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Thermal Properties
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Technical Bulletin No. 1413

Agricultural Research Service
UNITED STATES DEPARTMENT OF AGRICULTURE

ACKNOWLEDGMENTS

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Thermal Properties and Heat Transfer Characteristics of Marsh Grapefruit

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SUMMARY

Investigations of the thermal properties and heat transfer characteristics of Marsh grapefruit were conducted on 10 test fruits from each of five maturity groups. The harvest dates for maturity groups were: December 1963, April 1964, October 1964, February 1965, and May 1965. The investigations included tests to determine (1) effective thermal diffusivity of the whole fruit, (2) thermal conductivity of the rind and juice-vesicle components, (3) moisture content, (4) specific gravity, and (5) internal-temperature response—fundamental to response at specified conditions.

Of the properties measured, significant relations were found between rind thickness, moisture content of both rind and juice vesicles, thermal conductivity of both rind and juice vesicles, and specific gravity. By visual inspection these properties also appear to correlate with harvest season. As the harvest season is extended, the fruit ripens. During the ripening process, the fruit becomes more dense, its rind becomes thinner, and the rind moisture content decreases. These changes are consistent with a measured increase in effective thermal diffusivity as the fruit ripens. However, cooling rate is not significantly affected by differences in the maturing and ripening of the fruit.

INTRODUCTION

Refrigeration is the basic means for postharvest protection against decay and deterioration of horticultural crops. The extraction of heat from these living biological products slows their respiratory activity, reduces water loss, decreases the chance of invasion of new decay, and inhibits or slows the growth of incipient infections. The optimum rate and magnitude of heat extraction vary depending upon the physiological nature of the product. Some products are more perishable than others; thus they need to be cooled more rapidly. Marsh grapefruit, for example, differs from other citrus fruits in its physiological response to temperature. It also responds differently at various stages of maturity. In tests with Marsh Seedless and Ruby Red grapefruit, Chace and coworkers (*3*) found that the most desirable transit temperature for early-harvested fruit was 60° F. More mature midseason and late-season fruit kept best in transit at 50°. Their findings, substantiated by an abundance of previous

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research cited in their report, point out the need for postharvest, preshipment conditioning commensurate with the particular fruit requirements.

In a 1958 report on hydrocooling Florida citrus, Grierson and Hayward (4) stated that—

"The increasing use of automated methods, combined with the danger from such endemic post-harvest diseases as stem-end rot and *Penicillium* mold, make the use of such packing methods hazardous unless efficient refrigeration, post-harvest fungicides, or a combination of these two protective methods is used."

They further reported finding increased susceptibility to decay and rind injury on hydrocooled fruit. Yet, precooling, whether through hydrocooling or through the use of an air-cooling system, is one of the essential means of conditioning fruit in preparation for shipment. Precooling implies rapid heat removal, which may be done at the packinghouse in bulk, in pallet boxes, or in shipping containers. Generally it is done before shipment. Sometimes it is done after the fruit has been loaded into trucks or railcars. Either way, optimum precooling, i.e., the removal of a specified predetermined quantity of heat in a given period of time, depends upon knowledge of the thermal properties and characteristics of the fruit in question. This basic information will benefit the citrus industry in that it will eventually lead to the development of more efficient and effective precooling systems.

Published values of thermal properties of grapefruit, reviewed by Bennett (1), are inconclusive and questionable. Smith and coworkers (10) and Perry and coworkers (6) reported values of thermal diffusivity of Marsh grapefruit that are accepted by the authors to be accurate, and they are therefore compared with the results of the work reported herein.

This research was conducted to evaluate the basic heat transfer properties and characteristics of Marsh grapefruit and to investigate the possible relation of these properties to moisture content, density, and harvest season. Specifically, the research was designed to yield the following data:

1. Experimental temperature distribution.
2. Effective thermal diffusivity.
3. Thermal conductivity of the rind and juice vesicles.
4. Moisture content of the rind and juice vesicles.
5. Specific gravity of the whole fruit.
6. Correlations of the foregoing with seasonal effects.

INVESTIGATIONS

Test Fruit

Samples of Marsh grapefruit were harvested in December 1963, in April and October 1964, and in February and May 1965, from commercial groves in Indian River County, Fla. Five maturity groups, numbers 1 through 5, referred to hereinafter chronologically, correspond with the foregoing harvest dates. Ten test runs were

made for each maturity group on the basis of expected experimental variation of runs within a group.

The fruit was washed and waxed with a solvent-type wax to prevent loss of moisture during the short storage period before testing. Storage was at 50° F.

Experimental Procedure

Each test fruit was weighed and its diameter measured at several radial points in both the equatorial and polar planes. The fruit was brought to a uniform temperature of 85° F., and then it was immersed in an agitated water bath held at a constant temperature of approximately 35°. Fruit temperature was measured at ⅛-inch intervals along the radius in the equatorial plane by means of a thermal probe constructed of 36 a.w.g. (American wire gage) copper-constantan thermocouple wire connected to a 24-point recording potentiometer. The probe consisted of 12 individual thermocouples and was of sufficient length to permit insertion along the entire length of the diameter. This procedure compensated for conduction error, because the heat of conduction along the wire tended to flow in an opposite direction from that of the heat flux in the fruit. Surface temperature and temperature just beneath the rind were measured with individual thermocouples (fig. 1). The test fruit was cooled to a center temperature of approximately 40°, removed from the water bath,

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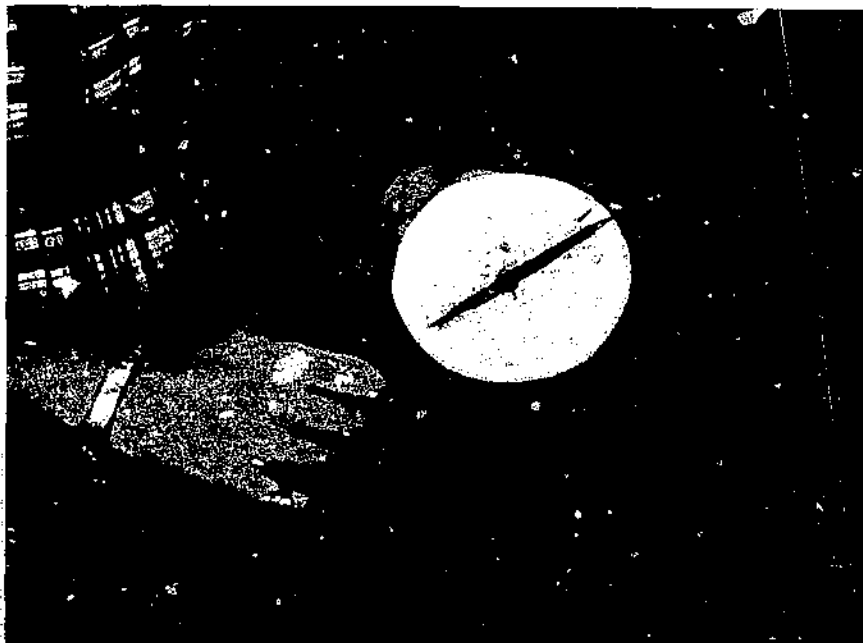
FIGURE 1.—Thermal probe positioned in test fruit. Note thermocouple to measure temperature at interface between rind and juice vesicle.

and reweighed for further tests. The test apparatus is shown in figure 2.

Specific gravity was measured by the water-displacement technique. Thermal conductivity and moisture content of the rind and the juice vesicle were measured from samples of each specimen.

The moisture content was measured by weighing the sample, extracting the water in a vacuum oven at 140° F. for 48 hours, then weighing the residual solids. Percentage moisture content was calculated on the basis of the wet weight of the sample.

The thermal conductivity was measured by use of an adaptation of the Fitch method (fig. 3) for measuring thermal conductivity of poor conductors. Equipment used included the adapted test unit, sensitive temperature controller, stopwatch, agitator, low-resistance microammeter, laboratory potentiometer, micrometer (fig. 4), and pressure meter (fig. 5). The experimental technique of Bennett and coworkers (2) was used.

Test specimens were removed from the fruit by use of a sharp-edged, hollow, stainless steel tube (fig. 6) having an inside diameter of 1.25 inches. The sample was cut to the same diameter as the heat sink to eliminate the possibility of heat energy radiating to the sink from the plate. Sample pressure was held standard at 1 p.s.i. (pound per square inch) for the rind and 0.5 p.s.i. for the juice vesicles. Juice-vesicle samples were wrapped in thin polyethylene to reduce moisture evaporation and juice losses.

Rind thickness was obtained from an average of nine micrometer readings taken before and after each run (fig. 7). An average of five measurements was used for juice-vesicle thickness.

