Technical Factors Influencing Investment in Crop Sowing Machinery

Ross Kingwell
Western Australian Department of Agriculture
Currently seconded to the Department of Agricultural Economics
University of Western Australia
39th Annual AAES Conference, Feb 14-16, Perth.
Abstract

Besides the purchase of additional land, the purchase of cropping machinery is perhaps the most important and expensive decision a farmer in the grains industry may make. This paper shows how various on-farm technical factors can influence levels of investment in crop sowing machinery. The paper examines the effect on investment in crop sowing machinery of discontinuities in sowing opportunities, varietal portfolios, soil type diversity and tillage technology. For the case of risk neutral management a simple model of profit from crop production is used to illustrate how these factors influence investment in crop sowing machinery. Work in progress is described that illustrates the model using farm survey data from the eastern wheatbelt of Western Australia.

Keywords: investment, farm machinery, uncertainty

Introduction

Besides the purchase of additional land, the purchase of cropping machinery is perhaps the most important and expensive decision a farmer in the grains industry may make. As Malcolm (1994) observes: "The keys to continuing to be a farmer is to get the big decisions on land purchase, machinery investment and resource improvement right;" (p.19) The nature and timing of machinery investment can crucially affect farm viability. In a survey of 1685 Western Australian wheatbelt farmers Ripley and Kingwell (1984) and Kingwell (1985) found that many farmers who were classed as unable to service their debts had in the last five years procured additional land, increased the size of their cropping operations and purchased more cropping machinery. Since making their purchases of additional land and machinery, poor seasons and adverse cost-price movements had worsened their financial situations to the point where their farm viability was questionable. In hindsight either less expansion or a more conservative cropping strategy with less investment in cropping machinery would have been a preferable more profitable strategy for these farmers.

In making decisions about machinery purchase often farmers rely on a panoply of advice and information from family and friends, accountants, consultants, machinery dealers and occasionally, decision support software. Agricultural economics researchers, although rarely direct participants in farmers' cropping machinery investment decisions, nonetheless potentially can facilitate these decisions. Economists have developed concepts and analytical tools that facilitate investment decisions. Occasionally these concepts are implicitly or explicitly incorporated in some of the advice and information farmers receive about machinery purchases.

This paper adds to the knowledge of agricultural economists about appropriate cropping machinery investment by showing how various on-farm technical factors can influence levels of investment in crop sowing machinery. The first section of the paper reports on some of the agricultural engineering and economics literature on investment in crop sowing machinery. The second section introduces a simple model of profit from crop production. This model is used to illustrate how several technical factors can influence investment in crop sowing machinery for the case of risk neutral management.
The particular technical factors examined are:
(i) discontinuities in sowing opportunities,
(ii) varietal portfolios,
(iii) soil type diversity and,
(iv) tillage technology.

The final section of this paper describes work in progress that seeks to illustrate the model using farm survey data from the eastern wheatbelt of Western Australia.

Investment in Cropping Machinery

In the capital intensive grain production systems of many countries machinery selection is an important and complex issue. Several ingredients of the complexity have been the subject of studies by agricultural economists, particularly in the late 1970s and early 1980s when interest in theoretical and applied research regarding machinery investment appeared to peak.

The issue that has attracted most attention in the literature on machinery selection has been the impact of timeliness costs on machinery use and investment (Van Kampen, 1971; Tulu et al., 1974; Hughes and Holtman, 1976; Danok et al., 1980; Edwards and Boehlje, 1980; Whan and Hammer, 1985; Wetzstein et al., 1990). In many grain growing regions timeliness costs arise through a combination of edaphic, weather and crop physiology effects. Uncertainty over weather events causes a variation in sowing and harvesting opportunities and resultant crop areas and yields. Soil conditions can deteriorate to delay or prevent crop sowing (eg waterlogging). Weather conditions can spoil crops not yet harvested and crops whose sowing or harvest is delayed can suffer yield loss.

