ij}) p(X_{kj}) - y_{ik}^+ + y_{ik}^- = 0 \quad (21)$$

The algorithm of equations (16) to (20) involves  $n(n-1)/2 + s + 1$  constraints and  $n(n-1) + m$  variables. Insofar as, first, the computational efficiency of solution is primarily a function of the number of constraints and, second,  $n$  (the number of periods) and  $s$  (the number of resources) are usually such that  $n(n-1)/2 + s$  is much larger than  $m$  (the number of activities), a substantial gain in the efficiency of solution can usually be achieved by solving the problem via its dual.

The dual to the primal of equations (16) to (20) is given by:

Maximize:

$$- \sum_{r=1}^s b_r V_r + \mu(V^+ - V^-) \quad (22)$$

subject to

$$\sum_{i=1}^n \sum_{k>i}^n (X_{ij} - X_{kj}) W_{ik} + \mu_i(V^+ - V^-) - \sum_{r=1}^s a_{rj} V_r \leq \sum_{i=1}^n \sum_{k>i}^n (X_{ij} - X_{kj}) \quad (23)$$

(for all  $j=1, 2, \dots, m$ )

and

$$W_{ik} \leq 2 \quad (\text{for all } i=1, 2, \dots, n \text{ and } k>i) \quad (24)$$

TABLE 1

Activity gross margins per acre for example problem (US\$)

Year	$X_{i1}$	$X_{i2}$	$X_{i3}$	$X_{i4}$
1	292	-128	420	579
2	179	560	187	639
3	114	648	366	379
4	247	544	249	924
5	426	182	322	5
6	259	850	159	569
Mean	253	443	284	516

acre  $\approx 0.404686$  ha  $\approx 4047$  m<sup>2</sup>.





TABLE 2 (continued)

Row <sup>1</sup>	$y_{12}^-$	$y_{13}^-$	$y_{14}^-$	$y_{15}^-$	$y_{16}^-$	$y_{23}^-$	$y_{24}^-$	$y_{25}^-$	$y_{26}^-$	$y_{34}^-$	$y_{35}^-$	$y_{36}^-$	$y_{45}^-$	$y_{46}^-$	$y_{56}^-$	Constraint
$F$ (US\$)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	Minimize
$b_1$ (acres)																$\leq 200$
$b_2$ (h)																$\leq 10\,000$
$b_3$ (acres)																$\leq 0$
$d_{12}$ (US\$)	+1															$= 0$
$d_{13}$ (US\$)		+1														$= 0$
$d_{14}$ (US\$)			+1													$= 0$
$d_{15}$ (US\$)				+1												$= 0$
$d_{16}$ (US\$)					+1											$= 0$
$d_{23}$ (US\$)						+1										$= 0$
$d_{24}$ (US\$)							+1									$= 0$
$d_{25}$ (US\$)								+1								$= 0$
$d_{26}$ (US\$)									+1							$= 0$
$d_{34}$ (US\$)										+1						$= 0$
$d_{35}$ (US\$)											+1					$= 0$
$d_{36}$ (US\$)												+1				$= 0$
$d_{45}$ (US\$)													+1			$= 0$
$d_{46}$ (US\$)														+1		$= 0$
$d_{56}$ (US\$)															+1	$= 0$
$\mu$ (US\$)																$= \mu$

<sup>1</sup>The symbol  $d_{ik}$  is used for  $X_{ij} - X_{kj}$ , where  $i = 1, 2, \dots, n$  and  $k > i$ .

where  $V_r$  represents the dual variable on the  $r$ th resource constraint ( $r=1, 2, \dots, s$ ),  $V^+ - V^-$  represents the dual variable of the return constraint,  $W_{ik}$  represents the dual variable of the  $ik$ th constraint of  $y_{ik}^+ - y_{ik}^-$ , and  $V_r, V^+, V^-, W_{ik} \geq 0$ .

Using a solution package with modified simplex algorithm dealing with bounded variables, thereby eliminating the need to specify the constraints of equation (24), the number of constraints in the linear programming tableau of the dual is reduced to  $m$ .