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FIGURE 2.—Equipment for evaluating temperature distribution and effective thermal diffusivity of Marsh grapefruit.
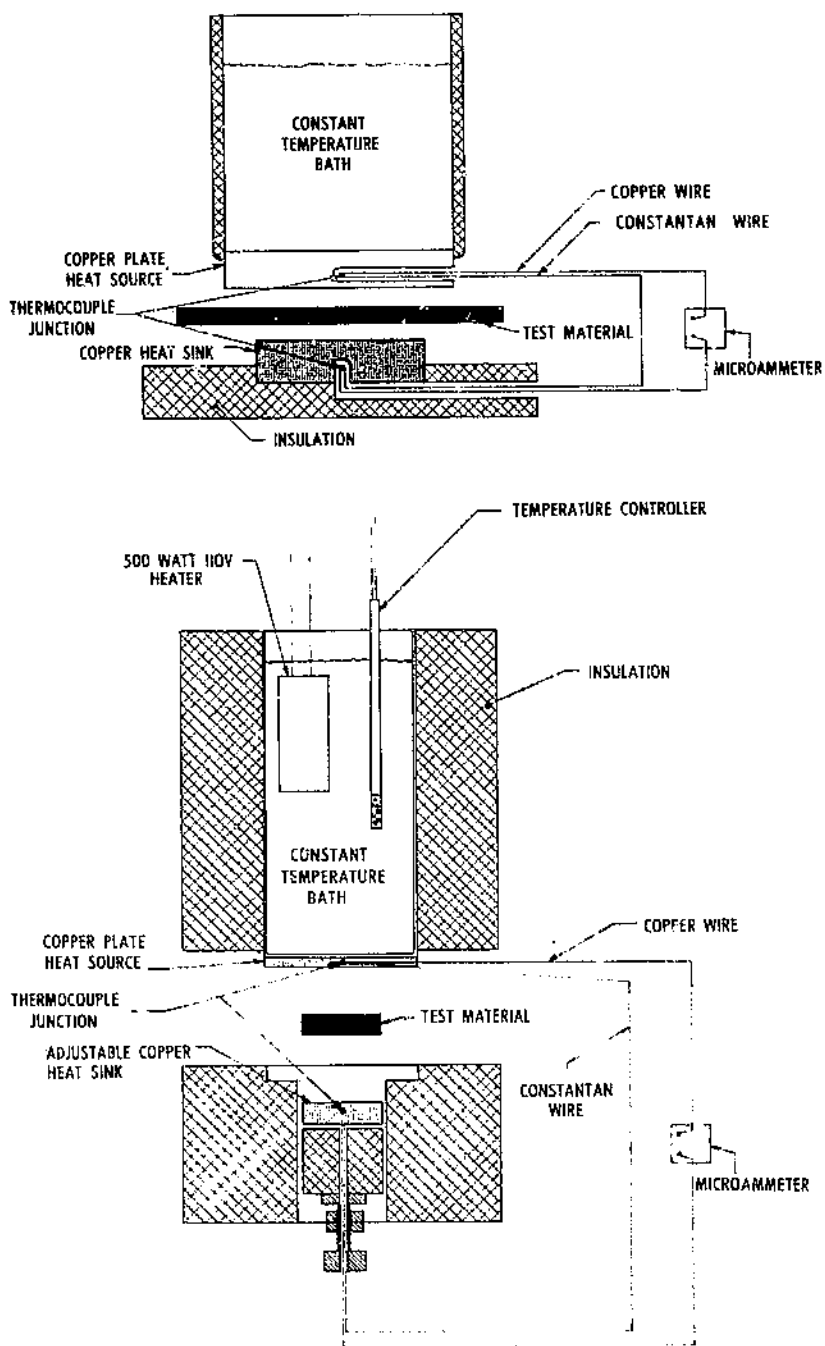
FIGURE 3.—Schematic cross-section of the standard Cenco-Fitch test unit for measuring thermal conductivity of poor conductors (above) and the adapted test unit (below).
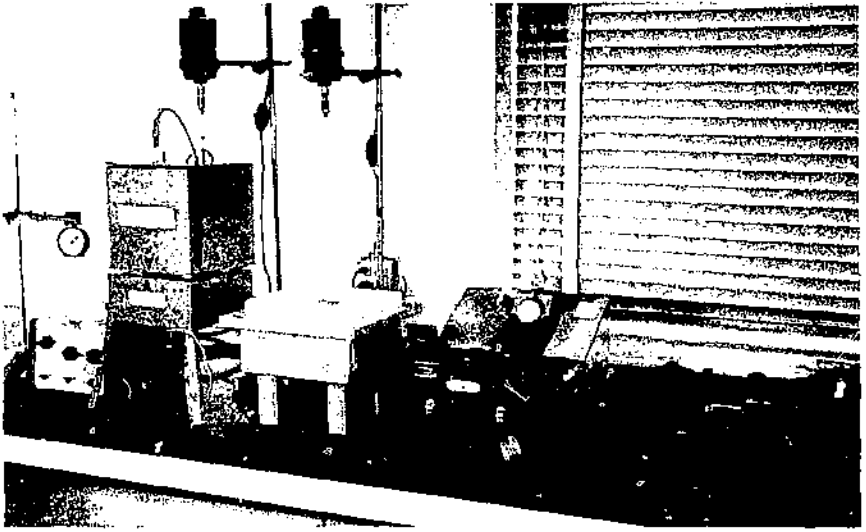
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FIGURE 4.— Laboratory equipment and instruments used to measure thermal conductivity of Marsh grapefruit rind and juice vesicles.
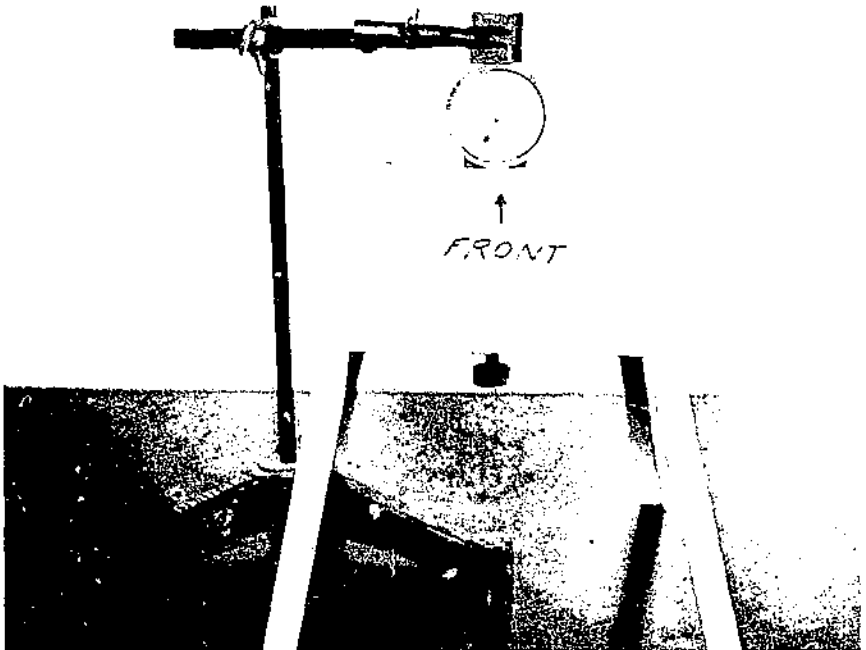
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FIGURE 5.— Pressure tester positioned over test sample in base of unit. Each sample was subjected to a standard predetermined pressure to minimize contact resistance.
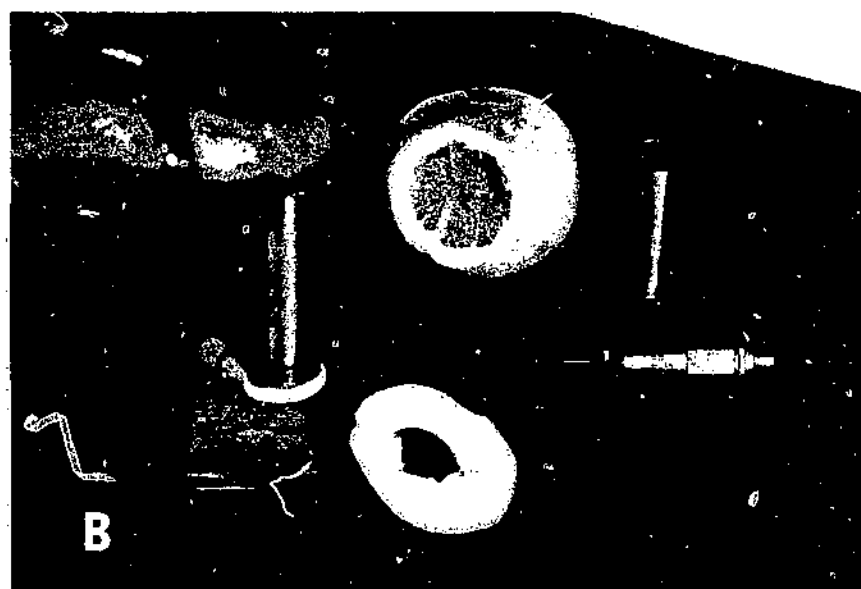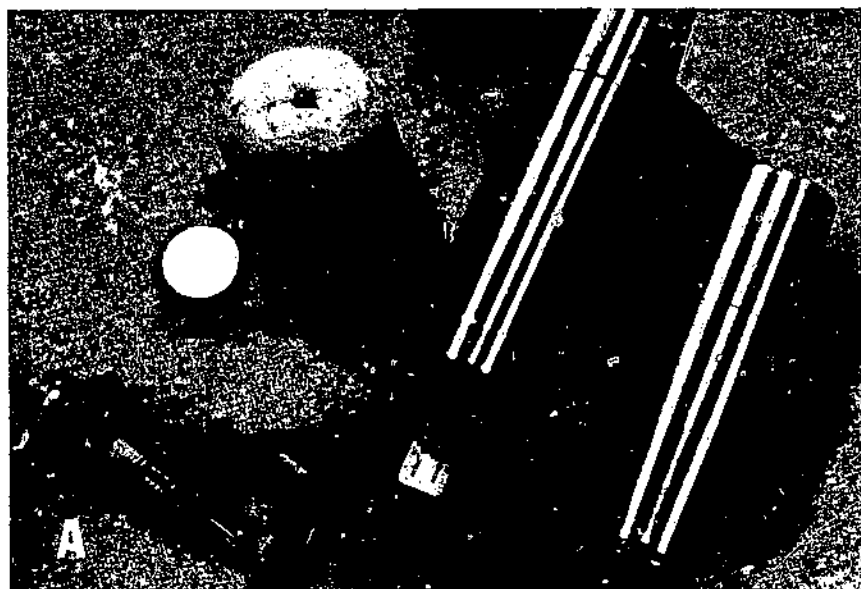
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FIGURE 6.—Equipment (A) and procedure (B) for obtaining rind and juice-vesicle test samples from the fruit for measuring moisture content and thermal conductivity.