Although timeliness costs have received the most attention in the literature, other influences upon investment decisions such as taxation (Reid and Bradford, 1983), the lumpiness of investment decisions (Danok et al., 1980), changes in technologies (Stoneham and Ockwell, 1981; Epplin et al., 1982), investment allowances (Vanzetti and Quiggin, 1985) and weather-year cropping tactics (Bathgate, 1993) have all received some attention. No model reported in the literature claims to examine all factors likely to affect machinery selection. The objective of most studies has been to identify the optimal set or level of investment in cropping machinery, usually given *ceteris paribus* assumptions about other factors known to affect machinery investment (eg McIsaac and Lovering, 1976; Danok et al., 1978). In this respect this paper is no different insofar as several factors known to influence investment in cropping machinery are overlooked.

The techniques employed to examine cropping machinery investment have been simulation (Sorenson and Gilheany, 1970; Van Kampen, 1971; Donaldson; 1975; Edwards and Boehlje, 1980; Epplin et al., 1982; Whan and Hammer, 1985; Wetzstein et al., 1990), a variety of programming methods (Boisvert, 1976; McCarl et al., 1977; Danok et al., 1980; Reid and Bradford, 1987; Bathgate, 1993) and econometric approaches (Vanzetti and Quiggin, 1985; Cooper, 1994). These techniques have been applied in normative and positive fashion to the study of machinery investment. This paper departs from these techniques by employing a simple algebraic model of decision choice. Although this model can be altered to include risk aversion, for simplicity's sake only the risk-neutral case is presented.
The Basic Model

A simple model of profit from crop production in weather-year \(i\) is as follows:

\[
\pi_i = Q_i \bar{p} - \bar{f} \bar{S} - \bar{g} - g
\]

where \(\bar{p}\) is the fixed or certain price of the crop,
\(\bar{f}\) is the certain production costs per hectare
\(\bar{S}\) is the fixed area in hectares to be sown to the crop
\(\bar{g}\) is the fixed overhead costs associated with the cropping programme
\(g\) is the opportunity cost of investment in seeding gear.

\(Q_i\) is the production of grain (tonnes) from \(\bar{S}\) hectares of crop sown in weather year \(i\).

In equation 1, both \(Q_i\) and \(g\) are functions of the seeding gear's work rate \((R)\) which is the average number of hectares sown each day during the cropping programme. When there are no discontinuities in crop sowing, crop production in year \(i\) can be represented by \(Q_i\)

\[
= \sum_{x=s}^{l} Y_{tx} R
\]

where \(Y_{tx}\) is the yield on day \(x\) in the set of working days \(\{s, \ldots, l\}\) in weather-year \(i\) where \(s\) is the starting day for crop sowing, \(l\) is the last day of crop sowing and all elements in the set of working days are consecutive days. The number of elements \((N)\) in the set of working days is determined by \(R\) (hectares per day) and \(S\), the size of the cropping programme (hectares): \(N = S / R\). For a given \(S\), changes to \(R\) effect \(N\) and \(Q_i\).

In the region to which this model is later applied, most empirical and simulation studies of crop yield, given continuous sowing opportunities, identify crop yield in year \(i\) as a linear function of the day of sowing, as shown in Figure 1.

**Figure 1: Typical wheat yields according to day of sowing in the eastern wheatbelt of Western Australia**

That is,

\[
Y_{xi|cont} = f(x) = Y_{xi} + d_x
\]

.......

\(Y_{xi}\) is the production of grain (tonnes) from \(S\) hectares of crop sown in weather year \(i\).
where \( Y_{xt\{con\}} \) is crop yield on calendar day \( x \) in weather-year \( i \) given continuous sowing opportunities. \( Y_{si} \) is the first day of sowing in weather-year \( i \) and \( d_i \) is the rate of yield decline per calendar day delay in crop sowing in weather-year \( i \) and \( d_i < 0 \).

