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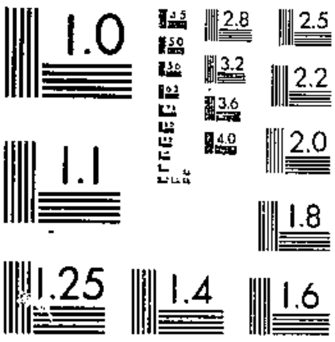
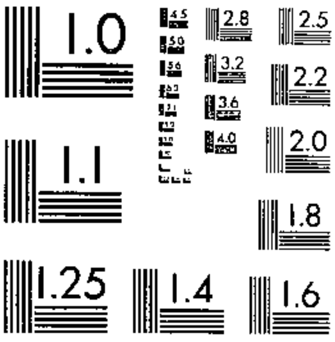


MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A



MICROCOPY RESOLUTION TEST CHART
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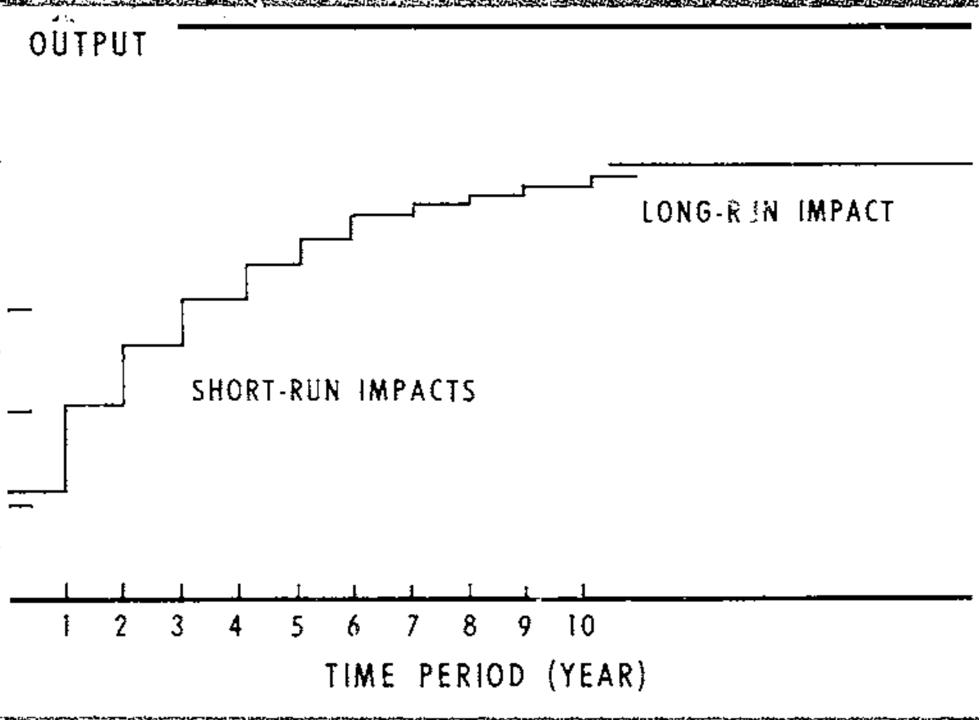
AN ECONOMIC ANALYSIS OF THE DYNAMICS OF THE UNITED STATES WHEAT SECTOR



TECHNICAL BULLETIN NO.1395 - ECONOMIC RESEARCH SERVICE
U. S. DEPARTMENT OF AGRICULTURE

PREFACE

The wheat industry is a major sector of U.S. agriculture. It currently accounts for about 5 percent of the gross income accruing to farmers in the United States. Moreover, wheat is the most important agricultural commodity exported and makes up about 20 percent of total U.S. agricultural exports.

The measurement of factors that affect domestic consumption and prices of wheat is necessary for carrying out programs to maintain equitable incomes to wheat producers and for determining the long-run economic outlook for the industry.

The analysis deals only with aggregate demands for wheat. The behavior of wheat supplies was not analyzed due to statistical and certain data problems. The analysis offers only a first approximation of the probable economic character of the total U.S. wheat industry and further studies are needed.

A significant part of the report is devoted to analysis of a particular econometric methodology. The mathematical background necessary for the reader to understand this report is some knowledge of vector and matrix operations, calculus, and probability theory. Several mathematical and technical derivations are included in the appendixes.

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Washington, D.C. APRIL 1968

SUMMARY

The econometric model described in this report can be used to make conditional long-run projections of utilization and average farm prices of wheat, and to estimate quantitatively the short-run and long-run impact of a change in the wheat support price on the wheat utilization in the United States. The model assumes that production is given, i.e., no attempt is made to explain changes in production.

The basic model, consisting of 6 equations, is a simple dynamic recursive system. Because of its recursive feature, the parameters of the structural relations were estimated by the ordinary least-squares and the two-round least-squares procedures. All signs of the estimated coefficients obtained from both procedures were in accordance with the theoretical and logical expectations. The estimated multiple correlation coefficients of the 6 equations ranged from 0.84 to 0.97.

The estimated short-run impact (single period) multipliers indicated that a dollar per bushel increase in the wheat support price, with all other predetermined variables in the system held constant, will lead to (1) an increase of 91 cents per bushel in average farm wheat price, (2) a decline of 0.21 bushel per capita per year in wheat food consumption, (3) a decline of 132 million bushels in wheat used for feed, (4) an increase of 116 million bushels in Government stocks, (5) a decline of 64 million bushels in commercial stocks, and (6) an increase of 24 million bushels in total U.S. wheat exports. The positive relation between exports and wheat price supports (and also the farm wheat price) results from the way in which wheat programs were operated during the analysis period 1928-64. During this period there usually was a considerable effort to reduce Government-held stocks through export programs.

The long-run impact (infinite period) multipliers are relevant only if the underlying model is a stable dynamic system. The stability of the dynamic system was established by a standard test. The estimated matrix of long-run multipliers revealed that a sustained increase of $1 per bushel in wheat support price will generate in the long-run (1) an increase in Government stocks of 453 million bushels--about 4 times as large as the short-run effect, (2) a decline in commercial stocks of 123 million bushels--about 2 times as large as the short-run effect, and (3) an increase in total U.S. wheat exports of 158 million bushels--7 times as large as the short-run effect.

Before using the estimated model for making predictions, retrospective analyses were made for testing the predictive performance of the model. The analyses indicated that (1) the predictions of average farm wheat prices and per capita wheat food consumption had a smaller prediction error in terms of absolute or relative magnitudes than the predictions of total wheat used for feed, Government stocks, commercial stocks, and U.S. wheat exports, and (2) the predictions of total Government wheat stocks, commercial wheat stocks, and total U.S. wheat exports have a very large relative overprediction error at the very low levels of the observed values.

Long-run projections were made under four alternative wheat support prices and other exogenous variables which were estimated by an auto-regressive time trend model.

AN ECONOMIC ANALYSIS OF THE DYNAMICS OF THE UNITED STATES WHEAT SECTOR

by
William Y. Mo
Economic Research Service

INTRODUCTION

The specific objectives of this study are (1) to identify and measure quantitatively the basic demand structure of the U.S. wheat sector, (2) to estimate the short-run and long-run impact multiplier matrix, and (3) to adapt the estimated model in making long-run projections of U.S. wheat utilization.

In attempting to quantify the important underlying demand structure of the U.S. wheat sector, a very simple and highly aggregate econometric model is formulated in the first part of the study. The reasons for formulating such a simple and highly aggregate model are: (1) Adequate empirical data with which to estimate a more complex and much disaggregated model are lacking and (2) the simple aggregate model is much easier to use in making future predictions than a more complex one. Of course, such a simple aggregative model has limitations because it does not reflect explicitly the many varieties of wheat produced and many different end products involved in the wheat market. The empirical results are presented in the second part of the study. Finally, the implications deduced from the estimated aggregate model and its applications in making predictions are analyzed and discussed in the last part of the study.

THE MODEL

To facilitate the presentation of the econometric model of the U.S. wheat sector, a simplified descriptive version of wheat supply and utilization is given in the following section.

SOURCES OF SUPPLY AND TYPES OF UTILIZATION

The sources of supply and types of utilization of wheat in the United States are shown, in simplified diagrammatic form, in figure 1. The physical variables are identified by a letter symbol in each box. The subscript "t" of letter symbols refers to the current time period, while "t-1" refers to the preceding time period.

As indicated in figure 1, the sources of total current supply (Q_t) consist of current domestic production (O_t), total import (M_t), and total carry-in stock from the preceding time period (C_{t-1}). During 1959-63, imports of wheat accounted for only 0.24 percent of the total supply in the United States. Hence, domestic production, together with carry-in stock, constitute the major sources of total supply.

The total available supply is distributed into different utilization outlets--current domestic consumption (q_t), export (q_{E_t}) and carryout stock (C_t). The total carryout stock (C_t) consists of commercial stock (C_{ct}) and Government stock (C_{gt}). There are four major types of domestic wheat consumption: (1) Wheat consumed as food (q_{ht}), (2) wheat used for feeding livestock (q_{ft}), (3) wheat used as seed (q_{st}),

U.S. WHEAT SUPPLY AND UTILIZATION

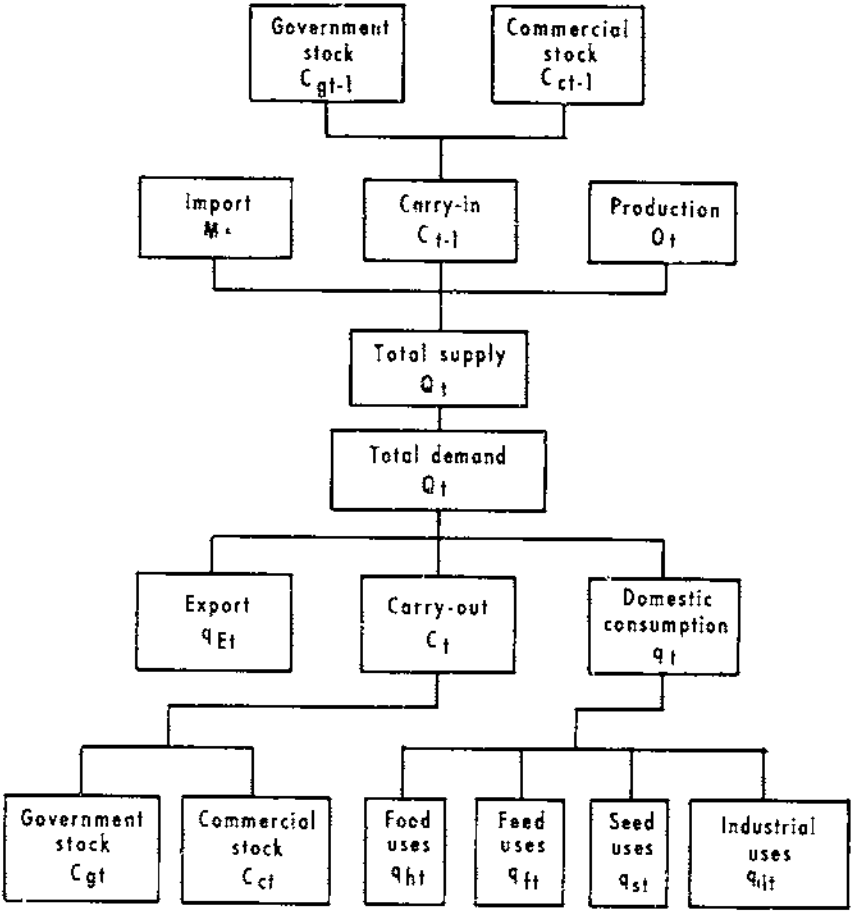


Figure 1

and (4) wheat diverted to industrial uses (q_{It}). The total aggregate wheat supply is normally in excess of domestic requirements for food or other domestic uses. The excess either enters export channels or is retained as a carryout stock. During 1959-63, approximately 24 percent of the total available supply was allocated to domestic consumption channels and 27 percent to the export market; 49 percent was retained as carryout stock. Of total domestic consumption, wheat consumed as food is the major component. During the marketing years 1959-63, wheat consumed as food averaged around 84 percent of total domestic consumption, and the remainder, approximately 16 percent of the total, was used for feed and seed. Only a negligible amount was diverted to industrial uses.

AN ECONOMETRIC MODEL

The market for wheat in the United States, of course, is much more complex than the one indicated in figure 1. On the production side, there are many varieties of wheat produced in different areas. Similarly, there are many different end products. However, figure 1 provides us with a simple macro framework which suggests (1) a reasonable partition of the consumption sector, and (2) some of the relevant variables entering in the system.

As indicated in the preceding section, imports of wheat and wheat diverted to industrial uses account for only a negligible portion of the total. The analyses of these two sectors, therefore, were excluded from the model. Also, no attempt was made to incorporate any analytical explanations of wheat used as seed.

Farm Price and Support Price Relation

To some extent, since the price support program was established in 1938, the U.S. domestic wheat market has not operated under a free competitive market situation. Under the price support program, average wheat prices received by farmers are influenced largely by the support prices. Before 1938, in the absence of price supports, average farm prices of wheat were closely related to farm prices of other feed grains. Therefore, in the model the following farm price and support price relation is postulated:

$$P_t = f_1(P_{st}, K_t P_{fot}) \qquad (2.1)$$

where

P_t = average wheat price received by farmers at time t (dol. per bu.)

P_{st} = average wheat support price at time t (dol. per bu.)

$K_t = \begin{cases} 1, \text{ if no price support program at time t,} \\ 0, \text{ otherwise} \end{cases}$

P_{fot} = farm price index of other feed grains (corn, oats, barley, and sorghum) at time t (1957-59=100)

Under normal conditions, the average wheat price received by farmers will change in the same direction as the change in support price. In other words, we would expect that the partial derivative of P_t with respect to P_{st} will be positive, i.e.:

$$\frac{\partial P_t}{\partial P_{st}} > 0 \tag{2.2}$$

In the absence of a support price, we would expect that the farm wheat price would change in the same direction as the change in other farm feed grain prices, i.e.:

$$\frac{\partial P_t}{\partial P_{fot}} > 0 \tag{2.3}$$
(provided $K_t = 1$)

Food Consumption Relation

On the basis of classical demand theory, the quantity demanded of a particular commodity is a function of its price, the prices of other commodities, and income. But classical theory gives no suggestion as to whether the demand function is linear or nonlinear. Engel's law suggests that the relation between food consumption and income tends to be curvilinear. Examination of U.S. per capita wheat consumption data reveals that the increase in per capita consumption of wheat is associated with a rise in per capita income only at lower income levels. Beyond a certain low income level, per capita wheat consumption declines as income rises. Therefore, in the model the relationship of per capita wheat consumption to income is postulated to be curvilinear as follows:

$$q^*_{ht} = f_2\ (P_t,\ P_{ct},\ G(I_t)\ ) \tag{2.4}$$

where

q^*_{ht} = domestic per capita use of wheat for food at time t (bu. per capita)

P_t = average wheat price received by farmers at time t (dol. per bu.)

P_{ct} = consumer price index at time t (1957-59=100)

I_t = per capita disposable income at time t (dol. per capita)

and,

$G(I_t)$ = a nonlinear transformation of variable I_t (the explicit form of transformation is discussed in detail in the estimation section)

In the above formulation, we would normally expect that changes in per capita food consumption of wheat, q^*_{ht}, will be (1)negatively related to changes in farm price of wheat, P_t, (2) positively related to changes in consumer price index, P_{ct}, and (3) positively or negatively related to changes in income depending upon the levels of income, i.e.:

$$\frac{\partial q^*_{ht}}{\partial P_t} < 0 \tag{2.5}$$

$$\frac{\partial q^*_{ht}}{\partial P_{ct}} > 0 \tag{2.6}$$

and

$$\frac{\partial q_{ht}}{\partial I_t} \lesseqgtr 0 \text{ (depending upon the income level)} \tag{2.7}$$

Feed Consumption Relation

The demand for wheat as feed for livestock is related to the price of wheat, the prices of other competing feed grains, and the numbers of livestock units fed annually. During World War II, the Government encouraged the use of wheat for feed in an effort to increase livestock production. To reflect this situation, a dummy variable is used in the following postulated feed consumption relation:

$$q_{ft} = f_3 (P_t, P_{fot}, L_t, D_t) \tag{2.8}$$

where

q_{ft} = domestic use of wheat for feed at time t (mil. bu.)

P_t = average wheat price received by farmers at time t (dol. per bu.)

