Labour on the Family Farm: a Theory under Uncertainty

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Abstract

Dawson, P.J., 1988. Labour on the family farm: a theory under uncertainty.
Agric. Econ., 1: 365–380.

This paper develops a theory of the family farm in conditions of uncertainty
where attention is focussed on the labour-input decisions. Specifically, the farm
family is faced with two labour decisions, namely with respect to hired labour
and to family labour. The framework for analysis is expected utility maximisa-
tion. The analysis has implications for agricultural policy since policy pronounce-
ments of economists are typically based on models which assume perfect certainty.
These pronouncements do not survive the incorporation of uncertainty.

Introduction

Agriculture is characterised by family farms and uncertainty.[1] Family farms
are those farms where the family has the opportunity to work on the land itself.
Hence, it is faced with a labour-leisure choice and impinging on this choice is
uncertainty. In this paper, a theory of the family farm in conditions of uncer-
tainty is developed where the focus of attention is on the labour input decisions
of family, hired and total labour.[2] Heady (1952, p. 453) cites two main types
of uncertainty in agriculture, first, price uncertainty for products or factors
and, second, yield uncertainty.[3] The type of uncertainty considered here is the

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[1]Notwithstanding the classic dictotomy of Knight (1921, p. 233) between 'risk' and 'uncertainty',
a number of authors (for example, Debreu, 1959, Arrow, 1970, and Arrow and Hahn, 1971) have
used the two terms synonymously and we will follow their example.

[2]For a similar analysis in conditions of subjective certainty, see Dawson (1984). Comparisons
made between the cases of uncertainty and subjective certainty use this analysis.

[3]See Heady and Jensen (1954, p. 516) for other types of uncertainty in agriculture.

simplest type, namely, uncertainty surrounding the future output price. The framework for the analysis is expected utility maximisation.

Policy pronouncements of agricultural economists are explicitly or implicitly (consciously or unconsciously) based on models which assume that farmers maximise profits in conditions of perfect certainty. As I have argued elsewhere (Dawson, 1984), a model, based on Nakajima's (1970, 1986) decomposition of labour into family and hired labour, which assumes utility maximisation may be more appropriate. While this is a most useful first stage in analysing family-farm behaviour, it may give rise to misleading conclusions for policy. Clearly, it is important to investigate whether policy pronouncements based on this subjective certainty model are 'robust', that is, whether they can survive the incorporation of uncertainty.

Section 1 considers the welfare function of the family farm. Section 2 deals with the decision process itself. Section 3 then examines some comparative static propositions which can be used to evaluate various agricultural policies. Since increasing farm incomes is the major aim of most agricultural policies in developed countries, it is clear that the distinction between hired and family labour is crucial in evaluating the effectiveness of such policies. Accordingly, this section also examines some of the policy implications of the analysis.

1. Farm family's welfare function

In conditions of uncertainty surrounding future output price, it follows that the income of the family is also uncertain. Decisions are made on the basis of expectations of the future output price. The fundamental assumption of the analysis is that the family's expected utility in any one production period depends on the family's expected utility of income, on the disutility of labour it supplies and on the composition of the family. In assuming that the family, as an entity, is an expected utility maximiser, it is implicitly assumed that all family members have the same subjective expectations of income. The expected utility function chosen is an adaptation of the one used by Sen (1966). For reasons of nomenclature, we will denote family utility by W and call it family welfare. The welfare function takes the form:

$$E[W] = (\alpha + \beta) E[U(m)] - \alpha D(l) \qquad (1)$$

where α is a number of family workers, β is the number of family dependents,[4] $U(m)$ is the von Neumann–Morgenstern utility of income per family member, $D(l)$ is the disutility of labour per family worker,[5] and E is the expectations operator. Assume that:

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[4]The classification of family members into 'workers' and 'dependants' is an 'ex ante' one. The former are 'potential' workers since the analysis allows for the possibility that workers may not actually work.