Figure 2.— [illegible caption]
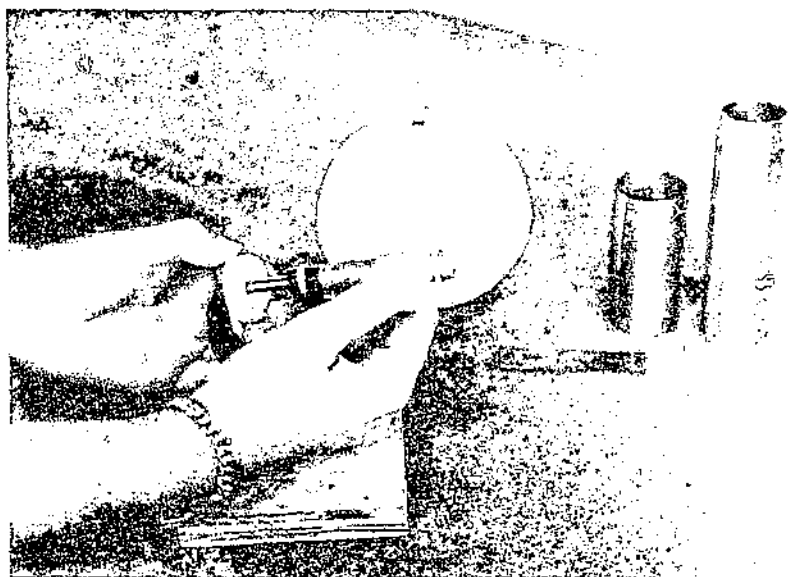
ANALYSIS AND RESULTS
Effective Thermal Diffusivity

The partial equations introduced by Fourier some 145 years ago permit an exact mathematical solution of problems involving heat transfer in homogeneous solids. He developed equations in the form of power series for objects having such basic shapes as the sphere, the cylinder, and the rectangle. Substitution of the boundary conditions for temperature, time, and object into the equation that describes temperature response within a particular geometric shape enables solution for temperature response as related to time and location within the object for a prescribed heating or cooling situation. And substitution of the empirically measured values of the temperature response of the object at time and location within a substance enables solution for the substance's thermal diffusivity, one of the basic heat transfer properties.

From Fourier's basic equations, several investigators have developed methods for the analysis of transient-state heat transfer problems in objects of various geometries. Most of these investigators are cited by Ingersoll and coworkers. In a valuable work describing and developing temperature-time curves for the three common geometries—the sphere, the infinite plate, and the infinite cylinder. Here the convergence of the various equations permits the temperature response in all geometries to be described in terms of a single

general form. If only the first term of the series and the straight-line approximation are used, a simple expression may be written. With application of the notation of Smith and coworkers (10), the equation takes the form

$$T = Ce^{-M_1^2 Fo} \tag{1}$$

where T is the dimensionless temperature ratio, $\dfrac{t - t_s}{t_i - t_s}$ (subscript i denotes initial temperature and subscript s denotes surface temperature), C and M_1 are functions of the object geometry, properties, and boundary conditions. The Fourier number, Fo, is a dimensionless ratio expressed in terms of $\alpha \tau / \ell^2$, where

$\alpha =$ thermal diffusivity, sq. ft. per (hr.)
$\tau =$ time, hours
$\ell =$ characteristic length, feet

When equation (1) is used to solve for the theoretical temperature response of an object in a specified heat transfer situation or to experimentally determine heat transfer properties of an object, it is necessary to evaluate M_1. The transcendental equation for a sphere is

$$N_{Bi} = 1 - M_1 \cot M_1 \tag{2}$$

where the Biot number, N_{Bi}, is a dimensionless ratio that describes the surface heat transfer capability as related to the heat transfer property and dimension of the object. It is mathematically expressed by $(hxr)/k$, where

$h =$ surface heat transfer coefficient, B.t.u. per (hr.) (sq. ft.) (° F.)
$r =$ radius of sphere, feet.
$k =$ thermal conductivity of object, B.t.u. per (hr.)(sq. ft.) (° F. per ft.)

Equation (1) is useful only when there is a temperature gradient within the object under consideration. There is a limiting boundary condition (surface heat transfer capability) that results in a negligible temperature gradient within an object being heated or cooled. For example, a small copper sphere being cooled in relatively still air will have an imperceptible thermal gradient along its radius at any time, which causes the Biot number to become infinitely small. Thus, M_1 approaches zero and T becomes equal to C for all values of time. When the temperature gradient is negligible, an evaluation of thermal diffusivity through the use of equation (1) is not possible.

As a contrast, a substance having relatively poor thermal conductance will exhibit a marked thermal gradient when heated or cooled if its surface suddenly becomes equal to the temperature of the surrounding fluid. Smith and coworkers (10) and Pflug and coworkers (7) describe the case for conditions of finite surface heat transfer resistance.

When Marsh grapefruit is suddenly immersed in a well-agitated water bath, the temperature on its surface can be assumed to rapidly approach the temperature of the cooling water. Thus, effective

thermal diffusivity can be evaluated on the basis of fruit temperature response. Because of the finite heat transfer resistance at the surface, the reciprocal of the Biot number is so small it can be neglected. However, acurate evaluation by the conventional method requires not only that specified boundary conditions be rigidly adhered to but also that the substance be homogeneous and that it conform to one of the conventional object geometries.

Whereas, for Marsh grapefruit, the specified boundary conditions can be easily satisfied, the requirement for homogeneity and object geometry is more difficult to satisfy. Actually, Marsh grapefruit is composed of constituency having widely varying properties and physical structure. In these evaluations, it must therefore be considered as a pseudohomogeneous material. In addition, it is shaped as an oblate spheroid. Failure to account for its departure from sphericity causes an error that is proportional to the magnitude of departure. Normally the equatorial diameter of Marsh grapefruit is 10 to 15 percent larger than the polar diameter. Therefore, the more accurate evaluation of effective thermal diffusivity of Marsh grapefruit is made by conceding its pseudohomogeneous composition and by making the necessary geometry correction.

Smith and coworkers (10) developed a technique of evaluating thermal diffusivity that corrects for deviation from the conventional shapes. The technique incorporates a geometry index, G, into the basic Fourier equation for a sphere. The geometry index was obtained "from a measure of two orthogonal areas of the shape." Use of this technique provides a means for a more accurate evaluation of effective thermal diffusivity of Marsh grapefruit than has previously been available. It is a significant breakthrough for investigators of thermal properties of biological materials having anomalous geometries. The equation, as presented by Smith and coworkers, is

$$\alpha = \frac{l^2}{M_1^2}\left[\frac{ln\,T_1 - ln\,T_2}{\tau_1 - \tau_2}\right]. \tag{3}$$

The ratio of the differences, $ln\,T$ vs. τ, is the slope, β, of the cooling curve for the corresponding time interval, or

$$\beta = \frac{ln\,T_1 - ln\,T_2}{\tau_1 - \tau_2}. \tag{4}$$

Therefore, the equation may be written

$$\alpha = \frac{l^2\beta}{M_1^2}. \tag{5}$$

When Marsh grapefruit is cooled in an agitated water bath, the surface heat transfer resistance is negligible. Hence,

$$M_1^2 = G\pi^2. \tag{6}$$

From 13 test runs, Smith and coworkers (10) measured an average thermal diffusivity of 3.63×10^{-3} sq. ft. per hr. for Marsh grapefruit.

