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GROWTH AND YIELD OF RED PINE IN MINNESOTA
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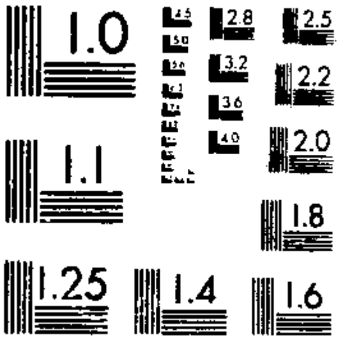

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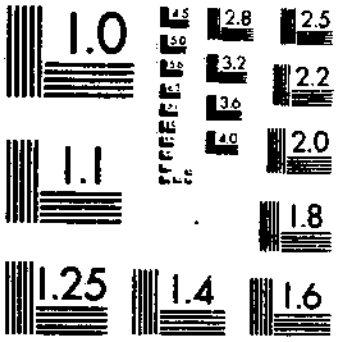

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Growth and Yield Of Red Pine In Minnesota

By ROBERT E. BUCKMAN, *Research Forester,*
Lake States Forest Experiment Station,
Forest Service, U.S. Department of Agriculture

(The Lake States Forest Experiment Station is maintained by the U.S. Department of Agriculture, at St. Paul 1, Minn., in cooperation with the University of Minnesota)

Technical Bulletin No. 1272 October 1962

ACKNOWLEDGMENT

The author expresses his appreciation to many people who helped make this report possible: Members of the Station staff, especially those who for many years have maintained the experiments that were the foundation for this study; faculty members of the School of Natural Resources, University of Michigan, who made helpful suggestions; and Mr. Sidney Rommel, Grand Rapids, Minn., who made available records for five permanent sample plots maintained on his private property.

CONTENTS

INTRODUCTION

Studies of growth and yield of trees began when forestry was first introduced in this country around the turn of the century; in Europe they began even earlier. It is not difficult to understand this early and sustained interest in the subject, for implicit in the process of forest management is the forecasting of anticipated growth and yield.

More important than forecasting is the decision-making function served by growth and yield predictions. Intensive forestry demands that alternatives be weighed in terms of the goals of management. Only when the forest manager knows the consequences of a number of alternatives can he realize the maximum in volume, quality, or monetary returns, or other aims he may have for owning forest property. This is especially important for red pine (*Pinus resinosa* Ait.) in Minnesota where there are many options that can be exercised in its management.

The traditional normal yield tables contained the variables of age and site. Recent trends in growth and yield predictions are toward the use of more variables—in particular, those subject to manipulation by forest managers. For example, some yield tables have appeared in Europe which incorporate not only age and site as variables, but thinning intensity and thinning methods (crown or low thinnings) as well (Wiedemann, 1949).[1] Stand density is being used as a variable in statistical approaches to growth predictions in the South (McClay, 1955; Wenger et al., 1958; Gruschow and Evans, 1959; Nelson et al., 1961). Those prediction systems that incorporate variables subject to manipulation by management provide the strongest base for decision making, and such systems are likely to appear in ever greater numbers.

The history of growth and yield predictions for red pine in Minnesota is similar to that for even-aged species elsewhere. First, there were normal yield tables prepared by Brown and Gevorkiantz (1934). Later, Eyre and Zehngraff (1948) presented normal yield tables that were thought to be derived from more realistic data. Eyre and Zehngraff also prepared a yield table from managed stands, based on 17 permanent sample plots.

Concurrent with the preparation of these growth and yield tables was the installation of several silvicultural study plots and other sets of permanent inventory plots. Sufficient time has elapsed that data from these plots can be drawn together in a more comprehensive treatment of growth and yield of red pine in Minnesota than has been possible in the past.

The purpose of this paper is to present growth and yield data based on 235 permanent sample plots with 324 periods of measured growth and mortality. For the benefit of research workers and students of mensuration, the methods used in obtaining those results are given more fully in the appendix.

[1] Names and dates in parentheses refer to Literature Cited, p. 42.

Unless otherwise specified, growth, for purposes of this paper, is periodic net annual increment. Growth can be in basal area, cubic feet, cordwood, or board feet. No deductions are made for defect. Yield is defined as the summation of a number of net annual growth increments. Yield may be represented by a change in the volume of a stand over a period of years, by harvested timber, or by a combination of both. This definition is more inclusive than the common use of the term, which refers simply to the amount of wood produced or harvested from the forest in a given number of years.

CHARACTERISTICS OF RED PINE

Red pine is relatively free from defect, windfirm, and moderately fast growing. In these respects it is superior to other coniferous sawtimber species in the Lake States. It can and does grow in pure stands, and the stands are essentially even aged (fig. 1). In Minnesota, at least, red pine has few insect and disease problems. Detailed silvical descriptions of red pine have been prepared by Woolsey and Chapman (1914), Eyre and Zehngraff (1948), and Rudolf (1957).

Red pine can be used for a great variety of products. Early thinnings are used for posts, pulpwood, and small poles. Intermediate thinnings are used for small saw logs, utility poles, larger posts, and mine timbers. Late thinnings produce larger utility poles, saw logs up to an 18-inch top diameter, and piling. At the final harvest, saw logs up to 24-inch top diameters, large piling, or utility poles can be cut. Of course, the size of products depends on site quality

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FIGURE 1.—Red pine typically grows in even-aged stands. This stand, on medium site, has been thinned several times to about 120 square feet of basal area per acre.

and on stand density. The larger products are generally the more valuable, but in Minnesota the premium product is piling. In some instances piling commands one-third to one-half more stumpage than large saw logs or poles.

AVAILABLE PLOT RECORDS

The growth and yield estimates are based on 14 sets of permanent sample plots, which contained 235 individual plots and 324 measurement periods. Three more sets of red pine plots are maintained in Minnesota by the forestry faculty of the University of Minnesota, but because of different utilization standards they are not included in the analytic part of the present study. A record of the 14 sets of permanent sample plots is given in table 1, and plot location is shown in figure 2.

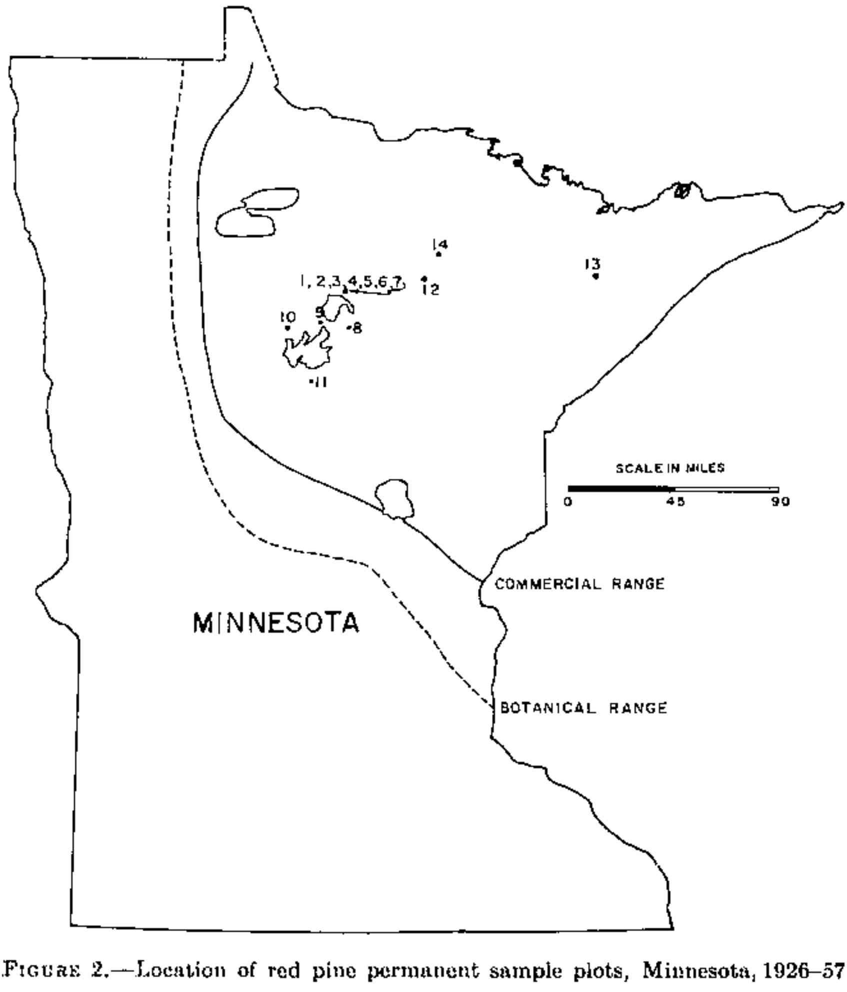

FIGURE 2.—Location of red pine permanent sample plots, Minnesota, 1926–57.

TABLE 1.—*Record of experiments used in the study of the growth and yield of red pine in Minnesota*

Permanent sample plots	Number of plots	Date of stand origin	Site index	Date experiment started	Dates experiment remeasured	Number of measurement periods
1. Cutting cycle study:						
3-year cycle	19	1870	46–48	1949	1953, 1955	2
6-year cycle	5			1949	1955	1
2. Cutting methods study	60	1870	50–54	1951	1956	1
3. Growing-stock levels study	45	1870	49–54	1950, 1951	1955, 1956	1
4. Growing-stock levels study	34	1905	46–50	1950	1956	1
5. Plots 18–21 "Common Sense Plots"	4	1870	48–52	1927	1932, 1937, 1943, 1949, 1954.	[1] 5
6. Plots 22–25 "Graveyard plots"	4	1905	47	1927	1932, 1937, 1944, 1949, 1953.	[1] 5
7. Miscellaneous inventory plots	10	1870	47–57	1940, 1945	1945, 1956	1
8. Bena plots 1–4	4	1820	52	1926	1931, 1936, 1941, 1945, 1949, 1954.	[1] 6
9. Portage Lake thinning plots	15	1902	54	1947	1956	1
10. Lake-13 plots	11	1820	52	1945	1957	1
11. Longville plots	2	1896	57	1941	1949, 1955	2
12. Marcell plots 1–14	14	1800	50–57	1944	1953, 1957	2
13. Birch Lake plantations	3	1913, 1914	57–63	1932	1937, 1944, 1949, 1956.	4
14. Permanent inventory plots [2]	5	1905	47–55	1949	1952, 1956	2

[1] Data from one of these measurement periods were not, for various reasons, used in the analysis.
[2] On private property of Sidney Rommel.

The measurement interval for the different plots varied from 3 to 12 years. In the subsequent analysis, net annual growth is the difference between two successive plot volumes divided by the number of years in the period. Likewise, the values of stand density and stand age corresponding to these growth estimates are taken at the year nearest the midpoint of the measurement interval.

Most of the plots were part of thinning studies to test different cutting intensities or cutting methods. On several of them, such as the "Common Sense Plots," the Marcell plots, and the Bena plots, regeneration studies were also made; these plots are now of principal interest because of the growth of the overstory. Two sets of the plots were part of permanent inventory systems, and one was part of an aspen release experiment.

Red pine was the chief component on all plots, but some plots had a rather large amount of aspen, jack pine, white pine, white spruce, and balsam fir. Plots were used only if more than 75 percent of the basal area was in red pine. This resulted in the elimination of a number of them from several experiments. Similarly, plots that had two or more distinct age classes were eliminated.

On the cutting cycle, growing-stock levels, and cutting methods plots at the Cutfoot Experimental Forest, measurements were taken on several smaller plots within each of the larger treated compartments. Growth from each plot in these compartments was treated as a separate observation because, even though age did not vary and site might vary only a little, there were marked differences in stand density, stand spacing, number of trees, and cutting intensity.

Treating successive measurements on the same plot and concurrent measurements on adjacent plots as separate estimates of growth results in serial (or auto) correlation of error residuals. Autocorrelation produces standard errors of estimate that are biased on the low side. It can only be hoped that such bias is inconsequential so far as the prediction of growth is concerned. This problem is discussed in the appendix section "Reliability of Growth Predictions."

FACTORS STUDIED—THE INDEPENDENT VARIABLES

When this analysis was begun, periodic annual basal area growth was plotted against each of seven proposed independent variables. Three of the variables—stand age, site, and stand density—appeared to be good predictors of basal area growth (and presumably cubic volume growth as well). Four of the variables—cutting methods, individual tree distribution as measured by variability in tree diameters, intensity of cutting, and numbers of trees—appeared to be poor predictors. It is helpful here to discuss each of the independent variables. Later, those variables that are related to growth will be presented in equation and tabular form; those that are rejected or set aside still have importance in forest management but not for purposes of estimating growth.

Stand Age

Stand age has been the prime independent variable in even-aged growth and yield studies since the beginning of such work. And indeed, implicit in the definition of growth and of yield is time, which is expressed as changes in stand age.

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Red pine stands are rarely perfectly even aged. There may be, as in the 90-year-old red pine stand at the Cutfoot Experimental Forest, a 10-year spread in total ages. Nevertheless, the stands are considered even aged, with the total age being determined from the average of a number of stump counts or increment borings, with appropriate corrections for stump or increment-boring height.

Growth increases rapidly after stand establishment, culminates, and then declines steadily through the remainder of the life of the stand.[2] For example, experimental evidence in Minnesota (Allison and Cole, 1956; Schantz-Hansen, 1956; plus data from the "graveyard plots" and the Birch Lake plantation) indicates that basal area growth of trees 3.6 inches d.b.h. and larger culminates between ages 20 to 30, and at about 6 to 8 square feet per acre per year. Basal area growth then declines: At age 130 it averaged 2.0 square feet in Bena plots 1 to 4; at age 145 it averaged 1.1 square feet in the Lake-13 stand; at age 160 it averaged 0.9 square foot in the Marcell plots. The average trend between basal area growth and age for the plots used in this study is shown in figure 3.

Site

Site has been the second traditional independent variable in growth and yield studies. For purposes of this study, conventional site index curves are used which relate height of dominants and codominants to an index age of 50 years.

Although other research workers have prepared site index curves for red pine, those of Eyre and Zehngraff (1948) and Gevorkiantz (1957) are used in this study. The two sets of curves are plotted on somewhat different scales but were drafted from the same basic data. They are reproduced in figure 4 and tabulated in table 2.