P_{fot} = farm price index of other feed grains (corn, oats, barley, and sorghum) at time t (1957-59=100)

L_t = grain-consuming animal units of livestock fed annually at time t (mil. units)

$$D_t = \begin{cases} 1, \text{ during World War II} \\ 0, \text{ otherwise} \end{cases}$$

The other feed grains are competitive with wheat as a livestock feed grain. The use of wheat for livestock feeding will decline if the price of wheat is substantially high in relation to other feed grain prices. As a consequence, we would normally expect (1) the partial derivative of q_{ft} with respect to P_{fot} to be positive, (2) the partial derivative of q_{ft} with respect to P_t to be negative, and (3) the partial derivative of q_{ft} with respect to L_t to be positive, i.e.:

$$\frac{\partial q_{ft}}{\partial P_t} < 0 \tag{2.9}$$

$$\frac{\partial q_{ft}}{\partial P_{fot}} > 0 \tag{2.10}$$

and

$$\frac{\partial q_{ft}}{\partial L_t} > 0 \tag{2.11}$$

Government Inventory Relation

Since 1938 a certain portion of wheat production has been delivered by producers to the Commodity Credit Corporation, (CCC), under the price support program. Under the operation of the price support program, a producer may obtain a loan from the CCC with his wheat as collateral, and may repay the loan by delivery of the wheat to the CCC. Thus, the amounts of wheat which are delivered to the CCC will be a function of the support price and production. Moreover, because wheat can be stored for relatively long periods, it is possible that certain amounts of wheat accumulated in a given year will be carried into the succeeding year. Therefore, a lagged stock variable is included in the following postulated Government inventory relation:

$$C_{gt} = f_4 \; (P_{st}, \; \overline{K}_t \overline{D}_{t-2} O_t, \; C_{gt-1}) \tag{2.12}$$

where

C_{gt} = Government wheat inventory at the end of time t (mil. bu.)

P_{st} = average wheat support price at time t (dol. per bu.)

$$\overline{K}_t = \begin{cases} 1, & \text{if there is a Government price support program at time t} \\ 0, & \text{otherwise} \end{cases}$$

$$\overline{D}_t = \begin{cases} 0, & \text{during World War II} \\ 1, & \text{otherwise} \end{cases}$$

O_t = total U.S. wheat production at time t (mil. bu.)

There are two dummy variables, $\overline{K}_t$ and $\overline{D}_{t-2}$, in the relation (2.12). The inclusion of the product of these two dummy variables means that wheat production will not have any effect on the accumulation of Government stocks if there are no Government price support programs or a major war similar to World War II occurs.

The method of operating the price support program for wheat since 1938 has been an implicit offer by the Government to buy the amount of wheat produced in excess of the quantity that would sell during the marketing year at the support price. As a result, we would expect the Government wheat inventory to be positively related to support prices, production, and lagged inventory, i.e.:

$$\frac{\partial C_{gt}}{\partial P_{st}} > 0 \tag{2.13}$$

$$\frac{\partial C_{gt}}{\partial \overline{K}_t \overline{D}_{t-2} O_t} > 0 \quad \text{(Provided } \overline{K}_t = 1 \text{ and } \overline{D}_{t-2} = 1\text{)} \tag{2.14}$$

and

$$\frac{\partial C_{gt}}{\partial C_{gt-1}} > 0 \tag{2.15}$$

Commercial Inventory Relation

The amount of wheat withheld as commercial stock is related to farm prices of wheat, Government stocks, and lagged commercial stocks in the following postulated relation:

$$C_{ct} = f_5 (P_t, C_{gt}, C_{ct-1}) \qquad (2.16)$$

where

C_{ct} = commercial wheat inventory at the end of time t (mil. bu.)

P_t = average wheat price received by farmers at time t (dol. per bu.)

C_{gt} = Government wheat inventory at the end of time t (mil. bu.)

C_{ct-1} = commercial wheat inventory at the end of time t-1 (mil. bu.)

The price support and storage operations of the CCC tend to stabilize farm prices of wheat. As a result, the amount of wheat stored as private commercial stocks tends to be low when Government stocks are high. Hence, we would normally expect the partial derivative of C_{ct} with respect to P_t and C_{gt} to be negative and with respect to C_{ct-1} to be positive, i.e.:

$$\frac{\partial C_{ct}}{\partial P_t} < 0 \qquad (2.17)$$

$$\frac{\partial C_{ct}}{\partial C_{gt}} < 0 \qquad (2.18)$$

and

$$\frac{\partial C_{ct}}{\partial C_{ct-1}} > 0 \qquad (2.19)$$

Export Relation

The variables entering the following postulated export relation are the current per capita domestic wheat consumption, the total lagged Government stocks plus total lagged commercial stocks, and the lagged exports.

$$q_{Et} = f_6 (q^*_{ht}, C_{ct-1} + C_{gt-1}, q_{Et-1}) \qquad (2.20)$$

where

q_{Et} = total U.S. export of wheat at time t (mil. bu.)

q^*_{ht} = domestic per capita use of wheat for food at time t (bu. per capita)

C_{ct-1} = commercial wheat inventory at the end of time t-1 (mil. bu.)

C_{gt-1} = Government wheat inventory at the end of time t-1 (mil. bu.)

q_{Et-1} = total U.S. export of wheat at time t-1 (mil. bu.)

Large portions of the U.S. exports of wheat were channeled through the various Government programs. As a consequence, we would normally expect total exports to be positively related to the total carry-in stocks and the lagged exports. On the other hand, if per capita domestic wheat consumption is increased, then we would expect smaller exports of wheat. Hence, we would expect the following relations:

$$\frac{\partial q_{Et}}{\partial q^*_{ht}} < 0 \tag{2.21}$$

$$\frac{\partial q_{Et}}{\partial (C_{ct-1} + C_{gt-1})} > 0 \tag{2.22}$$

and

$$\frac{\partial q_{Et}}{\partial q_{Et-1}} > 0 \tag{2.23}$$

STATISTICAL ESTIMATION

In the preceding section, six relations were formulated for the model. These six relations were expressed by a set of exact determinate, rather than stochastic, functional relationships among variables. The postulated relations do not include all the relevant variables in the system. On the contrary, only main variables for which reliable empirical data were available were considered in each relation, and other conceivable determining variables were left aside intentionally or unintentionally. The influences of such omitted variables are treated as distrubances and explicitly recognized by introducing a random disturbance term, u_{it}, into each of these six relations.

For simplifying the estimation procedures, all postulated stochastic relations in the formulated system are assumed to be linear in parameters. In estimating the parameters of the formulated system, the variables were divided into the following sets of endogenous variables and predetermined variables:

Endogenous Variables

P_t = average wheat price received by farmers at time t (dol. per bu.)

q^*_{ht} = domestic per capita use of wheat for food at time t (bu. per capita)

q_{ft} = domestic use of wheat for feed at time t (mil. bu.)

C_{gt} = Government wheat inventory at end of time t (mil. bu.)

C_{ct} = commercial wheat inventory at end of time t (mil. bu.)

q_{Et} = total U.S. wheat exports at time t (mil. bu.)

Predetermined Variables

The set of predetermined variables consists of the lagged endogenous variables in the system and the following exogenous variables:

P_{st} = average wheat support price at time t (dol. per bu.)

$K_t = \begin{cases} 1, \text{ if there is no price support program at time } t, \\ 0, \text{ otherwise} \end{cases}$

P_{fot} = farm price index of other feed grains (corn, oats, barley, and sorghum) at time t (1957-59=100)

$G(I_t)$ = a nonlinear transformation of U.S. per capita disposable income at time t (the unit of per capita disposable income is dollars per capita)

L_t = grain-consuming animal units of livestock fed annually at time t (mil. units)

$D_t = \begin{cases} 1, \text{ during World War II}, \\ 0, \text{ otherwise} \end{cases}$

$\overline{K}_t = \begin{cases} 1, \text{ if there is a Government price support program at time } t, \\ 0, \text{ otherwise} \end{cases}$

$\overline{D}_t = \begin{cases} 0, \text{ during World War II}, \\ 1, \text{ otherwise} \end{cases}$

O_t = total U.S. wheat production at time t (mil. bu.)

Structural System

Under the additional specifications discussed in this section, the complete structural system formulated in the preceding section can be summarized and written in the following simple matrix form:

$$Ay_t + By_{t-1} + Cx_t = u_t \qquad (3.1)$$

where

$$y_t = \text{vector of endogenous variables} = \begin{bmatrix} P_t \\ q^*_{ht} \\ q_{ft} \\ C_{gt} \\ C_{ct} \\ q_{Et} \end{bmatrix}$$

$$y_{t-1} = \text{vector of lagged endogenous variables} = \begin{bmatrix} P_{t-1} \\ q^{*}_{h\,t-1} \\ q_{ft-1} \\ C_{gt-1} \\ C_{ct-1} \\ q_{Et-1} \end{bmatrix}$$

$$x_t = \text{vector of exogenous variables} = \begin{bmatrix} P_{st} \\ K_t P_{fot} \\ P_{ct} \\ G(I_t) \\ P_{fot} \\ L_t \\ D_t \\ \overline{K}_t \overline{D}_{t-2} O_t \\ 1 \end{bmatrix}$$

A = matrix of the parameters associated with endogenous variables

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 & 0 & 0 \\ a_{31} & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ a_{51} & 0 & 0 & a_{54} & 1 & 0 \\ 0 & a_{62} & 0 & 0 & 0 & 1 \end{bmatrix}$$

B = matrix of the parameters associated with the lagged endogenous variables

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{55} & 0 \\ 0 & 0 & 0 & b_{64} & b_{65} & b_{66} \end{bmatrix}$$

C = matrix of the parameters associated with the exogenous variables

$$
= \begin{bmatrix}
c_{11} & c_{12} & 0 & 0 & 0 & 0 & 0 & 0 & c_{19} \\
0 & 0 & c_{23} & c_{24} & 0 & 0 & 0 & 0 & c_{29} \\
0 & 0 & 0 & 0 & c_{35} & c_{36} & c_{37} & 0 & c_{39} \\
c_{41} & 0 & 0 & 0 & 0 & 0 & 0 & c_{48} & c_{49} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{59} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{69}
\end{bmatrix}
$$

and

$$
u_t = \begin{bmatrix}
u_{1t} \\
u_{2t} \\
u_{3t} \\
u_{4t} \\
u_{5t} \\
u_{6t}
\end{bmatrix}
$$

There are two distinctive features of the formulated structural system (3.1): (1) The matrix of the parameters associated with the endogenous variables in the system, A, is a triangular matrix, and (2) the matrix of the parameters associated with the lagged endogenous variables in the system, B, is not a null matrix. By definition, a structural system is called (1) "recursive system" if the matrix of the parameters associated with the endogenous variables in the system is a triangular matrix, and (2) a "dynamic system" if the matrix of the parameters associated with the lagged endogenous variables in the system is not a null matrix. Therefore, the formulated structural system (3.1) is essentially a "dynamic recursive system."

ESTIMATION METHODS

The choice of an estimation method to be used in estimating the parameters of a stochastic structural system depends critically on the specifications of the model. It can be shown that the direct ordinary least-squares estimation of the coefficients of a structural relation in a system does yield, in general, inconsistent estimates. But it does yield consistent estimates for the case of a diagonal recursive system. A structural system is called a "diagonal recursive system" if it is recursive and if in addition the variance-covariance matrix of the disturbance terms, $U = E(u_t u_t')$, is a diagonal matrix, that is, the disturbances in all the structural relations are uncorrelated so that the variance-covariance matrix has only zeros off the diagonal. Furthermore, if we assume that the disturbance term u_t is serially independent and is multivariate normally distributed with zero mean and with a diagonal variance-covariance matrix, then it can be shown that the full-information maximum likelihood estimates are identical with the direct ordinary least-squares estimates. The detailed derivations are given in appendix A. If we adopt the preceding specifications in the formulated structural system (3.1), then we could apply the ordinary least-squares directly to each of the structural relations in the system, and obtain the full-information maximum likelihood estimates of the structural coefficients in the system. If we do not assume the variance-covariance matrix to be a diagonal matrix, then we can use the two-round least-squares estimation procedure. The two-round least-squares procedures of estimating the coefficients of the formulated system (3.1) are described in appendix B.

The two-round least-squares estimation procedure does yield consistent estimates. But it does not yield asymptotic efficiency estimates as the maximum likelihood estimation procedures do. The empirical estimates of the coefficients in the formulated structural system were estimated by both the least-squares and the two-round least-squares methods.

The properties of consistency and asymptotic efficiency are all large-sample properties. But we are working with only a small sample of data. Therefore, the small-sample properties of various estimation procedures should be an important criterion for making the choices among different alternative estimation procedures. But due to the mathematical difficulties, few analytic results of the small-sample properties of various estimation procedures are available. Hence, at the present time, we do not have enough information to use the small-sample properties as a criterion.

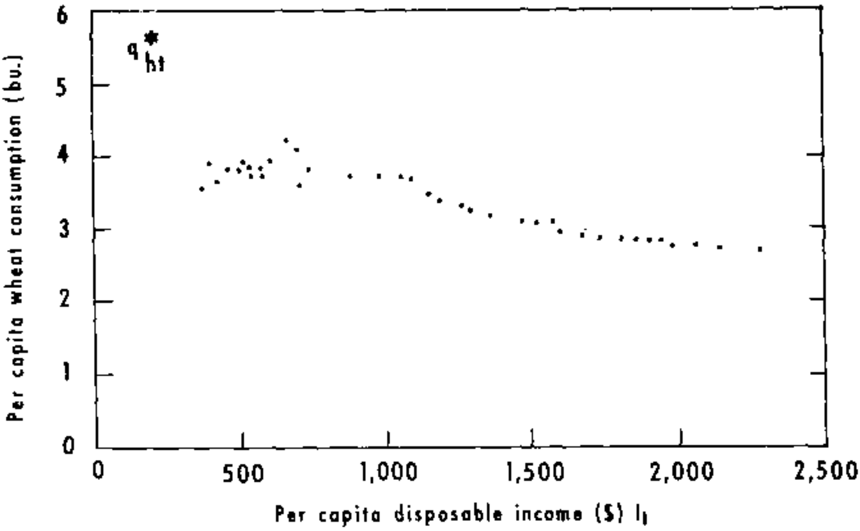
DATA ADJUSTMENTS

The basic sample data used in the estimation of the formulated structural system (3.1) were taken from U.S. Department of Agriculture publications. The detailed numerical sample data are given in appendix C. The sample period covers the marketing years from 1928 through 1964. The choice of the sample period was dictated largely by the availability of reliable data.

Among the eight exogenous variables included in the formulated structural system (3.1), there are three transformed variables K_tP_{fot}, $K_t\bar{D}_{t-2}O_t$, and $G(I_t)$. The adjustments of the two exogenous variables P_{fot} and O_t are straightforward and self-explanatory. But the transformation $G(I_t)$ requires a detailed explanation.

As indicated previously, there exists a nonlinear functional relation between U.S. per capita wheat consumption and per capita disposable income. The empirical data plotted in part A of figure 2 show that per capita consumption is highest when the income level is approximately $650. Consumption declines as income increases beyond that point, and approaches a relatively stable level as income approaches $2,000. To incorporate such a nonlinear relationship between per capita consumption and per capita disposable income into the model, there are two approaches which can be used: (1) Simply fitting a nonlinear relation to the original data; or (2) making a nonlinear transformation of the original data first and then fitting a linear relation with the transformed variable. We use the second approach because it is much simpler and more flexible.

PART A: U.S. PER CAPITA WHEAT CONSUMPTION AND INCOME RELATION



PART B: TRANSFORMATIONS OF THE U.S. PER CAPITA DISPOSABLE INCOME

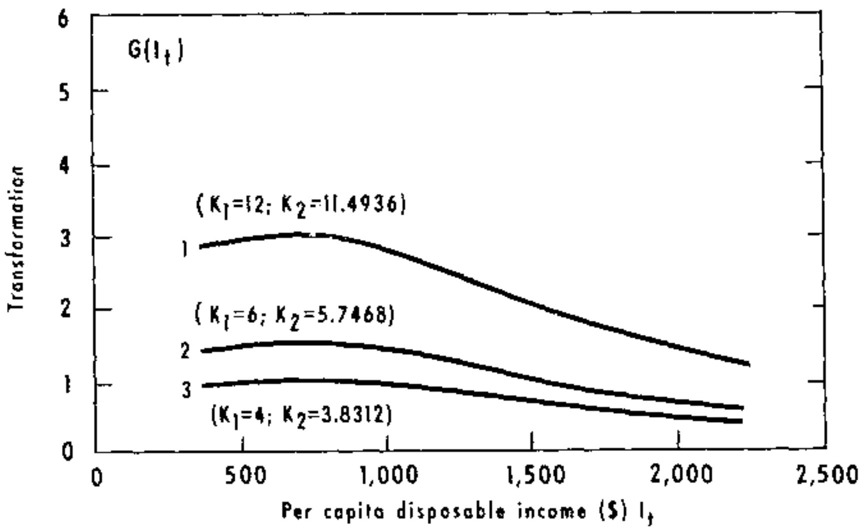


Figure 2

In performing the transformation of U.S. per capita disposable income I_t, the following family of nonlinear functions was selected:

$$G(I_t) = K_1 e^{-0.001 I_t} - K_2 e^{-0.002 I_t} \quad (3.2)$$

where K_1 and K_2 are parameters to be determined and they are assumed to be positive, and $e = 2.71828 =$ the base of the natural logarithm.