This value compares favorably with 3.54×10^{-3} sq. ft. per hr. from 15 tests runs as measured by Perry and coworkers (6) on Marsh grapefruit. By substitution of the values of slope, characteristic length, and function of geometry index, M_1, (as measured for each test fruit) into equation (5), the effective thermal diffusivity was evaluated for each individual test run. These values, averaged by maturity group, are listed in table 1. The method of evaluating slope is described in a later section (p. 22).

TABLE 1.—*Experimentally obtained effective thermal diffusivity of Marsh grapefruit; evaluated from temperature response at the center, one-half the radius, and three-fourths the radius*

[Average of 10 runs for each maturity group]

Maturity group [1]	Center		One-half radius		Three-fourths radius	
	Effective thermal diffusivity	Coefficient of variation	Effective thermal diffusivity	Coefficient of variation	Effective thermal diffusivity	Coefficient of variation
	10^{-3} sq. ft./hr.	Percent	10^{-3} sq. ft./hr.	Percent	10^{-3} sq. ft./hr.	Percent
1	3. 44	5. 43	3. 76	5. 97	4. 11	6. 67
2	3. 15	5. 12	3. 37	5. 42	3. 59	5. 81
3	2. 74	5. 00	2. 93	5. 39	3. 05	5. 65
4	2. 95	5. 10	3. 15	5. 45	3. 36	5. 94
5	3. 09	5. 47	3. 37	5. 96	3. 64	6. 55

[1] Harvest dates for maturity groups are given on p. 2.

If sufficient time is allowed for the rate of temperature change to be uniform throughout a solid homogeneous sphere, the slope of the linear temperature response will be equal at all points along any radial coordinate. Because of grapefruit characteristics and of ambiguity associated with heat conduction along the probe, temperature response of Marsh grapefruit produces a small difference in slope at the three points along the radius. This difference is reflected in the resulting values of effective thermal diffusivity as evaluated on the basis of temperature response at the center, one-half the radius in the equatorial plane, and three-fourths the radius in the same plane. We believe the more accurate values are taken at three-fourths the radius, which is approximately the point of mass-average temperature.

The effective thermal diffusivity of Marsh grapefruit might be arbitrarily assumed to vary with temperature in somewhat the same proportion as water; i.e., a decrease of about 10^{-5} sq. ft./hr. for each ° F. temperature reduction in the range considered. The averaging effect of the above described method—that of evaluating effective thermal diffusivity based on time-temperature response within a homogeneous sphere in an agitated bath—negates this phenomenon. But this method yields values for specified times corresponding to known values of fruit temperature, which may be statistically analyzed for correlations of temperature on thermal diffusivity.

From 631 observations in 50 test runs, the linear correlation of mass-average temperature with effective thermal diffusivity, based on tem-

perature response at the center, was found to be

$$t = -7.42 + 13592.24\alpha. \tag{7}$$

From a regression analysis, in which five types of curves were tested, this was the best fitting linear response obtained. Average values so obtained are approximately 22 percent greater than those found by applying the geometry correction to the first-term approximation. Interestingly, the deviation is of the same order of magnitude and in the same direction as that found when the first-term approximation is used without applying the geometry correction described above. The solid line of figure 8 illustrates the correlation as it stands without applying the geometry correction. A constant reduction of 22 percent over the temperature range yields a set of values comparable with those obtained by the geometry correction technique and that show the influence of temperature on diffusivity. The corrected values are illustrated with the dashed line in figure 8.

Dimensions and physical properties, averaged by maturity groups, are listed in table 2.

Evaluation of effective thermal diffusivity for the whole fruit was made on the basis that the fruit rind and juice-vesicle components comprise one homogeneous mass constituency. Actually, the two components contrast sharply in their texture and composition. The
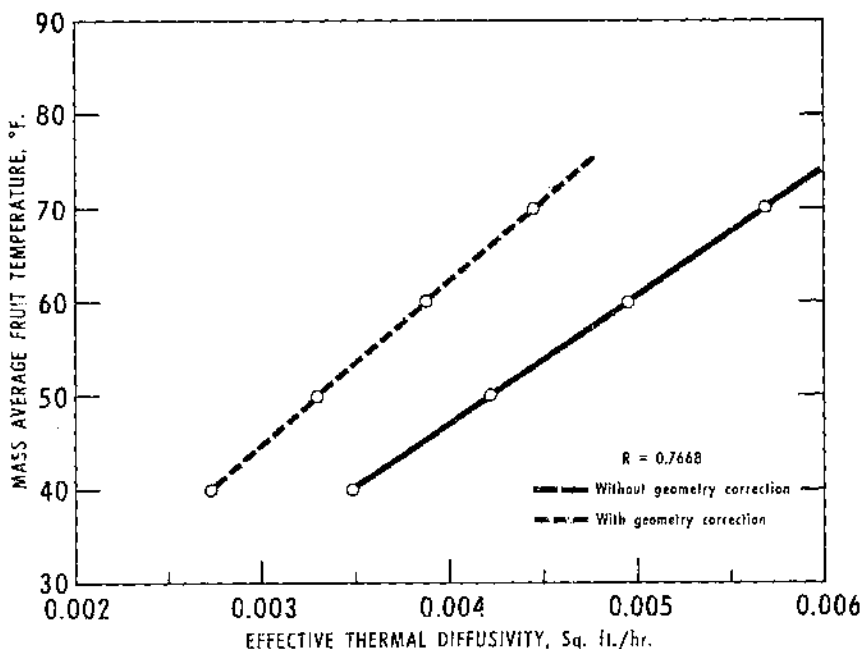

FIGURE 8.—Influence of temperature on experimentally evaluated effective thermal diffusivity of Marsh grapefruit.

TABLE 2.—*Diameter, specific gravity, thickness of rind, and moisture content of rind and juice-vesicle components for Marsh grapefruit*

[Average of 10 runs for each maturity group]

Maturity group [1]	Equatorial diameter	Coefficient of variation	Polar diameter	Coefficient of variation	Specific gravity (whole fruit)	Coefficient of variation
	Inches	*Percent*	*Inches*	*Percent*		*Percent*
1_____	4. 134	3. 3	3. 806	7. 9	0. 84	3. 2
2_____	4. 162	2. 8	3. 644	5. 4	. 86	3. 0
3_____	3. 895	3. 6	3. 431	6. 0	. 82	2. 8
4_____	3. 975	2. 5	3. 525	3. 5	. 85	2. 0
5_____	4. 038	5. 5	3. 557	5. 6	. 88	1. 8

Maturity group [1]	Rind thickness	Coefficient of variation	Moisture content of rind	Coefficient of variation	Moisture content of juice vesicle	Coefficient of variation
	Inches	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*
1_____	0. 306	13. 5	79. 3	3. 7	87. 8	0. 7
2_____	. 252	13. 4	77. 2	2. 0	87. 3	1. 2
3_____	. 307	21. 6	79. 4	2. 6	88. 4	. 6
4_____	. 253	15. 3	79. 6	1. 3	88. 8	. 8
5_____	. 224	15. 5	77. 8	1. 9	88. 9	1. 4

[1] Harvest dates for maturity groups are given on p. 2.

rind is composed of what is called the albedo and the flavedo. The albedo is a spongy layer of loosely arranged cells with many gas-filled intercellular spaces. The flavedo, or epidermal layer, contains numerous oil sacs and is more dense than the albedo. The juice-vesicle component is composed of numerous tightly arranged liquid-filled sacs. The density of the rind is about one-half that of the juice vesicles.

However, the difference in density does not affect the experimentally evaluated thermal diffusivity. As Perry and coworkers (6) described it, "in a given material where density changes because of changes in porosity, the thermal conductivity is approximately proportional to density, so that the diffusivity remains about constant." They confirmed their theory by a numerical solution involving two concentric spheres of known thermal properties. Further support of their theory was gained from the results of a more thorough scrutiny of the two separate components. The thermal conductivity, density, and specific heat were expressed in terms of a calculated diffusivity for the rind and for the juice vesicles separately to provide a comparison of the two respective values for each maturity group. The results are listed in table 3.

The equation for determining the calculated thermal diffusivity, α, is

$$\alpha = \frac{k}{\rho c_p}$$

where

> $k=$thermal conductivity, B.t.u. per (hr.) (sq.ft.) (° F. per ft.)
> $\rho=$density of substance, lbs. per (cu. ft.)
> $c_p=$specific heat of substance, B.t.u. per (lb.) (° F.)

Specific heat data were calculated from Siebel's equation [1] on the basis of moisture-content data reported in table 2. Density values were obtained by Otto Jahn,[2] during the 1964–65 season, from fruit other than that used in this work. Thermal-conductivity data were experimentally evaluated as described later (p. 15).