### An empirical example of MG analysis

Hazell (1971, p. 60) has presented MV and MOTAD analysis of a particular farm-planning problem. Anderson (1975) and Anderson et al. (1977) have presented a sample-based SSD analysis of the same problem. It is convenient, therefore, to illustrate MG analysis via this same problem which involves four vegetable activities: carrots ( $x_1$ ), celery ( $x_2$ ), cucumbers ( $x_3$ ) and capsicum ( $x_4$ ) together with three less-than or equal-to constraints on the available acreage of land ( $b_1$ ), hours of labour ( $b_2$ ) and a rotational and market outlet constraint ( $b_3$ ). The technical requirements coefficient matrix  $[a_{rj}]$  and constraint vector  $[b_r]$  are:

$$[a_{rj}] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 25 & 36 & 27 & 87 \\ -1 & 1 & -1 & 1 \end{bmatrix} \quad [b_r] = \begin{bmatrix} 200 \\ 10\,000 \\ 0 \end{bmatrix}$$

The constraint  $b_3$  requires that the total acreage of celery and capsicum be less than or equal to the total acreage of carrots and cucumbers. Unit (i.e., per acre) gross margins  $X_{ij}$  for each activity over the prior six years are presented in Table 1. The linear programming tableau of constraints and activities as per equations (16) to (20) for M $\Gamma$  and subsequent MG analysis of the above problem in its primal form is given in Table 2. The tableau for dual solution to the problem as per equations (22) to (24) is given in Table 3.

Table 4 presents cropping plans for the change-in-basis solutions as determined by M $\Gamma$  analysis. Analogously to MV and MOTAD analysis, linear segments between these basis-change solutions (beginning at the origin) specify the efficient set in  $[M, \Gamma]$  space. Inspection of the slope ( $\Delta\mu/\Delta\Gamma$ ) of each of these linear segments, as shown in Table 4, indicates all have a slope of greater than  $45^\circ$ . Thus, as per the discussion of Fig. 1 above, the only MG-efficient plan is that of solution VII.

Table 5 presents the  $\Gamma$  and  $\sigma$  (standard deviation) values and Table 6 the activity levels for some efficient plans as determined by MV, MOTAD, M $\Gamma$  and MG analysis of the example problem. Following Hazell (1971, p. 61), these plans are for the  $\mu$  levels corresponding to basis-change solutions in MV analysis of the problem. As would be expected, all four approaches specify the same

TABLE 3

Linear programming tableau for  $M\Gamma$  solution of example problem in its dual form

Row	$V_1$	$V_2$	$V_3$	$W_{12}$	$W_{13}$	$W_{14}$	$W_{15}$	$W_{16}$	$W_{23}$			
Objective	-200	-10 000										
	-1	-25	+1	113	178	45	-134	33	65			
	-1	-36	-1	-688	-776	-672	-310	-978	- 88			
	-1	-27	+1	233	54	171	98	261	-179			
	-1	-87	-1	- 60	200	-345	574	10	260			
$W_{24}$	$W_{25}$	$W_{26}$	$W_{34}$	$W_{35}$	$W_{36}$	$W_{45}$	$W_{46}$	$W_{56}$	$V^+$	$V^-$	Constraint	
										$\mu$	$-\mu$	Maximize
- 68	-247	- 80	-133	-312	-145	-179	- 12	167	253	-253	$\leq - 709$	
16	378	-290	104	466	-202	362	-306	-668	443	-443	$\leq -3652$	
- 62	-135	28	117	44	207	- 73	90	163	284	-284	$\leq 1017$	
-285	634	70	-545	374	-190	919	355	-564	516	-516	$\leq 1407$	