[5]We are concerned only with labour which is worked on the farm in the production of commodities: 'non-farm' employment is excluded.

$$\alpha \geq 0; \qquad \beta \geq 0; \qquad \alpha + \beta > 0 \tag{2}$$

Further, assume that the expected welfare function is continuous, twice differentiable and strictly concave. It immediately follows that the function exhibits diminishing marginal utility of income for each individual family member and increasing marginal disutility of labour for each worker (Chiang, 1974, p. 396), that is:

$$\begin{aligned} U_m > 0, \qquad U_{mm} < 0 \\ D_l > 0, \qquad D_{ll} > 0 \end{aligned} \tag{3}$$

where $U_m = dU/dm$, $U_{mm} = d^2U/dm^2$, and so on.[6]

As Arrow (1965, p. 31) has noted, diminishing marginal utility of income implies risk aversion.[7] There are two further, implicit assumptions in (1). First, the realised total income of the family, M, is shared equally amongst all family members. Second, if the total labour supplied by the family, F, is strictly positive, then this amount is divided equally amongst all family workers. Thus:

$$m = \frac{M}{\alpha + \beta}; \qquad l = \frac{F}{\alpha} \tag{4}$$

The expected welfare function is defined for strictly positive levels of income only.[8]

2. Decision-making process

The production of agricultural commodities is characterised by a production period. For simplicity, assume that the farm produces one product, Q, from one variable input labour, L, and one fixed input land, N. Output is realised at the end of the production period while the inputs are committed at the beginning and are used throughout that period. The non-stochastic production function showing the maximum output obtainable for any input-mix is given by:

$$Q = Q(L, N) \tag{5}$$

It is assumed to be continuous, twice differentiable and strictly concave over

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[6] We depart from Sen (1966) by assuming strict inequalities for U_m, D_l and D_{ll}.

[7] Most agricultural economists believe that the farm family is risk-averse — see, for example, Blandford and Currie (1975). There is some evidence to support this belief — see Young (1979) for a survey of this literature.

[8] Strictly, the Von Neumann–Morgenstern utility function is defined over 'wealth' which is clearly positive (except in the case of bankruptcy) — see, for example, Arrow (1965). However, Sandmo (1971) and Batra and Ullah (1974), inter alia, define the function over profits but do not appear to recognise that profits are restricted to be positive. Hey (1979, p. 48), on the other hand, does note the restriction.

the relevant output range. The latter implies, inter alia, diminishing marginal productivities, that is:

$$Q_L > 0, \qquad Q_{LL} < 0$$
$$Q_N > 0, \qquad Q_{NN} < 0$$

(6)

Consider the family's income. Profit from the farm is simply total revenue less total costs. Assume that the farm is a competitive price-taker and that all output is sold at the ruling market price, P. Total revenue $R = PQ$. Total costs are the variable costs of hired labour, wH — where w is the (given) wage rate — plus the fixed costs of land. Assume that payment for the inputs takes place at the end of the production period. Finally, we can incorporate into the analysis any non-farm income, that is unearned income, which is assumed to accrue at the end of the period. The family's income then is:

$$M = PQ(L, N) - wH + Y$$

(7)

where Y is net autonomous income and is defined as non-farm income less fixed costs and $L = F + H$. An implicit assumption here is that hired labour is a technologically perfect substitute for family labour.

When the family makes decisions at the beginning of the production period, the future output price is unknown. Decisions then must be based on price expectations. Assume that these expectations can be represented by a non-degenerate subjective probability density function which has a mean $E[P]$. Therefore, corresponding to each output level is an associated subjective probability density function of income.