The similarity in thermal diffusivity of the rind and juice-vesicle components calculated by equation (8) substantiates the logic of Perry and coworkers (6). Values so obtained are noticeably larger than those reported in table 1. This difference may be attributable to the influence of temperature on thermal diffusivity; it may be the result of convection heat transfer in the rind and juice-vesicle samples during thermal-conductivity tests; or possibly it may be the result of both. Table 1 values are based on measurements taken at an average temperature of approximately 55° F. Table 3 values are based on measurements taken at logarithmic mean temperature of 80° for the rind and 88° for the juice vesicle. Temperature effect is seen in figure 6. The texture and moisture content of the specimen is such that thermally induced fluid movement within the intercellular spaces of the rind and within each individual juice vesicle sac can be significant. The high temperature gradient across the sample is certain to induce some fluid movement. Consequently the convection component could cause the values to be larger than they would be if heat transfer were by conduction only.

TABLE 3.—*Specific heat, density, and calculated thermal diffusivity of the rind and juice-vesicle components of Marsh grapefruit*

Maturity group [1]	Specific heat,[2] B.t.u./lb./°F.		Density,[3] lb./cu. ft.		Thermal diffusivity,[4] 10^{-3} sq. ft./hr.	
	Rind	Juice vesicle	Rind	Juice vesicle	Rind	Juice vesicle
1_____	0. 835	0. 901	35. 1	63. 4	4. 80	4. 50
2_____	. 818	. 898	36. 1	63. 3	4. 65	4. 94
3_____	. 835	. 906	35. 2	63. 7	4. 75	4. 95
4_____	. 837	. 910	35. 6	62. 9	4. 79	4. 94
5_____	. 822	. 911	38. 7	62. 7	4. 34	4. 45

[1] Harvest dates for the maturity groups are given on p. 2.
[2] Based upon average of 10 runs within each maturity group.
[3] Personal communication from Otto Jahn.
[4] Calculated from experimentally obtained thermal conductivity.

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[1] $c_p=0.008$ X percent moisture content $+0.20$.
[2] Personal communication.

Characteristic Thermal Conductivity of Rind and Juice Vesicle

Characteristic thermal conductivity for the rind and juice-vesicle components was evaluated by use of the equation

$$k = \frac{C}{L\,b} \tag{9}$$

where the constant, C, is a measure of the heat-retaining capacity of the copper sink that is located in the base of the Cenco-Fitch apparatus. It has a value of 3.3 B.t.u. per (sq. ft.) (° F.). The slope, b, is the coefficient of linear regression of time (hours) on a function of the temperature difference between the heat source (upper copper plate) and the heat sink. Test specimen thickness, L, is expressed in inches. A more thorough description of the experimental and analytical procedure used may be found in the report of Bennett and co-workers (2). The results, shown as averages by maturity groups, are listed in table 4.

TABLE 4.—*Measured values of characteristic thermal conductivity, k, for the rind and juice-vesicles components of Marsh grapefruit*

[Average of 10 runs for each maturity group]

Maturity group [1]	Rind thermal conductivity	Coefficient of variation	Juice-vesicle thermal conductivity	Coefficient of variation
	B.t.u. per (hr.) (sq. ft.) (° F. per ft.)	Percent	B.t.u. per (hr.) (sq. ft.) (° F. per ft.)	Percent
1	0. 1412	11. 9	0. 2562	9. 3
2	. 1371	13. 1	. 2810	6. 2
3	. 1398	6. 5	. 2848	5. 6
4	. 1426	9. 6	. 2822	9. 0
5	. 1379	7. 9	. 2539	4. 0

[1] Harvest dates for maturity groups are given on p. 2.

Because of the conditions discussed in the preceding section, to simply call these values "thermal conductivity" would be a misnomer. Instead, it seems appropriate to use the term "characteristic thermal conductivity" to describe the heat transfer property obtained by the foregoing procedure.

Values of thermal conductivity, specific heat, and density that characterize the total mass constituency are needed for computation of an apparent value of thermal diffusivity for the whole Marsh grapefruit by use of equation (8). Such values would not be accurate, but they would provide a characteristic measure of the respective properties of the combined components. For this purpose, an apparent thermal conductivity for the whole fruit is calculated by utilizing the measured values obtained for the separate components. If the juice-vesicle and rind sections are assumed to be two hollow concentric

spheres (fig. 6), the total resistance to heat transfer is

$$\Omega_t = \Omega_{12} + \Omega_{23} \tag{10}$$

where, resistance through the juice vesicles is

$$\Omega_{12} = \frac{(r_2 - r_1)}{4\pi k_{12} r_1 r_2} \tag{11}$$

through the rind is

$$\Omega_{23} = \frac{(r_3 - r_2)}{4\pi k_{23} r_2 r_3} \tag{12}$$

and through both the juice vesicles and the rind is

$$\Omega_t = \frac{(r_3 - r_1)}{4\pi k_{app} r_1 r_3}. \tag{13}$$

Equations (11), (12), and (13) can be combined and simplified to

$$k_{app} = \frac{k_{12} k_{23} r_2 (r_3 - r_1)}{k_{12} r_1 (r_3 - r_2) + k_{23} r_3 (r_2 - r_1)}. \tag{14}$$

This technique is valid and is widely used for computing a single factor that denotes the heat transfer characteristic of a heterogeneous mass constituency consisting of several adjoining layers of different materials. Examination of values of apparent thermal conductivity, (table 5) indicates that the effect of the rind is almost negligible. A significant bias in favor of the juice-vesicle component is evident. When this factor is used in equation (8) to compute apparent thermal diffusivity, the inherent bias is reflected in the results.

The product of specific heat and density is a single factor that denotes the heat capacity per unit volume of a substance. Values given in table 5 represent the sum of partial capacities of the rind and juice-vesicle components based upon proportionate volumes of test fruit. The rind of Marsh grapefruit constitutes 30 to 40 percent

TABLE 5.—*Calculated values of apparent thermal conductivity and apparent thermal diffusivity for the whole Marsh grapefruit, based on measured thermal conductivity for the rind and juice-vesicle components*

Maturity group [1]	Heat capacity	Apparent thermal conductivity	Apparent thermal diffusivity
	B.t.u./cu.ft./° F.	*B.t.u./hr.ft.° F.*	*Sq.ft./hr. ×10⁻³*
1	46. 0	0. 2513	5. 5
2	45. 8	. 2755	6. 0
3	45. 8	. 2768	6. 0
4	47. 6	. 2764	5. 8
5	49. 2	. 2502	5. 1

[1] Harvest dates for the maturity groups are given on p. 2.

of its total volume. Its unit heat capacity is about half that of the juice vesicle and, therefore, is only 20 to 25 percent of the total. Consequently, the weighted heat capacity of Marsh grapefruit is approximately 20 percent less than it would be if the effect of the rind were neglected. Specific heat and density (table 3) were used for the weighted computations.

Apparent thermal diffusivity values reported in table 5 are 37.5 percent greater than those reported in table 1. Part of this discrepancy may be attributed to the reasons explained in the preceding section (p. 14). However, most of the error seems to be caused by the bias introduced through use of apparent thermal conductivity in relation to weighted heat capacity. If specific heat and density for the whole grapefruit are used as a basis for computing apparent thermal diffusivity, the results are comparable with those reported in table 3. Hence, the error probably is a product of bias attributable to the technique used for obtaining the results.

Correlations

Statistical analyses were made to ascertain correlations, by maturity groups and over all maturity groups, between the following variables: (1) Rind thickness, (2) rind thermal conductivity, (3) juice-vesicle thermal conductivity, (4) effective thermal diffusivity, (5) rind moisture content, (6) juice-vesicle moisture content, and (7) specific gravity. An analysis of all possible correlations over all maturity groups revealed five statistically significant correlations. In addition, four other relations reflected a strong tendency to correlate, but these were not significant at the 5-percent level of probability. (See following tabulation.)

	Correlation coefficients (r) [2]
Variable correlation: [1]	
Rind thickness on specific gravity	−0. 7823
Rind moisture content on specific gravity	−. 5914
Rind thickness on rind moisture content	. 4867
Rind moisture content on juice-vesicle moisture content	. 4059
Thermal conductivity of juice vesicle on specific gravity	−. 3049
Thermal conductivity of rind on juice-vesicle moisture content	. 2645
Thermal conductivity of rind on rind thickness	−. 2553
Thermal conductivity of rind on rind moisture content	. 2004
Rind thickness on fruit size	. 1991

[1] All maturity groups combined.
[2] Critical value of r, 5-percent level, 0.273.

From these correlations a considerable interaction between rind thickness, moisture content of the rind and juice vesicle, thermal conductivity of the rind and juice vesicle, and specific gravity is noted, with the specific gravity having the greatest influence. However, when maturity groups are considered separately, measured effective thermal diffusivity is found to correlate directly with specific gravity and inversely with rind thickness (table 6). In the absence of a clarifying explanation, the seemingly wayward results of maturity group 5 must be attributed to experimental error.