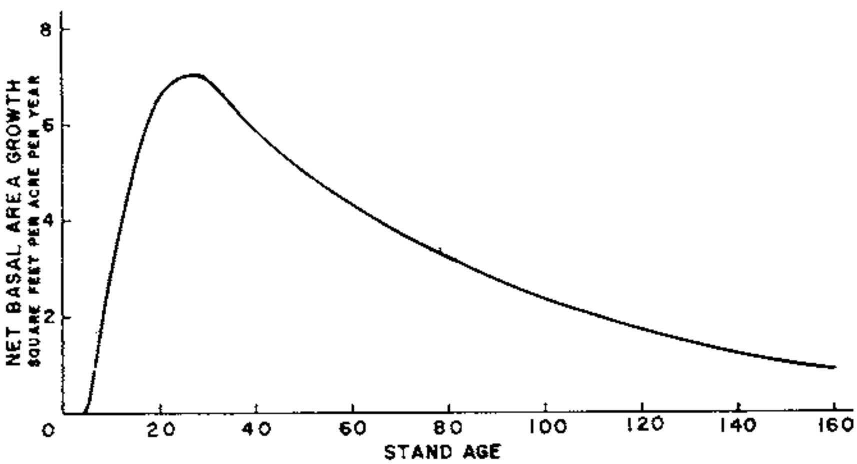

FIGURE 3.—Net periodic annual basal area growth in relation to age for average-site red pine in Minnesota, as postulated from published reports and scatter diagrams of permanent sample plot data.

[2] The age of culmination and rate of decline depend on utilization standards and whether growth is measured in basal area, cubic volume, or board feet.

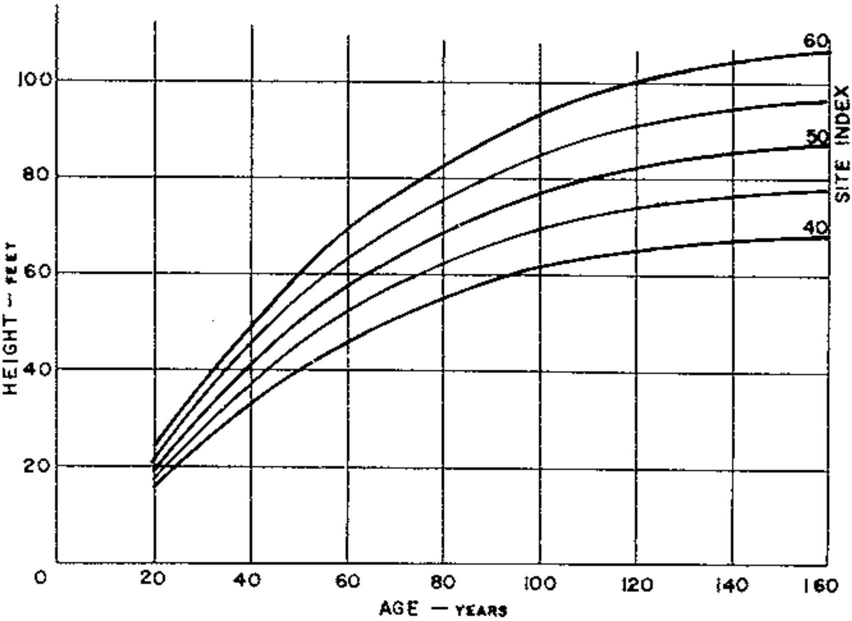

FIGURE 4.—Site index curves for red pine in Minnesota (after Eyre and Zehngraff, 1948).

A number of statistical studies have been done in which growth is related to one or more independent variables including site index. The correlation coefficients between growth and site index (height of dominants and codominants at given age) when considered alone are apt to be very low because the effects of other factors, primarily age and stand density, are not removed. When these effects are removed by multiple regression analysis, the correlation coefficient may still be low, especially if growth is predicted in basal area. For example, Simmons and Schnur (1937) found the multiple correlation coefficient of the percent basal area growth of loblolly pine was increased from 0.55 to only 0.57 when site index was added to the independent variables of stand density index and age.

Brock[3] found that site index did not make a significant contribution to the basal area growth of either planted or natural red pine after the effects of stand density and age had been removed. Likewise, Spurr (1952) found that site index did not materially improve the prediction of basal area growth of Douglas-fir when age and stand density were considered. Not surprisingly, Spurr found that site index was a helpful predictor of height and volume growth. Brender (1960) prepared a series of graphs which would indicate that basal area growth is not strongly influenced by site index (although the correlation was statistically significant), while total cubic volume growth is greatly changed by changes in site index.

What does this mean? Apparently site index is not highly correlated with basal area or basal area growth. But site index by definition

[3] Brock, Samuel M. Estimation of stand volume and basal area growth for red pine. Unpub. Master of Forestry thesis, School of Natural Resources, Univ. of Mich., 91 pp. 1956.

TABLE 2.—*Average height and annual height growth of dominant and codominant red pine (after Gevorkiantz, 1957)*

Age	Site index									
	40		45		50		55		60	
	Height	Annual height growth	Height	Annual height growth	Height	Annual height growth	Height	Annual height growth	Height	Annual height growth
	Feet	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
20	15.0	------	17.0	------	19.0	------	21.0	------	24.0	------
25	19.5	------	22.0	------	24.5	------	27.0	------	30.5	------
30	24.0	0.90	27.0	1.00	30.0	1.10	33.0	1.20	37.0	1.25
35	29.5	------	32.0	------	35.5	------	39.0	------	43.0	------
40	33.0	.80	37.0	.90	41.0	1.00	45.0	1.10	49.0	1.15
45	36.5	------	41.0	------	45.5	------	50.0	------	54.5	------
50	40.0	.65	45.0	.75	50.0	.80	55.0	.90	60.0	1.00
55	43.0	------	48.5	------	53.5	------	59.0	------	64.5	------
60	46.0	.55	52.0	.60	57.0	.65	63.0	.70	69.0	.80
65	48.5	------	54.5	------	60.0	------	66.0	------	72.5	------
70	51.0	.45	57.0	.50	63.0	.60	69.0	.60	76.0	.65
75	53.0	------	59.5	------	66.0	------	72.0	------	79.0	------
80	55.0	.40	62.0	.45	69.0	.50	75.0	.55	82.0	.60
85	57.0	------	64.0	------	71.0	------	77.5	------	85.0	------
90	59.0	.35	66.0	.35	73.0	.40	80.0	.50	88.0	.55
95	60.5	------	67.5	------	75.0	------	82.5	------	90.5	------
100	62.0	.25	69.0	.30	77.0	.35	85.0	.40	93.0	.45
105	63.0	------	70.5	------	78.5	------	86.5	------	95.0	------
110	64.0	.15	72.0	.25	80.0	.25	88.0	.30	97.0	.35
115	64.5	------	73.0	------	81.0	------	89.5	------	98.5	------
120	65.0	.10	74.0	.15	82.0	.20	91.0	.25	100.0	.25
125	65.5	------	74.5	------	83.0	------	92.0	------	101.0	------
130	66.0	.10	75.0	.10	84.0	.15	93.0	.15	102.0	.20
135	66.5	------	75.5	------	84.5	------	93.5	------	103.0	------
140	67.0	.05	76.0	.10	85.0	.10	94.0	.10	104.0	.15
145	67.3	------	76.5	------	85.5	------	94.5	------	104.5	------
150	67.5	.05	77.0	.10	86.0	.10	95.0	.10	105.0	.10
155	67.8	------	77.5	------	86.5	------	95.5	------	105.5	------
160	68.0	.05	78.0	.05	87.0	.05	96.0	.05	106.0	.05

is strongly related to height growth, and this dimension of stand growth gives the markedly greater volumes on good sites than on poor. For example, a normal yield table for red pine (Eyre and Zehngraff, 1948) shows a difference in basal area at 160 years between good and poor sites of only 24 square feet per acre, but a difference in board-foot volume of 22,000 feet.

Two more factors can account for some of the relatively poor correlations between periodic annual basal area growth and site index. The first of these is caused by fluctuations in climate, insect attacks, mortality, inexact measurement techniques, and so forth, all of which tend to be amplified by short measurement periods. The second is caused by the fact that site index at best is only an imperfect measure of site quality. Indeed, in the same stand one is often confronted with an appreciable change in predicted site index when the two determinations are made from heights taken many years apart. For purposes of this paper, site index on a particular plot is determined from the most recent height and age measurements.

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FIGURE 5.—This stand of red pine is about 30 years old and has been thinned twice. Trees of this size are used for pulpwood, fence posts, and small poles. Site index here exceeds 60 feet at 50 years.

Stand Density

The growth of stands in relation to different densities has received an enormous amount of attention in world forestry literature. Out of this mass of reports are emerging principles of growth that fit reasonably well with the results of the red pine experiments in Minnesota. It seems that stand density, once regarded as critical in maximizing forest growth, is not so important after all. This does not say that stand density is unimportant in the control of size and quality of timber trees (fig. 5); rather, that over a comparatively wide range of stocking conditions volume growth varies but little.

It would serve no purpose to cite the individual studies that have demonstrated these findings. Excellent summaries of growth in relation to stand density have been written in this country by Baker (1950), Spurr (1952), and Hall.[4] European work has been discussed by Assman (1953), Möller and Holmsgaard (1947), Heiberg (1954), Braathe (1957), and Holmsgaard (1958).

Individual experiments are cited in these papers to show that volume increment is higher at higher stand densities, that increment is the same over a wide range of stand densities, or even that increment is higher at lower stand densities. When a number of these experiments are compared, it is obvious that there is a substantial experimental error involved within individual experiments.

[4] Hall, Otis F. The growth patterns of thinned and unthinned forest stands and their value in the management of the forest. Unpub. Ph. D. thesis, Univ. of Minn., 288 pp., illus. 1954.

This error could well mask differences of say 5 to 10 percent in growth if such differences in fact exist. Most of these summary papers adopt the attitude that within wide limits density has little effect on total growth—more specifically, that there is no important difference in growth from about 50 percent to 100 percent of full stocking.

In general, red pine growth studies support the noncritical nature of stand density. Examples of such studies are those reported by Engle and Smith (1952), and Spurr and Allison (1956), and the stand density experiments in 40- and 90-year timber at the Cutfoot Experimental Forest (fig. 6). As will be shown later, however, there is some evidence in this study and in other published sources that young red pine stands, those with rapid height growth, produce somewhat more cubic volume growth at high than at low densities. (See "Cubic-Foot and Cordwood Growth Tables," p. 15.)

Except for poor sites, or extremely open or dense stands, total height growth of the pines is apparently unaffected by density

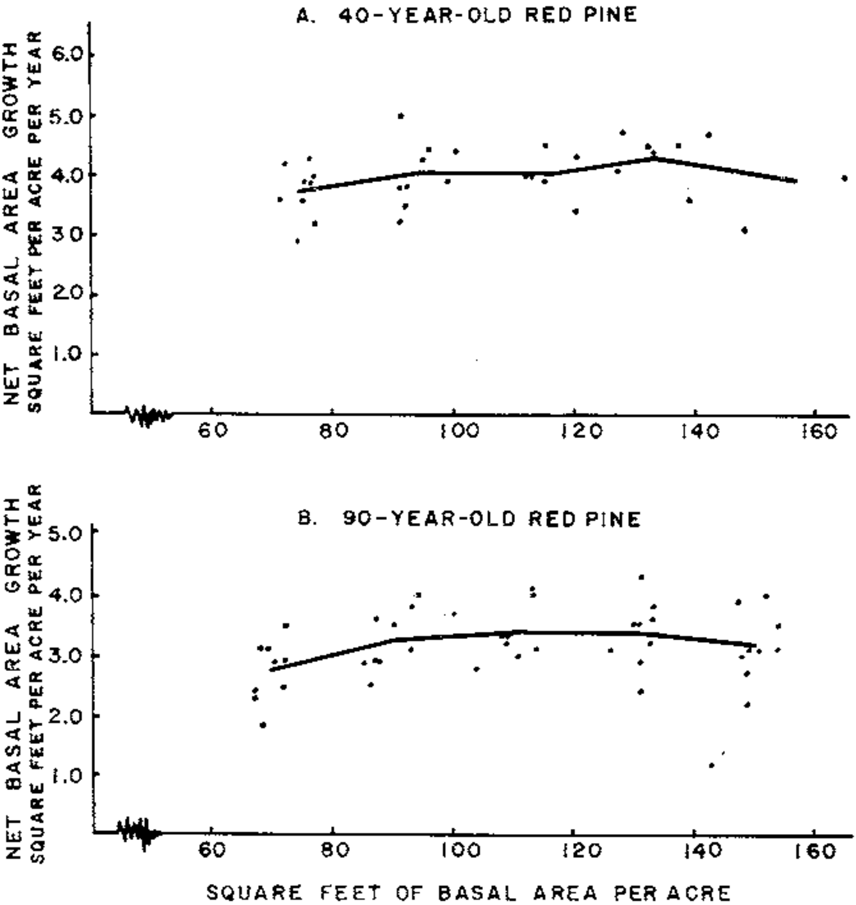

FIGURE 6.—Basal area growth in relation to stand density for two experiments at the Cutfoot Experimental Forest.

(Baker, 1950; Spurr, 1952). Recent research in red pine lends additional support to this conclusion.[5] The fact that total height growth is little affected by density is relied upon heavily in the development of red pine growth and yield tables presented later.

Basal area has been selected as a measure of stand density for this paper. There are disadvantages with basal area, the chief one being that it is only an abstract or index measure of stand density. Nevertheless, basal area is easy to use; it is measured in absolute rather than relative units; it is closely related to stand volume when height is considered, a relationship employed in the subsequent development of this paper; and it is the traditional measure of stand density used in Minnesota.

Cutting Methods

One of the 14 experiments used in this analysis is a cutting methods study. The experiment is designed specifically to test three cutting methods (cutting from above, from below, and from above and below) with the stand densities held in the range of 100 to 120 square feet of basal area per acre. The experiment is remeasured and recut each 5 years. The first 5-year results of the three replications[6] are available. The growth per acre per year has been:

Cutting method:	*Basal area (square feet)*	*Volume (cubic feet)*
Cut from above	3.0, 3.2, 2.2	98, 101, 76.
Cut from below	2.7, 2.3, 2.1	84, 77, 70.
Cut from above and below	2.8, 2.8, 2.5	92, 88, 83.

The F ratio for basal area growth is 1.16; for cubic-foot growth it is 2.02. Neither of these is significant. The degrees of freedom for the analysis are few, making this a rather insensitive experiment.

European work (Möller and Holmsgaard, 1947; Heiberg, 1954; Braathe, 1957; and Holmsgaard, 1958) would indicate little or no difference in cubic volume growth between crown and low thinnings. Since there is as yet no significant difference between treatments in the cutting methods study, and there is little published evidence to suggest that there will be a marked difference, it is grouped together with the other thinning experiments.

The fact that there is some latitude in cutting methods introduces flexibility heretofore little used in red pine management in Minnesota. It will mean that larger or smaller trees or some of both can be cut with no concern about losing cubic volume growth. It will also mean that the size of trees in the residual stand will change depending on whether larger or smaller components are removed (fig. 7).

Individual Tree Distribution

The distribution of trees in a stand is difficult to describe and quantify, and yet it seems that the arrangement or spacing of trees on a

[5] Buckman, Robert E. A progress report of three growing stock density experiments in Minnesota. A manuscript to be published as a paper of the Lake States Forest Experiment Station, probably in 1962.