Let us now consider the decision-making process. The family's decision is to maximise its expected welfare function of (1) subject to the income constraint in (7) when (4) is invoked, that is:

$$\underset{H,l}{\text{Max}} \, E[W] = \underset{H,l}{\text{Max}} \, [(\alpha + \beta) \, E[U(m)] - \alpha \, D(l)]$$

subject to

$$m = \frac{1}{\alpha + \beta} [PQ(L, N) - wH + Y] \qquad (H, l \geq 0)$$

(8)

where $L = \alpha l + H$. The inequality constraints require that hired labour and family labour be non-negative. The first-order, necessary conditions for a maximum are:

$$\frac{\partial E[W]}{\partial H} = E[U_m (PQ_L - w)] \leq 0$$

$$H \geq 0; \qquad H \, E[U_m (PQ_L - w)] = 0$$

(9)

$$\frac{\partial E[W]}{\partial l} = \alpha E[U_m PQ_L] - \alpha D_l \leq 0$$

$$l \geq 0; \qquad F E[U_m PQ_L] - FD_l = 0 \tag{10}$$

The corresponding second-order conditions for a maximum are:

$$\frac{\partial^2 E[W]}{\partial H^2} = A_1 = E\left[\frac{1}{\alpha+\beta} U_{mm}(PQ_L - w)^2 + U_m PQ_{LL}\right] < 0 \tag{11}$$

$$\frac{\partial^2 E[W]}{\partial l^2} = A_2 = E\left[\frac{\alpha^2}{\alpha+\beta} U_{mm}(PQ_L)^2 + \alpha^2 U_m PQ_{LL}\right] - \alpha D_{ll} > 0 \tag{12}$$

and

$$\Delta = A_1 A_2 - B^2 > 0 \tag{13}$$

where

$$B = \frac{\partial^2 E[W]}{\partial H \partial l} = E\left[\frac{\alpha}{\alpha+\beta} U_{mm} PQ_L(PQ_L - w) + \alpha U_m PQ_{LL}\right] \tag{14}$$

$\Delta > 0$ can be shown to hold by substituting (11), (12) and (14) into (13).

Consider the first-order conditions. Assume $H > 0$ in (9). Following Batra and Ullah (1974, p. 541), it can be shown that hired labour is employed such that the wage rate is strictly less than the expected marginal value product of labour, that is:

$$w < E[P] Q_L \tag{15}$$

Assume $l > 0$ in (10).[9] By dividing (10) by (9) and invoking (15) then:

$$E\left[\frac{D_l}{U_m}\right] = w < E[P] Q_L \tag{16}$$

that is, the expected marginal rate of substitution of income for work for each individual worker is equal to the wage rate but strictly less than the expected marginal value product of labour. If $H = 0$, it can be shown using the method of Batra and Ullah that:

$$E\left[\frac{D_l}{U_m}\right] < E[P] Q_L \tag{17}$$

Equations (16) and (17) imply that less labour will be used and hence less

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[9]The corner-solution where $F = 0$ is not without interest since the decision criterion effectively becomes one of maximising the expected utility of income. Further, when $\alpha > 0$ but $F = 0$, a sufficiently large increase in the wage rate say will bring forth a positive own labour supply. This case would appear to be rare and we will discuss it no further.

output produced in an uncertain situation when compared to the case of subjective certainty.[10]

3. Some comparative static propositions

In this section, we examine some comparative static propositions. In particular, we consider those which are concerned with policy issues. Direct support policies affect net autonomous income (through either non-farm income by direct income supplements, or fixed costs) and output prices. Structural policy is designed to change the operating/ownership structure of agriculture by giving incentives to workers and families to leave the land. This affects the numbers of family dependants and workers and the land size of the farm.

There are two types of equilibria that are the most interesting, namely, 'labour-hiring' farms when $H > 0$ and 'family-labour-only' farms when $H = 0$. The comparative statics depend on whether labour is hired or not. Therefore, we consider each type of farm.[11]

3.1. Change in net autonomous income

For labour-hiring farms, consider the effects on family, hired and total labour of a change in net autonomous income. Specifically, consider an increase in Y by an amount of δ which could result from an increase in non-farm income or a fall in fixed costs. The problem is given in (8) where Y is augmented by δ. The first- and second-order conditions are given in (9)–(14). Now, totally differentiating (9) and (10) with respect to δ and evaluating at $\delta = 0$ while noting that $H, F > 0$ gives:

$$A_1 \frac{\partial H}{\partial \delta} + B \frac{\partial l}{\partial \delta} = -Z_1$$

$$B \frac{\partial H}{\partial \delta} + A_2 \frac{\partial l}{\partial \delta} = -Z_2 \qquad (18)$$

where

$$Z_1 = \frac{\partial^2 \mathrm{E}[W]}{\partial H \partial \delta} = \frac{1}{\alpha + \beta} \mathrm{E}[U_{mm}(PQ_L - w)] \qquad (19)$$