We selected this family of nonlinear functions because it has the desired properties: (1) For all nonnegative values of I_t, K_1 and K_2, and $K_1 \geq K_2$, the values of the function are positive; and (2) at the lower levels of I_t the values of the function $G(I_t)$ will increase as I_t increases and reaches its maximum value at a certain level of I_t. Beyond such a level, the values of the function will decline as I_t increases and will asymptotically approach a constant level.

As indicated in part A of figure 2, U.S. per capita wheat consumption approaches its maximum level when per capita disposable income is approximately \$650. Hence, in making a choice of the transformation function, we would like to select two parameters K_1 and K_2 such that the transformation function $G(I_t)$ reaches its maximum value at $I_t =$ \$650, i.e., Max. $G(I_t) = G(I_t = \$650)$. This can be done by the following simple steps:

(1) Taking the derivative of $G(I_t)$ with respect to I_t and setting it equal to zero, i.e.,

$$\frac{dG(I_t)}{d\,I_t} = \frac{d}{d\,I_t}\left(K_1 e^{-0.001\,I_t} - K_2 e^{-0.002\,I_t}\right)$$

$$= -0.001K_1 e^{-0.001\,I_t} + 0.002K_2 e^{-0.002\,I_t}$$

$$= 0 \quad (3.3)$$

(2) Solving for the ratio of K_2 and K_1 and evaluating (3.3) at $I_t = \$650$, i.e.,

$$-0.001K_1 e^{-0.001\,I_t} + 0.002K_2 e^{-0.002\,I_t} = 0$$

$$-0.001K_1 e^{-0.001(650)} + 0.002K_2 e^{-0.002(650)} = 0$$

$$-0.000522046K_1 + 0.000545064K_2 = 0$$

$$K_2/K_1 = 0.9578 \quad (3.4)$$

By the preceding derivations, we found that $K_2 = 0.9578K_1$. Therefore, we should select the transformation function from a small subfamily of functions $G(I_t)$ which satisfies the condition $K_2 = 0.9578K_1$. To select a particular transformation function among this subfamily of functions, we use a second criterion. The second criterion is simply selecting a particular transformation function from the subfamily of functions so that the curvature of the selected transformation function is approximately similar to the average curvature of the empirical data as indicated in part A of figure 2. The procedures of the selection are: (1) Plotting several curves of this subfamily of functions, and

(2) comparing the curvatures of these plotted curves with the curvature of the empirical data, and selecting the particular function which has approximately the similar curvature as the curvature of the empirical data. For example, in part B of figure 2, we plotted three curves of this subfamily of functions. In each curve, there are two parameters. The parameters of curve 1 are $K_1 = 12$ and $K_2 = 0.9578(12) = 11.4936$. Similarly, the parameters are $K_1 = 6$, $K_2 = 5.7468$ for curve 2, and $K_1 = 4$, $K_2 = 3.8312$ for curve 3. The curvature of curve 2 is similar to the curvature of the empirical data. Therefore, we used the transformation function (3.5) in the formulated structural system (3.1):

$$G(I_t) = 6\ e^{-0.001\ I_t} - 5.7468\ e^{-0.002\ I_t} \qquad (3.5)$$

where $e = 2.71828 =$ the base of the natural logarithm.

ESTIMATED STRUCTURES

The empirical estimates of the formulated structural system (3.1) were obtained by ordinary least-squares and two-round least-squares procedures. The following are these two sets of estimated empirical structures. The figures in parentheses below the estimated parameters are the standard errors of the estimates, and R is the estimated coefficient of the multiple correlation:

ORDINARY LEAST-SQUARES RESULTS

Farm Price and Support Price Relation

$$P_t = 0.1492 + \underset{(0.0448)}{0.9189}\ P_{st} + \underset{(0.0014)}{0.0108}\ K_t P_{fot} \qquad R = 0.9747 \qquad (3.6)$$

Food Consumption Relation

$$q^{*}_{ht} = 1.1989 - \underset{(0.0678)}{0.2284}\ P_t + \underset{(0.0042)}{0.0077}\ P_{ct} + \underset{(0.2254)}{1.6005}\ G(I_t) \qquad (3.7)$$

$$R = 0.9747$$

Feed Consumption Relation

$$q_{ft} = -137.8420 - \underset{(37.4650)}{143.7966}\ P_t + \underset{(0.5804)}{1.6302}\ P_{fot} + \underset{(0.8894)}{1.7860}\ L_t + \underset{(34.3702)}{159.4989}\ D_t$$

$$R = 0.8775 \qquad (3.8)$$

Government Inventory Relation

$$C_{gt} = -182.9923 + \underset{(78.0566)}{115.6075}\ P_{st} + \underset{(0.0913)}{0.1806}\ \overline{K}_t\overline{D}_{t-2}O_t + \underset{(0.0974)}{0.7446}\ C_{gt-1}$$

$$R = 0.9381 \qquad (3.9)$$

Commercial Inventory Relation

$$C_{ct} = 200.2999 - \underset{(24.5510)}{64.4016}\ P_t - \underset{(0.0270)}{0.0422}\ C_{gt} + \underset{(0.1538)}{0.3635}\ C_{ct-1}$$

$$R = 0.8367 \qquad (3.10)$$

Export Relation

$$q_{Et} = \underset{}{433.5437} - \underset{(80.3589)}{112.0979}\, q^{*}_{ht} + \underset{(0.0695)}{0.0967}\, (C_{ct-1} + C_{gt-1})$$

$$+ \underset{(0.1361)}{0.6494}\, q_{Et-1}$$

$$R = 0.9274 \qquad (3.11)$$

TWO-ROUND LEAST-SQUARES RESULTS

Farm Price and Support Price Relation

$$P_t = 0.1492 + \underset{(0.0448)}{0.9189}\, P_{st} + \underset{(0.0014)}{0.0108}\, K_t P_{fot} \qquad R = 0.9747 \qquad (3.12)$$

Food Consumption Relation

$$q^{*}_{ht} = 1.1339 - \underset{(0.0704)}{0.2835}\, P_t + \underset{(0.0040)}{0.0093}\, P_{ct} + \underset{(0.2105)}{1.6237}\, G(I_t)$$

$$R = 0.9747 \qquad (3.13)$$

Feed Consumption Relation

$$q_{ft} = -140.5809 - \underset{(35.0491)}{113.1157}\, P_t + \underset{(0.5096)}{1.0316}\, P_{fot} + \underset{(0.9790)}{1.8828}\, L_t$$

$$+ \underset{(37.6104)}{160.2604}\, D_t$$

$$R = 0.8660 \qquad (3.14)$$

Government Inventory Relation

$$C_{gt} = -182.9923 + \underset{(78.0566)}{115.6075}\, P_{st} + \underset{(0.0913)}{0.1806}\, \overline{K}_t \overline{D}_{t-2} O_t$$

$$+ \underset{(0.0974)}{0.7446}\, C_{gt-1}$$

$$R = 0.9381 \qquad (3.15)$$

Commercial Inventory Relation

$$C_{ct} = 199.0957 - \underset{(25.4078)}{79.5079}\, P_t - \underset{(0.0317)}{0.0084}\, C_{gt} + \underset{(0.1458)}{0.4231}\, C_{ct-1}$$

$$R = 0.8426 \qquad (3.16)$$

Export Relation

$$q_{Et} = 543.5585 - \underset{(75.6830)}{139.7191}\, q^{*}_{ht} + \underset{(0.0706)}{0.0809}\, (C_{ct-1} + C_{gt-1})$$

$$+ \underset{(0.1362)}{0.6190}\, q_{Et-1} \qquad R = 0.9327 \qquad (3.17)$$

In the above estimated structures, all the estimated standard errors are smaller than their corresponding estimated coefficients, except the coefficient of C_{gt} in (3.16). The estimated multiple correlation coefficients range from 0.8367 for (3.10) to 0.9747 for (3.6), (3.7), (3.12), and (3.13). The signs of all the estimated coefficients obtained from ordinary least-squares or two-round least-squares methods are in accordance with the theoretical and logical expectations specified in the formulation of the model in the section beginning on page 3.

ANALYSES, IMPLICATIONS, AND PROJECTIONS

In making economic policy decisions, one usually asks questions such as these: (1) If a policy instrument were changed during a given time period, what would be the effects on the related policy target or on other important variables in the system during the same time period? (2) What would be the impacts of such a change during each of the successive time periods? (3) What would be the total impacts over a long period of time? To answer such questions, we need a knowledge of the underlying economic structure. In the framework of our formulated model, the above questions are equivalent to the general problem of finding the solutions of the short-run and long-run impact multiplier matrices of a dynamic recursive system.

SHORT-RUN IMPACT MULTIPLIERS

To derive the short-run impact multiplier matrix for the dynamic recursive system (3.1), we consider the following reduced form of the underlying structure (3.1):

$$y_t = D_1 y_{t-1} + D_2 x_t + v_t \tag{4.1}$$

where

$$D_1 = -A^{-1}B \tag{4.2}$$

$$D_2 = -A^{-1}C \tag{4.3}$$

and

$$v_t = A^{-1}u_t \tag{4.4}$$

By taking the partial derivative of y_t (4.1) with respect to x_t, we get the derived reduced-form coefficient matrix D_2, that is,

$$\frac{\partial y_t}{\partial x_t} = D_2 = \left[d_{2ij}\right] \tag{4.5}$$

The element d_{2ij} in the above matrix D_2 is the partial derivative of the i-th endogenous variable, y_{it}, in the vector y_t with respect to the j-th exogenous variable, x_{jt}, in the vector x_t. By definition, the short-run multiplier is defined as the impact of a unit change in the j-th exogenous variable during a given time period on the i-th endogenous variable during the same time period. Therefore, the derived reduced-form coefficient matrix D_2 is precisely the deduced short-run impact multiplier matrix for the dynamic recursive system (3.1).

Detailed calculations of the estimated reduced form obtained from the ordinary least-squares results are given in appendix E.

One important policy instrumental variable among the exogenous variables of the system (3.1) is the support price variable (P_{st}). The short-run impact multipliers of the support price are summarized in table 1. These short-run impact multipliers correspond to the first column of the estimated derived reduced-form coefficient matrix D_2.

Table 1.--Short-run impact multipliers: Impacts of the support price on the endogenous variables in the system

Endogenous variables	Identification	Short-run impact multipliers of support price (P_{st}) 1/ Ordinary least-squares estimate	Short-run impact multipliers of support price (P_{st}) 1/ Two-round least-squares estimate
Average farm price of wheat (dol. per bu.)	(P_t)	0.9189	0.9189
Per capita wheat consumption (bu.)	(q^*_{ht})	-0.2099	-0.2605
Wheat used for feed (mil. bu.)	(q_{ft})	-132.1347	-103.9420
Government wheat inventory (mil. bu.)	(C_{gt})	115.6075	115.6075
Commercial wheat inventory (mil. bu.)	(C_{ct})	-64.0573	-74.0309
Total U.S. wheat exports (mil. bu.)	(q_{Et})	23.5266	36.3980

1/ Dollar per bushel.

These estimates of the short-run support price impact multipliers obtained from the ordinary least-squares results imply that an increase of $1 per bushel in support price, with all other predetermined variables in the system held constant, will lead to (1) an increase of 91 cents per bushel in average farm wheat price, (2) a decline of 0.21 bushel per capita in wheat food consumption, (3) a decline of 132 million bushels in wheat used for feed, (4) an increase of 116 million bushels in Government stocks, (5) a decline of 64 million bushels in commercial stocks, and (6) an increase of 24 million bushels in total U.S. wheat exports during the same time period.

The apparent paradox of increased price supports (and also domestic farm prices) leading to higher exports is explained by the operation of the price support program. When Government stocks mounted to relatively high levels, due to the operation of the price support program, the policy usually was to reduce them through export programs. The same interpretations can be applied to estimates of the short-run multipliers of the other exogenous variables. The estimated numerical values of these multipliers are given in appendix E.

Given the dynamic system (4.1) and given the initial conditions of endogenous variables (y_o) and the time-path of exogenous variables (x_t, for $t = 1,2,...,k$) in the system, then the time-path of the endogenous variables (y_t, for $t = 1,2,...,k$) can be determined as follows:

$$y_1 = D_1 y_o + D_2 x_1$$

$$y_2 = D_1^2 y_o + D_2 x_2 + D_1 D_2 x_1$$

$$\dots\dots\dots\dots\dots\dots\dots\dots$$

$$y_k = D_1^k y_o + D_2 x_k + D_1 D_2 x_{k-1} + \dots\dots + D_1^{k-1} D_2 x_1 \qquad (4.6)$$

Stability Conditions of a Dynamic System

Within the framework of our analysis, the analysis of stability condition is important because the long-run impact multipliers are relevant only if the underlying dynamic system is stable. The dynamic system (4.1) will be stable if the matrix D_1^k in (4.6)approaches a null matrix as k increases. The matrix D_1^k will approach a null matrix if the latent roots of the matrix D_1 are all in the interior of the unit circle. 1/ Hence, the stability of the dynamic system (4.1) is determined by the magnitude of the maximum (dominant) latent root of the matrix D_1. The numerical method of determining the stability of a dynamic system is presented in Appendix D. Examination of the stability conditions shows that the two estimated structures are both stable.

Long-Run Impact Multipliers

Having established the stability of a dynamic system, we are now able to analyze the following question: If an exogenous variable is raised by one unit and remains at its new level in successive time periods, then what would be the impact of such a change on the endogenous variables in the system during the successive time periods and over a long period of time?

Let us consider the case where the exogenous variables remain at a constant level, i.e.,

$$x_1 = x_2 = x_3 = \dots = x_k = x^* \qquad (4.7)$$

Given (4.7), the relation (4.6) becomes:

$$y_k = D_1^k y_o + (I + D_1 + D_1^2 + \dots\dots + D_1^{k-1})\, D_2 x^* \qquad (4.8)$$

It is clear from (4.8) that the effect of a sustained unit increase in an exogenous variable on endogenous variables of a dynamic system over successive years can be obtained by simply taking the partial derivative of y_k with respect to x^*, i.e.,

$$\frac{\partial y_k}{\partial x^*} = (I + D_1 + D_1^2 + D_1^3 + \dots + D_1^{k-1})\, D_2, \text{ for } k=1,2,\dots,n \qquad (4.9)$$

1/ For detailed derivations, see Arthur S. Goldberger, Econometric Theory, John Wiley & Sons, Inc., New York, 1964, pp. 376-78.

The empirical ordinary least-squares estimates of the k-periods impact multiplier matrices (4.9) are given in appendix F. The effects of a sustained unit increase in wheat support price on Government and commercial wheat inventory for successive time periods are summarized in table 2.

Table 2.--The k-periods impact multipliers: The impact of a sustained increase in wheat support price by one dollar per bushel on Government commercial wheat inventory in successive time periods

Time period (k)	The k-period impact multipliers: Government wheat inventory 1/ (C_{gt})	Commercial wheat inventory (C_{ct}): Ordinary least-squares estimate	Two-round least-squares estimate
	Mil. bu.	Mil. bu.	Mil. bu.
1	115.61	-64.06	-74.03
2	201.69	-90.97	-106.08
3	265.78	-103.46	-120.19
4	313.50	-110.01	-126.56
5	349.04	-113.89	-129.55
6	375.50	-116.41	-131.03
7	395.20	-118.15	-131.83
8	409.87	-119.40	-132.28
9	420.80	-120.33	-132.56
10	428.94	-121.00	-132.75

1/ Ordinary and two-round least-squares estimates.