[6] Each treatment within a replication contains ten ⅕-acre plots. Some 60 of these ⅕-acre plots have been treated as separate observations for the regression analysis described in the appendix. While cutting methods do not vary within treatments, each plot represents a somewhat different stand density, cutting intensity, and number of trees per acre.

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FIGURE 7. Cutting in older red pine stands produces round timber piles, larger utility poles, saw logs, and a small quantity of pulpwood.

particular area should influence growth. Staebler,[7] for example, attempted to measure the competition of trees with their neighbors by relating the growth of those trees to two different methods of describing or measuring spacing. (One method was called the overlap hypothesis, the other the competition curve hypothesis.) Neither measure had a significant relationship to growth, and both were extremely difficult to use.

In the present study, the standard deviation of tree diameters was used as an indirect measure of spacing. It is conceivable that plots with more variable tree diameters could have a relatively less uniform spacing with more of the trees in clumps. On the other hand, in plantations or well-managed stands the trees might be uniformly distributed, and this would be reflected in less variable tree diameters. The irregular stands would use the growing space less efficiently, hence have a lower growth rate. Conversely, well-spaced natural stands and plantations would be using more of the sunlight, water, and soil nutrients because they occupy the site more uniformly.

The null hypothesis, "there is no relationship between basal area growth residuals and the standard deviation of tree diameters," was tested by the following analysis. The residuals, or the amount of growth left over after the effects of age, site, and stand density had been removed by multiple regression, were calculated and listed by an

[7] Staebler, George. Growth and spacing in an even-aged stand of Douglas-fir. Unpub. Master of Forestry thesis, Univ. of Mich., 46 pp. 1951.

electronic computer.[8] The standard deviations of tree diameters for 296 growth periods were listed beside their respective growth residuals and a correlation coefficient calculated.

The correlation coefficient between the standard deviation of tree diameters and growth residuals was 0.110, which is not significantly different from zero. Thus it is concluded that the variability of tree diameters, as measured by their standard deviations, is not a useful predictor of basal area growth.

Perhaps the standard deviation of tree diameters is not a good measure of tree spacing. It is certainly not difficult to imagine a stand with such poor distribution of growing stock that growth is impaired. Because the areal distribution of trees is difficult to describe and quantify, thinning recommendations are often accompanied by the simple admonition that the trees be reasonably well spaced.

Intensity of Cutting

For some forest types in this country, there is evidence of an adverse reaction to thinning. Some of the hardwoods, yellow birch in particular, apparently suffer sunscald when stands are opened too much. Other forest types are subject to bending, breaking, or uprooting by too heavy thinning. It may be asked, "Is any red pine growth (or lack of it) accounted for by intensity of cutting after the effects of age, site, and stand density have been removed?"

This hypothesis may again be tested by calculating the correlation coefficients between intensity of cut (as measured by the amount of basal area cut) and growth residuals (the amount of basal area growth unaccounted for after the effects of age, site, and stand density have been removed). Some 156 of the measurement periods that had at least some cutting were selected. Thinning intensity ranged from very light up to that which removed about half the basal area. Stand density after cutting was as low as 60 square feet of basal area per acre.

The correlation coefficient between the growth residuals and intensity of cut was −0.054, again not significantly different from zero. This means that intensity of cut is not a factor to be used in estimating growth of red pine stands with a cutting history of the kind used in this study; that any reduction in growth associated with heavy cutting may be accounted for by the reduction in stand density and not by the intensity of cut.

The fact that none of the variation in growth residuals could be accounted for by intensity of cutting is important in forest management. It means that stands can be cut from a high to a relatively low basal area in a single cut—that stands with a high proportion of crooked, defective, and otherwise poor-quality growing stock can be put in a good condition in one cut—all without fear of losing growth because of the heavy cutting.

It is always necessary to add the qualifying restriction that some stands, because of high density, may be too tall for the diameters of individual stems; that the individual stems may not be strong enough

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[8] One interaction term in the regression equation used to compute these residuals later turned out to be nonsignificant. The contribution made by this term was so slight that recomputing all of the residuals with this term removed did not seem justified.

to support the tree after heavy cutting. Such stands are found, but much less commonly than one would think. For example, three red pine growing-stock levels studies have been installed in Minnesota in recent years, the results of which are not included here. Initial basal area stocking in all three of these dense stands was about 200 square feet per acre. Some of the study compartments were cut down to 60 and even 30 square feet of basal area per acre. No appreciable breakage or bending has occurred in any of these, with the exception of the 30-square-foot compartment cut from above in the Birch Lake plantation. This treatment has suffered heavily from uprooting and bending as a result of windstorms.

Number of Trees Per Acre

The importance of number of trees per acre in accounting for basal area growth after the effects of age, site, and stand density are removed, can also be tested. It is recognized that number of trees per acre for any given age tends to be correlated with stand density as measured in basal area, so some of the variability associated with number of trees should also be accounted for by stand density.

The number of trees per acre was calculated for 272 out of the total 324 measurement periods. Number of trees ranged from slightly over 1,000 per acre for the more dense plots in young stands to less than 50 per acre in open plots of the older stands. The correlation coefficient between growth residuals and number of trees per acre is −0.039, not significantly different from zero.

The apparent inability of number of trees per acre to account for basal area growth residuals is useful in forest management. It means that for a given stand density, age, and site, there is some flexibility in how many trees need be on each acre. If there are few trees, they tend to be relatively large; if there are many trees, they tend to be smaller and in both cases the basal area growth rates per acre are unchanged. This is especially useful because the utility of red pine is closely related to size, whether it be for mine timbers, poles, piling, or sawtimber.

If, however, the idiosyncrasies of log rules place a premium on large trees, then number of trees does indeed make a difference in estimated volume growth. For example, the Doyle and Scribner log rules underestimate the volumes in small trees. As the trees increase in size there is an apparent increase in growth simply because the log rules more closely estimate the lumber volume that can be sawed from the trees.

GROWTH OF RED PINE

Based on the preliminary analysis of the importance of various independent variables, growth equations were derived which considered age, site index, and stand density. From these equations, tables were made for basal area, cubic-foot, cordwood, and board-foot growth.

Basal Area Growth

Basal area growth was derived by statistical methods. (See appendix section "Fitting the Regression Surface.") A degree of uncertainty or error is associated with each basal area growth prediction;

this is discussed in the appendix section "Reliability of Growth Predictions."

Basal area growth is given by the following equation:

$$Y = 1.6889 + .041066X_1 - .00016303X_1^2 - .076958X_2 + .0002274X_2^2 + .06441X_3$$

where Y = periodic net annual basal area increment
X_1 = basal area in square feet per acre
X_2 = age in years
X_3 = site index

The equation is plotted in three dimensions in figure 8 for site index 50 and for the approximate range of age and stand densities for which information is available.

Basal area growth could be calculated from the equation just given. For most purposes it is more convenient to tabulate the values, and this has been done in table 3. In general, the tabulated values correspond to the range of ages, sites, and stand densities found in the underlying experiments. Some entries are extrapolated for short distances beyond the original data, especially at high and low site indexes and at low stand densities. None of these extrapolated values are inconsistent with theoretical considerations or with published data for red pine in other regions. Because information is needed at these extremes, the most recent thinning experiments of the Lake States Forest Experiment Station cover a still greater range of densities and sites than shown in table 3.

Cubic-Foot and Cordwood Growth Tables

The development of volume growth tables requires (1) basal area growth equations given in the previous section, (2) the height growth table (table 2, p. 8), and (3) stand volume equations, which are given in the appendix. The construction of volume growth tables is discussed in the appendix.

Cubic-foot growth is given in table 4, and cordwood growth in table 5. These tables have been prepared in the same detail as basal area growth tables.

At young ages, maximum cubic volume and cordwood growth occur at stand densities of 180 square feet of basal area. As the stand gets older, the maximum gradually shifts to 150 and then to 120 square feet of basal area per acre. This shifting in maximum volume growth is associated with the height development of the stand. At early ages, rapid height growth contributes a great deal to volume. This component of growth is more important in high-density stands, because there are more stems on which to add height growth.

As height growth slows down, the basal area component of growth becomes more important. Since basal area growth appears to be slightly higher at densities of 120 to 130 square feet, this has the effect of moving maximum volume growth toward those densities as height growth declines.

The data given in tables 4 and 5, and the essentially deductive argument that cubic volume growth is somewhat greater in high-density stands at young ages, require further study for confirmation. They also require study of stand form in relation to stand density. It would seem that trees in higher density stands would have relatively less taper; this, if true, would be another reason for high-density red pine management, provided one is interested only in maximizing cubic

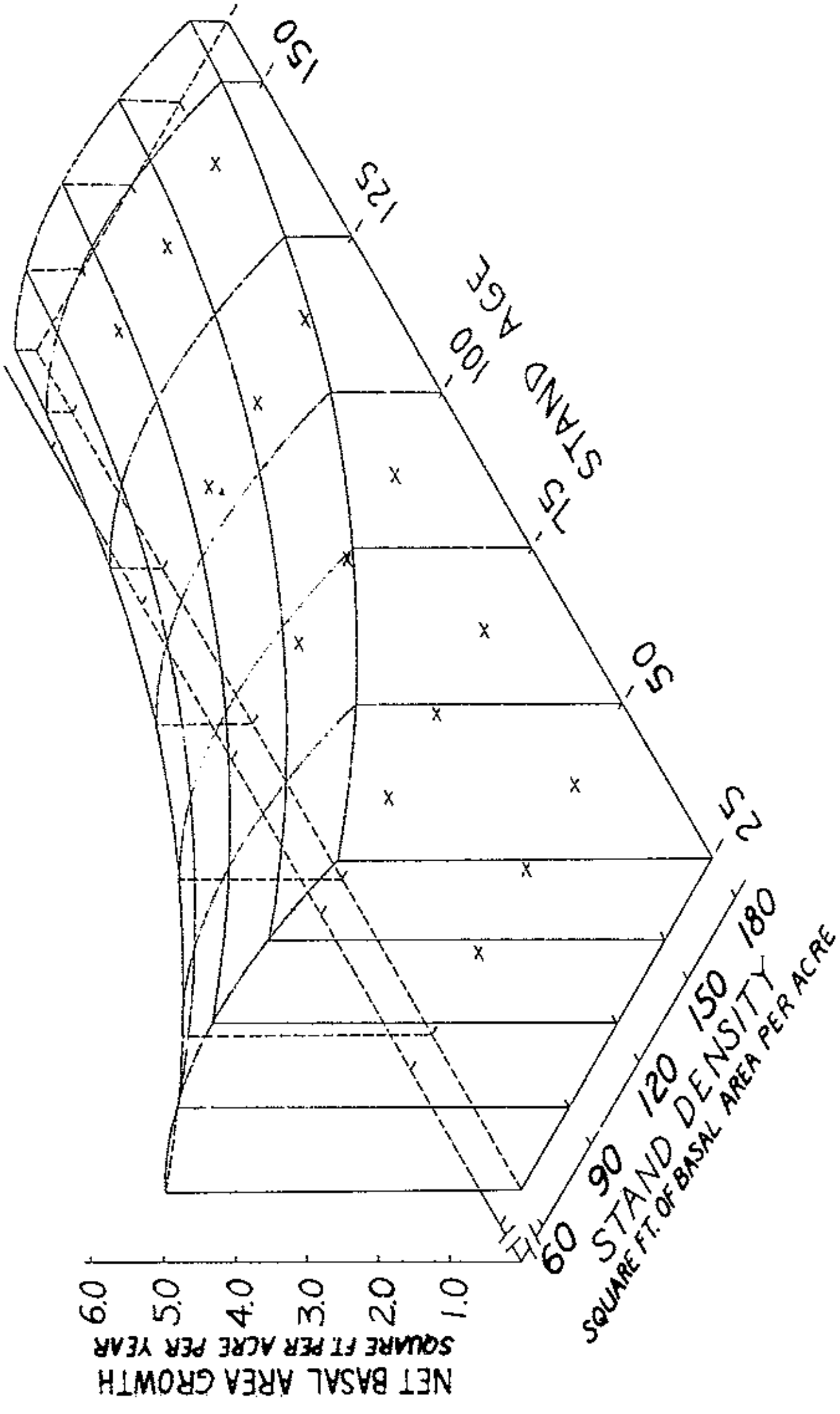

FIGURE 8.—Basal area growth of red pine in Minnesota in relation to age and stand density (site index 50).

volume growth. However, the evidence presented in these tables accounts for a common observation in the published results of thinning in young red pine stands: that the control and high-density plots often have a higher gross cubic volume growth, and sometimes a higher net cubic volume growth as well, than do the lower density plots (Eyre and Zehngraff, 1948; Smithers, 1954; Wilson, 1955; Baldwin, 1959; and Nelson et al., 1961). Obviously, if density levels become high enough, suppression losses are likely to occur. If a forest manager desires the maximum cubic volume growth in young stands, his aim is then to maintain high densities but at the same time avoid suppression losses. (See section on "Maximizing Growth".)

Board-Foot Growth Tables

Except for omission of growth estimates during early ages, the board-foot growth table (table 6) covers the same range of age, stand density, and site as the basal area, cubic-foot, and cordwood tables.

Board-foot growth was obtained from the same underlying basal area growth equation as cubic-foot and cordwood growth. But the basal area growth function predicts growth of trees 3.6 inches d.b.h. and larger, whereas board-foot growth is for trees 7.6 inches and larger. An approximate method for overcoming these differences in utilization standards is used: Prediction of board-foot growth is deferred until all or nearly all of the trees can reasonably be expected to have grown to sawtimber size. Roughly this occurs at about age 40 in more open stands (up to 80 square feet of basal area per acre); at about age 50 in medium-density stands (80 to 120 square feet); and at about age 60 for high-density stands. Thus, board-foot growth is only approximate during the transition from cordwood to sawtimber. But once the stand has attained sawtimber size, the board-foot predictions should be as reliable as those for cordwood or cubic feet.

A constant conversion ratio is maintained between cubic feet, cordwood, and board feet. This occurs because the stand volume equations assume a constant form for the stand throughout the period for which growth predictions are made. (See "Stand Volume Equations," p. 44.) If regression coefficients different than those given in the appendix were developed for the stand volume equations, or if height curves different than those given in table 2 were used, volume growth tables could still be developed from the basal area growth function. The only requirement is that the new volume equations be for trees 3.6 inches d.b.h. and larger for cordwood and cubic feet, and for trees 7.6 inches d.b.h. and larger for saw logs.

MAXIMIZING GROWTH

The basal area growth function (p. 14) itself can be used to find the stand density at which *basal area* growth is maximized. This is done by setting the first derivative of growth with respect to stand density X_1 equal to zero. Once specified, age and site become constants, and the derivative of a constant is zero. Hence, for this particular growth equation maximum basal area growth occurs at the same density for all combinations of age and site.