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[10]Sandmo (1971, p. 67) shows explicitly that output will be less in conditions of uncertainty. It is worthy of note however that Hartman (1975) in a comment to Batra and Ullah's paper shows that for a two-variable input production function, the smaller output under uncertainty does not necessarily imply smaller levels of both inputs.

[11]The 'formal' comparative static propositions relate to infinitesimally small changes in the exogenous variables. For discrete changes, a farm may move from a family-labour-only farm to a labour-hiring farm or vice versa. This possibility is not admitted.

$$Z_2 = \frac{\partial^2 \mathrm{E}[W]}{\partial l \, \partial \delta} = \frac{\alpha}{\alpha + \beta} \, \mathrm{E}[U_{mm} \, PQ_L] \tag{20}$$

Solving (18) simultaneously yields:

$$\frac{\partial l}{\partial \delta} = \frac{-A_1 Z_2 + B Z_1}{\varDelta} \tag{21}$$

and

$$\frac{\partial H}{\partial \delta} = \frac{-A_2 Z_1 + B Z_2}{\varDelta} \tag{22}$$

Substituting (11)–(14), (19) and (20) into (21) and rearranging gives:

$$\frac{\partial l}{\partial \delta} = -\frac{\dfrac{\alpha}{\alpha + \beta} \, \mathrm{E}[U_{mm} U_m \, PQ_{LL}] \, w}{\varDelta} \tag{23}$$

For family-labour-only farms, the problem is the same except that all terms in H disappear so that (18) reduces to a single equation, namely:

$$A_2 \frac{\partial l}{\partial \delta} = -Z_2 \tag{24}$$

Substituting (20) into (24) and rearranging gives:

$$\frac{\partial l}{\partial \delta} = -\frac{\dfrac{\alpha}{\alpha + \beta} \, \mathrm{E}[U_{mm} \, PQ_L]}{A_2} \tag{25}$$

From (2)–(4), (6), (12) and (13) and for both farm types:

$$\frac{\partial l}{\partial \delta} \equiv \frac{\partial l}{\partial Y} < 0 \tag{26}$$

Also:

$$\frac{\partial F}{\partial Y} = \alpha \frac{\partial l}{\partial Y} < 0 \tag{27}$$

that is, an increase in net autonomous income leads to a fall in individual and total family labour.[12] Equations (23) and (25) represent 'wealth' effects: the introduction of uncertainty leaves the sign unchanged for both farm types.

Returning to labour-hiring farms, substituting (11)–(14), (19) and (20) into (22) and rearranging gives:

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[12] A similar result is shown by Block and Heineke (1973).

$$\frac{\partial H}{\partial \delta} = -\frac{\partial F}{\partial Y} + \frac{\dfrac{\alpha}{\alpha+\beta} D_{ll}\, \mathrm{E}\,[\,U_{mm}(PQ_L - w)\,]}{\Delta} \tag{28}$$

The first term on the right-hand side of (28) is positive from (27). Using (2), (3) and (13), the sign of the second term depends upon the sign of $\mathrm{E}\,[\,U_{mm}(PQ_L - w)\,]$. Sandmo (1971, pp. 68–69) has shown that decreasing absolute risk aversion (Arrow, 1965, pp. 33–35 or Pratt, 1964) is sufficient to show that:

$$\mathrm{E}\,[\,U_{mm}(PQ_L - w)\,] > 0 \tag{29}$$

Hence, the second term is also positive. Thus:

$$\frac{\partial H}{\partial \delta} \equiv \frac{\partial H}{\partial Y} > 0 \tag{30}$$

that is, an increase in net autonomous income leads to an increase in hired labour: the introduction of uncertainty leaves the sign of $\partial H / \partial Y$ unchanged. Now, since $L = F + H$:

$$\frac{\partial L}{\partial Y} = \frac{\partial F}{\partial Y} + \frac{\partial H}{\partial Y} > 0 \tag{31}$$

by substituting (28) whose second term on the right-hand side is positive. Thus, an increase in net autonomous income leads to total labour increasing. Under conditions of subjective certainty, $\partial L / \partial Y = 0$.