When sowing opportunities are continuous, crop production in year \( i \) can be represented as a sum of an arithmetic series:

\[
Q_i = R \left( \frac{N}{2R} (2Y_{si} + \left( \frac{N}{R} - 1 \right) d_i) \right) \quad \ldots \ldots (3)
\]

Hence, across weather years \( E(Q) \) based on equation (3) is:

\[
E(Q) = \bar{S} \left( E(Y_{si}) + \frac{1}{2} \left( \frac{N}{R} - 1 \right) E(d) \right) \quad \ldots \ldots (4)
\]

Having described \( Q_i \) in equation 1, we now consider \( g \), the opportunity cost of investment in seeding gear which is also a function of \( R \). Currently data is being collected for an empirical estimation of \( g = f(R) \). To facilitate exposition of the model we assume for now a simple linear relationship; \( g = a + bR \). This assumes the marginal cost of work rate \( b \) is constant, so in effect the opportunity cost of acquiring an additional unit of seeding capacity is the same across different sizes of seeding gear. Note that \( g \) could be modified to include the effects of marginal tax rates and investment allowances (see Vanzetti and Quigg, 1985).

For a risk neutral decision-maker with a fixed size of cropping programme \( \bar{S} \), the decision problem is to discover the optimal investment in \( R \) such that expected profit \( (\pi) \) is maximized, i.e.

\[
\text{Max } E(\pi) = E(Q - P \bar{S} - f - a - bR) \quad \ldots \ldots (5)
\]

The first-order condition of equation (5) is that

\[
\frac{\partial E(\pi)}{\partial R} = \bar{P} - \frac{\partial E(Q)}{\partial R} - h = 0
\]

and after substituting into equation (5) \( E(Q) \) from equation (4) and the expression \( g = f(R) \) the first-order condition for optimality is:

\[
\frac{\partial E(\pi)}{\partial R} = -\frac{1}{2} \bar{P} \bar{E}(d) \bar{S} \bar{R}^2 - b = 0
\]

\[
\bar{R}^2 = -\frac{\bar{P} \bar{E}(d) \bar{S}^2}{2b}
\]

Remembering that \( E(d) < 0 \),

\[
\bar{R}_{\text{opt}} = \bar{S} \sqrt{\bar{P} \bar{E}(d)} \frac{\bar{S}^2}{2b} \quad \ldots \ldots (6)
\]

In equation (6), for the case of risk neutrality when sowing opportunity is continuous, the optimal work rate is a direct function of the size of cropping programme \( \bar{S} \), the price of the crop \( \bar{P} \), the marginal cost of the machinery work rate \( b \) and the expected marginal
cost (as forgone yield) of late-sowing (E(d)). Increases in the size of the cropping programme or the price of the crop necessitate an increase in $R_{opt}$. Increases in the marginal cost of the machinery work rate or decreases in the expected marginal cost of late-sowing (E(d) is less negative) cause a decrease in $R_{opt}$.

The outcome of equation (6) can be illustrated with a numerical example. Suppose $\bar{S} = 1000$ hectares, $\bar{P} = $125 per tonne, $b = $250 ha per day and $E(|d|) = 15$ kg per ha per day; then $R_{opt} = 61$ ha per day. If the wheat price was 20 per cent greater then $R_{opt}$ would be 67 ha per day. If the size of the cropping programme was 20 per cent less then $R_{opt}$ would be 49 ha per day.

**Technical Factors affecting Investment in Crop Sowing Machinery**

*Discontinuities in the Opportunity to Sow Crops*

In dryland environments the sowing of crops depends on rainfall events that signal the start of the growing season. These rainfall events provide opportunities for tillage of the soil and control of germinating weeds. However, in practice rarely are rainfall patterns of sufficient regularity to provide continuous opportunities for sowing of all crops. Sometimes the amount and pattern of rainfall allows only a few days of crop sowing before soil profiles become so wet that paddocks cannot be trafficked by farm machinery, so a delay in crop sowing occurs. Conversely, rainfall can be inadequate, only permitting a few days of crop sowing before soil profiles become dry or hard, again prohibiting adequate seed bed preparation (Wetzstein et al., 1990).

Such discontinuities in sowing opportunities are typical of dryland agriculture and complicate a farmer’s decision about investment in crop sowing machinery. For example, investing in larger gear will enable more crop to be sown sooner, resulting often in higher yields but at cost of a significant investment in machinery. However, a farmer may be prepared to invest in smaller and therefore less expensive gear albeit at cost of lower yields because the farmer experiences more delays in seeding or seeding takes longer because of the lesser work rate of this machinery.