The first derivative of the basal area growth function is:

$$\frac{dY}{dX_1} = .041066 - .00032606X_1$$

TABLE 3.—*Basal area growth of red pine in Minnesota in relation to age, stand density, and site index*

SITE INDEX 45

Stand density basal area per acre (sq. ft.)	Net growth per acre per year when stand age in years is—													
	30	40	50	60	70	80	90	100	110	120	130	140	150	160
	Sq. ft.	*Sq. ft.*	*Sq. ft.*	*Sq. ft.*	*Sq. ft.*	*Sq. ft.*	*Sq. ft.*	*Sq. ft.*	*Sq. ft.*	*Sq. ft.*	*Sq. ft.*	*Sq. ft.*	*Sq. ft.*	*Sq. ft.*
60	4.36	3.75	3.18	2.66	2.19	1.76	1.38	1.04	0.75	0.50	------	------	------	------
90	4.86	4.25	3.68	3.16	2.69	2.26	1.88	1.54	1.25	1.00	0.80	0.64	0.54	0.47
120	5.06	4.45	3.89	3.37	2.89	2.47	2.08	1.74	1.45	1.21	1.01	.85	.74	.67
150	4.97	4.36	3.80	3.28	2.81	2.38	1.99	1.66	1.36	1.12	.92	.76	.65	.59
180	4.59	3.98	3.42	2.90	2.42	2.00	1.61	1.27	.98	.74	.54	.38	.27	.20

SITE INDEX 50

Stand density basal area per acre (sq. ft.)	30	40	50	60	70	80	90	100	110	120	130	140	150	160
60	4.68	4.07	3.50	2.98	2.51	2.08	1.70	1.36	1.07	0.82	0.62	------	------	------
90	5.18	4.57	4.00	3.48	3.01	2.58	2.20	1.86	1.57	1.32	1.12	0.96	0.86	0.79
120	5.38	4.77	4.21	3.69	3.21	2.79	2.40	2.06	1.77	1.53	1.33	1.17	1.06	.99
150	5.29	4.68	4.12	3.60	3.13	2.70	2.31	1.98	1.68	1.44	1.24	1.08	.97	.91
180	4.91	4.30	3.74	3.22	2.74	2.32	1.93	1.59	1.30	1.06	.86	.70	.59	.52

SITE INDEX 55

60	5.00	4.39	3.82	3.30	2.83	2.40	2.02	1.68	1.39	1.14	0.94	0.79	-----	-----
90	5.50	4.89	4.32	3.80	3.33	2.90	2.52	2.18	1.89	1.64	1.44	1.28	1.18	1.11
120	5.70	5.09	4.53	4.01	3.53	3.11	2.72	2.38	2.09	1.85	1.65	1.49	1.38	1.31
150	5.61	5.00	4.44	3.92	3.45	3.02	2.63	2.30	2.00	1.76	1.56	1.40	1.29	1.23
180	5.23	4.62	4.06	3.54	3.06	2.64	2.25	1.91	1.62	1.38	1.18	1.02	.91	.84

SITE INDEX 60

60	5.33	4.72	4.15	3.63	3.16	2.73	2.35	2.01	1.72	1.47	1.27	1.12	1.01	-----
90	5.83	5.22	4.65	4.13	3.66	3.23	2.85	2.51	2.22	1.97	1.77	1.61	1.51	1.44
120	6.03	5.42	4.86	4.34	3.86	3.44	3.05	2.71	2.42	2.18	1.98	1.82	1.71	1.64
150	5.94	5.33	4.77	4.25	3.78	3.35	2.96	2.63	2.33	2.09	1.89	1.73	1.62	1.56
180	5.56	4.95	4.39	3.87	3.39	2.97	2.58	2.24	1.95	1.71	1.51	1.35	1.24	1.17

TABLE 4.—*Cubic-foot growth of red pine in Minnesota in relation to age, stand density, and site index*[1]

SITE INDEX 45

Stand density basal area per acre (sq. ft.)	Net growth per acre per year when stand age in years is—													
	30	40	50	60	70	80	90	100	110	120	130	140	150	160
	Cu. ft.	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*
60	74	80	78	72	64	56	46	37	28	19				
90	92	99	96	90	82	74	64	55	46	36	28	24	21	17
120	107	113	109	102	92	85	74	64	55	44	36	31	28	24
150	118	123	117	107	97	88	76	65	56	43	34	30	27	22
180	126	128	119	107	94	84	69	58	47	33	24	19	16	10

SITE INDEX 50

Stand density basal area per acre (sq. ft.)	30	40	50	60	70	80	90	100	110	120	130	140	150	160
60	87	94	92	86	80	71	61	51	41	32	25			
90	106	115	112	106	100	92	80	71	60	52	44	37	34	30
120	122	131	127	119	113	104	92	82	70	61	53	46	42	38
150	135	142	134	125	118	107	94	84	70	60	52	44	40	36
180	143	147	136	124	115	103	87	76	61	50	40	32	28	22

SITE INDEX 55

60	99	109	109	103	95	87	79	68	58	49	40	33	-----	-----
90	121	132	132	125	117	110	101	91	79	70	60	53	49	45
120	138	150	147	139	130	123	114	103	90	81	70	62	58	54
150	152	162	156	145	135	127	117	105	91	81	69	60	56	51
180	161	168	159	143	131	122	111	96	80	70	56	47	43	37

SITE INDEX 60

60	114	125	128	123	115	107	98	88	77	66	58	51	46	-----
90	137	149	153	147	138	131	123	112	101	90	81	74	69	64
120	155	167	170	163	153	145	137	125	114	102	92	85	78	74
150	170	180	180	170	158	150	141	128	114	101	91	83	76	71
180	179	186	183	169	154	144	134	118	103	88	78	69	60	54

[1] Based on table 3, U.S. Dept. Agr. Tech. Bul. 1104, by Gevorkiantz and Olsen (1955). Includes total peeled volume of stem on all trees 3.6 inches d.b.h. and larger.

TABLE 5.—*Cordwood growth of red pine in Minnesota in relation to age, stand density, and site index* [1]

Stand density basal area per acre (sq. ft.)	Net growth per acre per year when stand age in years is—													
	30	40	50	60	70	80	90	100	110	120	130	140	150	160
SITE INDEX 45														
	Cords	*Cords*	*Cords*	*Cords*	*Cords*	*Cords*	*Cords*	*Cords*	*Cords*	*Cords*	*Cords*	*Cords*	*Cords*	*Cords*
60	0.72	0.78	0.75	0.70	0.62	0.54	0.45	0.36	0.27	0.18	------	------	------	------
90	.89	.96	.93	.87	.79	.72	.62	.53	.45	.35	0.27	0.23	0.20	0.16
120	1.04	1.10	1.06	.99	.89	.82	.71	.62	.53	.43	.35	.30	.27	.23
150	1.14	1.19	1.13	1.04	.94	.85	.73	.63	.54	.42	.33	.29	.26	.21
180	1.22	1.24	1.15	1.03	.91	.82	.67	.56	.46	.32	.23	.19	.15	.10
SITE INDEX 50														
60	0.84	0.91	0.89	0.84	0.78	0.69	0.59	0.50	0.40	0.31	0.24	------	------	------
90	1.03	1.12	1.09	1.03	.97	.89	.78	.69	.59	.50	.43	0.36	0.33	0.29
120	1.18	1.27	1.23	1.15	1.09	1.01	.89	.80	.68	.59	.51	.44	.41	.36
150	1.31	1.37	1.30	1.21	1.14	1.04	.91	.82	.68	.59	.50	.42	.39	.34
180	1.39	1.43	1.32	1.20	1.12	.99	.85	.74	.59	.49	.39	.31	.27	.21

SITE INDEX 55

60	0.96	1.06	1.06	1.00	0.92	0.85	0.76	0.66	0.56	0.47	0.38	0.32	------	------
90	1.17	1.28	1.27	1.21	1.13	1.06	.98	.88	.77	.68	.59	.51	0.48	0.44
120	1.34	1.45	1.43	1.34	1.26	1.19	1.10	.99	.87	.79	.68	.60	.57	.52
150	1.47	1.57	1.52	1.41	1.31	1.23	1.14	1.01	.88	.78	.66	.58	.55	.50
180	1.56	1.63	1.54	1.39	1.27	1.18	1.07	.97	.78	.68	.54	.45	.42	.36

SITE INDEX 60

60	1.10	1.21	1.24	1.19	1.11	1.03	0.95	0.85	0.74	0.64	0.56	0.50	0.44	------
90	1.33	1.44	1.48	1.42	1.34	1.27	1.20	1.09	.98	.87	.79	.72	.66	0.62
120	1.51	1.62	1.65	1.58	1.48	1.41	1.33	1.22	1.10	.99	.89	.82	.76	.71
150	1.64	1.74	1.75	1.65	1.53	1.45	1.37	1.24	1.10	.98	.88	.80	.73	.68
180	1.73	1.80	1.77	1.64	1.49	1.40	1.29	1.15	1.00	.85	.75	.66	.59	.53

[1] Based on table 5, U.S. Dept. Agr. Tech. Bul. 1104, by Gevorkiantz and Olsen (1955). Includes rough cords for trees 3.6 inches d.b.h. and larger to a 3-inch top d.i.b.

TABLE 6. *Board-foot growth of red pine in Minnesota in relation to age, stand density, and site index* [1]

SITE INDEX 45

Stand density basal area per acre (sq. ft.)	Net growth per acre per year when stand age in years is—													
	30	40	50	60	70	80	90	100	110	120	130	140	150	160
	Bd. ft.	*Bd. ft.*	*Bd. ft.*	*Bd. ft.*	*Bd. ft.*	*Bd. ft.*	*Bd. ft.*	*Bd. ft.*	*Bd. ft.*	*Bd. ft.*	*Bd. ft.*	*Bd. ft.*	*Bd. ft.*	*Bd. ft.*
60	------	410	400	370	330	290	240	190	140	100	------	------	------	------
90	------	------	490	460	420	380	320	280	240	180	140	120	110	90
120	------	------	560	520	470	430	380	330	280	230	180	160	140	120
150	------	------	------	550	490	450	390	330	280	220	180	150	140	110
180	------	------	------	540	480	430	350	300	240	170	120	100	80	50

SITE INDEX 50

Stand density basal area per acre (sq. ft.)	30	40	50	60	70	80	90	100	110	120	130	140	150	160
60	------	480	470	440	410	360	310	260	210	160	130	------	------	------
90	------	------	570	540	510	470	410	360	310	260	230	190	170	150
120	------	------	650	610	580	530	470	420	360	310	270	230	210	190
150	------	------	------	640	600	550	480	430	360	310	260	220	210	180
180	------	------	------	630	590	520	450	390	310	260	210	160	140	110

SITE INDEX 55														
60		560	560	530	490	450	400	350	290	250	200	170		
90			670	640	600	560	520	460	400	360	310	270	250	230
120			750	710	660	630	580	520	460	410	360	320	300	280
150				740	690	650	600	530	460	410	350	310	290	260
180				730	670	620	560	490	410	360	290	240	220	190
SITE INDEX 60														
60		640	650	630	590	540	500	450	390	340	300	260	230	
90			780	750	710	670	630	570	520	460	410	380	350	330
120			870	830	780	740	700	640	580	520	470	430	400	380
150				870	810	760	720	650	580	510	460	420	390	360
180				860	790	740	680	600	530	450	400	350	310	280

[1] Based on table 108, Univ. of Minn. Agr. Expt. Sta. Tech. Bul. 39, by Brown and Gevorkiantz (1934). Includes board-foot volume by Scribner Dec. C log rule for trees 7.6 inches d.b.h. and larger to a 6-inch top d.i.b.

This is set equal to zero and $X_1=125.9$ square feet of basal area per acre. The practical application of this level must be tempered by consideration of the experimental error associated with it. Also, since stand density has been represented by a relatively flat-topped parabola, deviations of 15 square feet of basal area on either side of the maximum represent a growth loss of less than one-tenth of a square foot of basal area growth per year.

Finding the maxima for cubic-foot, cordwood, and board-foot growth is somewhat more complicated because volume growth is not expressed mathematically. Then too, volume growth maxima differ by ages and sites as was evident in the tables introduced earlier. The best way to determine the maximum volume growth for a particular age and site combination is to plot a curve of growth over stand density from the data given in table 4 for cubic feet, table 5 for cordwood, and table 6 for board feet. These points can be smoothed, and the density giving maximum growth can be read with acceptable accuracy from the graph.

Care must be used with high-density management even if it does maximize cubic volume growth, for some of these levels approach the threshold where suppression losses are likely to occur. Although some plots used in this study reach 200 square feet of basal area per acre without suppression losses, this paper treats 180 square feet of basal area as an upper limit of stand density for managed stands, and this is reflected in the yield tables given later. As additional growth information becomes available, modifications can be made in the upper density limits.

YIELDS OF RED PINE

For purposes of this paper, yield has been defined as the summation of a number of net annual growth increments. This yield may be stored on the stump until final harvest, or part or all of it may be removed in thinnings. If thinning is done, the yield may be harvested in short or irregular or long cutting cycles.

It is evident that yields can be obtained in many ways. To represent these in tabular or graphic form would require many tables and graphs. Instead, one series of yield tables will be presented. Some of these tables approximate the red pine management practices used by the national forests in Minnesota and by other forestry agencies as well. No claim is made that these yield tables outline an optimum management; rather, they represent a convenient form of tabulation that follows reasonably closely present-day thinning practices in red pine. If one chooses to depart from the schedule of thinnings shown in the tables, he may still obtain some information about yields by interpolation. The methods of construction will be sufficiently illustrated in the appendix so that he can synthesize his own tables.

Cutting Cycles

For these yield tables a 10-year cutting cycle has been used except in young stands that are maintained at 150 square feet of basal area per acre. For them, a 5-year cutting cycle is used. The 5-year cycle prevents stands from growing to a density where suppression losses are possible. Thinning in red pine is assumed to begin at age 25.

Stand Density Control

Three levels of stand density have been chosen for the yield tables. They are 90, 120, and 150 square feet of basal area per acre. At the end of each cutting cycle the stand is thinned back to the appropriate density. This process is illustrated in figure 9.

These are but three of the many density options that can be exercised by a forest manager. He could, for example, have his stands average a given density midway through a cutting cycle rather than begin the cycle with the given density. Or he could choose to maximize cubic-foot and cordwood growth, in which case densities would change with each cutting cycle.

Yield Tables

The yield tables (tables 7, 8, 9, and 10) are given for site indexes 45, 50, 55, and 60. The three major subdivisions in the tables give basal area and volume remaining after each cut (columns 3 to 6), the basal area and volume cut (columns 7 to 10), and the cumulative basal area and volumes, which include increases in the residual stand plus the sum of all cuts (columns 11 to 14).

The net periodic annual growth in cubic feet is given in column 15. For the same age and site index, net periodic annual growth differs somewhat from that shown in the cubic volume growth table given previously (table 4). This occurs because the yield tables average a somewhat higher stand density during the cutting cycle than is assumed in the growth tables. It is possible to construct growth tables that closely approximate the kind of thinning schedule shown in the yield tables.