Direct income supplements, which increase net autonomous income by increasing non-farm income, are often put forward by economists as an alternative to product price support. The rationale for this is that in textbook profit maximisation models, direct income supplements have no undesirable output-increasing effects. However, for labour-hiring farms, the introduction of direct income supplements decreases family labour but increases hired and total labour thereby leading to an increase in output and increased rural employment. For family-labour-only farms, the introduction of direct income supplements will decrease family labour and hence decrease output. Hence it appears that most of the benefit of direct income supplements are received by larger farms in the sense that labour-hiring farms can increase expected income by more than the increase in net autonomous income.

3.2. Change in expected output price

Consider the effects of a change in expected output price. In particular, consider an increase in the mathematical expectation of the price where the dis-

tribution of P remains constant. By a similar method of derivation to that above, it can be shown that for labour-hiring farms:

$$\frac{\partial l}{\partial E[P]} = \frac{\partial l}{\partial Y}Q + \frac{\frac{\alpha}{\alpha+\beta} E[U_{mm}(PQ_L - w)]w E[U_m Q_L]}{\Delta} \gtreqless 0 \qquad (32)$$

and for family-labour-only farms:

$$\frac{\partial l}{\partial E[P]} = \frac{\partial l}{\partial Y}Q - \frac{\alpha E[U_{mm} Q_L]}{A_2} \gtreqless 0 \qquad (33)$$

The first terms on the right-hand side of both (32) and (33) are 'wealth' effects and are negative from (26). The second terms are substitution effects. In (32), decreasing absolute risk aversion is sufficient to show that it is positive. It is also positive in (33) from (2), (3), (6) and (12). Hence, for both farm-types, the effect of a change in expected price on individual family labour in indeterminate. Moreover, the effect on total family labour is indeterminate, that is:

$$\frac{\partial E}{\partial E[P]} = \alpha \frac{\partial l}{\partial E[P]} \gtreqless 0 \qquad (34)$$

For family-labour-only farms, the introduction of uncertainty leaves this sign unchanged but this contrasts with the case of subjective certainty for those hiring labour where only the 'wealth' effect operates.

Similarly for labour-hiring farms, it can be shown that:

$$\frac{\partial H}{\partial E[P]} = -\frac{\partial F}{\partial Y}Q + \frac{\frac{\alpha}{\alpha+\beta} D_{ll} E[U_{mm}(PQ_L - w)] Q}{\Delta}$$

$$- \frac{\frac{\alpha^2}{\alpha+\beta} E[U_{mm} U_m PQ_L{}^2]w}{\Delta} + \frac{\alpha D_{ll} E[U_m Q_L]}{\Delta} > 0 \qquad (35)$$

The first and second terms on the right-hand side of (35) equal $\partial H/\partial Y Q$. Decreasing absolute risk aversion is sufficient for them to be positive. The third and fourth terms are both positive from (2), (3), (6) and (13). Hence, an increase in expected price leads to an increase in hired labour: the introduction of uncertainty leaves this sign unchanged. Further, from (32), (34) and (35) an increase in expected price leads to an increase in total labour,[13] that is:

[13]Batra and Ullah (pp. 544–554) show a similar result given two inputs, labour and capital where maximisation of the expected utility of profits is the decision-making criterion.

$$\frac{\partial L}{\partial \mathrm{E}[P]} > 0 \tag{36}$$

This implies that an increase in expected price leads to an increase in output.[14] The introduction of uncertainty leaves the signs in (35) and (36) unchanged.