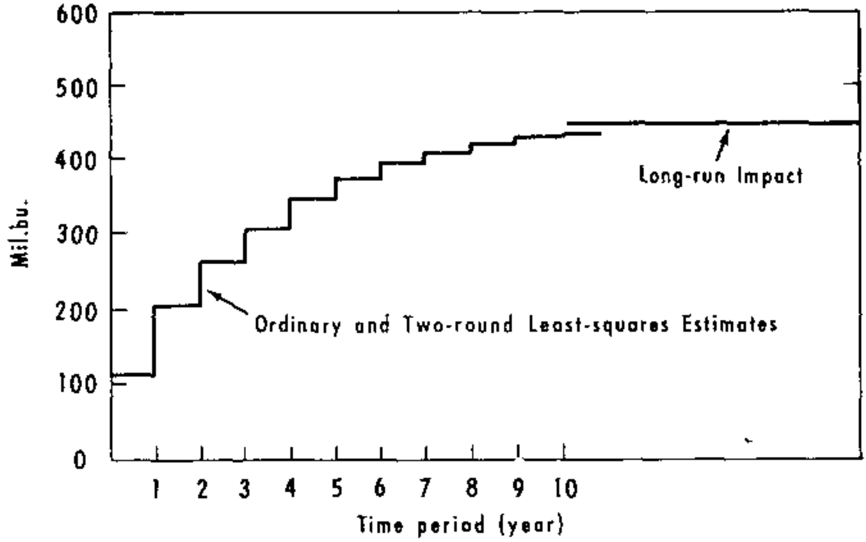
The empirical estimates in table 2 imply that: If the wheat support price is raised by $1 per bushel and remains at its new level during successive time periods, then it will lead to (1) an increase of 116 million bushels in Government stocks in the first year, 202 million bushels in the following 2 years, 266 million bushels in the following 3 years, and so on; and (2) a decline of 64 million bushels in commercial stocks in the first year, 91 million bushels in the following 2 years, 103 million bushels in the following 3 years, and so on.

The time-paths of these estimated k-periods impact multipliers are shown in figure 3. The magnitudes of the absolute differences of impact multipliers between two successive time periods decrease as the time increases. This implies that the response of the change in stock to a change in support price will be much larger in the immediate time periods than in future time periods.

The first-period impact multipliers are in fact the same as the short-run impact multipliers which were obtained in the preceding section. It can be shown by setting k=1 in (4.9), i.e.,

$$\left.\frac{\partial y_k}{\partial x^*}\right|_{k=1} = D_2 \qquad (4.10)$$

PART A: TOTAL INCREASE IN GOVERNMENT WHEAT INVENTORY IN SUCCESSIVE YEARS WHEN SUPPORT PRICE RISES BY ONE DOLLAR PER BUSHEL



PART B: TOTAL CHANGE IN COMMERCIAL WHEAT INVENTORY RESULTING FROM A ONE DOLLAR PER BUSHEL INCREASE IN THE SUPPORT PRICE

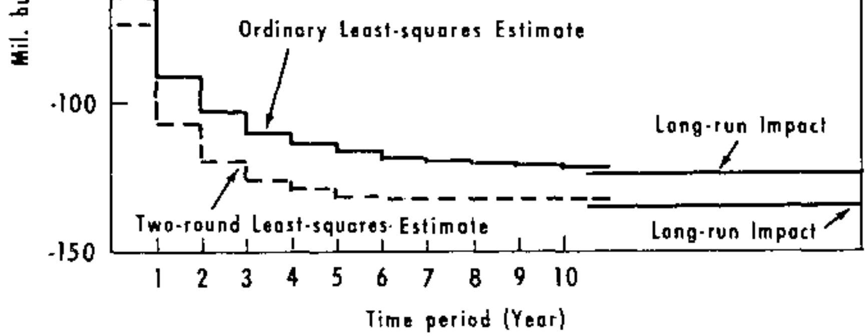


Figure 3

Comparing (4.10) with (4.5) reveals that they are the same. Thus, the short-run impact multiplier matrix is a special case of the k-periods impact multiplier matrix, namely the 1-period impact multiplier matrix. As a result, the numerical values in the first row of table 2 are the same as the ones in the fourth and fifth rows of table 1.

Since the sum of series $I + D_1 + D_1^2 + \dots + D_1^{k-1}$ can be expressed as $(I - D_1)^{-1}(I - D_1^k)$, the relation (4.8) can be rewritten as:

$$y_k = D_1^k y_o + (I - D_1)^{-1}(I - D_1^k)D_2 x^* \tag{4.11}$$

For a stable dynamic system, the matrix D_1^k will approach a null matrix as k increases. Hence, if the exogenous variable vector is indefinitely sustained at the level of x^*, then the endogenous variable vector will approach the long-run stationary equilibrium state:

$$\lim_{k \to \infty} y_k = (I - D_1)^{-1} D_2 x^* \tag{4.12}$$

Therefore, the long-run impact multiplier matrix of a stable dynamic system is:

$$(I - D_1)^{-1} D_2 \tag{4.13}$$

The elements of the matrix (4.13) measure the ultimate or long-run response of endogenous variables to a sustained unit change of exogenous variables.

Estimates of the long-run impact multiplier matrix obtained from the ordinary least-squares results are given in appendix E. The long-run impact multipliers of the wheat support price are summarized in table 3.

The estimated long-run multipliers derived from the ordinary least-squares results indicate that a sustained increase of $1 per bushel in wheat support price will generate in the long run (1) an increase in Government stocks of 453 million bushels, (2) a decline in commercial stocks of 123 million bushels, and (3) an increase in total U.S. wheat exports of 158 million bushels.

Long-run impact multipliers are not very meaningful for a pure static system. In a static system the matrix $D_1 = O$, and the long-run impact multiplier matrix given by (4.13) becomes D_2, which is the same as the short-run impact multiplier matrix given by (4.5). In our formulated model, there are no lagged endogenous variables involved in the first three relations. As a result, the estimated long-run impact multipliers are identical to the estimated short-run impact multipliers in the first three rows of tables 1 and 3.

In making a policy decision, it is useful to know the relative magnitudes between the long-run impact multipliers and their corresponding short-run impact multipliers resulting from a change in policy instrumental variable. Table 4 gives the estimated relative effects of an increase in support price.

Table 3.--The long-run impact multipliers: Impacts of the support price on the endogenous variables in the system

Endogenous variables	Identification	Long-run impact multipliers of support price (P_{st}) 1/	
		Ordinary least-squares estimate	Two-round least-squares estimate
Average farm price of wheat (dol. per bu.)	P_t	0.9189	0.9189
Per capita wheat consumption (bu.)	$\dot{q}_{ht}$	-0.2099	-0.2605
Wheat used for feed (mil. bu)	q_{ft}	-132.1347	-103.9420
Government wheat inventory (mil. bu.)	C_{gt}	452.6496	452.6496
Commercial wheat inventory (mil. bu.)	C_{ct}	-122.9642	-133.2732
Total U.S. wheat exports (mil. bu.)	q_{Et}	158.0206	163.3471

1/ Dollar per bushel.

Table 4.--Relative magnitudes between the long-run and short-run impact multipliers: Relative effects of support price on the endogenous variables in the system

Endogenous variables	Identification	Relative effect of support price	
		Ordinary least-squares estimate	Two-round least-squares estimate
Government wheat inventory	C_{gt}	3.92	3.92
Commercial wheat inventory	C_{ct}	1.92	1.80
Total U.S. wheat exports	q_{Et}	6.72	4.49

Based on the results derived from the ordinary least-squares estimated structure, the long-run effects of a change in wheat support price on Government wheat stocks, commercial wheat stocks, and total U.S. wheat exports will be 3.9, 1.9, and 6.7 times as large as the corresponding short-run effects.

PROJECTIONS

An important application of an econometric model is in predicting future values of endogenous variables. An econometric model is valid for predictive purposes only if the structure of the system, the estimates of the parameters, can be assumed to be unchanged in the future. Thus, before using the model for making predictions, the extent to which the model is able to simulate the past should be examined.

Retrospective Analysis of Predictive Performance

Predicted values of endogenous variables from 1928 to 1964, based on the ordinary least-squares estimated structures, are compared with observed values for the same period in figures 4 and 5.

In evaluating the past predictive performance of a model, one often uses the correlation coefficient between the predicted and observed values as a criterion. But a high correlation coefficient between predicted and observed values does not always imply a good prediction, therefore an alternative measure of predictive accuracy was proposed by Theil: 2/

$$U = \frac{\sqrt{(1/n) \sum_{t=1}^{n} (F_t^* - F_t)^2}}{\sqrt{(1/n) \sum_{t=1}^{n} F_t^{*2}} + \sqrt{(1/n) \sum_{t=1}^{n} F_t^2}} \qquad (4.14)$$

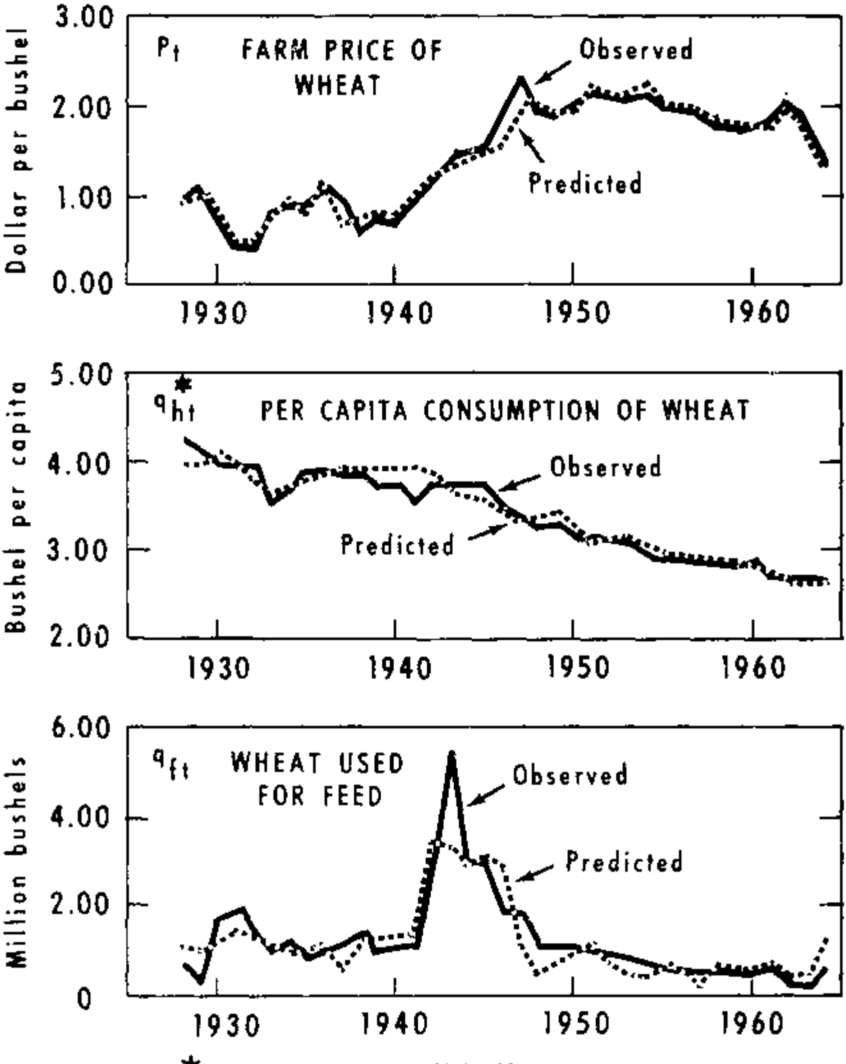
where F_t^* = the predicted value at time t, and F_t = the observed value at time t.

The Theil-U measure (4.14) has the property of varying between zero and one; and the higher the overall predictive accuracy, the closer is U to zero. The computed Theil-U measures for all the endogenous variables in the system are given in table 5. The computed values of U vary between 0.0150 and 0.1616. Table 5 indicates that (1) predictions of per capita wheat consumption and average farm prices of wheat in the sample period were more accurate than predictions of other endogenous variables in the system, and (2) the overall accuracy of predictions based on ordinary least-squares estimated structures is more or less the same as that of predictions obtained from two-round least-squares estimated structures.

The Theil-U values measure only the overall accuracy of the predictions. They do not provide detailed information about the direction of prediction errors. Therefore, it is useful to plot the predicted values (F_t^*) against the corresponding observed values (F_t) as in figure 6.

2/ H. Theil, Economic Forecasts and Policy, North-Holland Publishing Company, Amsterdam, 1961.

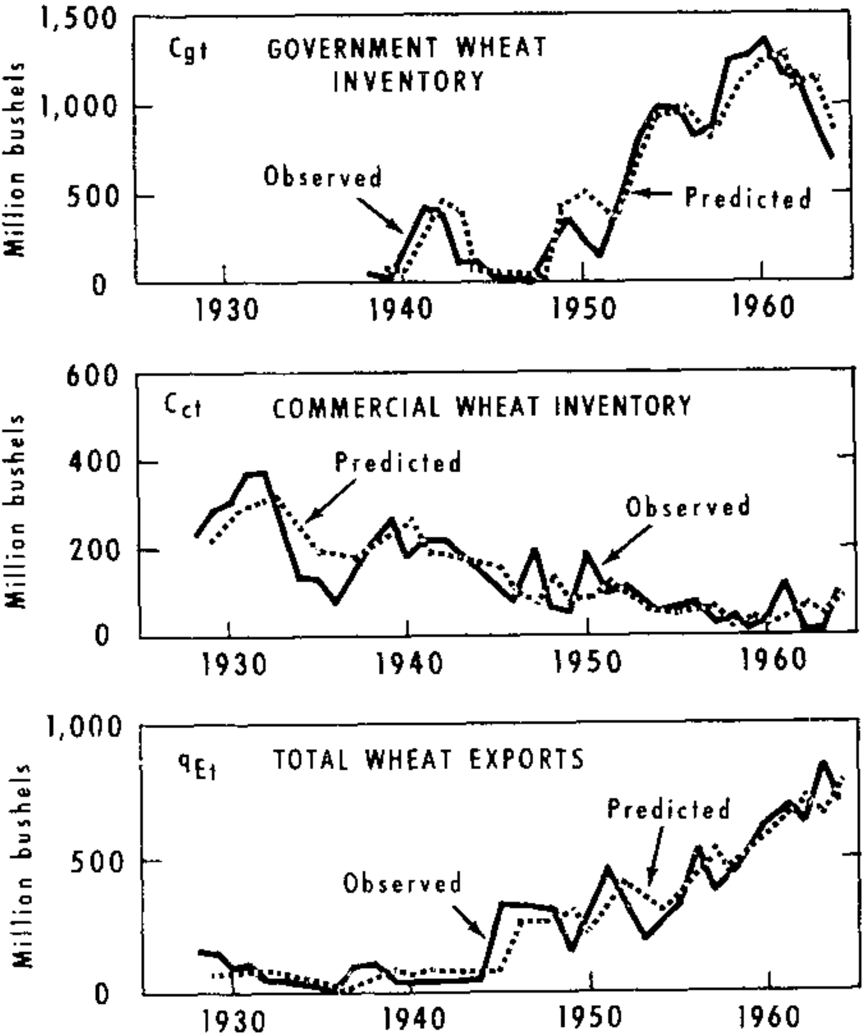
PREDICTED* AND OBSERVED VALUES OF ENDOGENOUS VARIABLES



* ORDINARY LEAST-SQUARES ESTIMATES.

Figure 4

PREDICTED* AND OBSERVED VALUES OF ENDOGENOUS VARIABLES



* ORDINARY LEAST-SQUARES ESTIMATES.