Mean annual increment is given in column 16. The culmination of mean annual increment occurs between the ages 75 and 95, the later culmination being on better sites. Column 17 gives the proportion of stand basal area cut at the end of each cutting cycle.

It is evident from the tables that the higher densities produce higher cubic-foot volume yields, although the advantage is not as great as might once have been thought. For example, if the forest manager chose a rotation of 165 years on site index 50 land, the 150-square-foot density would produce about 1,800 cubic feet more volume than would the 90-square-foot level, a difference of about 15 percent. If he should choose a density that would maximize cubic volume growth, there would be an additional increase in yields. (See "Maximizing Growth," p. 17.)

A major advantage of the three density levels, or variations of them, is that they permit considerable control of size and radial growth of trees. To explore these questions adequately, assumptions or additional studies must be made as to mortality, the effects of cutting on tree size and tree diameter distribution, and so forth. These considerations are beyond the scope of this paper. However, the 90-square-foot basal area density represents a rather open red pine stand and is the kind of control that might be used to obtain large red pine saw logs. The 120-square-foot level is a middle-of-the-road management closely paralleling present thinning practices for red pine. By age 100 this density will produce saw logs up to an 18-inch top diameter and numerous large poles and piling.

(Continued on page 41)

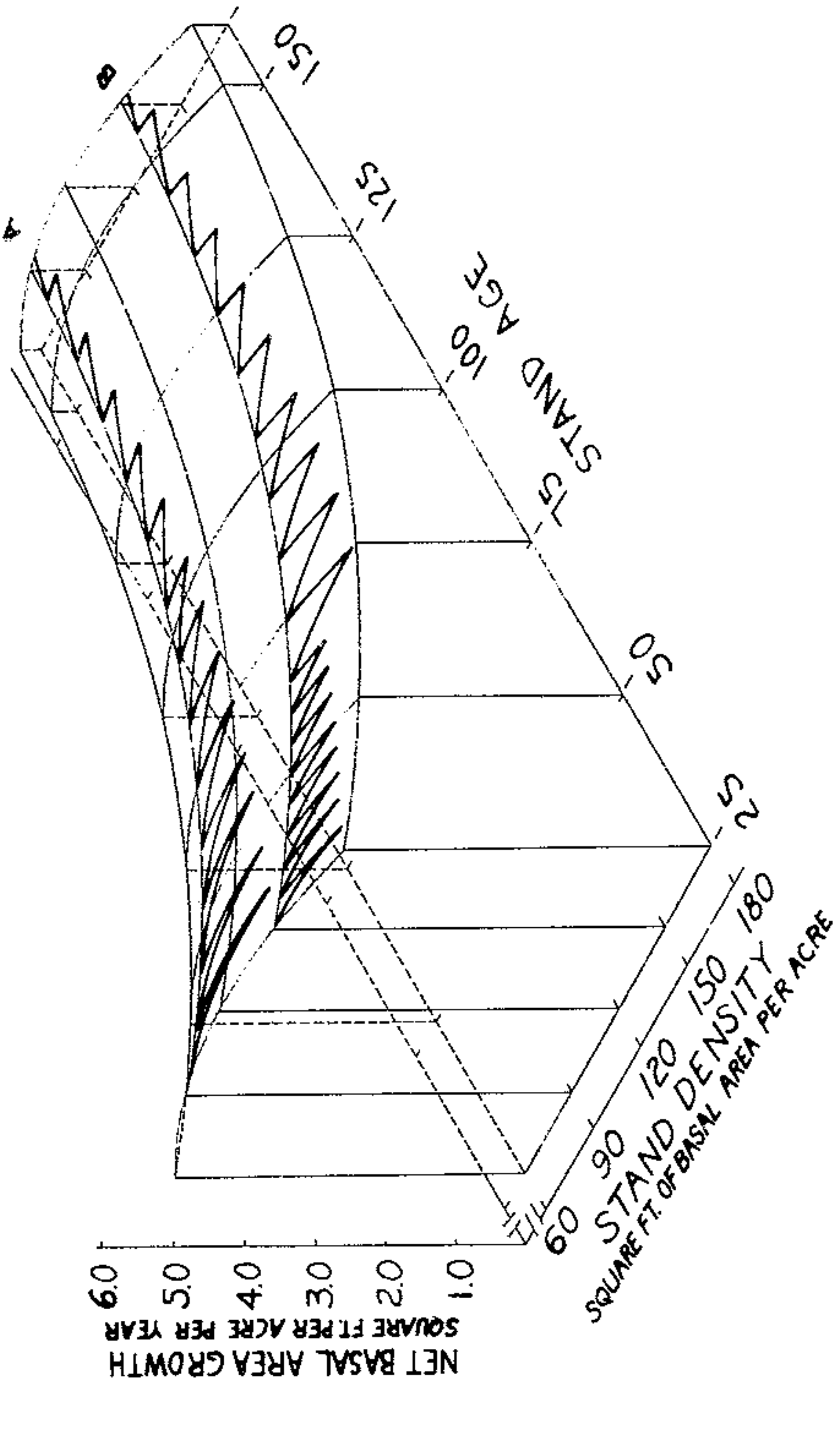

FIGURE 9.—Basal area growth of red pine stands with stand density: *A*, 90 square feet of basal area at beginning of each cutting cycle; *B*, 150 square feet of basal area at beginning of each cutting cycle (site index 50).

TABLE 7.—*Yield table for site index 45 red pine in Minnesota (all data are per acre): Thinning schedule—maintain given stand density at beginning of each cutting cycle*

90 SQUARE FEET OF BASAL AREA AT BEGINNING OF CUTTING CYCLE

Age, years	Height of dominants and codominants, feet	Remaining stand				Cut stand				Cumulative total volume—cut stand plus remaining stand				Periodic net annual growth, cubic feet	Mean net annual growth, cubic feet	Proportion of stand basal area cut, percent	Age, years
		Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(1)
25	22.0	[1]90.0	810	7.8	----	----	----	----	----	90.0	810	7.8	----		32	----	25
35	32.0	90.0	1,180	11.4	----	50.4	660	6.4	----	140.4	1,840	17.8	----	103	52	35.9	35
45	41.0	90.0	1,510	14.6	7,700	44.1	740	7.2	----	184.5	2,910	28.2	7,700	107	65	32.9	45
55	48.5	90.0	1,780	17.3	9,100	38.5	760	7.4	3,900	223.0	3,940	38.3	13,000	103	72	30.0	55
65	54.5	90.0	2,000	19.4	10,200	33.3	740	7.2	3,800	256.3	4,900	47.6	17,900	96	75	27.0	65
75	59.5	90.0	2,190	21.2	11,200	28.2	690	6.6	3,500	284.5	5,780	56.0	22,400	88	77	23.9	75
85	64.0	90.0	2,350	22.8	12,000	23.9	630	6.1	3,200	308.4	6,570	63.7	26,400	79	77	21.0	85
95	67.5	90.0	2,480	24.0	12,700	19.7	540	5.3	2,800	328.1	7,240	70.2	29,900	67	76	18.0	95
105	70.5	90.0	2,590	25.1	13,200	16.3	470	4.5	2,400	344.4	7,820	75.8	32,800	58	74	15.3	105
115	73.0	90.0	2,680	26.0	13,700	13.1	390	3.8	2,000	357.5	8,300	80.5	35,300	48	72	12.7	115
125	74.5	90.0	2,740	26.5	14,000	10.4	320	3.1	1,600	367.9	8,680	84.1	37,200	38	69	10.4	125
135	75.5	90.0	2,780	26.9	14,200	8.6	270	2.6	1,400	376.5	8,990	87.1	38,800	31	67	8.7	135
145	76.5	90.0	2,810	27.2	14,400	6.9	220	2.1	1,100	383.4	9,240	89.5	40,100	25	64	7.1	145
155	77.5	90.0	2,850	27.6	14,500	5.7	180	1.7	900	389.1	9,460	91.6	41,100	22	61	6.0	155
165	78.0	[2]	[2]	[2]	[2]	95.0	3,030	29.3	15,400	394.1	9,640	93.3	42,000	18	58	100.0	165

See footnotes at end of table.

TABLE 7.—*Yield table for site index 45 red pine in Minnesota (all data are per acre): Thinning schedule—maintain given stand density at beginning of each cutting cycle*—Continued

120 SQUARE FEET OF BASAL AREA AT BEGINNING OF CUTTING CYCLE

Age, years	Height of dominants and codominants, feet	Remaining stand				Cut stand				Cumulative total volume—cut stand plus remaining stand				Periodic net annual growth, cubic feet	Mean net annual growth, cubic feet	Proportion of stand basal area cut, percent	Age, years
		Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(1)
25	22.0	[1] 120.0	1,080	10.4	----	----	----	----	----	120.0	1,080	10.4	----	----	43	----	25
35	32.0	120.0	1,570	15.2	----	50.3	660	6.4	----	170.3	2,230	21.6	----	115	64	29.5	35
45	41.0	120.0	2,010	19.5	----	44.1	740	7.2	----	214.4	3,410	33.1	----	118	76	26.9	45
55	48.5	120.0	2,380	23.0	12,100	38.6	760	7.4	----	253.0	4,540	44.0	12,100	113	83	24.3	55
65	54.5	120.0	2,670	25.9	13,600	33.8	750	7.3	3,800	286.8	5,580	54.2	17,400	104	86	22.0	65
75	59.5	120.0	2,920	28.3	14,900	28.9	700	6.8	3,600	315.7	6,530	63.4	22,300	95	87	19.4	75
85	64.0	120.0	3,140	30.4	16,000	24.8	650	6.3	3,300	340.5	7,400	71.8	26,700	87	87	17.1	85
95	67.5	120.0	3,310	32.1	16,900	20.9	580	5.6	2,900	361.4	8,150	79.1	30,500	75	86	14.8	95
105	70.5	120.0	3,460	33.5	17,600	17.5	500	4.9	2,600	378.9	8,800	85.4	33,800	65	84	12.7	105
115	73.0	120.0	3,580	34.7	18,300	14.7	440	4.2	2,200	393.6	9,360	90.8	36,700	56	81	10.9	115
125	74.5	120.0	3,650	35.4	18,600	12.1	370	3.6	1,900	405.7	9,800	95.1	38,900	44	78	9.2	125
135	75.5	120.0	3,700	35.9	18,900	9.9	310	3.0	1,600	415.6	10,160	98.6	40,800	36	75	7.6	135
145	76.5	120.0	3,750	36.3	19,100	8.6	270	2.6	1,400	424.2	10,480	101.6	42,400	32	72	6.7	145
155	77.5	120.0	3,800	36.8	19,400	7.3	230	2.2	1,200	431.5	10,760	104.3	43,900	28	69	5.7	155
165	78.0	[2]	[2]	[2]	[2]	126.8	4,040	39.1	20,600	438.3	11,000	106.6	45,100	24	67	100.0	165

150 SQUARE FEET OF BASAL AREA AT BEGINNING OF CUTTING CYCLE

25	22.0	[1] 150.0	1,350	13.1						150.0	1,350	13.1			54		25
30	27.0	150.0	1,650	16.0		25.3	280	2.7		175.3	1,930	18.7		[3] 120		14.4	30
35	32.0	150.0	1,960	19.0		23.8	310	3.0		199.1	2,550	24.7			73	13.7	35
40	37.0	150.0	2,270	22.0		22.2	340	3.2		221.3	3,200	30.9		124		12.9	40
45	41.0	150.0	2,510	24.3		20.8	350	3.4		242.1	3,790	36.6			84	12.2	45
50	45.0	150.0	2,760	26.7		19.5	360	3.5		261.6	4,400	42.5		118		11.5	50
55	48.5	150.0	2,970	28.8	15,200	18.0	360	3.5		279.6	4,970	48.1	15,200		90	10.7	55
60	52.0	150.0	3,190	30.9	16,300	16.0	360	3.5	1,800	296.5	5,550	53.7	18,100	108		10.1	60
65	54.5	150.0	3,340	32.4	17,000	15.6	350	3.4	1,800	312.1	6,050	58.6	20,600		93	9.4	65
														96			
75	59.5	150.0	3,650	35.3	18,600	26.7	650	6.3	3,300	338.8	7,010	67.8	25,500		93	15.1	75
														87			
85	64.0	150.0	3,920	38.0	20,000	22.8	600	5.8	3,000	361.6	7,880	76.3	29,900		93	13.2	85
														74			
95	67.5	150.0	4,140	40.1	21,100	19.0	520	5.1	2,700	380.6	8,620	83.5	33,700		91	11.2	95
														64			
105	70.5	150.0	4,320	41.9	22,000	15.8	460	4.4	2,300	396.4	9,260	89.7	36,900		88	9.5	105
														54			
115	73.0	150.0	4,470	43.3	22,800	13.0	390	3.8	2,000	409.4	9,800	94.9	39,700		85	8.0	115
														42			
125	74.5	150.0	4,560	44.2	23,300	10.7	330	3.2	1,700	420.1	10,220	99.0	41,900		82	6.7	125
														34			
135	75.5	150.0	4,630	44.8	23,600	8.8	270	2.6	1,400	428.9	10,560	102.2	43,600		78	5.5	135
														29			
145	76.5	150.0	4,690	45.4	23,900	7.2	230	2.2	1,100	436.1	10,850	105.0	45,000		75	4.6	145
														26			
155	77.5	150.0	4,750	46.0	24,200	6.2	200	1.9	1,000	442.3	11,110	107.5	46,300		72	4.0	155
														20			
165	78.0	(2)	(2)	(2)	(2)	155.4	4,950	48.0	25,300	447.7	11,310	109.5	47,400		69	100.0	165

[1] All wood harvested prior to this age not included in yield table.
[2] End of rotation.
[3] Periodic and mean annual growth given by 10-year intervals. Linear interpolation of height by 5-year periods produces slight anomalies which are avoided by the 10-year interval.