The main plank of most agricultural policies in the developed world is price support. The *introduction* of support for product prices generally reduces the level of price uncertainty rather than guarantees prices. However, we can consider policies which change product prices without changing the probability distribution, that is, we can consider changing current price support policies. For labour-hiring farms, an increase in the product price has an indeterminate effect on family labour but leads to increases in both hired and total labour. Hence, the output of these farms increases and this is often thought to be undesirable. However, such policies do act to stimulate rural employment. For family-labour-only farms the effect on family labour of an increase in the product price is indeterminate. Only when the substitution effect outweights the 'wealth' effect will total labour, and hence output, increase. This is consistent with the commonly-held view that price support for agricultural commodities gives most benefit to the larger (labour-hiring) farms.

3.3. Changes in the constituents of the family

Let us now consider the effects of changes in the constituents of the family. In particular, consider the effects of changes in the number of dependants and workers. It can be shown that for labour-hiring farms:

$$\frac{\partial l}{\partial \beta} = \frac{\partial l}{\partial \alpha} = \frac{\dfrac{\alpha}{\alpha+\beta} \mathrm{E}[U_{mm} U_m PQ_{LL} m] w}{\varDelta} > 0 \tag{37}$$

and for family-labour-only farms:

$$\frac{\partial l}{\partial \beta} = \frac{\partial l}{\partial \alpha} = \frac{\dfrac{\alpha}{\alpha+\beta} \mathrm{E}[U_{mm} PQ_L m]}{A_2} > 0 \tag{38}$$

Further, for both farm types:

$$\frac{\partial F}{\partial \beta} = \alpha \frac{\partial l}{\partial \beta} > 0 \tag{39}$$

and

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[14]Sandmo (p. 69) shows this result explicitly.

$$\frac{\partial F}{\partial \alpha} = \frac{\partial F}{\partial \beta} + l > 0 \tag{40}$$

Thus, an increase in the number of dependants and workers increases both individual and total family labour. The terms $\partial l/\partial \beta$ and $\partial l/\partial \alpha$ are 'wealth' effects: the introduction of uncertainty leaves their signs unchanges. Further $\partial F/\partial \beta$ can be thought of as a 'demand side' effect. Thus, there are two effects of an increase in family workers on the total labour supply of the family: the first is the 'demand side' effect while the second, l, is the 'supply side' effect.

Similarly for farms hiring labour, it can be shown that:

$$\frac{\partial H}{\partial \beta} = \frac{\partial H}{\partial \alpha} = -\frac{\partial F}{\partial \beta} = -\frac{\dfrac{\alpha}{\alpha + \beta} D_{ll} \, \mathrm{E}\,[\,U_{mm}(PQ_L - w)m\,]}{\Delta} \tag{41}$$

The first-term on the right-hand side of (41) is negative from (39). Using (2), (3) and (13), the sign of the second-term depends upon the sign of $-\mathrm{E}\,[\,U_{mm}(PQ_L - w)m\,]$. Sandmo (1971, p. 70) has shown that increasing relative risk aversion (Arrow, 1965, pp. 33–35, or Pratt, 1964) is sufficient to show that:

$$\mathrm{E}\,[\,U_{mm}(PQ_L - w)m\,] < 0 \tag{42}$$

Thus, the second term in (41) is positive and the effect of a change in the numbers of dependants or workers on hired labour is indeterminate, that is:

$$\frac{\partial H}{\partial \beta} = \frac{\partial H}{\partial \alpha} \gtrless 0 \tag{43}$$

This contrasts with the case of subjective certainty where both these signs are negative. Further, from (37) and (39)–(41):

$$\frac{\partial L}{\partial \beta} > 0 \tag{44}$$

and

$$\frac{\partial L}{\partial \alpha} = \frac{\partial L}{\partial \beta} + l > 0 \tag{45}$$

that is, the effect of an increase in the number of dependants or workers leads to an increase in total labour. This contrasts with the case of subjective certainty where both these effects are zero.