Figure 5

GRAPHIC ANALYSIS OF PREDICTION ERRORS

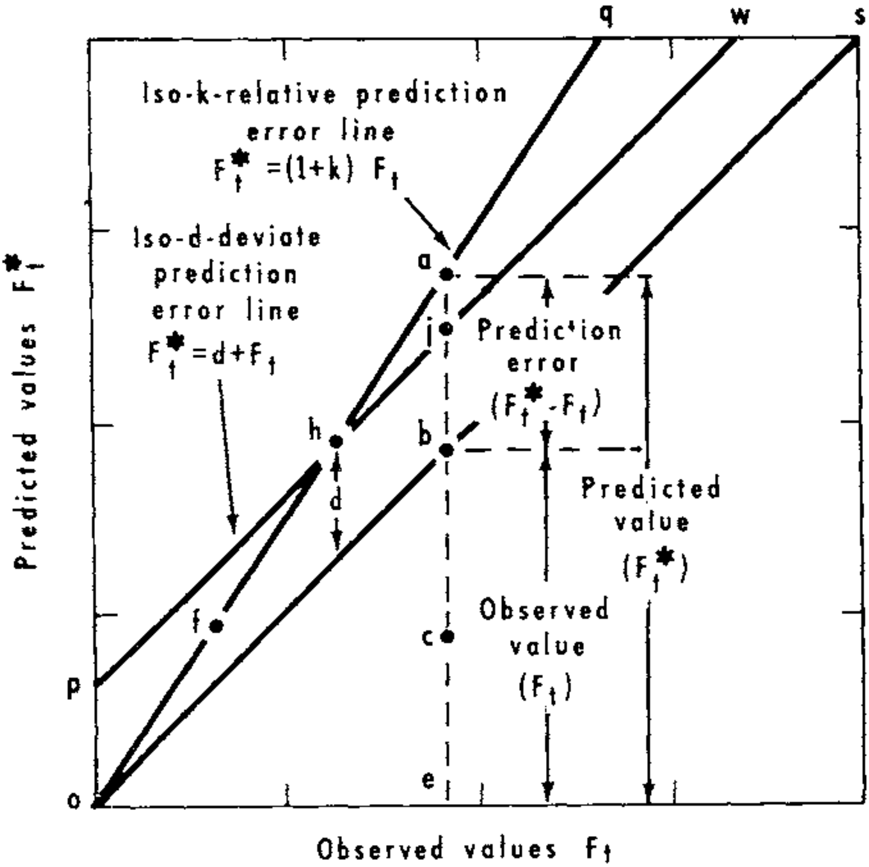


Figure 6

The deviations between the predicted values (F_t^*) and their corresponding observed values (F_t) are prediction errors ($F_t^* - F_t$). If $F_t^* - F_t = O$, then we have perfect prediction. Otherwise, we will have overprediction ($F_t^* - F_t > O$) or underprediction ($F_t^* - F_t < O$). Therefore, the line os (45° line) in figure 6 is the line of perfect prediction, i.e., all points on the line are points of perfect prediction, points below the line are underprediction, and points above it are overprediction. For example, point b is perfect prediction, point c is underprediction, and points a, j, h, and f are all overprediction. Line pw is the iso-d-deviate prediction error line. All the points on line pw have the same vertical deviation from line os. Thus, points h and j have the same magnitude of prediction error d. Line oq is the iso-k-relative prediction error line. The points on the same iso-k-relative prediction error line have the same magnitude of relative prediction error k. Hence, points a, h, and f have the same relative prediction error, but they have different magnitudes of absolute prediction errors. On the other hand, points h and j have the same magnitude of absolute prediction error, but point j has a smaller relative prediction error than point h. Therefore, the graphic analysis of prediction errors is a useful tool because it provides detailed information as to the frequency of overprediction or underprediction in terms of both relative and absolute errors. The results obtained from the ordinary least-squares estimated structures are shown in figures 7 through 9. Careful examination of figures 7 through 9 indicates: (1) The predictions of average farm prices of wheat and per capita wheat consumption have smaller prediction errors in terms of absolute or relative magnitudes than the predictions of total wheat used for feed, Government wheat inventory, commercial wheat inventory, and total U.S. wheat exports, and (2) the predictions of total Government wheat inventory, commercial wheat inventory, and total U.S. wheat exports have very large relative overprediction errors at the very low levels of the observed values.

Table 5.--Measures of overall predictive accuracy: Computed Theil-U measures for endogenous variables, 1928-64

Endogenous variables	Identification	Theil-U measures: Ordinary least-squares estimate	Theil-U measures: Two-round least-squares estimate
Average farm price of wheat	P_t	0.0424	0.0424
Per capita wheat consumption	q_{ht}^*	0.0158	0.0150
Wheat used for feed	q_{ft}	0.1535	0.1616
Government wheat inventory	C_{gt}	0.1068	0.1068
Commercial wheat inventory	C_{ct}	0.1597	0.1577
Total U.S. wheat exports	q_{Et}	0.1196	0.1217

This retrospective analysis has suggested that the model's predictions of average farm prices of wheat and per capita wheat consumption can be expected to have smaller prediction errors than predictions of other endogenous variables in the system.

GRAPHIC ANALYSIS OF PREDICTION ERRORS

Ordinary Least-Squares Estimates
1928-64

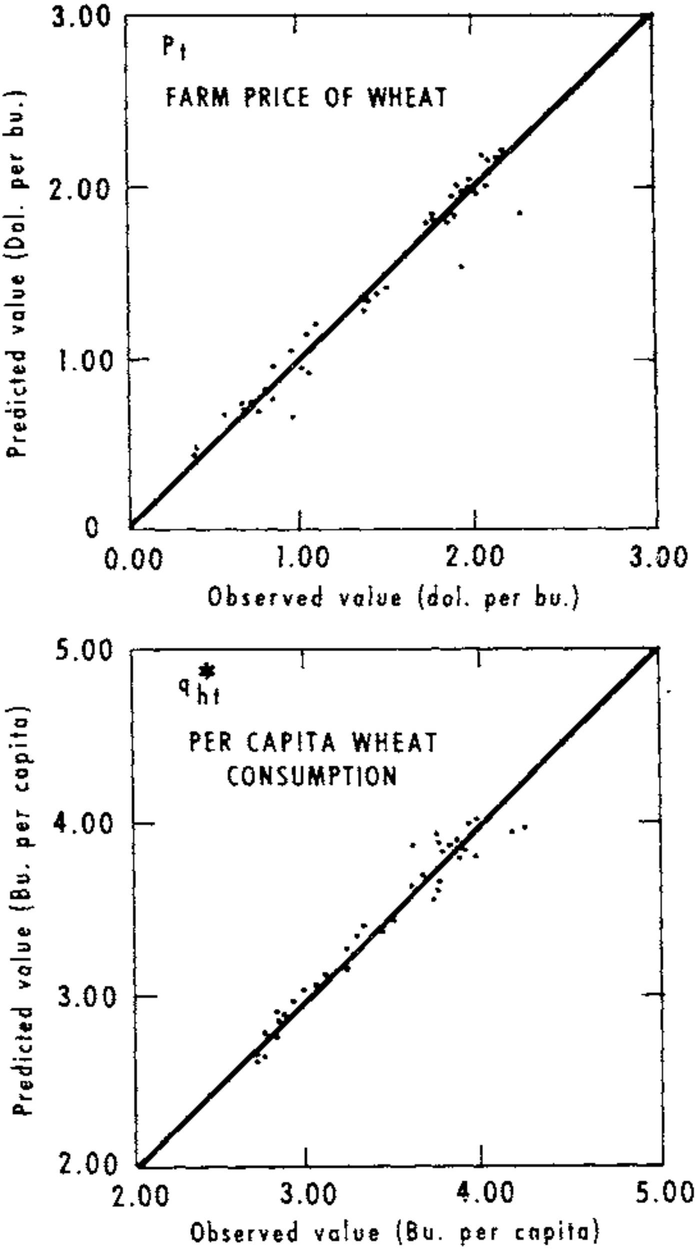
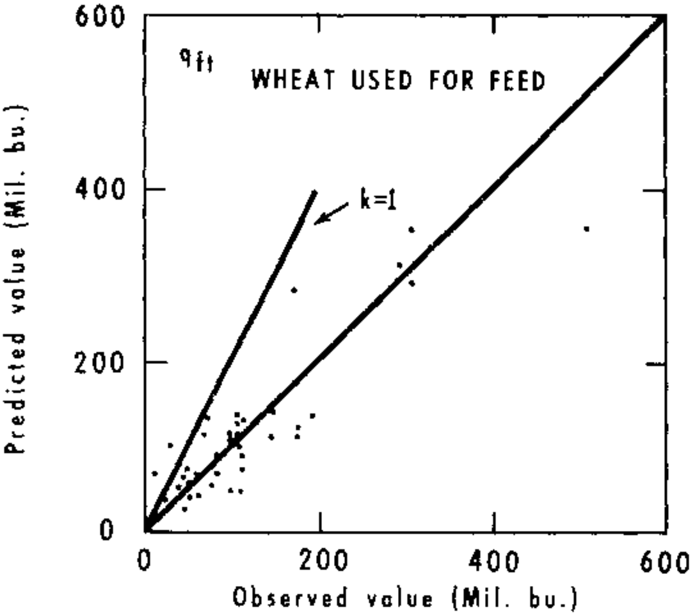


Figure 7

GRAPHIC ANALYSIS OF PREDICTION ERRORS

Ordinary Least-Squares Estimates

1928-64



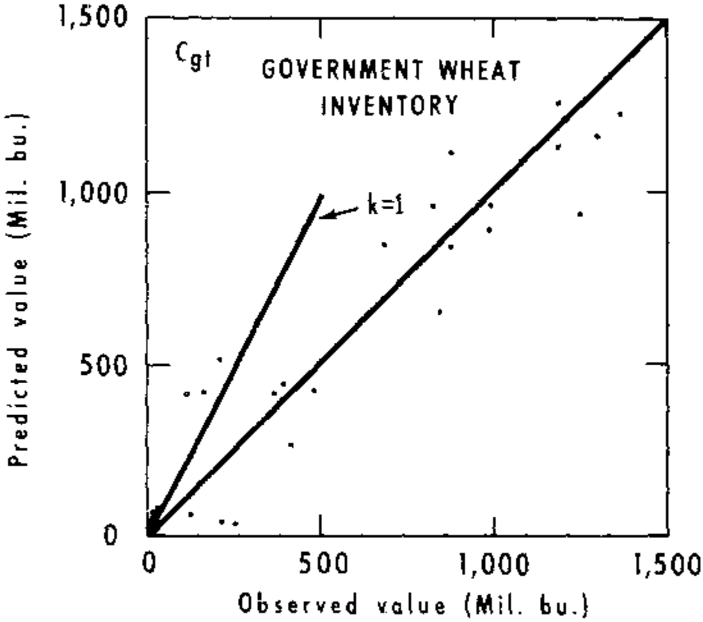
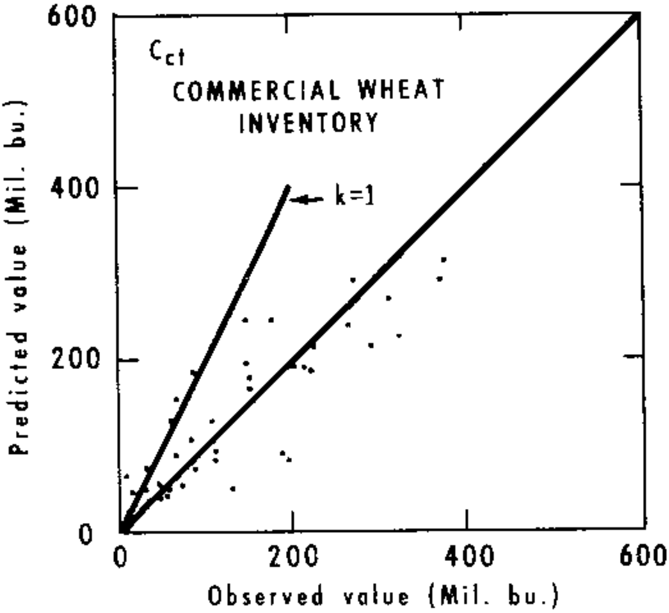


Figure 8

GRAPHIC ANALYSIS OF PREDICTION ERRORS

Ordinary Least-Squares Estimates
1928-64



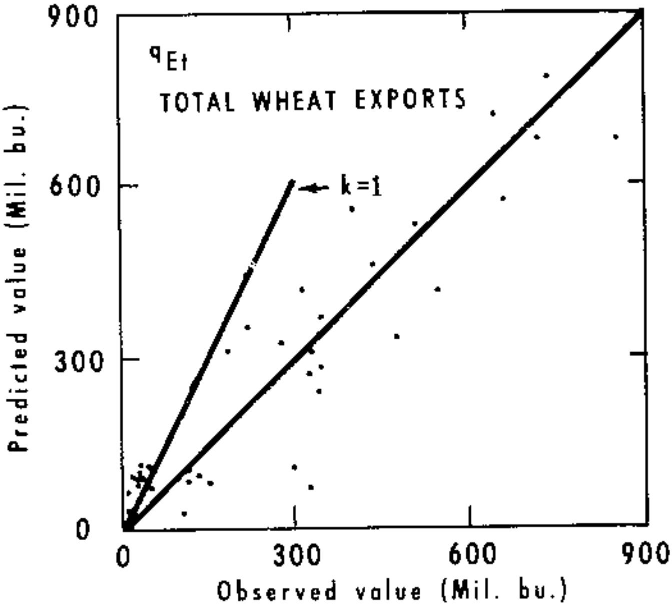


Figure 9

Conditional Predictions

One of the purposes of this study is to demonstrate how the model could be used to make alternative projections based upon different sets of conditions and assumptions. In using the estimated dynamic recursive structure in making predictions, four alternative wheat support prices--\$1.00, \$1.25, \$1.50, and \$1.75 per bushel--are considered and the other exogenous variables in the system are estimated by using the following autoregressive model:

$$X_{it} = a_i + b_i T_t + u_{it} \tag{4.15}$$

$$T_t = T_{t-1} + 1 \tag{4.16}$$

$$u_{it} = r_i u_{it-1} + e_{it} \tag{4.17}$$

where

X_{it} = i-th exogenous variable at time t

T_t = time (T_{1928} = 28)

u_{it} = disturbance terms

e_{it} = a non-serially-correlated disturbance term, and a_i, b_i, and r_i are the parameters to be estimated.

Because of the nature of the relation (4.16), the estimation procedure can be simplified as follows:

From (4.15) and (4.17), we have:

$$X_{it} - a_i - b_i T_t = r_i X_{it-1} - a_i r_i - b_i r_i T_{t-1} + e_{it}$$

or

$$X_{it} = a_i(1 - r_i) + r_i X_{it-1} + b_i T_t - b_i r_i T_{t-1} + e_{it} \tag{4.18}$$

Substituting (4.16) into (4.18), we get:

$$X_{it} = [a_i(1-r_i) + b_i] + r_i X_{it-1} + b_i(1-r_i)\ T_{t-1} + e_{it}$$

$$= s_{oi} + s_{1i} X_{it-1} + s_{2i} T_{t-1} + e_{it} \tag{4.19}$$

The coefficients s_{oi}, s_{1i}, and s_{2i} can be estimated by the ordinary least-squares method. And the parameters a_i, b_i, and r_i can be uniquely estimated by the following formulas:

$$r_i = s^*_{1i} \tag{4.20}$$

$$a_i = s^*_{oi}/(1-s^*_{1i}) - s^*_{2i}/(1-s^*_{1i})^2 \tag{4.21}$$

and

$$b_i = s^*_{2i}/(1-s^*_{1i}) \tag{4.22}$$

where s^*_{0i}, s^*_{1i}, and s^*_{2i} are the ordinary least-squares estimates of s_{0i}, s_{1i}, and s_{2i}, respectively.

The following relations were obtained by applying the above procedures to the 1928-64 data:

$$P_{fot} = 2.3008 + 0.7426\ P_{fot-1} + 0.4900\ T_t \quad (4.23)$$

$$P_{ct} = -5.6133 + 0.8464\ P_{ct-1} + 0.3908\ T_t \quad (4.24)$$

$$L_t = 36.7710 + 0.7282\ L_{t-1} + 0.1560\ T_t \quad (4.25)$$

$$O_t = 131.6000 + 0.4370\ O_{t-1} + 9.3000\ T_t \quad (4.26)$$

$$I_t = -324.3000 + 0.8162\ I_{t-1} + 12.4000\ T_t \quad (4.27)$$

From a predictive point of view, the above relations are better than the following simple ordinary least-squares non-autoregressive time trend model:

$$P_{fot} = -8.1955 + 2.2316\ T_t \quad (4.28)$$

$$P_{ct} = -14.6210 + 1.9257\ T_t \quad (4.29)$$

$$L_t = 134.5753 + 0.5676\ T_t \quad (4.30)$$

$$O_t = 308.1900 + 14.9800\ T_t \quad (4.31)$$

$$I_t = -1273.1400 + 53.0300\ T_t \quad (4.32)$$

To indicate the overall predictive performance of both models during the sample period, the Theil-U measures are computed and summarized in table 6.

Table 6 clearly indicates that the autoregressive model is substantially better than the non-autoregressive model as a prediction device. Therefore, in making conditional predictions, the future values of exogenous variables are estimated by using the autoregressive relations (4.23) through (4.27). The estimated exogenous variables, based on the 1964 initial conditions, are given in table 7.

Given the estimated dynamic recursive structure (3.6) through (3.11), and the estimated exogenous variables in table 7, the conditional predictions of the endogenous variables implied by the model are summarized in table 8.

The conditional predictions given in table 8 can be properly interpreted only by keeping in view several limitations:

(1) The reliability of the conditional predictions rests on the validity of the assumption that the structure of the system, or the estimates of the parameters, will not change in the future. If there is reason to believe that they will change, then the required changes should be incorporated into the model before making predictions.