TABLE 8.—*Yield table for site index 50 red pine in Minnesota (all data per acre): Thinning schedule—maintain given stand density at beginning of each cutting cycle*

90 SQUARE FEET OF BASAL AREA AT BEGINNING OF CUTTING CYCLE

Age, years	Height of dominants and codominants, feet	Remaining stand				Cut stand				Cumulative total volume—cut stand plus remaining stand				Periodic net annual growth, cubic feet	Mean net annual growth, cubic feet	Proportion of stand basal area cut, percent	Age, years
		Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(1)
25	24.5	[1] 90.0	900	8.7	-----	-----	-----	-----	-----	90.0	900	8.7	-----	-----	36	-----	25
														118			
35	35.5	90.0	1,300	12.6	-----	53.7	780	7.5	-----	143.7	2,080	20.1	-----		59	37.4	35
														125			
45	15.5	90.0	1,670	16.2	8,500	47.4	880	3.5	-----	191.1	3,330	32.2	8,500		74	34.5	45
														122			
55	53.5	90.0	1,970	19.1	10,000	41.9	920	8.9	4,700	233.0	4,550	44.0	14,700		83	31.8	55
														113			
65	60.0	90.0	2,210	21.4	11,200	36.4	890	8.6	4,600	269.4	5,680	54.9	20,500		87	28.8	65
														107			
75	66.0	90.0	2,430	23.5	12,400	31.4	850	8.2	4,300	300.8	6,750	65.2	26,000		90	25.9	75
														96			
85	71.0	90.0	2,610	25.3	13,300	27.0	780	7.6	4,000	327.8	7,710	74.6	30,900		91	23.1	85
														86			
95	75.0	90.0	2,760	26.7	14,100	23.2	710	6.9	3,600	351.0	8,570	82.9	35,300		90	20.5	95
														76			
105	78.5	90.0	2,890	28.0	14,700	19.6	630	6.1	3,200	370.6	9,330	90.3	39,100		89	17.9	105
														64			
115	81.0	90.0	2,980	28.9	15,200	16.6	550	5.3	2,800	387.2	9,970	96.5	42,400		87	15.6	115
														54			
125	83.0	90.0	3,050	29.6	15,600	13.8	470	4.5	2,400	401.0	10,510	101.7	45,200		84	13.3	125
														47			
135	84.5	90.0	3,110	30.1	15,800	11.9	410	4.0	2,100	412.9	10,980	106.2	47,500		81	11.7	135
														38			
145	85.5	90.0	3,140	30.5	16,000	10.0	350	3.4	1,800	422.9	11,360	110.0	49,500		78	10.0	145
														36			
155	86.5	90.0	3,180	30.8	16,200	9.0	320	3.1	1,600	431.9	11,720	113.4	51,300		76	9.1	155
														30			
165	87.0	[2]	[2]	[2]	[2]	98.0	3,480	33.7	17,800	439.9	12,020	116.3	52,900	-----	73	100.0	165

120 SQUARE FEET OF BASAL AREA AT BEGINNING OF CUTTING CYCLE

25	24.5	[1] 120.0	1,200	11.6	......	......	......	......	......	120.0	1,200	11.6	......	......	48	......	25
35	35.5	120.0	1,740	16.9		53.4	770	7.5	......	173.4	2,510	24.4	......	131	72	30.8	35
45	45.5	120.0	2,230	21.6	......	47.2	880	8.5	......	220.6	3,880	37.6	......	137	86	28.2	45
55	53.5	120.0	2,620	25.4	13,400	41.9	920	8.9	......	262.5	5,190	50.3	13,400	131	94	25.9	55
65	60.0	120.0	2,940	28.5	15,000	36.7	900	8.7	4,600	299.2	6,410	62.1	19,600	122	99	23.4	65
75	66.0	120.0	3,240	31.3	16,500	32.1	870	8.4	4,400	331.3	7,580	73.3	25,500	117	101	21.1	75
85	71.0	120.0	3,480	33.7	17,700	27.8	810	7.8	4,100	359.1	8,630	83.5	30,800	105	102	18.8	85
95	75.0	120.0	3,680	35.6	18,800	24.1	740	7.2	3,800	383.2	9,570	92.6	35,700	94	101	16.7	95
105	78.5	120.0	3,850	37.3	19,600	20.7	660	6.4	3,400	403.9	10,400	100.7	39,900	83	99	14.7	105
115	81.0	120.0	3,970	38.5	20,200	17.8	590	5.7	3,000	421.7	11,110	107.6	43,500	71	97	12.9	115
125	83.0	120.0	4,070	39.4	20,700	15.2	520	5.0	2,600	436.9	11,730	113.5	46,600	62	94	11.2	125
135	84.5	120.0	4,140	40.1	21,100	13.2	460	4.4	2,300	450.1	12,260	118.6	49,300	53	91	9.9	135
145	85.5	120.0	4,190	40.6	21,400	11.6	410	3.9	2,100	461.7	12,720	123.0	51,700	46	88	8.8	145
155	86.5	120.0	4,240	41.1	21,600	10.4	370	3.6	1,900	472.1	13,140	127.1	53,800	42	85	8.0	155
165	87.0	(2)	(2)	(2)	(2)	130.0	4,620	44.8	23,600	482.1	13,520	130.8	55,800	38	82	100.0	165

See footnotes at end of table.

TABLE 8.—*Yield table for site index 50 red pine in Minnesota (all data per acre): Thinning schedule—Maintain given stand density at beginning of each cutting cycle*—Continued

150 SQUARE FEET OF BASAL AREA AT BEGINNING OF CUTTING CYCLE

Age, years	Height of dominants and codominants, feet	Remaining stand				Cut stand				Cumulative total volume—cut stand plus remaining stand				Periodic net annual growth, cubic feet	Mean net annual growth, cubic feet	Proportion of stand basal area cut, percent	Age, years
		Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(1)
25	24.5	[1] 150.0	1,500	11.5						150.0	1,500	14.5			60		25
30	30.0	150.0	1,840	17.8		26.9	330	3.2		176.9	2,170	21.0		[3] 137		15.2	30
35	35.5	150.0	2,170	21.1		25.2	370	3.5		202.1	2,870	27.8			82	14.4	35
40	41.0	150.0	2,510	24.3		23.8	400	3.9		225.9	3,610	34.9		144		13.7	40
45	45.5	150.0	2,790	27.0		22.4	420	4.0		248.3	4,310	41.6			96	13.0	45
50	50.0	150.0	3,060	29.7		21.0	430	4.2		269.3	5,010	48.5		135		12.3	50
55	53.5	150.0	3,280	31.8	16,700	19.5	430	4.1		288.8	5,660	54.7	16,700		103	11.5	55
60	57.0	150.0	3,400	33.8	17,800	18.4	430	4.2	2,200	307.2	6,300	60.9	20,000	125		10.9	60
65	60.0	150.0	3,680	35.6	18,800	17.0	420	4.0	2,100	324.2	6,910	66.7	23,100		106	10.2	65
														117			
75	66.0	150.0	4,040	39.2	20,600	29.9	810	7.8	4,100	354.1	8,080	78.1	29,000		108	16.7	75
														106			
85	71.0	150.0	4,350	42.2	22,200	25.9	750	7.3	3,800	380.0	9,140	88.4	31,400		108	14.7	85
														93			
95	75.0	150.0	4,600	44.5	23,400	22.2	680	6.6	3,500	402.2	10,070	97.3	39,100		106	12.9	95
														82			
105	78.5	150.0	4,810	46.6	24,500	18.9	610	5.9	3,100	421.1	10,890	105.3	43,300		104	11.2	105
														69			
115	81.0	150.0	4,960	48.1	25,300	16.2	540	5.2	2,700	437.3	11,580	112.0	46,800		101	9.7	115
														59			
125	83.0	150.0	5,090	49.3	26,000	13.6	460	4.5	2,400	450.9	12,170	117.7	49,900		97	8.3	125
														50			
135	84.5	150.0	5,180	50.2	26,400	12.0	410	4.0	2,100	462.9	12,670	122.6	52,400		94	7.4	135
														41			
145	85.5	150.0	5,240	50.8	26,700	10.1	350	3.4	1,800	473.0	13,080	126.6	54,500		90	6.3	145
														39			
155	86.5	150.0	5,300	51.4	27,000	9.2	330	3.2	1,700	482.2	13,470	130.4	56,500		87	5.8	155
														31			
165	87.0	(2)	(2)	(2)	(2)	158.6	5,640	54.6	28,800	490.8	13,810	133.6	58,300		84	100.0	165

[1] All wood harvested prior to this age not included in yield table.
[2] End of rotation.
[3] Periodic and mean annual growth given by 10-year intervals. Linear interpolation of height by 5-year periods produces slight anomalies which are avoided by the 10-year interval.

TABLE 9. *Yield table for site index 55 red pine in Minnesota (all data are per acre): Thinning schedule—maintain given stand density at beginning of each cutting cycle*

90 SQUARE FEET OF BASAL AREA AT BEGINNING OF CUTTING CYCLE

Age, years	Height of dominants and codominants, feet	Remaining stand				Cut stand				Cumulative total volume—cut stand plus remaining stand				Periodic net annual growth, cubic feet	Mean net annual growth, cubic feet	Proportion of stand basal area cut, percent	Age, years
		Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(1)
25	27.0	[1] 90.0	990	9.6	------	------	------	------	------	90.0	990	9.6	------		40	------	25
35	39.0	90.0	1,430	13.9	------	56.8	900	8.8	------	146.8	2,330	22.7	------	134	67	38.7	35
45	50.0	90.0	1,840	17.8	9,400	50.6	1,030	10.0	------	197.4	3,770	36.6	9,400	144	84	36.0	45
55	59.0	90.0	2,170	21.0	11,100	44.9	1,080	10.5	5,500	242.3	5,180	50.3	16,600	141	94	33.3	55
65	66.0	90.0	2,430	23.5	12,400	39.5	1,060	10.3	5,400	281.8	6,500	63.1	23,300	132	100	30.5	65
75	72.0	90.0	2,650	25.6	13,500	34.7	1,020	9.9	5,200	316.5	7,740	75.1	29,600	124	103	27.8	75
85	77.5	90.0	2,850	27.6	14,500	30.2	960	9.3	4,900	346.7	8,900	86.4	35,500	116	105	25.1	85
95	82.5	90.0	3,030	29.4	15,500	26.6	900	8.7	4,600	373.3	9,980	96.9	41,100	108	105	22.8	95
105	86.5	90.0	3,180	30.8	16,200	22.8	810	7.8	4,100	396.1	10,940	106.1	45,900	96	104	20.2	105
115	89.5	90.0	3,290	31.9	16,800	19.8	720	7.0	3,700	415.9	11,770	114.2	50,200	83	102	18.0	115
125	92.0	90.0	3,380	32.8	17,300	17.2	650	6.3	3,300	433.1	12,510	121.4	54,000	74	100	16.0	125
135	93.5	90.0	3,440	33.3	17,500	15.0	570	5.5	2,900	448.1	13,140	127.4	57,100	63	97	14.3	135
145	94.5	90.0	3,470	33.7	17,700	13.3	510	5.0	2,600	461.4	13,680	132.8	59,900	54	94	12.9	145
155	95.5	90.0	3,510	34.0	17,900	12.0	470	4.5	2,400	473.4	14,190	137.6	62,500	51	92	11.8	155
165	96.0	(2)	(2)	(2)	(2)	101.5	3,980	38.6	20,300	484.9	14,660	142.2	64,900	47	89	100.0	165

See footnotes at end of table.

TABLE 9.—*Yield table for site index 55 red pine in Minnesota (all data are per acre): Thinning schedule—maintain given stand density at beginning of each cutting cycle*—Continued

120 SQUARE FEET OF BASAL AREA AT BEGINNING OF CUTTING CYCLE

Age, years	Height of dominants and codominants, feet	Remaining stand				Cut stand				Cumulative total volume—cut stand plus remaining stand				Periodic net annual growth, cubic feet	Mean net annual growth, cubic feet	Proportion of stand basal area cut, percent	Age, years
		Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(1)
25	27.0	[1] 120.0	1,320	12.8	----------	----------	----------	----------	----------	120.0	1,320	12.8	----------	----------	53	----------	25
35	39.0	120.0	1,910	18.5	----------	56.3	900	8.7	----------	176.3	2,810	27.2		149	80	31.9	35
45	50.0	120.0	2,450	23.7	----------	50.3	1,030	10.0	----------	226.6	4,380	42.4	----------	157	97	20.5	45
55	59.0	120.0	2,800	28.0	14,700	45.0	1,080	10.5	----------	271.6	5,900	57.2	14,700	152	107	27.3	55
65	66.0	120.0	3,230	31.3	16,500	39.8	1,070	10.4	5,500	311.4	7,310	70.9	22,000	141	112	24.9	65
75	72.0	120.0	3,530	34.2	18,000	35.2	1,040	10.0	5,300	346.6	8,650	83.8	28,800	134	115	22.7	75
85	77.5	120.0	3,800	36.8	19,400	31.0	980	9.5	5,000	377.6	9,900	95.9	35,200	125	116	20.5	85
95	82.5	120.0	4,040	39.2	20,600	27.2	920	8.9	4,700	404.8	11,060	107.2	41,100	116	116	18.5	95
105	86.5	120.0	4,240	41.1	21,600	23.8	840	8.1	4,300	428.6	12,100	117.2	46,400	104	115	16.6	105
115	89.5	120.0	4,390	42.5	22,400	20.9	760	7.4	3,900	449.5	13,010	126.0	51,100	91	113	14.8	115
125	92.0	120.0	4,510	43.7	23,000	18.5	700	6.7	3,500	468.0	13,830	133.9	55,200	82	111	13.4	125
135	93.5	120.0	4,580	44.4	23,400	16.6	630	6.1	3,200	484.6	14,530	140.7	58,800	70	108	12.2	135
145	94.5	120.0	4,630	44.9	23,600	14.9	580	5.6	2,900	499.5	15,160	146.8	61,900	63	105	11.0	145
155	95.5	120.0	4,680	45.4	23,900	13.7	530	5.2	2,700	513.2	15,740	152.5	64,900	58	102	10.2	155
165	96.0	(2)	(2)	(2)	(2)	133.0	5,220	50.5	26,600	526.2	16,280	157.6	67,600	54	99	100.0	165

150 SQUARE FEET OF BASAL AREA AT BEGINNING OF CUTTING CYCLE

25	27.0	[1] 150.0	1,650	16.0						150.0	1,650	16.0			66		25
30	33.0	150.0	2,020	19.6		28.3	380	3.8		178.3	2,400	23.4		[3] 155		15.9	30
35	39.0	150.0	2,390	23.2		26.8	430	4.1		205.1	3,200	31.1			91	15.2	35
40	45.0	150.0	2,760	26.7		25.4	470	4.5		230.5	4,040	39.1		163		14.5	40
45	50.0	150.0	3,060	29.7		23.9	490	4.7		254.4	4,830	46.8			107	13.7	45
50	55.0	150.0	3,370	32.7		22.5	510	4.9		276.9	5,650	54.7		158		13.0	50
55	59.0	150.0	3,620	35.0	18,400	21.3	510	5.0		298.2	6,410	62.0	18,400		117	12.4	55
60	63.0	150.0	3,860	37.4	19,700	20.0	510	5.0	2,600	318.2	7,160	69.4	22,300	144		11.8	60
65	66.0	150.0	4,040	39.2	20,600	18.8	510	4.9	2,600	337.0	7,850	76.1	25,800		121	11.1	65
70	69.0	150.0	4,230	41.0	21,600	17.5	490	4.8	2,500	354.5	8,530	82.7	29,300	134		10.4	70
75	72.0	150.0	4,410	42.7	22,500	16.3	480	4.6	2,400	370.8	9,190	89.0	32,600		123	9.8	75
														125			
85	77.5	150.0	4,750	46.0	24,200	28.7	910	8.8	4,600	399.5	10,440	101.1	38,900		123	16.1	85
														115			
95	82.5	150.0	5,050	49.0	25,800	25.2	850	8.2	4,300	424.7	11,590	112.3	44,800		122	14.4	95
														102			
105	86.5	150.0	5,300	51.4	27,000	21.9	770	7.5	3,900	446.6	12,610	122.2	49,900		120	12.7	105
														88			
115	89.5	150.0	5,480	53.1	28,000	19.1	700	6.8	3,600	465.7	13,490	130.7	54,500		117	11.3	115
														80			
125	92.0	150.0	5,640	54.6	28,800	16.9	640	6.2	3,200	482.6	14,290	138.4	58,500		114	10.1	125
														66			
135	93.5	150.0	5,730	55.5	29,200	14.9	570	5.5	2,900	497.5	14,950	144.8	61,800		111	9.0	135
														57			
145	94.5	150.0	5,790	56.1	29,500	13.3	510	5.0	2,600	510.8	15,520	150.4	64,700		107	8.1	145
														54			
155	95.5	150.0	5,850	56.7	29,800	12.2	480	4.6	2,400	523.0	16,060	155.6	67,400		104	7.5	155
														49			
165	96.0	(2)	(2)	(2)	(2)	161.6	6,340	61.4	32,300	534.6	16,550	160.3	69,900		100	100.0	165

[1] All wood harvested prior to this age not included in yield table.
[2] End of rotation.
[3] Periodic and mean annual growth given by 10-year intervals. Linear interpolation of height by 5-year periods produces slight anomalies which are avoided by the 10-year interval.