TABLE 6.—*Correlation of measured effective thermal diffusivity on specific gravity and rind thickness for Marsh grapefruit by maturity groups*

Maturity group [1]	Correlation coefficients (r) [2]	
	Thermal diffusivity on specific gravity	Thermal diffusivity on rind thickness
1	0. 5218	—0. 3814
2	. 6339	—. 4690
3	. 7914	—. 5598
4	. 8295	—. 6084
5	—. 1436	. 4290

[1] Harvest dates for the maturity groups are given on p. 2.
[2] Critical value of r, 5-percent level, 0.602.

Another interesting result of this investigation of correlations is the effect of harvest date on certain of the physical properties. This effect was not evaluated statistically, but it is apparent from the data given in table 2. Results ndicate that as the harvest season advances the fruit becomes more dense, its rind becomes thinner, and the moisture content of the rind decreases. Harvest date apparently did not affect moisture content of the juice vesicles. A notable tendency for measured effective thermal diffusivity to correlate with harvest date if maturity group 1 is omitted from consideration (table 1). This omission is considered valid because of experimental error encountered in maturity group 1, which caused the values to be significantly larger than those in the other four groups.

Temperature Distribution

Multiple regression analysis

As described under "Experimental Procedure," the temperature history of each test fruit was measured at $\frac{1}{4}$-inch intervals along the radius in the equatorial plane. The multiple regression technique of curve fitting was used to compute polynomial coefficients that express temperature distribution within the fruit as related to time for each maturity group. The model for the prediction equation is of the form

$$Y = a + b_1 x_1 + b_2 x_2 + b_3 x_1^2 + b_4 x_2^2 + b_5 x_1^3 + b_6 x_2^3 + b_7 x_1 x_2$$

where, in this example,

$Y =$ the dimensionless temperature variable
$x_1 =$ time
$x_2 =$ position in the fruit
$a =$ intercept
b_1 to $b_7 =$ constant coefficients

when the temperature and position variables are expressed as nor-

malized ratios, the resulting coefficients should be essentially equal in all planes. Computed values for the respective maturity groups are listed in table 7. The coefficient of variation among maturity-group means for temperature was 5.22 percent.

Temperature distribution during cooling does not differ statistically between maturity groups or between runs within a maturity group. Consequently, for practical application, predicted values from any of the five groups will adequately describe temperature distribution within Marsh grapefruit during cooling with negligible surface heat transfer resistance. Figure 9 illustrates the internal temperature distribution within Marsh grapefruit, initially at 85° F., being cooled in agitated ice water at 35°. Data for similar curves may be calculated by substituting the appropriate constant coefficients into the regression equation and solving for temperature ratio for any number of specified conditions.
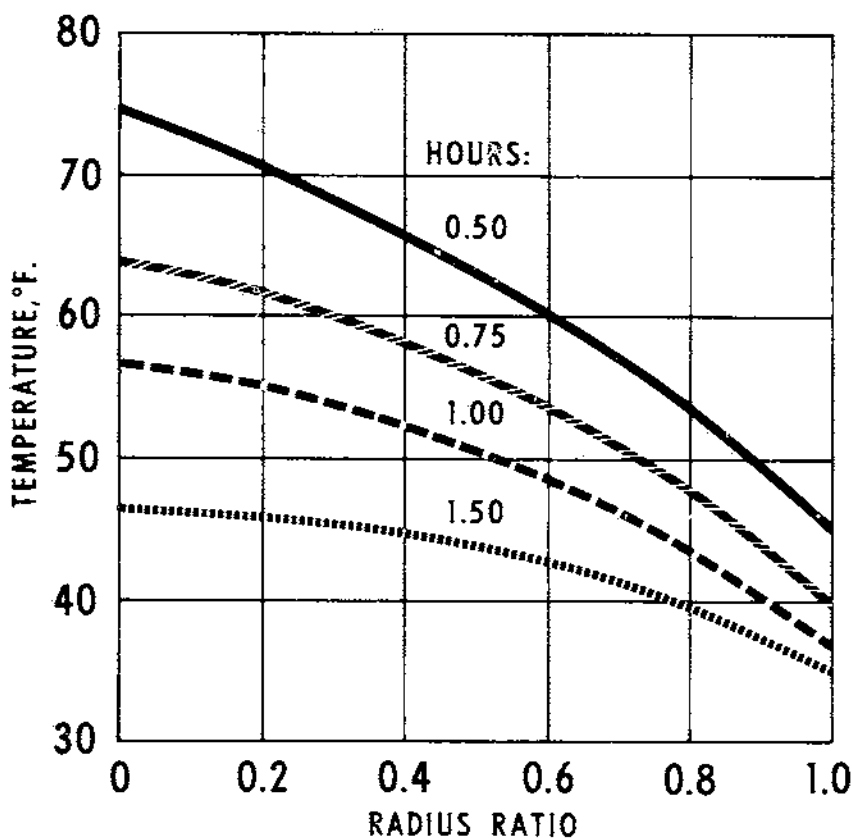

Figure 9.—Temperature response within a Marsh grapefruit, initially at 85° F., cooled in agitated water at 35°. Calculated from prediction equation for maturity group 4.

TABLE 7.—*Intercepts and polynomial coefficients for temperature distribution in each of 5 maturity groups of Marsh grapefruit*

Maturity group [1]	a	b_1	b_2	b_3	b_4	b_5	b_6	b_7	Coefficient of variation
									Percent
1	1.2424	−1.1601	−0.4334	0.3973	−0.5532	−0.0562	0.1841	0.3852	13.96
2	1.2621	−1.1340	−.4159	.3647	−.1710	−.0460	−.1410	.3202	4.45
3	1.2326	−1.1063	−.4588	.3713	−.0786	−.0508	−.1801	.3093	3.80
4	1.2762	−1.2196	−.5117	.4540	−.0858	−.0731	−.1548	.3454	5.10
5	1.2436	−1.2058	−.3590	.4313	−.3293	−.0597	−.0247	.3243	4.60

[1] Harvest dates for the maturity groups are given on p. 2.

Cooling curves plotted from prediction data do not precisely conform to actual temperature distribution patterns. The curve-fitting analysis is an approximating procedure; hence, small irregularities, whether actually present or experimentally induced, are not shown. From figure 9 it appears that the temperature distribution from center to surface is smooth, with no sudden change in gradient between the juice-vesicle and rind components. Actually, because of the relative resistance of heat transfer, there is a noticeable change in gradient at the interface between the two components. This phenomenon is discussed in the following section.

Effect of rind on temperature distribution

Findings reported in earlier sections have shown that the three Marsh grapefruit rind properties, (1) thermal conductivity, (2) specific heat, and (3) density, are not equal to the corresponding juice-vesicle properties. Heat, during transient cooling, flows through these two different materials, the rind and the juice vesicles, along radial coordinates moving from center to surface (fig. 10).

According to laws of heat transfer, it is possible to have a discontinuity of temperature and of temperature gradient at the interface between the two materials. It has been shown that the thermal diffusivity of the two components is essentially equal. From Schneider's "Temperature Response Charts" (8), it is seen that the temperature
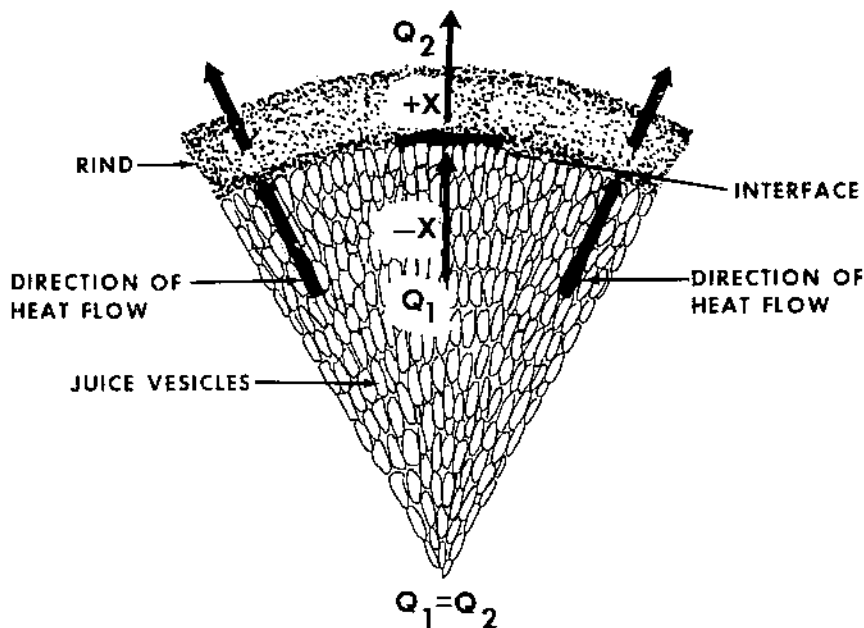

FIGURE 10.—Section of Marsh grapefruit illustrating the different properties of the rind and juice-vesicle components and the flow of heat through the interface between the two components.

at the interface of the two adjoining materials is continuous. Therefore, there is no change in rate of heat transfer through the boundary between the two materials. At an instantaneous time, the heat, dQ_1, that leaves the juice vesicles in the direction of positive x, is equal to the heat, dQ_2, that enters the rind. When these two elemental heat quantities are equated, the equation is

$$k_1 \left(\frac{dt}{dx}\right)_1 = k_2 \left(\frac{dt}{dx}\right)_2. \tag{15}$$

From table 4 it is noted that $k_1 \neq k_2$. Therefore, to satisfy the condition of equation (15), the temperature gradient $(dt/dx)_1$ does not equal the temperature gradient $(dt/dx)_2$. As Gröber and coworkers (5) observed: "Since there is a discontinuity in k at the point considered, it follows therefore that there must also be a discontinuity in the temperature gradient, and this is true not only for the steady state but also for the unsteady state."