Table 6.--Measure of overall predictive accuracy: Computed Theil-U measures for autoregressive and non-autoregressive models of exogenous variables in the system, 1928-64

Exogenous variables	Identification	Theil-U measures: (1) Autoregressive model	(2) Non-autoregressive model	(2)/(1)
Other feed grains, farm price index	P_{fot}	0.1001	0.1503	1.50
Consumer price index	P_{ct}	0.0153	0.0525	3.43
Grain-consuming livestock animal units	L_t	0.0237	0.0341	1.44
U.S. wheat production	O_t	0.0693	0.0781	1.13
U.S. per capita disposable income	I_t	0.0179	0.0582	3.25

Table 7.--Estimated exogenous variables, based on autoregressive model, 1965-80

Year (t)	Estimated exogenous variables: Other feed grains, farm price index (P_{fot})	Consumer price index (P_{ct})	Grain-consuming livestock animal units (L_t)	U.S. wheat production (O_t)	U.S. per capita disposable income (I_t)
	1957-59= 100	1957-59= 100	Mil. units	Mil. bu.	Dol. per capita
1965	113.61	111.28	169.00	1300.04	2332.84
1966	119.01	114.37	170.13	1313.52	2398.17
1967	123.51	117.37	171.11	1328.71	2463.88
1968	127.34	120.31	171.98	1344.64	2529.92
1969	130.67	123.18	172.77	1360.91	2596.22
1970	133.64	126.00	173.51	1377.32	2662.74
1971	136.33	128.78	174.19	1393.79	2729.43
1972	138.82	131.52	174.85	1410.29	2796.26
1973	141.16	134.24	175.49	1426.79	2863.20
1974	143.38	136.92	176.10	1443.31	2930.25
1975	145.53	139.59	176.71	1459.83	2997.37
1976	147.61	142.24	177.31	1476.34	3064.55
1977	149.64	144.87	177.90	1492.86	3131.79
1978	151.65	147.48	178.48	1509.38	3199.06
1979	153.62	150.09	179.07	1525.90	3266.38
1980	155.58	152.69	179.65	1542.42	3333.72

Table 8.--Predictions of endogenous variables, based on the ordinary least-squares estimated structure and the autoregressive estimated exogenous variables, 1970, 1975, and 1980

Year (t)	Per capita wheat consumption (q^{*}_{ht})	Wheat used for feed (q_{ft})	Government wheat inventory (C_{gt})	Commercial wheat inventory (C_{ct})	Total U.S. wheat exports (q_{Et})
	Bu. per capita	Mil. bu.	Mil. bu.	Mil. bu.	Mil. bu.
		Wheat support price = $1.75 (Per bu.)			
1970	2.39	136.82	975.10	73.10	662.55
1975	2.33	161.92	1064.65	66.61	713.17
1980	2.31	183.56	1130.17	62.16	748.61
		Wheat support price = $1.50 (Per bu.)			
1970	2.45	169.90	881.23	102.24	633.96
1975	2.38	195.00	955.90	97.03	676.19
1980	2.36	216.63	1018.01	92.68	707.97
		Wheat support price = $1.25 (Per bu.)			
1970	2.50	202.97	787.35	131.37	605.36
1975	2.43	228.07	847.15	127.44	638.63
1980	2.41	249.70	905.86	123.54	669.12
		Wheat support price = $1.00 (Per bu.)			
1970	2.55	236.04	693.47	160.51	577.77
1975	2.48	261.14	738.41	157.85	600.24
1980	2.46	282.78	803.71	153.81	628.54

(2) The predictions are based on the assumption that all future values of the disturbances are equal to their expected value, zero. But we have every reason to believe that the future disturbances will in fact take on some nonzero values even though they might be distributed around zero as in the past. As a result, there are some errors involved in our predictions.

(3) The prediction errors also result from errors in estimating the exogenous variables in the system. When using such a model for making long-run projections and policy analysis, more sophisticated methods than simple extrapolative models are needed for projecting the exogenous variables. The extrapolative method was used here to demonstrate the mechanics of using the model to make long-run projections. Because of these limitations, the projections are conditioned by the data, assumptions, and model used.

APPENDIX A

SOME PROPERTIES OF ESTIMATION PROCEDURES OF A RECURSIVE SYSTEM

A system of linear stochastic structural relations can be written in the following simple matrix form:

$$Ay_t + B^{*}z_t = u_t \tag{A-1}$$

where

- A = a G by G matrix of coefficients of current endogenous variables,
- B^{*} = a G by K matrix of coefficients of predetermined variables,
- y_t = a 1 by G column vector of endogenous variables in time period t,
- z_t = a 1 by K column vector of predetermined variables in time period t,

and

- u_t = a 1 by G column vector of random disturbance terms in time period t.

DEFINITIONS

A system of linear stochastic structural relations (A-1) is called a recursive system if the matrix (A) of the coefficients associated with the current endogenous variables in the system is a triangular matrix.

A system of linear stochastic structural relations (A-1) is called a diagonal recursive system if (1) the matrix (A) is a triangular matrix, and (2) the variance-covariance matrix is a diagonal matrix, that is, $U = E\ (u_t u_t')$ is a diagonal matrix.

PROPERTIES OF ESTIMATION

Nonparametric Case

The direct ordinary least-squares estimation of the coefficients of a structural equation in a system of structural relations (A-1) does not, in general, yield consistent estimates. But it does yield consistent estimates for the case of the diagonal recursive system. An estimate Θ^{*} of a parameter Θ is said to be consistent if the probability that the absolute deviation between the estimate Θ^{*} and the true parameter Θ be less than any given arbitrarily small number d, approaches one as the size of the sample approaches infinity, that is,

$$\lim_{t \to \infty} P \left[\left| \Theta^{*}(t) - \Theta \right| < d \right] = 1 \tag{A-2}$$

where P is probability and t is the sample size.

If $\Theta^*(t)$ has the property (A-2) it is said to possess the probability limit Θ, and the relation (A-2) is also denoted as:

$$\underset{t \to \infty}{\text{plim}}\ \Theta^*(t) = \Theta \qquad \text{(A-3)}$$

For simplifying the exposition, let us consider the following simple case: 1/

Structural System

Demand relation:

$$P_t + a_{12} Q_t + b_{1o} = u_{1t} \qquad \text{(A-4)}$$

Supply relation:

$$a_{21} P_t + Q_t + b_{20} + b_{21} z_{1t} = u_{2t} \qquad \text{(A-5)}$$

where

P_t = price at time t,

Q_t = quantity at time t,

z_t = a supply shifter, and

u_{it} = disturbance terms

Rewriting the above system of structural relations (A-4) and (A-5) in the matrix form (A-1), we have:

$$Ay_t + B^* z_t = u_t$$

or

$$\begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix} \begin{bmatrix} P_t \\ Q_t \end{bmatrix} + \begin{bmatrix} b_{1o} & O \\ b_{2o} & b_{21} \end{bmatrix} \begin{bmatrix} 1 \\ z_{1t} \end{bmatrix} = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \qquad \text{(A-6)}$$

In this model, we assume: (1) The expected value of the disturbance term is zero; and (2) the variance-covariance matrix is $U = E(u_t u_t')$, i.e.:

$$E(u_t) = \begin{bmatrix} E(u_{1t}) \\ E(u_{2t}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0 \qquad \text{(A-7)}$$

and

$$U = E(u_t u_t') = E\left(\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} [u_{1t}\ u_{2t}] \right) = E\left(\begin{bmatrix} u_{1t}u_{1t} & u_{1t}u_{2t} \\ u_{2t}u_{1t} & u_{2t}u_{2t} \end{bmatrix} \right)$$

$$= \begin{bmatrix} E(u_{1t})^2 & E(u_{1t}u_{2t}) \\ E(u_{2t}u_{1t}) & E(u_{2t})^2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \qquad \text{(A-8)}$$

1/ The principal derivations of a general model are, of course, similar to the derivations of this simple model.

If the matrix (A) is assumed to be nonsingular, then the structural system (A-6) can be expressed in terms of the following reduced form:

Reduced Form

$$y_t = Dz_t + v_t \tag{A-9}$$

where

$$D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} = -A^{-1}B^*$$

$$= -\begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} b_{1o} & 0 \\ b_{2o} & b_{21} \end{bmatrix}$$

$$= \begin{bmatrix} (-b_{1o} + b_{2o}a_{12})/(1-a_{12}a_{21}) & (b_{21}a_{12})/(1-a_{12}a_{21}) \\ (-b_{2o} + b_{1o}a_{21})/(1-a_{12}a_{21}) & (-b_{21})/(1-a_{12}a_{21}) \end{bmatrix} \tag{A-10}$$

and

$$v_t = \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} = A^{-1}u_t$$

$$= -\begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

$$= \begin{bmatrix} (u_{1t} - a_{12}u_{2t})/(1-a_{12}a_{21}) \\ (-a_{21}u_{1t} + u_{2t})/(1-a_{12}a_{21}) \end{bmatrix} \tag{A-11}$$

Now, if the demand relation (A-4) is fitted directly by the ordinary least-squares method, then the ordinary least-squares estimate a^*_{12} of the coefficient a_{12} is:

$$a^*_{12} = \frac{\sum_t (P_t - \overline{P})(Q_t - \overline{Q})}{\sum_t (Q_t - \overline{Q})^2} \tag{A-12}$$

where $\overline{P}$ is the mean value of P_t and $\overline{Q}$ is the mean value of Q_t.

From the reduced form (A-9), we have the following relations:

$$P_t = d_{11} + d_{12}z_{1t} + v_{1t} \tag{A-13}$$

or

$$\overline{P} = d_{11} + d_{12}\overline{z}_1 + \overline{v}_1 \tag{A-14}$$

and

$$Q_t = d_{21} + d_{22}z_{1t} + v_{2t} \tag{A-15}$$

or

$$\overline{Q} = d_{21} + d_{22}\overline{z}_1 + \overline{v}_2 \qquad \text{(A-16)}$$

where $\overline{z}_1$ is the mean value of z_{1t}, $\overline{v}_1$ is the mean value of v_{1t}, and $\overline{v}_2$ is the mean value of v_{2t}.

By substituting (A-13), (A-14), and (A-15) into the numerator of (A-12), and (A-15) and (A-16) into the denominator, we obtain:

$$\sum_t (P_t-\overline{P})(Q_t-\overline{Q}) = d_{12}d_{22}\sum_t(z_{1t}-\overline{z}_1)^2 + d_{12}\sum_t(v_{2t}-\overline{v}_2)(z_{1t}-\overline{z}_1)$$
$$+ d_{22}\sum_t(z_{1t}-\overline{z}_1)(v_{1t}-\overline{v}_1) + \sum_t(v_{1t}-\overline{v}_1)(v_{2t}-\overline{v}_2) \qquad \text{(A-17)}$$

and

$$\sum_t(Q_t-\overline{Q})^2 = d_{22}^2\sum_t(z_{1t}-\overline{z}_1)^2 + 2d_{22}\sum_t(z_{1t}-\overline{z}_1)(v_{2t}-\overline{v}_2)$$
$$+ \sum_t(v_{2t}-\overline{v}_2)^2 \qquad \text{(A-18)}$$

Now, we define the following notations:

$$\operatorname*{plim}_{t\to\infty} \sum_t (z_{1t}-\overline{z}_1)^2 = H_{11} \qquad \text{(A-19)}$$

$$\operatorname*{plim}_{t\to\infty} \sum_t (v_{1t}-\overline{v}_1)^2 = w_{11} \qquad \text{(A-20)}$$

$$\operatorname*{plim}_{t\to\infty} \sum_t (v_{2t}-\overline{v}_2)^2 = w_{22} \qquad \text{(A-21)}$$

$$\operatorname*{plim}_{t\to\infty} \sum_t (v_{1t}-\overline{v}_1)(v_{2t}-\overline{v}_2) = w_{21} \text{ or } w_{12} \qquad \text{(A-22)}$$

$$\operatorname*{plim}_{t\to\infty} \sum_t(z_{1t}-\overline{z}_1)(v_{it}-\overline{v}_i) = 0 \text{ for } i = 1,2. \qquad \text{(A-23)}$$

By taking the probability limit of the direct ordinary least-squares estimate a_{12}^*, and noticing the relations from (A-17) through (A-23), we have:

$$\operatorname*{plim}_{t\to\infty} a_{12}^* = \operatorname*{plim}_{t\to\infty} \frac{\sum_t(P_t-\overline{P})(Q_t-\overline{Q})}{\sum_t (Q_t-\overline{Q})^2}$$

$$= (d_{12}d_{22}H_{11} + w_{12})/(d_{22}^2H_{11} + w_{22}) \qquad \text{(A-24)}$$

Furthermore, the values of w_{12} and w_{22} can be expressed in terms of the disturbance terms u_{it} and the coefficients a_{ij} of the structural system as following:

$$E(v_t v_t') = E\left(\begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} [v_{1t} \; v_{2t}] \right) = \begin{bmatrix} E(v_{1t})^2 & E(v_{1t}v_{2t}) \\ E(v_{2t}v_{1t}) & E(v_{2t})^2 \end{bmatrix}$$

$$= \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \qquad \text{(A-25)}$$

By substituting the relations of (A-11) and (A-8) into (A-25), we obtain:

$$w_{21} = E(v_{1t}v_{2t}) = E\left[\left(\frac{u_{1t} - a_{12}u_{2t}}{1 - a_{12}a_{21}} \right) \left(\frac{-a_{21}u_{1t} + u_{2t}}{1 - a_{12}a_{21}} \right) \right]$$

$$= (1-a_{12}a_{21})^{-2} E(u_{1t} - a_{12}u_{2t})(-a_{21}u_{1t} + u_{2t})$$

$$= (1-a_{12}a_{21})^{-2} E(-a_{21}u_{1t}^2 + a_{12}a_{21}u_{1t}u_{2t} + u_{1t}u_{2t} - a_{12}u_{2t}^2)$$

$$= (-a_{21}k_{11} + a_{12}a_{21}k_{12} + k_{12} - a_{12}k_{22})/(1-a_{12}a_{21})^2 \qquad \text{(A-26)}$$

and

$$w_{22} = E(v_{2t})^2$$

$$= (a_{21}^2 k_{11} - 2a_{21}k_{12} + k_{22})/(1-a_{12}a_{21})^2 \qquad \text{(A-27)}$$

By substituting the relations (A-10), (A-26), and (A-27) into (A-24), we finally obtain:

$$\operatorname*{plim}_{t\to\infty} a_{12}^* = \frac{-a_{12}(b_{21}^2 H_{11} + k_{22}) - a_{21}k_{11} + (1 + a_{12}a_{21})k_{12}}{(b_{21}^2 H_{11} + k_{22}) + a_{21}^2 k_{11} - 2a_{21}k_{12}} \qquad \text{(A-28)}$$

It is clearly indicated in the above relation (A-28) that the direct ordinary least-squares estimate a_{12}^* is not a consistent estimate of the coefficient a_{12}.

Now, if we assume that the structural system (A-6) is a diagonal recursive system, that is, (1) $a_{21} = 0$ in the matrix (A) of (A-6) and (2) both k_{12} and k_{21} are zero in the matrix U of (A-8), then the last two terms of both numerator and denominator in (A-28) are equal to zero. Hence, we have:

$$\operatorname*{plim}_{t\to\infty} a_{12}^* = -a_{12} \qquad \text{(A-29)}$$

Therefore, we have shown that the direct ordinary least-squares estimation of the coefficients of a structural relation in a structural system does not, in general, yield consistent estimates; but it does yield consistent estimates for the case of a diagonal recursive system.

<u>Parametric Case</u>

In the previous section, we considered a nonparametric case in which the distribution of the disturbance term u_t is not specified or restricted to be one of a certain class of probability distributions. In this section we consider a parametric case and assume the disturbance term u_t to have a multivariate normal distribution with a zero mean vector and with a constant variance-covariance matrix U, where $U = E(u_t u_t')$ for t = 1, 2, .., n. Moreover, we assume: (1) the disturbance terms are serially independent, and (2) all exogenous variables in the system are independent of the disturbance terms. Under these assumptions, the likelihood function for the endogenous variables, conditional upon the values of predetermined variables z_t, is: 2/

$$L = P(y_t \mid z_t \text{ and } t = 1,2,..,n)$$

$$= \left|\det A\right|^n (1/2\pi)^{1/2Gn} (\det U)^{-1/2n} e^{-1/2 \sum_{t=1}^{n} u_t' U^{-1} u_t} \qquad \text{(A-30)}$$

where

$u_t = Ay_t + B^* z_t$,

$\left|\det A\right|$ = the absolute value of the determinant of the matrix A,

det U = the determinant of the variance-covariance matrix U,

$\pi = 3.1416$

e = 2.7183, and

G = the number of endogenous variables in the system.