TABLE 10.—*Yield table for site index 60 red pine in Minnesota (all data are per acre): Thinning schedule—maintain given stand density at beginning of each cutting cycle*

90 SQUARE FEET OF BASAL AREA AT BEGINNING OF CUTTING CYCLE

Age, years	Height of dominants and codominants, feet	Remaining stand				Cut stand				Cumulative total volume—cut stand plus remaining stand				Periodic net annual growth, cubic feet	Mean net annual growth, cubic feet	Proportion of stand basal area cut, percent	Age, years
		Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(1)
25	30.5	[1] 90.0	1,120	10.9	----	----	----	----	----	90.0	1,120	10.9	----	----	45	----	25
35	43.0	90.0	1,580	15.3	----	59.9	1,050	10.2	----	149.9	2,630	25.5	----	151	75	40.0	35
45	54.5	90.0	2,000	19.4	10,200	53.9	1,200	11.6	----	203.8	4,250	41.2	10,200	162	94	37.5	45
55	64.5	90.0	2,370	23.0	12,100	48.1	1,270	12.3	6,500	251.9	5,890	57.1	18,600	164	107	34.8	55
65	72.5	90.0	2,660	25.8	13,600	42.9	1,270	12.3	6,500	294.8	7,450	72.2	26,600	156	115	32.3	65
75	79.0	90.0	2,900	28.1	14,800	38.2	1,230	11.9	6,300	333.0	8,920	86.4	34,100	147	119	29.8	75
85	85.0	90.0	3,130	30.3	15,900	33.5	1,160	11.3	5,900	366.5	10,310	99.9	41,100	139	121	27.1	85
95	90.5	90.0	3,330	32.2	17,000	29.6	1,090	10.6	5,600	396.1	11,600	112.4	47,800	129	122	24.7	95
105	95.0	90.0	3,490	33.8	17,800	26.1	1,010	9.8	5,200	422.2	12,770	123.8	53,800	117	122	22.5	105
115	98.5	90.0	3,620	35.1	18,500	23.0	930	9.0	4,700	445.2	13,830	134.1	59,200	106	120	20.4	115
125	101.0	90.0	3,710	36.0	18,900	20.3	840	8.1	4,300	465.5	14,760	143.1	63,900	93	118	18.4	125
135	103.0	90.0	3,790	36.7	19,300	18.2	770	7.4	3,900	483.7	15,610	151.2	68,200	85	116	16.8	135
145	104.5	90.0	3,840	37.2	19,600	17.0	730	7.0	3,700	500.7	16,390	158.7	72,200	78	113	15.9	145
155	105.5	90.0	3,880	37.6	19,800	15.7	680	6.6	3,500	516.4	17,110	165.7	75,900	72	110	14.9	155
165	106.0	[2]	[2]	[2]	[2]	104.8	4,540	44.0	23,200	531.2	17,770	172.1	79,300	66	108	100.0	165

120 SQUARE FEET OF BASAL AREA AT BEGINNING OF CUTTING CYCLE

25	30.5	[1] 120.0	1,480	14.5	----------	----------	----------	----------	----------	120.0	1,480	14.5	----------	---------- 165	59	----------	25
35	43.0	120.0	2,090	20.4	----------	59.2	1,040	10.1	----------	179.2	3,130	30.5	----------	---------- 175	89	33.0	35
45	54.5	120.0	2,650	25.9	----------	53.4	1,190	11.5	----------	232.6	4,880	47.5	----------	---------- 175	108	30.8	45
55	64.5	120.0	3,140	30.6	16,100	48.0	1,260	12.3	----------	280.6	6,630	64.5	16,100	---------- 166	121	28.6	55
65	72.5	120.0	3,530	34.4	18,100	42.9	1,270	12.3	6,500	323.5	8,290	80.6	24,600	---------- 156	128	26.3	65
75	79.0	120.0	3,850	37.5	19,800	38.3	1,240	12.0	6,300	361.8	9,850	95.7	32,600	---------- 147	131	24.1	75
85	85.0	120.0	4,140	40.4	21,300	34.0	1,180	11.4	6,000	395.8	11,320	110.0	40,100	---------- 139	133	22.1	85
95	90.5	120.0	4,410	43.0	22,600	30.3	1,120	10.9	5,700	426.1	12,710	123.5	47,100	---------- 126	134	20.2	95
105	95.0	120.0	4,630	45.1	23,800	26.9	1,040	10.1	5,300	453.0	13,970	135.7	53,600	---------- 114	133	18.3	105
115	98.5	120.0	4,800	46.8	24,600	24.1	970	9.4	4,900	477.1	15,110	146.8	59,300	---------- 100	131	16.7	115
125	101.0	120.0	4,920	48.0	25,300	21.4	880	8.6	4,500	498.5	16,110	156.6	64,500	---------- 92	129	15.1	125
135	103.0	120.0	5,020	48.9	25,800	19.6	820	8.0	4,200	518.1	17,030	165.5	69,200	---------- 84	126	14.0	135
145	104.5	120.0	5,090	49.6	26,100	18.1	770	7.5	3,900	536.2	17,870	173.7	73,400	---------- 78	123	13.1	145
155	105.5	120.0	5,140	50.1	26,400	16.9	730	7.1	3,700	553.1	18,650	181.3	77,400	---------- 75	120	12.3	155
165	106.0	([2])	([2])	([2])	([2])	136.1	5,890	57.1	30,100	569.2	19,400	188.3	81,100	----------	118	100.0	165

See footnotes at end of table.

TABLE 10.—*Yield table for site index 60 red pine in Minnesota (all data are per acre): Thinning schedule—maintain given stand density at beginning of each cutting cycle*—Continued

150 SQUARE FEET OF BASAL AREA AT BEGINNING OF CUTTING CYCLE

Age, years	Height of dominants and codominants, feet	Remaining stand				Cut stand				Cumulative total volume—cut stand plus remaining stand				Periodic net annual growth, cubic feet	Mean net annual growth, cubic feet	Proportion of stand basal area cut, percent	Age, years
		Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet	Basal area	Cubic feet	Cords	Board feet				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(1)
25	30.5	[1] 150.0	1,870	18.1						150.0	1,870	18.1			75		25
30	37.0	150.0	2,270	22.0		29.8	450	4.4		179.8	2,720	26.4		[3] 171		16.6	30
35	43.0	150.0	2,630	25.5		28.3	500	4.8		208.1	3,580	31.7			102	15.9	35
40	49.0	150.0	3,000	29.1		26.9	540	5.2		235.0	4,490	43.5		182		15.2	40
45	54.5	150.0	3,340	32.4		25.5	570	5.5		260.5	5,400	52.3			120	14.5	45
50	60.0	150.0	3,680	35.6		24.0	590	5.7		284.5	6,330	61.2		180		13.8	50
55	64.5	150.0	3,950	38.3	20,200	22.7	600	5.8		307.2	7,200	69.7	20,200		131	13.1	55
60	69.0	150.0	4,230	41.0	21,600	21.5	610	5.9	3,100	328.7	8,090	78.3	24,700	170		12.5	60
65	72.5	150.0	4,440	43.0	22,700	20.1	600	5.8	3,000	348.8	8,900	86.1	28,800		137	11.8	65
70	76.0	150.0	4,660	45.1	23,800	19.0	590	5.7	3,000	367.8	9,710	93.9	32,900	157		11.2	70
75	79.0	150.0	4,840	46.9	24,700	18.0	580	5.6	3,000	385.8	10,470	101.3	36,800		140	10.7	75
80	82.0	150.0	5,020	48.7	25,600	16.9	570	5.5	2,900	402.7	11,220	108.6	40,600	149		10.1	80
85	85.0	150.0	5,210	50.5	26,600	15.8	550	5.3	2,800	418.5	11,960	115.7	44,400		141	9.5	85
95	90.5	150.0	5,540	53.7	28,300	28.2	1,040	10.1	5,300	446.7	13,330	129.0	51,400	137	140	15.8	95
105	95.0	150.0	5,820	56.4	29,700	25.0	970	9.4	4,900	471.7	14,580	141.1	57,700	125	139	14.3	105
115	98.5	150.0	6,040	58.5	30,800	22.2	890	8.7	4,600	493.9	15,690	151.9	63,400	111	136	12.9	115
125	101.0	150.0	6,190	60.0	31,600	19.7	810	7.9	4,100	513.6	16,650	161.3	68,300	96	133	11.6	125
135	103.0	150.0	6,310	61.2	32,200	17.9	750	7.3	3,800	531.5	17,520	169.8	72,700	87	130	10.7	135
145	104.5	150.0	6,400	62.0	32,700	16.4	700	6.8	3,600	547.9	18,310	177.4	76,800	79	126	9.9	145
155	105.5	150.0	6,460	62.6	33,000	15.2	660	6.3	3,300	563.1	19,030	184.3	80,400	72	123	9.2	155
165	106.0	(2)	(2)	(2)	(2)	164.7	7,130	69.1	36,400	577.8	19,700	190.8	83,800	67	119	100.0	165

[1] All wood harvested prior to this age not included in yield table.
[2] End of rotation.
[3] Periodic and mean annual growth given by 10-year intervals. Linear interpolation of height by 5-year periods produces slight anomalies which are avoided by the 10-year interval.

(Continued from page 27)

The 150-square-foot level permits the harvest of more trees late in the rotations, but the trees are slender by comparison with those produced at lower densities. This kind of management produces utility poles, small sawlogs, and small piling. Of course, not only the stand density but also the rotation age selected by the forest manager will have a powerful effect on the size of trees harvested in the final cut.

SUMMARY

This study presents a method of predicting the growth and yield of red pine in Minnesota. Growth is here considered synonymous with net periodic annual increment. Yield is the summation of any number of these annual increments. The yield may be harvested, it may be added to the growing stock, or it may be some of both.

Some 324 growth periods are available from 235 individual plots contained in 14 sets of permanent sample plots. Measurement intervals varied from 3 to 12 years; net annual increment and values for stand density and stand age as used in the analysis are average for the interval. A number of plots have been measured more than once, but each measurement period has been treated as a separate observation. Likewise a number of plots adjacent to one another in the same experiment have been treated as separate observations. This produces significant autocorrelation of residuals, the end result of which is that the calculated standard errors are not as high as those found in the population of red pine. The prediction equation itself is exposed to bias, but it is hoped that the bias is of little consequence.

Net periodic annual basal area increment was predicted from the independent variables of age, site index, and stand density. Intensity of cutting, number of trees per acre, cutting method, and variability of tree diameter were also considered. None of these appeared to add to the prediction of basal area growth after the effects of age, site index, and stand density were removed. While not important for the prediction of basal area growth, several of these rejected variables are of great importance in the control of size and quality of red pine trees.

Basal area growth tables were constructed by 10-year intervals for ages 30 to 160; for stand densities of 60, 90, 120, 150, and 180 square feet of basal area per acre; and for site indexes of 45, 50, 55, and 60. Total cubic-foot, cordwood, and board-foot growth tables were also prepared for the same range of ages, stand densities, and site indexes, except that board-foot growth figures were omitted at early ages.

Maximum basal area growth occurs at about 126 square feet of basal area per acre for the sites and ages studied. There is considerable experimental error involved, however, and deviations of 15 square feet on either side of this maximum are of little practical consequence. Maxima for cubic-foot, cordwood, and board-foot growth were found to vary by age and site and can be best determined by plotting growth over stand density for the various combinations of ages and sites.

Yield tables were constructed for thinning schedules in which the stand is cut back to densities of 90, 120, or 150 square feet of basal area each 10 years (except 5 years for young stands with the 150-square-foot density). The 120-square-foot schedule approximates present-day thinning practice in Minnesota. Differences in yields

between the three thinning schedules are not great; the major advantage of one over the other is in the control of size and radial growth rates of trees. Methods used to prepare the growth and yield tables are presented in the appendix.

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APPENDIX

When this study was begun, a prediction scheme was sought that would be flexible enough to describe the growth and yield of red pine under the wide range of silvicultural options that are or could be exercised in Minnesota. These might include cutting cycles of different or irregular lengths; stand densities of different or varying basal areas at the beginning of those cutting cycles; and stand ages that may terminate in short or long rotations.

Several key considerations in the development of the scheme were (1) that basal area, because of its long and satisfactory use in Minnesota, continue to be used as a measure of stand density; (2) that growth also be predicted in basal area so that annual increments could be added to stand density—this would permit stand density to be traced through short or long cutting cycles; (3) that growth and yield be given not only for basal area but for cubic-foot, cordwood, and board-foot volumes as well. These considerations gave rise to the basal area growth equations, stand volume equations, volume growth tables, and yield summations.

The chief weakness of the approach developed here is that it does not provide for an estimate of the errors associated with volume growth (tables 4, 5, and 6) or with yield (tables 7, 8, 9, and 10). Only the basal area growth equation has a sampling error associated with it, and this is biased because of autocorrelation. Of course, limitations in the underlying experimental data also place restrictions on the range and accuracy of inference that could be made.

An advantage of the approach is that it permits the forest manager to synthesize and study a number of different silvicultural options that could be exercised in stands of red pine. For example, in this paper growth tables were made for 10-year intervals, but they could have been made for other intervals as well. Likewise, for the yield tables, cutting cycles were assumed to be 5 or 10 years, but they could just as well have been for different or irregular lengths of time. A second advantage of the approach is that it permits the mensurationist to study independently some of the elements that go into stand growth—the basal area and height components, the site index contribution, and the stand volume estimators.