The cooling of Marsh grapefruit involves unsteady state heat transfer through the juice-vesicle and rind components. Since the properties of these components are different, there is a difference in the temperature gradient between the two. There is, therefore, a "break" in the temperature distribution curve at the interface between the two materials. This "break" is not noticeable when the temperature distribution is plotted from the prediction equation. However, when raw experimental data is plotted, the response is evident. This response, representing an average of 10 test runs from maturity group 4, is shown in figure 11.

Linear regression analysis

Solution of the regression equation containing the polynomial coefficients listed in table 7 produces a nonlinear response when plotted on rectangular coordinates. For specified values of time and distance ratio, a family of curves is generated. When these data points are plotted on semilogarithmic coordinates, the result will be a family of straight lines whose slopes are a function of time, position, product geometry, and heat transfer property. This linear response may also be evaluated by converting the dimensionless temperature variable to logarithms and computing a linear regression. If there is a close "fit" of the data points to a straight line in the graphical analysis, the results of the two methods will be comparable; i.e., essentially equal slopes.

A linear regression of temperature as related to time—at the center, at one-half the radius, and at three-fourths the radius—was computed for each Marsh grapefruit test run. The general form of the equation is

$$Y = a + \beta x_1$$

where, for this application, Y is $\log_e T$, a is the intercept, and β is the constant coefficient corresponding to the slope. The slope β is used in equation (5) for computing effective thermal diffusivity. The
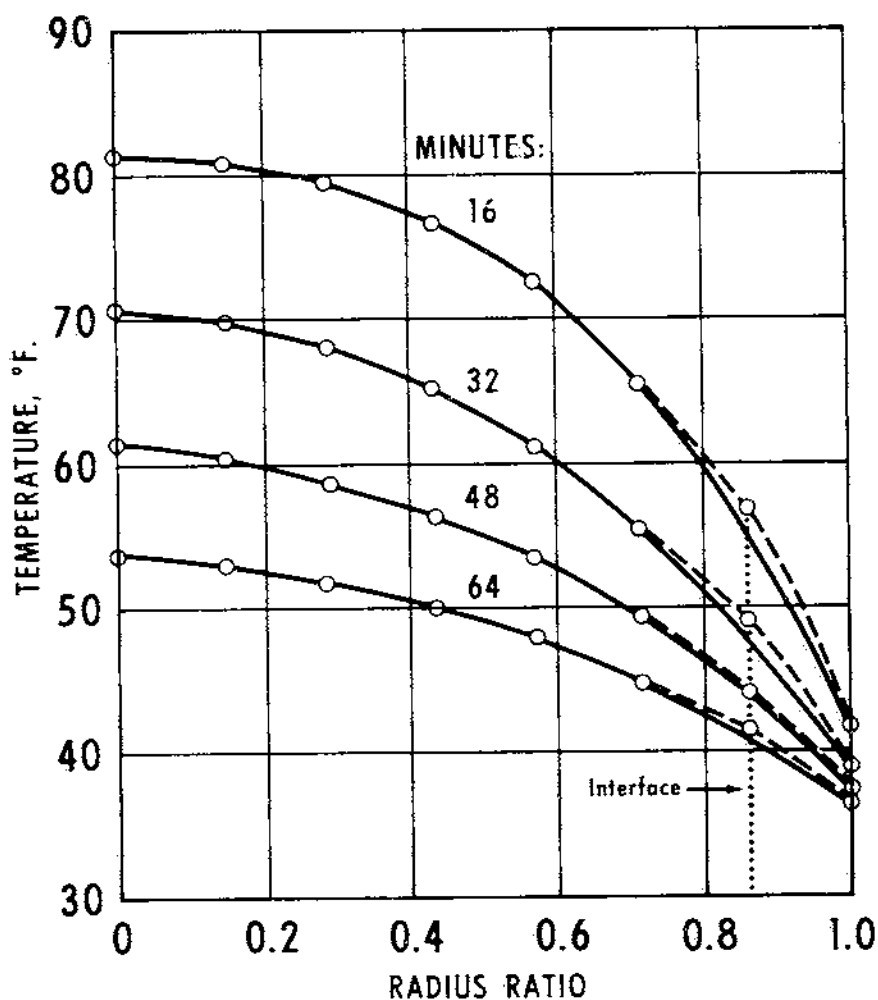
FIGURE 11.—Internal temperature distribution in Marsh grapefruit plotted from raw experimental data recorded during transient cooling test runs. Dotted lines show discontinuity in temperature gradient at interface. Average of 10 test runs from maturity group 4.

intercepts and coefficients are listed in table 8 as maturity-group means. The linear response of maturity group 4 is shown in figure 12.

Pflug and coworkers (7) plotted the actual product temperatures on a logarithmic scale so that the change of temperature per unit change of time could be read directly. This procedure provides a simple and easily understood way of showing the temperature response within a product during cooling. It, however, has the disadvantage of applying to specific conditions of the product and the surrounding fluid temperature. In practice, these conditions are

TABLE 8.—*Linear temperature response parameters for Marsh grapefruit cooled in agitated ice water*

[Maturity group means]

Maturity group and location of response [1]	Slope	Intercept	Coefficient of variation	Correlation coefficient
1:				
A	−1.23	0.29	5.44	−0.96
B	−1.35	.11	5.97	−.95
C	−1.47	−.32	6.67	−.93
2:				
A	−1.17	.29	5.12	−.97
B	−1.25	.20	5.42	−.98
C	−1.33	−.12	5.81	−.98
3:				
A	−1.14	.26	5.00	−.97
B	−1.22	.17	5.39	−.97
C	−1.27	−.16	5.65	−.96
4:				
A	−1.17	.29	5.10	−.99
B	−1.26	.19	5.45	−.99
C	−1.34	−.14	5.94	−.97
5:				
A	−1.23	.28	5.47	−.96
B	−1.34	.19	5.96	−.97
C	−1.46	−.15	6.55	−.95

[1] Harvest dates for maturity groups are given on p. 2. A, Center of fruit; B, one-half of radius; C, three-fourths of radius.

generally confined to a narrow range, and two curves of upper and lower limits may be used to encompass a band of normally expected conditions. Temperature-time curves for specified points within the fruit may be plotted from the empirically based coefficients listed in table 7. The parameters listed in table 8 may also be used to plot temperature response at the locations shown.

Mass-average temperature

The concept of mass-average temperature is receiving increased acceptance among engineers and scientists engaged in the design of, or research toward, development of refrigeration systems for cooling (or heating) perishable food products. This concept is particularly important with rapid precooling where there is likely to be a temperature gradient from the center to the surface of the substance being cooled.

The amount of heat stored in or released from a substance in a given time is ascertained by measuring the temperature. If the temperature throughout the substance is uniform, measurement may be made without regard to location within the substance. If the temperature is not uniform, however, an average temperature must be used. If the gradient is linear, an average is easily obtained. When the gradient is not linear, which is usual, a mass-average tempera-
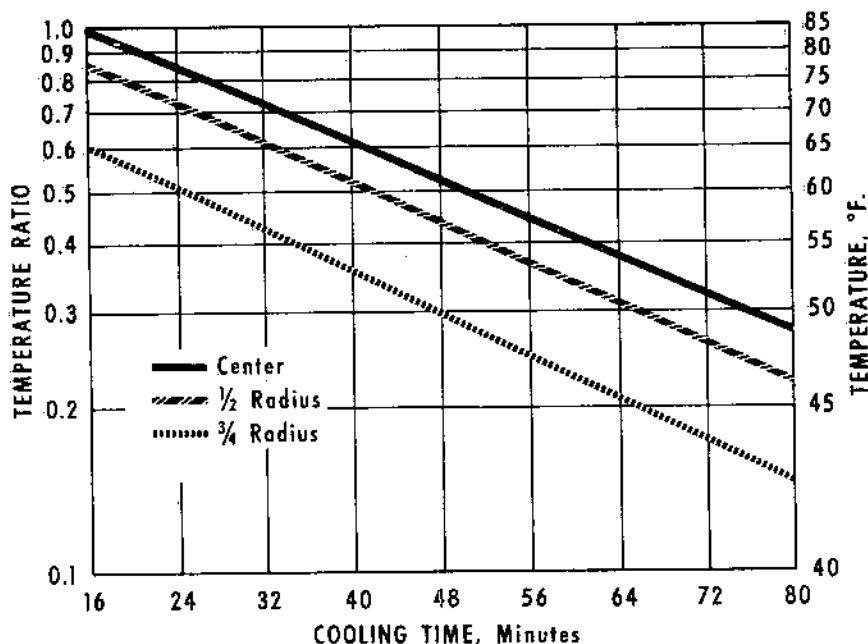
FIGURE 12.—Linear temperature response of Marsh grapefruit, initially at 85° F., cooled in agitated ice water at 35°. Fruit harvested in February 1965 (maturity group 4).

ture, based on the nonlinear temperature distribution as related to time, should be evaluated.