To show the equivalence between the full-information maximum likelihood estimates and the direct ordinary least-squares estimates in a diagonal recursive system, we consider again the structural system (A-6) foumulated in the previous section and assume that the system (A-6) is a diagonal recursive system, i.e.,

$$A = \begin{bmatrix} 1 & a_{12} \\ 0 & 1 \end{bmatrix} \qquad \text{(A-31)}$$

and

$$U = E(u_t u_t') = \begin{bmatrix} k_{11} & 0 \\ 0 & k_{22} \end{bmatrix} \qquad \text{(A-32)}$$

Under the assumptions of (A-31) and (A-32), we have the following simple relations:

$$\det A = 1 \qquad \text{(A-33)}$$

$$\det U = k_{11} k_{22} \qquad \text{(A-34)}$$

and

2/ For detailed discussion, see J. Johnston, Econometric Method, McGraw-Hill, New York, 1960, pp. 240-42 and pp. 264-65; and also see W. C. Hood and T. C. Koopmans (eds.), Studies in Econometric Method, Cowles Commission Monograph No. 14, Willey, New York, 1953, pp. 190-91.

and

$$-1/2 \sum_{t=1}^{n} u_t' U^{-1} u_t = -1/2 \sum_{t=1}^{n} \left\{ [u_{1t} \; u_{2t}] \begin{bmatrix} k_{11} & 0 \\ 0 & k_{22} \end{bmatrix}^{-1} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \right\}$$

$$= -1/2 \sum_{t=1}^{n} (u_{1t}^2/k_{11} + u_{2t}^2/k_{22}) \qquad \text{(A-35)}$$

For this diagonal recursive system, we have the log of the likelihood function as following:

$$L^* = \log_e L = n\log_e \left|\det A\right| + n\log_e(1/2\pi) - 1/2n\log_e(\det U) -1/2 \left(\sum_{t=1}^{n} u_t' U^{-1} u_t \right)\log_e e$$

$$= n\log_e(1/2\pi) - 1/2n\log_e k_{11} - 1/2n\log_e k_{22} - 1/2 \sum_{t=1}^{n} (u_{1t}^2/k_{11}) -1/2 \sum_{t=1}^{n} (u_{2t}^2/k_{22}) \qquad \text{(A-36)}$$

where

$$\sum_{t=1}^{n} u_{1t}^2 = \sum_{t=1}^{n} (P_t + a_{12}Q_t + b_{1o})^2$$

$$= \sum_{t=1}^{n} (P_t^2 + 2a_{12}P_tQ_t + 2b_{1o}P_t + a_{12}^2Q_t^2 + 2b_{1o}a_{12}Q_t + b_{1o}^2) \qquad \text{(A-37)}$$

and

$$\sum_{t=1}^{n} u_{2t}^2 = \sum_{t=1}^{n} (Q_t + b_{2o} + b_{21}z_{1t})^2 \qquad \text{(A-38)}$$

Therefore, the maximum likelihood estimates, a_{12}^{**}, of the coefficient a_{12} can be obtained by following steps:

(1) Taking the partial derivative of L^* with respect to a_{12} and setting it equal to zero:

$$\frac{d\,L^*}{d\,a_{12}} = -(1/k_{11}) \sum_{t=1}^{n} (P_tQ_t + a_{12}Q_t^2 + b_{1o}\,Q_t) = 0 \qquad \text{(A-39)}$$

(2) Taking the partial derivative of L^* with respect to b_{1o} and setting it equal to zero:

$$\frac{d\,L^*}{d\,b_{1o}} = -(1/k_{11}) \sum_{t=1}^{n} (P_t + a_{12}Q_t + b_{1o}) = 0 \qquad \text{(A-40)}$$

(3) Solving b_{1o} from (A-40):

$$\sum_{t=1}^{n} P_t + a_{12} \sum_{t=1}^{n} Q_t + nb_{1o} = 0$$

$$b_{1o} = -(1/n) \sum_{t=1}^{n} P_t - a_{12}(1/n) \sum_{t=1}^{n} Q_t$$

$$= -(\overline{P} + a_{12}\overline{Q}) \tag{A-41}$$

where $\overline{P}$ is the mean value of P_t and $\overline{Q}$ is the mean value of Q_t.

(4) Substituting (A-41) into (A-39) for b_{1o} and solving for a_{12}:

$$\sum_{t=1}^{n} P_tQ_t + a_{12} \sum_{t=1}^{n} Q_t^2 - (\overline{P} + a_{12}\overline{Q}) \sum_{t=1}^{n} Q_t = 0$$

$$\sum_{t=1}^{n} P_tQ_t + a_{12} \sum_{t=1}^{n} Q_t^2 - n\,(\overline{P} + a_{12}\overline{Q})\,\overline{Q} = 0$$

$$\sum_{t=1}^{n} P_tQ_t - n\,\overline{P}\,\overline{Q} + a_{12}\left(\sum_{t=1}^{n} Q_t^2 - n\,\overline{Q}^2\right) = 0$$

$$a_{12}^{**} = -\left(\sum_{t=1}^{n} P_tQ_t - n\,\overline{P}\,\overline{Q}\right)\Big/\left(\sum_{t=1}^{n} Q_t^2 - n\,\overline{Q}^2\right)$$

$$= -\frac{\sum_{t=1}^{n} (P_t - \overline{P})\,(Q_t - \overline{Q})}{\sum_{t=1}^{n} (Q_t - \overline{Q})^2} \tag{A-42}$$

Comparing (A-42) with (A-12), we find that the maximum likelihood estimate a_{12}^{**} is exactly equal to the direct ordinary least-squares estimate a_{12}^{*}. Therefore, we have shown that the full-information maximum likelihood estimates of a diagonal recursive system are identical to the direct ordinary least-squares estimates.

APPENDIX B

TWO-ROUND LEAST-SQUARES ESTIMATION PROCEDURES

The two-round least-squares procedures of estimating the coefficients of the formulated system (3.1) can be summarized by the following steps:

(1) Estimating the farm price and support price relation directly by the ordinary least-squares procedures to obtain the estimated values $(\hat{P}_t)$ of P_t, i.e.:

$$\hat{P}_t = c^*_{19} + c^*_{11}P_{st} + c^*_{12}K_tP_{fot} \tag{B-1}$$

where c^*_{ij} are the least-squares estimates of the coefficients c_{ij}.

(2) Substituting the estimated values $\hat{P}_t$ obtained from (B-1) for P_t in the food consumption relation and applying least-squares procedures again to obtain the estimated values $(\hat{q}^*_{ht})$ of q^*_{ht}, i.e.:

$$\hat{q}^*_{ht} = c^*_{29} + a^*_{21}\hat{P}_t + c^*_{23}P_{ct} + c^*_{24}G(I_t) \tag{B-2}$$

where a^*_{ij} are the least-squares estimates of a_{ij}.

(3) Substituting the estimated values $\hat{P}_t$ obtained from (B-1) into the feed consumption relation and applying the least-squares procedures again, i.e.:

$$q_{ft} = c^*_{39} + a^*_{31}\hat{P}_t + c^*_{35}P_{fot} + c^*_{36}L_t + c^*_{37}D_t \tag{B-3}$$

(4) Estimating the Government inventory relation by applying directly the ordinary least-squares procedures to obtain the estimated values $(\hat{C}_{gt})$ of C_{gt}, i.e.:

$$\hat{C}_{gt} = c^*_{49} + c^*_{41}P_{st} + c^*_{48}\overline{K}_t\overline{D}_{t-2}O_t + b^*_{44}C_{gt-1} \tag{B-4}$$

where b^*_{ij} are the least-squares estimates of b_{ij}.

(5) Substituting the estimated values $\hat{P}_t$ obtained from (B-1) and the estimated values $\hat{C}_{gt}$ obtained from (B-4) into the commercial inventory relation, and applying the ordinary least-squares procedures again, i.e.:

$$C_{ct} = c^*_{59} + a^*_{51}\hat{P}_t + a^*_{54}\hat{C}_{gt} + b^*_{55}C_{ct-1} \tag{B-5}$$

(6) Substituting the estimated values $\hat{q}^*_{ht}$ obtained from (B-2) into the export relation and applying the ordinary least-squares procedures again, i.e.:

$$q_{Et} = c^*_{69} + a^*_{62}\hat{q}^*_{ht} + b^*_{64}(C_{ct-1} + C_{gt-1}) + b^*_{66}q_{Et-1} \tag{B-6}$$

APPENDIX C

BASIC SAMPLE DATA

Table C-1.--Basic sample data, endogenous variables

Year (t)	Average farm price of wheat (P_t)	Per capita wheat consumption (q^*_{ht})	Wheat used for feed (q_{ft})	Government wheat inventory (C_{gt})	Commercial wheat inventory (C_{ct})	Total U.S. wheat exports (q_{Et})
	Dol. per bu.	Bu. per capita	Mil. bu.	Mil. bu.	Mil. bu.	Mil. bu.
1928	0.99	4.22	63.87	0	226.82	141.22
1929	1.03	4.14	28.90	0	291.12	140.35
1930	0.66	3.98	179.50	0	312.51	112.43
1931	0.38	3.90	190.24	0	375.26	122.90
1932	0.38	3.95	142.81	0	377.75	31.87
1933	0.74	3.58	102.36	0	272.89	25.60
1934	0.84	3.64	113.49	0	145.89	10.53
1935	0.83	3.86	83.34	0	140.43	4.44
1936	1.02	3.86	100.15	0	83.17	9.58
1937	0.96	3.81	114.86	0	153.11	103.89
1938	0.56	3.83	141.69	28.10	221.92	108.08
1939	0.69	3.74	101.13	11.90	267.82	45.26
1940	0.67	3.72	111.77	207.80	176.93	33.87
1941	0.94	3.59	114.25	419.20	211.58	27.77
1942	1.09	3.78	305.77	398.00	220.90	30.96
1943	1.35	3.74	511.23	117.10	199.46	42.73
1944	1.41	3.73	300.10	125.70	153.48	49.11
1945	1.49	3.71	296.55	32.50	67.59	320.03
1946	1.90	3.46	177.53	0.70	83.14	328.05
1947	2.29	3.39	178.31	0.80	195.14	340.22
1948	1.98	3.25	105.35	243.50	63.79	327.83
1949	1.88	3.28	111.26	361.20	63.51	179.21
1950	2.00	3.19	108.81	207.60	192.27	334.51
1951	2.11	3.18	102.40	154.90	101.08	470.35
1952	2.09	3.09	82.48	492.50	113.04	315.65
1953	2.04	3.03	76.64	849.90	83.61	215.70
1954	2.12	2.97	60.07	990.00	46.18	273.42
1955	1.98	2.89	53.14	976.60	53.89	346.27
1956	1.97	2.84	47.40	836.70	72.13	549.54
1957	1.93	2.82	41.98	853.10	28.27	402.92
1958	1.75	2.83	46.86	1,242.70	52.37	443.29
1959	1.76	2.80	40.78	1,287.40	26.12	510.24
1960	1.74	2.80	45.73	1,367.90	43.28	661.95
1961	1.83	2.71	54.39	1,191.60	130.27	719.86
1962	2.04	2.69	21.38	1,188.90	6.03	642.30
1963	1.85	2.67	12.84	881.50	19.69	858.70
1964	1.38	2.67	70.02	705.50	113.41	728.00

Table C-2.--Basic sample data, exogenous variables

Year	Average support price of wheat (P_{st})	Other feed grains farm price index (P_{fot})	Consumer price index (P_{ct})	U.S. per capita disposable income (I_t)	Grain-consuming livestock animal units (L_t)	U.S. wheat production (O_t)	Dummy variables (D_t)	(K_t)	($\overline{K_t}\overline{D_{t-2}}$)
	Dol. per bu.	1957-59 = 100	1957-59 = 100	Dol. per capita	Mil. units	Mil. bu.			
1928	0	76	59.7	653	153.16	914.37	0	1	0
1929	0	71	59.7	684	154.07	824.18	0	1	0
1930	0	51	58.2	605	152.75	886.52	0	1	0
1931	0	31	53.0	516	156.44	941.54	0	1	0
1932	0	30	47.6	390	159.74	756.31	0	1	0
1933	0	51	45.1	363	153.95	552.22	0	1	0
1934	0	76	46.6	414	131.19	526.05	0	1	0
1935	0	59	47.8	460	138.66	628.23	0	1	0
1936	0	93	48.3	518	137.83	629.88	0	1	0
1937	0	48	50.0	553	137.81	873.91	0	1	0
1938	0.59	43	49.1	504	148.78	919.91	0	0	1
1939	0.63	52	48.4	537	156.14	741.21	0	0	1
1940	0.64	56	48.8	573	155.75	814.65	0	0	1
1941	0.98	70	51.3	695	167.12	941.97	0	0	1
1942	1.14	89	56.8	867	192.23	969.38	1	0	1
1943	1.23	110	60.3	976	193.05	843.81	1	0	1
1944	1.35	103	61.3	1,057	172.56	1,060.11	1	0	0
1945	1.38	125	62.7	1,074	167.26	1,107.62	1	0	0
1946	1.49	152	68.0	1,132	159.62	1,152.12	1	0	0
1947	1.84	191	77.8	1,179	153.10	1,358.91	0	0	0
1948	2.00	113	83.8	1,290	158.60	1,294.91	0	0	0
1949	1.95	117	83.0	1,264	163.84	1,098.42	0	0	1
1950	1.99	142	83.8	1,364	168.10	1,019.34	0	0	1
1951	2.18	152	90.5	1,468	167.33	988.16	0	0	1
1952	2.20	136	92.5	1,518	158.94	1,306.44	0	0	1
1953	2.21	129	93.2	1,582	156.85	1,173.07	0	0	1
1954	2.24	122	93.6	1,585	161.60	983.90	0	0	1
1955	2.08	113	93.3	1,666	165.26	937.09	0	0	1
1956	2.00	110	94.7	1,743	160.93	1,005.40	0	0	1
1957	2.00	97	98.0	1,801	159.91	955.74	0	0	1
1958	1.82	98	100.7	1,831	167.73	1,457.44	0	0	1
1959	1.81	95	101.5	1,905	165.75	1,121.12	0	0	1
1960	1.78	92	103.1	1,937	167.56	1,357.27	0	0	1
1961	1.79	95	104.2	1,983	168.99	1,234.74	0	0	1
1962	2.00	99	105.4	2,064	172.80	1,093.67	0	0	1
1963	1.82	101	106.7	2,132	172.26	1,142.01	0	0	1
1964	1.32	107	108.1	2,268	167.66	1,290.47	0	0	1

APPENDIX D

METHOD OF DETERMINING THE STABILITY OF A DYNAMIC SYSTEM

The dynamic system (D-1) is a stable system, if the matrix D_1^k approaches a null matrix as k increases.

$$y_t = D_1 y_{t-1} + D_2 x_t \qquad \text{(D-1)}$$

where

y_t = a vector of endogenous variables in the system at time t,

y_{t-1} = a vector of lagged endogenous variables in the system at time t-1,

x_t = a vector of exogenous variables in the system at time t,

D_1 and D_2 are coefficient matrices.

The matrix D_1^k will approach a null matrix if the latent roots of the matrix D_1 are all in the interior of the unit circle. Hence, the stability of the dynamic system (D-1) is determined by the magnitude of the maximum (dominant) latent root of the matrix D_1. The latent root of matrix D_1 is defined as a scalar w such that the determinant $|D_1 - wI| = 0$, where I is the identity matrix. The determinant $|D_1 - wI| = 0$ can be expressed in terms of a polynomial $F_o(w)$ of n-th degree in w as (D-2), n is the rank of the matrix D_1 and the roots of (D-2) are the latent roots of D_1:

$$F_o(w) = d_n w^n + d_{n-1} w^{n-1} + \dots + d_1 w + d_o = 0 \qquad \text{(D-2)}$$

where

$d_n > 0$

Without the use of a computer, finding the roots of (D-2) may be quite time consuming, especially if n is large. But fortunately, there is a relatively simple method with which we can determine whether or not all the roots of (D-2) are in the interior of the open unit circle without solving for the roots themselves. This method was introduced recently by Jury. 1/ The necessary and sufficient conditions for the roots of (D-2) being in the interior of unit circle are simply the following three conditions, (D-3), (D-4), and (D-5):

$$F_o(w=1) > 0 \qquad \text{(D-3)}$$

$F_o(w=-1) < 0$, if n is an odd number; or

$F_o(w=-1) > 0$, if n is an even number, and (D-4)

$|s_i| < 1$, for $i = 0,1,2,\dots, n-2$. (D-5)

1/ E. I. Jury, "A Stability Test for Linear Discrete Systems Using a Simple Division," Institute of Radio Engineers Proceeding, Vol. 49, No. 2, December 1961, pp. 1948-49.