Fitting the Regression Surface

The main statistical problem of the study was to relate the independent variables of age, site, and stand density to periodic net annual basal area growth, using multiple regression techniques. Originally it was thought that the work would be done by desk computer, hence great care was exercised in the selection of independent variables. The analysis was actually done with an IBM 650 drum computer.

Basal area growth in relation to age appears curvilinear, and for the range of data available a second-degree polynomial was postulated. Had growth data been available at ages of less than 25, a third-degree or higher term would have been needed (see fig. 3). Growth in relation to stand density also appeared curvilinear and a second-degree equation was postulated (see fig. 6). A linear relationship was proposed for the relation between basal area growth and site index. Only one interaction term, stand density times age, was tried.

Thus the proposed growth equation was as follows:

$$Y=A+B_1X_1+B_2X_1^2+B_3X_2+B_4X_2^2+B_5X_3+B_6X_1X_2$$

where Y=periodic net annual basal area increment
X_1=basal area in square feet per acre
X_2=average stand age in years
X_3=site index

All variables made a significant contribution (at the 1-percent level) to the prediction equation except the interaction term. The final regression equation, adjusted for the deleted variables, is given on page 45. The standard error of estimate associated with the regression equation is given later in the section "Reliability of Growth Predictions."

Stand Volume Equations

Stand volume equations provide a direct estimate of volume (V) from some measurable stand characteristics, usually the product of basal area and height (BH). Cordwood and cubic-foot stand volume equations used in this growth and yield study were based on the composite tree volume table of Gevorkiantz and Olsen (1955). The board-foot equation was based on a sawtimber table by Brown and Gevorkiantz (1934). The development of the stand volume equations is given elsewhere.[9]

[9] Buckman, Robert E. Development and use of three stand volume equations in Minnesota. Jour. Forestry 59: 573–575. 1961.

These equations, derived both by ratio and standard least squares regression techniques, are summarized below.

Ratio method	*Equation*	*Standard error (percent)*
Cubic feet	$V=.4085\ B{\cdot}H$	5.42
Cordwood	$V=.003958\ B{\cdot}H$	9.05
Board feet	$V=2.084\ B{\cdot}H$	9.02
Standard least squares method		
Cubic feet	$V=15.03+.4062\ B{\cdot}H$	5.45
Cordwood	$V=-.807+.004133\ B{\cdot}H$	8.77
Board feet	$V=-1707+2.283\ B{\cdot}H$	8.38

The equations prepared by the ratio method are used in this study although the equations by least squares could be used for many purposes.[10] Utilization standards for these equations are summarized as footnotes in the appropriate volume growth tables (table 4, cubic feet; table 5, cordwood; table 6, board feet).

The regression equations given above convert $B{\cdot}H$ to stand volume in cubic feet, cordwood, or board feet. For purposes of this paper the same equation is used for all combinations of stand density and sites from age 25 through 165. This presents no serious problem for total cubic-foot estimates of volume because the underlying volume table (Gevorkiantz and Olsen, 1955) demonstrates that a constant form can be assumed for all but the shortest of trees. Using the same equation throughout most of the life of the stand for either cordwood or board feet is an oversimplification, however, and requires further study.

Construction of Volume Growth Tables

The stand volume equations given previously are of the form:

$$V=kBH$$

where V=volume

k=a constant representing average form for trees in the stand

B=basal area per acre

H=average height of dominant and codominant trees

Volume growth is made up of changes in basal area and height. If d represents a change or increment, then the initial volume plus growth can be written:

$$V+dV=k(B+dB)(H+dH) \qquad (1)$$

Expanding and subtracting the initial volume gives:

$$dV=k(dB{\cdot}H+dH{\cdot}B+dB{\cdot}dH) \qquad (2)$$

This relationship, discussed by Wiedemann [11] is helpful in under-

[10] Except for the preparation of the growth tables where a relatively large negative or positive Y-axis intercept may greatly distort growth for such short periods of time as one year. This does not occur with the ratio equations, which must pass through the origin.

[11] Wiedemann, Eilhard. Ertragskundliche und Waldbauliche Grundlagen der Forstwirtschaft. Teil I. Das Wachstum des Einzelstammes und des Gleichaltrigen Reinbestandes unter dem Einfluss von Standort und Bestandespflege. [Silvicultural and growth studies as bases of forestry. Pt. 1. The growth of individual trees and even-aged stands as affected by site and tending.] J. D. Sauerlander's Verlag, Frankfurt A.M. 128 pp., illus. 1950.

standing how stands of timber grow. For example, if height growth dH is rapid and if stand density B is high, height growth makes an important contribution to volume growth. This component of growth can contribute up to half the volume increase in dense young stands, and is the principal reason why basal area growth is not directly proportional to volume growth. The simple practical explanation for this is that height growth is being made on more stems in stands of higher density. The last term in the equation is difficult to visualize, but it is the volume increment contributed by new growth in both height and basal area. This component of growth is usually small, and when basal area and height growth slow down it can be ignored altogether.

Equation (2) is used to develop the volume growth tables. The values needed to satisfy the equation are readily available: basal area growth dB is given in table 3; height H and height growth dH can be found in table 2; B will be arbitrarily chosen at intervals of 60, 90, 120, 150, and 180 square feet of basal area per acre. And k comes from the stand volume equations given previously. With these values at hand, volume growth tables (tables 4, 5, and 6) can be constructed.

Construction of Yield Tables

Yield has been defined as the summation of a number of net growth increments. The main problem of obtaining yields reduces itself to one of summing basal area growth increments. Basal area yields can then be converted to volume yields with the aid of height tables (table 2) and stand volume equations.

Summation of Growth Increments

Basal area growth increments can be summed by (1) differential equations, (2) repeated solution of the basal area growth function given on page 15, or (3) approximations from the basal area growth table itself (table 3).

The summation of basal area growth increments by differential equation can be briefly illustrated. If a particular site is chosen and stand density is set at some level of basal area stocking, then the stand density (X_1) and site index (X_3) terms in the growth equation (p. 15) reduce to constants. Further, growth can be defined as a change in volume with respect to time. This entire relationship can be written:

$$Y=\frac{dV}{dX_2}=C_1-.076958X_2+.00022741X_2^2$$

where d is read as "a change in," V is yield, Y is net basal area growth per acre per year, X_2 is age, and C_1 is a constant containing the a, X_1, and X_3 terms of the growth equation. Since this equation contains a derivative $\frac{dV}{dX_2}$, it is by definition a differential equation.[12]

[12] In a derivative, the increment of the independent variable dX_2 is assumed to be very small. The increment for age (X_2) is 1 year, which is sufficiently small to produce errors of no consequence.

Integration of the relationship gives a yield function

$$Y = C_1X_2 - .038479X_2^2 + .00007580X_2^3 + C_2$$

with C_2 the constant of integration to be determined by substitution of some known value in the equation.

As a specific example, suppose that a stand is to be managed at a density of 120 square feet of basal area per acre, with management beginning at age 25. The land is capable of supporting site-index-50 red pine. For the moment let us assume a cutting cycle of 1 year. What yields can be expected?

The conditions have been given to evaluate the constant C_1 and the constant of integration C_2. Substitution in the growth equation of stand density 120 (X_1) and site index 50 (X_3) gives a C_1 of 7.490. Further, at age 25 the stand has a density of 120 square feet of basal area. Substitution of this value in the yield function gives a constant of integration of −44.385. Thus, the yield equation for this schedule of management is

$$Y = -44.385 + 7.490X_2 - .038479X_2^2 + .00007580X_2^3$$

Basal area yields between any two ages can be found by $Y = Yb - Ya$ where b and a are the terminal and beginning ages, respectively. Between ages 25 and 165, for example, 364.4 square feet of basal area would have been produced under the conditions given above.

This is a very brief development of the most elementary of differential equations. The differential equation illustrated assumes cutting cycles of a length that does not change stand density appreciably. It would be far more useful to have a differential equation containing the two independent variables of age and stand density, but to investigate these possibilities is beyond the scope of this paper. Perhaps it cannot be done at all.

The second method of summing growth increments is by a repeated solution of the basal area growth equation given on page 15. This can be a tedious process. However, if one uses a systematic form of tabulation, and takes advantage of the terms that become constants, a 10-year summation of growth increments can be done with a desk computer in about 10 minutes, depending on the skill of the operator. Electronic computers could easily be used for this job; in fact electronic computers would be essential if the growth equation were very much more complex.

Essentially this method of summing growth increments involves solving of the basal area growth equation for a particular site, age, and stand density. Basal area growth is then added back into stand density, one year is added to age, and the equation is solved again. The results of this process are illustrated in the tabulation on page 48 for site index 50 red pine beginning at age 25 with 120 square feet of basal area.

The yield tables given earlier (tables 7, 8, 9, and 10) were developed by the repeated solution of the basal area growth equation. The 5- or 10-year basal area growth is shown in column 7 of the yield tables. (Note that this column shows basal area cut, but basal area cut and basal area growth are identical.)

Age	Stand density	Basal area growth	Age	Stand density	Basal area growth
25	120.0	5.7	31	153.1	5.2
26	125.7	5.6	32	158.3	5.1
27	131.3	5.6	33	163.4	5.0
28	136.9	5.5	34	168.4	4.8
29	142.4	5.4	35	173.2	
30	147.8	5.3			

The third method of summing basal area growth increment is to develop approximations from the basal area growth table itself (table 3). In effect, these approximations are capitalizing on the relative flatness of short segments of the growth surface. A comparison of 10-year basal area increments by repeated solution of the growth function, by differential equations, and by approximations from the growth tables is shown in table 11.

It may be noted that the differential equations and growth table approximations underestimate basal area increments at low densities and overestimate at high densities when compared with increments summed by repeated solution of the growth function. The more distant the summations from the top of the parabola representing stand density (fig. 8), the less exact the comparisons. For 90-, 120-, and 150-square-foot thinning schedules, with early 5-year cutting cycles, the growth tables generally provide acceptable approximations of basal area growth. The natural variability in the growth of red pine stands probably exceeds errors introduced from growth-table approximations by severalfold.

TABLE 11.—*A comparison of 10-year basal area growth increments, by three methods of summation (site index 50 red pine)*

Stand density	Age	Increments as summed by—: (1) Repeated solution of basal area growth equation	(2) Differential equation: Predicted basal area growth	(2) Differential equation: Difference from (1)	(3) Growth table approximation: Predicted basal area growth	(3) Growth table approximation: Difference from (1)
	Years	*Sq. ft.*	*Sq. ft.*	*Percent*	*Sq. ft.*	*Percent*
60	25–35	50.8	46.8	−7.9	46.8	−7.9
	95–105	15.0	13.7	−8.7	13.6	−9.3
90	25–35	53.7	51.8	−3.5	51.8	−3.5
	95–105	19.6	18.6	−5.1	18.6	−5.1
120	25–35	53.4	53.9	.9	53.8	.7
	95–105	20.7	20.7	.0	20.6	−.5
150	[1]25–30	26.9 } 52.1	27.3 } 53.0	1.7	52.9	1.5
	30–35	25.2	25.7			
	95–105	18.9	19.8	4.8	19.8	4.8

[1] Two 5-year summations are used for this density and age to prevent the stand from growing to excessively high density (see p. 26).

Format of Yield Tables

The basic format for yield tables 7, 8, 9, and 10 is the same. The following tabulation shows column numbers used in the tables and the source of information for each.

Column number	*Source of information*
1. Age	Specified by forest manager, for these tables ages 25, 35, 45, ...
2. Height of dominants and codominants.	Read from table 2 or from height curves (fig. 4), for appropriate age and site.
3. Basal area at beginning of each cutting cycle.	Specified by forest manager, in this case 90, 120, or 150 square feet of basal area per acre at beginning of each cutting cycle.
4, 5, 6. Cubic feet, cordwood, and board feet in residual stand.	Obtained from product of basal area (col. 3) × height (col. 2) × appropriate stand volume equation coefficient.
7. Basal area cut	See appendix section "Summation of Growth Increments." For this problem, basal area cut (and growth) were obtained by a repeated solution of basal area growth function (p. 15).
8, 9, 10. Cubic feet, cordwood, and board feet cut.	Obtained from product of basal area cut (col. 7) × height (col. 2) × appropriate stand volume equation coefficient.
11, 12, 13, 14. Cumulative total yield of basal area, cubic feet, cordwood, and board feet.	Obtained by cumulative addition of increases in residual stand plus volume of cut.
15. Periodic annual increment.	10-year increase in cubic-foot yield (col. 12) divided by number of years in cutting cycle.
16. Mean annual increment.	Cumulative cubic-foot yield up to that date (col. 12) divided by total years elapsed (col. 1).
17. Proportion of stand basal area cut.	Column 7 divided by sum of columns 3 and 7.

Reliability of Growth Predictions

Confidence in the Basal Area Growth Surface

The standard error of estimate, $S_{y\cdot 12\cdots k}$, a measure of scatter of the individual observations around the regression surface (see fig. 7), is 0.786 square feet of basal area growth per acre per year. This is 27.7 percent of $\bar{y}$. The regression surface accounts for 69.5 percent of the total variation of y.

The standard error of estimate of an individual predicted growth rate, $S_{\bar{y}}$, is 0.0437 square feet of basal area growth per acre per year at the mean of the independent variables. This is 1.5 percent of $\bar{y}$.

Autocorrelation of Growth Residuals

As was pointed out in the section "Available Plot Records," p. 3, successive measurements on the same plot and concurrent measurements on adjacent plots in the same experiment were treated as separate observations in the development of the basal area growth prediction equation (p. 15). This causes serial (or auto) correlation of error residuals. Tests are available to determine the presence or absence of autocorrelation, but if such a correlation is found appropriate adjustments are not well defined.

The residuals, or the amount of growth remaining after the effects of age, site, and stand density had been removed, were calculated. Some 308 of the 324 observations came from experiments that contained closely grouped plots. The autocorrelation of residuals is +0.534 and is significantly different from zero.

Likewise, some 145 out of the 324 observations were obtained from two or more successive measurements on each of 55 plots. The coefficient of autocorrelation between residuals is +0.282 and is also significantly different from zero.

What does autocorrelation mean? First of all the prediction equation itself is exposed to bias that may or may not be serious. The chief importance of autocorrelation is that the error term is higher than actually calculated, or conversely, there are fewer degrees of freedom available than the number of observations would indicate. This could mean that some of the regression coefficients might turn out to be nonsignificant. So far as the prediction equation developed earlier is concerned, however, the overall relationship and all of the regression coefficients would still be significant at the 1-percent level with but one-tenth the number of degrees of freedom.

U.S. GOVERNMENT PRINTING OFFICE: 1962 O—641730

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