Structural policies aim basically to increase farm incomes by reducing the numbers which earn a living from agriculture. This changes the operating structure of agriculture by influencing outmigration and increasing farm size. We can comment on the case where such policies have influenced some, but

not all, family workers to leave the farm. For both types of farm, family labour will fall. For labour-hiring farms, reducing the number of family workers has an indeterminate effect on hired labour but reduces total labour. Therefore, for both farm types, output also falls. The case of family dependants leaving the farm is similar to that of out-migrating workers.

3.4. Change in land size

Consider the effects of an increase in the land size of the farm. It can be shown that for labour-hiring farms:

$$
\frac{\partial l}{\partial N} = -\frac{\dfrac{\alpha}{\alpha+\beta}\,\mathrm{E}\,[\,U_{mm}\,U_m\,PQ_N\,PQ_{LL}\,]\,w}{\varDelta}
$$

$$
+\frac{\dfrac{\alpha}{\alpha+\beta}\,\mathrm{E}\,[\,U_{mm}\,(PQ_L-w)\,]\,w\,\mathrm{E}\,[\,U_m\,PQ_{LN}\,]}{\varDelta} \gtrless 0 \tag{46}
$$

and for family-labour-only farms:

$$
\frac{\partial l}{\partial N} = -\frac{\dfrac{\alpha}{\alpha+\beta}\,E\,[\,U_{mm}\,PQ_L\,PQ_N\,]}{A_2} - \frac{\alpha\,\mathrm{E}\,[\,U_m\,PQ_{LN}\,]}{A_2} \gtrless 0 \tag{47}
$$

The first-term on the right-hand sides of both (46) and (47) are 'wealth' effects and are negative. The second-terms are substitution effects. In (46), decreasing absolute risk aversion is sufficient to show that it is positive if the not unreasonable assumption that $Q_{LN} > 0$ is invoked. Again by invoking the assumption that $Q_{LN} > 0$, the substitution effect in (47) is positive. Thus, for both farm types, an increase in land has an indeterminate effect on individual family labour and hence has an indeterminate effect on total family labour, that is:

$$
\frac{\partial F}{\partial N} = \alpha\frac{\partial l}{\partial N} \gtrless 0 \tag{48}
$$

The impact of uncertainty leaves the sign in (48) unchanged for the family labour-only farm but this proposition contrasts with the case of subjective certainbty for the farm hiring labour where an increase in land leads to a fall in own labour.

Similarly, for the labour-hiring farm, it can be shown that:

$$\frac{\partial H}{\partial N} = \frac{\frac{\alpha^2}{\alpha+\beta} \, \mathrm{E}\,[\,U_{mm}\,U_m\,PQ_{LL}\,PQ_N\,]\,w}{\varDelta} - \frac{\frac{\alpha^2}{\alpha+\beta} \, \mathrm{E}\,[\,U_{mm}\,U_m\,PQ_L\,PQ_{LN}\,]\,w}{\varDelta}$$

$$+ \frac{\frac{\alpha}{\alpha+\beta}\,D_{ll}\,\mathrm{E}\,[\,U_{mm}(PQ_L - w)\,PQ_N\,]}{\varDelta} + \frac{\alpha D_{ll}\,\mathrm{E}\,[\,U_m\,PQ_{LN}\,]}{\varDelta} \gtrless 0 \qquad (49)$$

This first, second and fourth terms are all positive. However, the third-term appears to be indeterminate since $\mathrm{E}\,[\,U_{mm}(PQ_L - w)PQ_N\,]$ can be decomposed into two parts, each having the opposite sign. This can be shown formally. Let $\hat{PQ}_N$ be the marginal value product of land when $PQ_L = w$. Therefore, by adding and subtracting $\mathrm{E}\,[\,U_{mm}(PQ_L - w)\,]\,\hat{PQ}_N$, $\mathrm{E}\,[\,U_{mm}(PQ_L - w)PQ_N\,] = \mathrm{E}\,[\,U_{mm}(PQ_L - w)(PQ_N - \hat{PQ}_N)\,] + \mathrm{E}\,[\,U_{mm}(PQ_L - w)\,]\,\hat{PQ}_N$. Now if $PQ_L \geq w$, then $PQ_N \geq \hat{PQ}_N$, and if $PQ_L \leq w$, $PQ_N \leq \hat{PQ}_N$ so that $(PQ_L - w)(PQ_N - \hat{PQ}_N) \geq 0$. Hence, $\mathrm{E}\,[\,U_{mm}(PQ_L - w)(PQ_N - \hat{PQ}_N)\,] \leq 0$ and $\mathrm{E}\,[\,U_{mm}(PQ_L - w)\,]\,\hat{PQ}_N > 0$ from (29). Therefore, the third term is indeterminate and, at the present level of generality, the effect of a change in land on hired labour is also indeterminate. This proposition contrasts with that under subjective certainty which implies a positive response of hired labour to an incease in land. Further, a consideration of (46), (48) and (49) reveals that an increase in land appears to have an indeterminate effect on total labour,[15] that is:

$$\frac{\partial L}{\partial N} \gtrless 0 \qquad (50)$$

Again, this contrasts with the case of subjective certainty which implies that the effect of an increase in land is an increase in total labour.

Structural policy which increases farm size aims to increase farm incomes by distributing the aggregate income of the agricultural sector amongst fewer individuals. Increasing the area of the farm (land payments, that is fixed costs, remaining unchanged) has an indeterminate effect on all labour inputs of both farm types so that qualitatively, there is little to recommend such policies.

Summary and conclusions

In this paper, we have examined the labour inputs of the family farm in conditions of price uncertainty. The fundamental assumption is that the aim

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[15]Batra and Ullah (pp. 546-547) demonstrate that an increase in the wave rate has an indeterminate effect on land use.

TABLE 1

Summary of comparative static propositions

Increase in	Non-farm income Y	Expected product price $E[P]$	Number of family workers α	Number of family dependants β	Land N
Labour-hiring farms					
Family labour F	−	?	+	+	?
	(−)	(−)	(+)	(+)	(−)
Hired labour H	+	+	?	?	?
	(+)	(+)	(−)	(−)	(+)
Total labour L	+	+	+	+	?
	(0)	(+)	(0)	(0)	(+)
Family-labour-only farms					
Family labour F	−	?	+	+	?
	(−)	(?)	(+)	(+)	(?)

Note: Signs in parentheses indicate the propositions under subjective certainty.

of the family is to maximise expected welfare which is defined over the expected utility of income, the disutility of labour and the composition of the family. A constraint on maximising expected welfare is expected income which embodies the expected revenues and costs of the farm. From the analysis, two types of farm can be identified: those which use hired labour as well as family labour and those which use family labour only. From this conceptual framework, we examined some comparative static properties for each farm type which are summarised in Table 1.

The dichotomy of agricultural labour into family labour and hired labour is an important one for two reasons. First, we are able to examine the labour-leisure choice which faces the farm family. Second, agricultural policy affects family and hired labour differently. Thus, we are able to assess the impact on rural employment through the latter. Furthermore, we are able to examine the effects of agricultural policy on the total labour input of the farm and hence on output.

Three conclusions result from the analysis. First, price support for agricultural commodities leads to increased output and increased rural employment for those farms which do hire labour, but has an indeterminate effect on those which do not. Second, and contrary to conventional wisdom, direct income supplements lead to increased total labour and hence increased output for those farms which hire labour. Rural employment will also increase. For those farms which do not hire labour, family labour and hence output will fall. Qualitatively, therefore, there is little to choose between direct income supplements and price support for products. Third, structural policy aimed at increasing out-migration reduces the total labour input and hence the output of labour-

hiring farms. The effect on rural employment is indeterminate. However, for those farms which do not hire labour, family labour and output increase. Structural policy aimed at increasing farm amalgamations appears to have indeterminate effects on labour inputs for both farm types.

Finally, we stressed the comparison of this family farm model under uncertainty with one which assumes perfect certainty. A number of propositions whose signs were previously determinate now become indeterminate. Moreover, demonstrating that certain comparative effects are qualitatively indeterminate emphasises the need for reliable empirical analyses.

Acknowledgement

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