Conditions (D-3) and (D-4) can be easily checked by substituting 1 or -1 for w in (D-2). If they are not satisfied, then it can be established that (D-2) has at least one root that is not in the interior of the unit circle and, consequently, the dynamic system associated with (D-2) is an unstable dynamic system. There is then no need to check condition (D-5). Condition (D-5) has to be checked only when (D-3) and (D-4) are satisfied. The values of s_i in (D-5) can be obtained as follows:

Given a polynomial such as (D-2), we define the reverse polynomial $F_o^{-1}(w)$ of (D-2) as (D-6):

$$\begin{aligned} F_o^{-1}(w) &= w^n F_o(1/w) \\ &= w^n[d_n(1/w)^n + d_{n-1}(1/w)^{n-1} + \ldots + d_1(1/w) + d_o] \\ &= d_o w^n + d_1 w^{n-1} + \ldots + d_{n-1}w + d_n \end{aligned} \qquad \text{(D-6)}$$

By comparing (D-2) with (D-6), we can see that the only difference is that the coefficients are reversed.

Dividing (D-6) by (D-2), we obtain one quotient term s_o and a remainder $F_1^{-1}(w)$ as indicated in (D-7).

$$\frac{F_o^{-1}(w)}{F_o(w)} = s_o + \frac{F_1^{-1}(w)}{F_o(w)} \qquad \text{(D-7)}$$

The remainder $F_1^{-1}(w)$ will be a polynomial of degree n-1, and the quotient term s_o is simply equal to d_o/d_n. The other successive quotient term s_i (for i = 1,2,..., n-2) can be obtained by the following recursive relation (D-8):

$$\frac{F_i^{-1}(w)}{F_i(w)} = s_i + \frac{F_{i+1}^{-1}(w)}{F_i(w)} \quad \text{, for } i = o,1,2,\ldots, n-2. \qquad \text{(D-8)}$$

The estimated empirical reduced-form coefficient matrix D_1 obtained from the ordinary least-squares estimated structure is given in D_1 of appendix E. For the purpose of checking the stability of the estimated dynamic system, we delete the first three rows and the first three columns of the estimated matrix D_1 because the rank of the matrix D_1 is three in this case, and consider the following characteristic polynomial:

$$F_o(w) = \begin{vmatrix} 0.7446 - w & 0.0000 & 0.0000 \\ -0.0314 & 0.3635 - w & 0.0000 \\ 0.0967 & 0.0967 & 0.6494 - w \end{vmatrix} = 0$$

or

$$F_o(w) = w^3 - 1.7575\ w^2 + 0.9903\ w - 0.1758 = 0 \qquad \text{(D-9)}$$

By substituting 1 and -1 for w in (D-9), we obtain:

$$F_o(w=1) = 0.0570 > 0 \qquad \text{(D-10)}$$

$$F_o(w=-1) = -3.9236 < 0 \qquad (n = 3, \text{ odd number}) \qquad \text{(D-11)}$$

Since both conditions (D-3) and (D-4) are satisfied in (D-10) and (D-11), we therefore proceed to check condition (D-5):

$$\frac{F_0^{-1}(w)}{F_0(w)} = \frac{-0.1758\ w^3 + 0.9903\ w^2 - 1.7575\ w + 1}{w^3 - 1.7575\ w^2 + 0.9903\ w - 0.1758}$$

$$= -0.1758 + \frac{0.6813\ w^2 - 1.5834\ w + 0.9691}{w^3 - 1.7575\ w^2 + 0.9903\ w - 0.1758}$$

and

$$\frac{F_1^{-1}(w)}{F_1(w)} = \frac{0.6813\ w^2 - 1.5834\ w + 0.9691}{0.9691\ w^2 - 1.5834\ w + 0.6813}$$

$$= 0.7030 + \frac{-0.4703\ w + 0.4901}{0.9691\ w^2 - 1.5834\ w + 0.6813}$$

Because $n = 3$, we have to check the absolute value of s_0 and s_1. In our case, the absolute value of s_0 is 0.1758 and the absolute value of s_1 is 0.7030. Since they are both less than one, condition (D-5) is satisfied. Hence, the estimated dynamic system obtained from the ordinary least-squares method is a stable dynamic system. Similarly, it can be shown that the estimated dynamic system obtained from the two-round least-squares method is also stable.

APPENDIX E

ESTIMATION OF THE DERIVED REDUCED-FORM COEFFICIENT MATRIX OF THE STRUCTURE AND THE SHORT-RUN AND LONG-RUN IMPACT MULTIPLIER MATRICES

Structural System:

$$Ay_t + By_{t-1} + Cx_t = u_t$$

Derived Reduced Form:

$$y_t = D_1 y_{t-1} + D_2 x_t + v_t$$

where

$$D_1 = -A^{-1}B$$

$$D_2 = -A^{-1}C$$

and

$$v_t = A^{-1}u_t$$

Short-Run Impact Multiplier Matrix:

$$D_2 = -A^{-1}C$$

Long-Run Impact Multiplier Matrix:

$$(I - D_1)^{-1} D_2 = -(I + A^{-1}B)^{-1} A^{-1}C$$

Ordinary Least-Squares Estimates (1928-1964):

Structural Coefficient Matrix: A

	P_t	$\dot{q}_{ht}$	q_{ft}	C_{gt}	C_{ct}	q_{Et}
P_t	1.0000					
$\dot{q}_{ht}$	0.2284	1.0000				
q_{ft}	143.7966		1.0000			
C_{gt}				1.0000		
C_{ct}	64.4016			0.0422	1.0000	
q_{Et}		112.0979				1.0000

Structural Coefficient Matrix: B

	P_{t-1}	q^*_{ht-1}	q_{ft-1}	C_{gt-1}	C_{ct-1}	q_{Et-1}
P_t						
q^*_{ht}						
q_{ft}						
C_{gt}				-0.7446		
C_{ct}					-0.3635	
q_{Et}				-0.0967	-0.0967	-0.6494

Structural Coefficient Matrix: C

	P_{st}	K_tP_{fot}	P_{ct}	$G(I_t)$	P_{fot}	L_t	D_t	$\overline{K}_t\overline{D}_{t-2}0_t$	1
P_t	-0.9189	-0.0108							-0.1492
q^*_{ht}			-0.0077	-1.6005					-1.1989
q_{ft}					-1.6302	-1.7860	-159.4989		137.8420
C_{gt}	-115.6075							-0.1806	182.9923
C_{ct}									-200.2999
q_{Et}									-433.5437

Inverse Matrix: $-A^{-1}$

	P_t	q^*_{ht}	q_{ft}	C_{gt}	C_{ct}	q_{Et}
P_t	-1.0000					
q^*_{ht}	0.2284	-1.0000				
q_{ft}	143.7966		-1.0000			
C_{gt}				-1.0000		
C_{ct}	64.4016			0.0422	-1.0000	
q_{Et}	-25.6032	112.0979				-1.0000

Short-Run Impact Multiplier Matrix: $D_2 = -A^{-1}C$

	P_{st}	K_tP_{fot}	P_{ct}	$G(I_t)$	P_{fot}	L_t	D_t	$\overline{K}_t\overline{D}_t-20_t$	1
P_t	0.9189	0.0108							0.1492
q^*_{ht}	-0.2099	-0.0025	0.0077	1.6005					1.1648
q_{ft}	-132.1347	-1.5530			1.6302	1.7860	159.4989		-159.2965
C_{gt}	115.6075							0.1806	-182.9923
C_{ct}	-64.0573	-0.6955						-0.0076	198.4135
q_{Et}	23.5268	0.2765	-0.8632	-179.4127					302.9695

Derived Reduced-Form Coefficient Matrix: $D_1 = -A^{-1}B$

	P_{t-1}	q^*_{ht-1}	q_{ft-1}	C_{gt-1}	C_{ct-1}	q_{Et-1}
P_t						
q^*_{ht}						
q_{ft}						
C_{gt}				0.7446		
C_{ct}				-0.0314	0.3635	
q_{Et}				0.0967	0.0967	0.6494

Matrix: $(I - D_1)$

	P_{t-1}	q^*_{ht-1}	q_{ft-1}	C_{gt-1}	C_{ct-1}	q_{Et-1}
P_t	1.0000					
q^*_{ht}		1.0000				
q_{ft}			1.0000			
C_{gt}				0.2554		
C_{ct}				0.0314	0.6365	
q_{Et}				-0.0967	-0.0967	0.3506

Inverse Matrix: $(I - D_1)^{-1}$

	P_{t-1}	q^*_{ht-1}	q_{ft-1}	C_{gt-1}	C_{ct-1}	q_{Et-1}
P_t	1.0000					
q^*_{ht}		1.0000				
q_{ft}			1.0000			
C_{gt}				3.9154		
C_{ct}				-0.1931	1.5711	
q_{Et}				1.0265	0.4333	2.8523

Long-Run Impact Multiplier Matrix: $(I - D_1)^{-1}D_2$

	P_{st}	K_tP_{fot}	P_{ct}	$C(I_t)$	P_{fot}	L_t	D_t	$\overline{K}_t\overline{D}_t-2O_t$	I
P_t	0.9189	0.0108							0.1492
q^*_{ht}	-0.2099	-0.0025	0.0077	1.6005					1.1648
q_{ft}	-132.1347	-1.5530			1.6302	1.7860	159.4989		-159.2965
C_{gt}	-452.6496							0.7071	-716.4881
C_{ct}	-122.9642	-1.0927						-0.0468	347.0633
q_{Et}	158.0206	0.4873	-2.4621	-511.7388				0.1821	762.2909

APPENDIX F

ESTIMATION OF THE k-PERIODS IMPACT MULTIPLIER MATRICES

k-periods Impact Multiplier Matrix:

$$(I + D_1 + D_1^2 + \dots + D_1^{k-1})D_2$$

Ordinary least-squares estimates (1928-64):

$$(I + D_1)D_2$$

	P_{st}	K_tP_{fot}	P_{ct}	$G(I_t)$	P_{fot}	L_t	D_t	$\overline{K}_t\overline{D}_{t-2}O_t$	1
P_t	0.9189	0.0108							0.1492
q^*_{ht}	-0.2099	-0.0025	0.0077	1.6005					1.1648
q_{ft}	-132.1347	-1.5530			1.6302	1.7860	159.4989		-159.2965
C_{gt}	201.6888							0.3151	-319.2484
C_{ct}	-90.9722	-0.9483						-0.0160	276.2828
q_{Et}	43.7900	0.3888	-1.4238	-295.9233				0.0167	501.2091

$$(I + D_1 + D_1^2)D_2$$

	P_{st}	K_tP_{fot}	P_{ct}	$G(I_t)$	P_{fot}	L_t	D_t	$\overline{K}_t\overline{D}_{t-2}O_t$	1
P_t	0.9189	0.0108							0.1492
q^*_{ht}	-0.2099	-0.0025	0.0077	1.6005					1.1648
q_{ft}	-132.1347	-1.5530			1.6302	1.7860	159.4989		-159.2965
C_{gt}	265.7816							0.4152	-420.6993
C_{ct}	-103.4573	-1.0402						-0.0233	308.8613
q_{Et}	62.6771	0.4373	-1.7878	-371.5816				0.0398	624.2777

$$(I + D_1 + D_1^2 + D_1^3)D_2$$

	P_{st}	K_tP_{fot}	P_{ct}	$G(I_t)$	P_{fot}	L_t	D_t	$\overline{K}_t\overline{D}_{t-2}O_t$	1
P_t	0.9189	0.0108							0.1492
q^*_{ht}	-0.2099	-0.0025	0.0077	1.6005					1.1648
q_{ft}	-132.1347	-1.5530			1.6302	1.7860	159.4989		-159.2965
C_{gt}	313.5044							0.4898	-496.2385
C_{ct}	-110.0119	-1.0736						-0.0291	323.8932
q_{Et}	79.9266	0.4599	-2.0242	-420.7228				0.0637	697.5694

$(I + D_1 + D_1^2 + \dots + D_1^4)D_2$

	P_{st}	K_tP_{fot}	P_{ct}	$G(L_t)$	P_{fot}	L_t	D_t	$\bar{K}_t\bar{D}_{t-2}0_t$	1
P_t	0,9189	0,0108							0,1492
$\dot{q}_{ht}$	-0,2099	-0,0025	0,0077	1,6005					1,1648
q_{ft}	-132,1347	-1,5530			1,6302	1,7860	159,4989		-159,2965
C_{gt}	349,0422							0,5453	-552,4904
C_{ct}	-113,8895	-1,0857						-0,0336	331,7191
q_{Et}	95,1078	0,4713	-2,1778	-452,6403				0,0859	739,3309

$(I + D_1 + D_1^2 + \dots + D_1^5)D_2$

	P_{st}	K_tP_{fot}	P_{ct}	$G(L_t)$	P_{fot}	L_t	D_t	$\bar{K}_t\bar{D}_{t-2}0_t$	1
P_t	0,9189	0,0108							0,1492
$\dot{q}_{ht}$	-0,2099	-0,0025	0,0077	1,6005					1,1648
q_{ft}	-132,1347	-1,5530			1,6302	1,7860	159,4989		-159,2965
C_{gt}	375,5047							0,5866	-594,3773
C_{ct}	-116,4086	-1,0901						-0,0369	336,3179
q_{Et}	108,0280	0,4776	-2,2775	-473,3625				0,1053	761,7541

$(I + D_1 + D_1^2 + \dots + D_1^6)D_2$

	P_{st}	K_tP_{fot}	P_{ct}	$G(L_t)$	P_{fot}	L_t	D_t	$\bar{K}_t\bar{D}_{t-2}0_t$	1
P_t	0,9189	0,0108							0,1492
$\dot{q}_{ht}$	-0,2099	-0,0025	0,0077	1,6005					1,1648
q_{ft}	-132,1347	-1,5530			1,6302	1,7860	159,4989		-159,2965
C_{gt}	395,2042							0,6174	-625,5592
C_{ct}	-118,1514	-1,0917						-0,0394	339,2996
q_{Et}	118,7369	0,4812	-2,3422	-486,8184				0,1215	772,7055

$$(1 + D_1 + D_1^2 + \dots + D_1^7)D_2$$

	P_{st}	K_tP_{fot}	P_{ct}	$G(I_t)$	P_{fot}	L_t	D_t	$\bar{K}_t\bar{D}_{t-2}O_t$	1
P_t	0.9189	0.0108							0.1492
$\dot{q}_{ht}$	-0.2099	-0.0025	0.0077	1.6005					1.1648
q_{ft}	-132.1347	-1.5530			1.6302	1.7860	159.4989		-159.2965
C_{gt}	409.8748							0.6403	-648.7809
C_{ct}	-119.4049	-1.0922						-0.0413	341.3614
q_{Et}	127.4230	0.4834	-2.3842	-495.5558				0.1348	777.0964

$$(1 + D_1 + D_1^2 + \dots + D_1^8)D_2$$

	P_{st}	K_tP_{fot}	P_{ct}	$G(I_t)$	P_{fot}	L_t	D_t	$\bar{K}_t\bar{D}_{t-2}O_t$	1
P_t	0.9189	0.0108							0.1492
$\dot{q}_{ht}$	-0.2099	-0.0025	0.0077	1.6005					1.1648
q_{ft}	-132.1347	-1.5530			1.6302	1.7860	159.4989		-159.2965
C_{gt}	420.7997							0.6574	-666.0737
C_{ct}	-120.3259	-1.0924						-0.0427	342.8483
q_{Et}	134.3661	0.4848	-2.4115	-501.2253				0.1455	777.8854

$$(1 + D_1 + D_1^2 + \dots + D_1^9)D_2$$

	P_{st}	K_tP_{fot}	P_{ct}	$G(I_t)$	P_{fot}	L_t	D_t	$\bar{K}_t\bar{D}_{t-2}O_t$	1
P_t	0.9189	0.0108							0.1492
$\dot{q}_{ht}$	-0.2099	-0.0025	0.0077	1.6005					1.1648
q_{ft}	-132.1347	-1.5530			1.6302	1.7860	159.4989		-159.2965
C_{gt}	428.9385							0.6701	-678.9563
C_{ct}	-121.0028	-1.0925						-0.0438	343.9295
q_{Et}	139.8400	0.4857	-2.4292	-504.9032				0.1539	776.8646

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