When Marsh grapefruit at room temperature is immersed in a bath of agitated ice water, its surface temperature suddenly becomes essentially equal to the temperature of water. Initially, there is a steep temperature gradient near the surface of the fruit. As cooling progresses, the gradient moves toward the center and diminishes as it moves inward. Eventually, the gradient vanishes. The fruit is then at the temperature of the water throughout. This characteristic relation of surface to internal heat transfer is discussed in the section on evaluation of effective thermal diffusivity. The application of this phenomenon in the evaluation of the magnitude and location of mass-average temperature during rapid precooling of Marsh grapefruit is described in this section.

The method of Smith and Bennett (9) for evaluating the mass-average temperature within a substance during transient cooling makes use of the expression for internal temperature distribution as related to time. By substituting specified time values into the polynomial prediction equation, they obtained a set of equations (one for each time) that express the temperature ratio, $Y(R)$, in terms of the radius ratio, R. The equation takes the form

$$Y(R) = a + bR + cR^2 + dR^3 \tag{16}$$

where a, b, c, and d are the constant coefficients of R.

The mass-average temperature of a substance denotes a measure of the heat contained in the substance, above some reference level, at any time during heating or cooling. The heat content of a sphere, when $R=1$, was expressed by Smith and Bennett (9) in the form

$$Q = \frac{K\,Y_{ma}}{3} \tag{17}$$

where K is a constant for $4\pi\,\rho\,c_p$. By equating this equation to an equation that expresses the same heat content in terms of the temperature distribution along the radial coordinate, they obtained a function that expresses temperature ratio at the mass-average point for a specified time.

The heat content of the substance is a direct function of its volume, or mass. The volume of an oblate spheroid is less than the volume of a sphere whose diameter is equal to the major axis of the spheroid. Based on the average equatorial and polar diameters of the 50 test fruit used in this study, the volume coefficient of Marsh grapefruit is 0.883. Volume coefficient is the ratio of the volume of the average fruit specimen used in this study to the volume of a sphere whose diameter is equal to the equatorial diameter of the fruit, which means that the heat content of Marsh grapefruit is 88.3 percent that of a corresponding sphere. Introduction of this volume coefficient into equation (17) yields a measure of the heat content of an oblate spheroid whose dimensions correspond to the representative fruit of this study; that is,

$$Q = \frac{0.883\,K\,Y_{ma}}{3} = 0.2943\,K\,Y_{ma}. \tag{18}$$

When equation (18) is equated to the expression of heat content in terms of the temperature distribution along the equatorial plane of Marsh grapefruit, the equation obtained is

$$Y_{ma} = 1.132a + 0.848b + 0.679c + 0.566d. \tag{19}$$

Substitution of the above constant coefficients into equation (19) yields the solution of mass-average temperature ratio for the specified time. The point on the radius where this particular mass-average temperature occurs may be found by either graphical or analytical solution of equation (16).

Application of this method for maturity group 4 takes the following procedure:

Insert the appropriate polynomial coefficients from table 7 into the model equation and solve for temperature ratio, Y_{ma}, in terms of radius ratio, R. For a cooling time of 30 minutes (0.5 hours), compute

$$Y(R) = 0.7708 - 0.3390R - 0.0858R^2 - 0.1548R^3 \tag{20}$$

Thus

$$a = 0.7708;\ b = -0.3390;\ c = -0.858;\ d = -0.1548.$$

Substitute the respective coefficients into equation (19) and solve for Y_{ma}.
Result:

$$Y_{ma} = 0.4391.$$

Problem: Initial fruit temperature is 90° F.
 Surface temperature during cooling is 35°.
Solution:

$$Y_{ma} = \frac{t - ts}{t_t - ts}$$

$$0.4391 = \frac{t - 35°}{90° - 35°}$$

$$t = 59.15°F.$$

The point along the radius in the equatorial plane where this mass-average temperature occurs can be found by substituting the value obtained for Y_{ma} into $Y(R)$ of equation (29) and solving for R. The result is

$$R = 0.699.$$

This process is one of trial and error. Values of predicted magnitude and location of mass-average temperature may be similarly computed for other cooling times. Values so computed for maturity group 4 are illustrated in figure 13.

CONCLUSIONS

The effective thermal diffusivity of Marsh grapefruit may be accurately evaluated by using the first-term approximation of Fourier's Sine-Series temperature response and applying Smith's correction factor.

There is no appreciable difference in the thermal diffusivity of the rind and juice vesicles when calculated from separately measured values of thermal conductivity.

The thermal conductivity of the rind of a Marsh grapefruit is about one-half that of the juice vesicles. When the thermal conductivity is measured by the method cited in this bulletin, a small convection component of heat transfer causes the resulting values to be higher than if conduction heat transfer alone were involved.

Apparent thermal conductivity, computed by summing the resistances of the rind and the juice-vesicle components, does not provide a meaningful measure of the heat transfer characteristic of a whole Marsh grapefruit unless the error introduced by inherent bias is corrected.

Findings have shown a significant interaction between rind thickness, moisture content of the rind and juice vesicles, thermal conductivity of both rind and juice vesicle, and specific gravity; and, also that as the harvest season advances from October through May fruit becomes more dense, its rind becomes thinner, the moisture content of the rind decreases. These findings are consistent with the
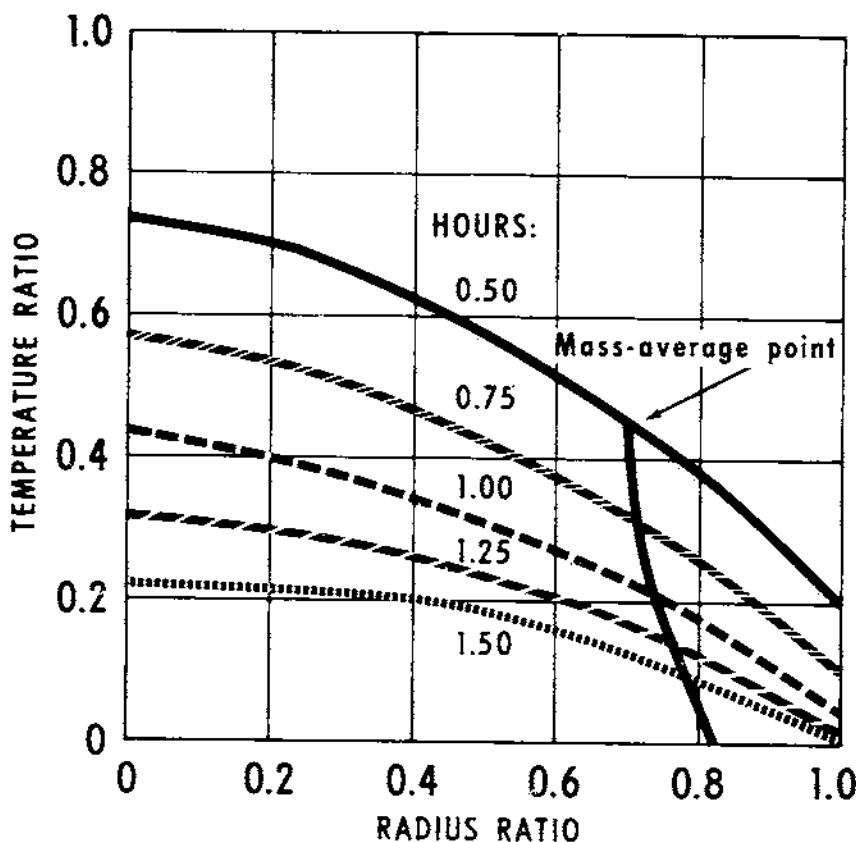
FIGURE 13.—Magnitude and location of mass-average temperature of Marsh grapefruit cooled in agitated ice water. From temperature response of maturity group 4.

observed increase in effective thermal diffusivity in relation to length of harvest season.

The effect of time of harvest on experimental temperature distribution was not considered statistically significant. However, the analytical technique was probably not sufficiently critical to detect the effect of harvesttime on temperature distribution.

The influence of the greater heat transfer resistance through the rind than through the juice vesicles is noticeable in experimental data recorded during transient cooling. This factor could possibly have a slight effect on cooling rate, but it would not be sufficient to be considered of significance for commercial application.

Because Marsh grapefruit is an oblate spheroid, its mass-average temperature is slightly nearer to the center than it would be if it were a perfect sphere.

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