TABLE 4

 $M\Gamma$ -efficient plans for example problem

Variable	Change-in-basis solution						
	I	II	III	IV	V	VI	VII
$\mu$ (\$)	62 160	62 347	65 217	77 772	77 825	77 978	77 996
$\Gamma$ (\$)	1 977	1 986	2 964	9 580	9 615	9 745	9 763
$\Delta\mu/\Delta\Gamma$	31.4	20.8	2.9	1.9	1.5	1.2	1.01
$x_1$ (acres)	69.30	69.93	57.78	4.65	4.29	0.64	0.00
$x_2$ (acres)	26.50	26.16	26.23	26.52	26.89	27.43	27.45
$x_3$ (acres)	90.00	88.58	89.96	95.99	96.04	99.36	100.00
$x_4$ (acres)	14.20	15.33	26.03	72.84	72.78	72.57	72.55

plan for  $\mu$  at its highest feasible level (i.e., plan 4 with  $\mu=77996$ ). This plan is both the only  $M\Gamma$ -efficient one and the risk-neutral solution to the problem, thereby supporting the contention of Buccola and Subaei (1984) that  $M\Gamma$  analysis is most appropriate for weakly risk-averse decision makers. Comparing the MV, MOTAD and  $M\Gamma$  plans in terms of their  $\Gamma$  and  $\sigma$  values (Table 5), it is apparent that they are not very different. As would be expected,  $M\Gamma$  analysis yields plans with the lowest  $\Gamma$  values while MV analysis gives plans with the lowest  $\sigma$  values; values for MOTAD generally being intermediate between those for MV and  $M\Gamma$  analysis. From Table 6 it is apparent that the MV, MOTAD and  $M\Gamma$  plans differ somewhat more in their activity levels than in

TABLE 5

Comparison of  $\Gamma$  and  $\sigma$  values of efficient plans based on MV, MOTAD,  $M\Gamma$  and MG analysis of example problem

Statistic	Model	Plan <sup>1</sup>			
		1	2	3	4
$\mu$ (\$)		62609	77142	77354	77996
$\Gamma$ (\$)	MV	2149	9798	9865	9763
	MOTAD	2160	9679	9801	9763
	$M\Gamma$	2115	9236	9349	9763
	MG	na <sup>2</sup>	na	na	9763
$\sigma$ (\$)	MV	4624	20882	21182	22372
	MOTAD	4694	21187	21485	22372
	$M\Gamma$	4690	21251	21539	22372
	MG	na	na	na	22372

<sup>1</sup>At  $\mu$  levels for change-of-basis solutions in MV analysis.

<sup>2</sup>Not applicable (since 4 is the only MG-efficient plan).

TABLE 6

Comparison of activity mixes of efficient plans based on MV, MOTAD,  $M\Gamma$  and MG analysis of example problem

Activity	Model	Plan <sup>1</sup>			
		1	2	3	4
$x_1$ (acres)	MV	68.67	4.32	0.00	0.00
	MOTAD	72.08	19.83	18.83	0.00
	$M\Gamma$	60.70	7.32	6.41	0.00
	MG	na <sup>2</sup>	na	na	0.00
$x_2$ (acres)	MV	28.26	37.21	36.14	27.45
	MOTAD	26.73	28.44	28.25	27.45
	$M\Gamma$	28.15	26.50	26.51	27.45
	MG	na	na	na	27.45
$x_3$ (acres)	MV	88.23	95.68	100.00	100.00
	MOTAD	83.71	80.89	81.17	100.00
	$M\Gamma$	97.29	95.69	95.79	100.00
	MG	na	na	na	100.00
$x_4$ (acres)	MV	14.85	62.77	63.85	72.55
	MOTAD	16.97	70.84	71.75	72.55
	$M\Gamma$	13.86	70.49	71.28	72.55
	MG	na	na	na	72.55

<sup>1</sup>At  $\mu$  levels for change-of-basis solutions in MV analysis.

<sup>2</sup>Not applicable (since 4 is the only MG-efficient plan).

their  $\Gamma$  and  $\sigma$  values. Though not invariably so, the MV and  $M\Gamma$  plans are the most similar. Overall, the general similarity of the efficient plans however determined is not surprising. On the one hand, MOTAD is an approximation to MV analysis while, on the other hand, the MV- and  $M\Gamma$ - (and SSD-) efficient sets are identical in the case of normally distributed prospects (Yitzhaki, 1982, p. 181) – a situation which approximately prevails in the present problem where only the distributions of returns for two of the four activities appear to be somewhat non-normal (Anderson, 1975, p. 99).

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