It is worth noting that discontinuities in sowing opportunities may also arise, not just from weather events, but also from machinery breakdown. If crop sowing machinery is unreliable, by virtue of its age or the inexperience of its users, then discontinuities in sowing opportunities are introduced.

The impact of discontinuities in sowing opportunities is to alter the set of feasible days for crop sowing. For a given investment in crop sowing machinery ($\bar{R}$) and size of cropping programme ($\bar{S}$), the effect of discontinuities in crop sowing opportunities is to shift leftwards the production ($Q$) distribution and increase its variance. The increase in variance is due to areas of crop being sown later than would otherwise be the case in the absence of the discontinuities. Crops sown on these later dates typically are lower yielding and cause overall production to display a lower mean and greater variance. That is, $E[Q_d] > E[Q_s]$ and $\text{Var}[Q_d] < \text{Var}[Q_s]$, where $Q_d$ refers to production when crop sowing is continuous and $Q_s$ refers to production when crop sowing is discontinuous. The impact of these discontinuities on the optimal level of investment in seeding gear ($R_{opt}$), given a fixed size of cropping programme, fixed costs of production and certain crop prices is examined in the following section.
In the presence of discontinuities in crop sowing opportunities, profit from crop production in weather-year $i$ is:

$$\pi = \pi_i = Q_{\text{e},i} \bar{p} - \bar{r} \bar{S} - \bar{r} - g$$

where $\bar{p}$, $\bar{r}$, $\bar{S}$, $\bar{r}$ and $g$ are the same as in equation (1).

For a risk neutral decision-maker with a fixed size of cropping programme ($\bar{S}$), the decision problem is the same as equation (5) except that $E[Q]$ is replaced by $E[Q_{\text{e}}]$. The impact of discontinuities in crop sowing on crop production ($Q_{\text{e}}$) in year $i$ can be represented by

$$Q_i = \sum_{x=s}^{l} Y_x \cdot R$$

where $s$ is the starting day for crop sowing, $l$ is the last day of crop sowing and elements in the set of working days are not all consecutive days. An illustration of the effect of discontinuities in crop sowing on crop yields within a sowing program is given in Figure 2 in which yields are expressed as a function of the effective day of sowing rather than the calendar day.

**Figure 2: Typical wheat yields according to effective day of sowing in the eastern wheatbelt of Western Australia**

After considering the discontinuity possibilities in all weather-years expected crop production can be represented as a linear function of the effective day of sowing:

$$E(Y_{x|\text{disc}}) = f(x) = E(Y_x) + E(e) \cdot x$$

where $E(Y_{x|\text{disc}})$ is the expected yield on effective day $x$ of crop sowing given the likely presence of discontinuities in crop sowing opportunities. $E(Y_x)$ is the expected first day of sowing and $E(e)$ is the expected rate of yield decline per effective day of crop sowing. $E(Y_x)$ in equation (7) is the same as $E(Y_x)$ in equation (4) and $E(e) < 0$ and $E(e) < E(d)$ in equation (4).

As in equation (4), expected crop production, given discontinuous sowing opportunities, can be represented as a sum of an arithmetic series:

$$E(Q_{\text{e}}) = \frac{\bar{S}}{2} (2E(Y_x) + \frac{\bar{S}}{R} \cdot E(e)) \quad \text{and}$$
after substituting $E(Q_i)$ into the equivalent of equation (5) the first-order condition for optimality is:

$$
\frac{\partial E(\pi)}{\partial R} = -\frac{1}{2} \bar{p} E(e) S^2 R^{-2} - b = 0
$$

$$
R^2 = -\frac{p E(e) S^2}{2b}
$$

Remembering that $E(e) < E(d) < 0$.

$$
R_{\text{opt}} = \frac{S \sqrt{\frac{p E(e)}{2b}}}{15 \text{ kg per ha per day}}
$$

Comparing equations (8) and (6), in the case of risk neutrality when sowing opportunity is discontinuous, the optimal work rate ($R_{\text{opt}}$) is greater than that for risk neutrality when sowing opportunity is continuous. A practical inference is that if a farmer operated in an environment where discontinuities in sowing opportunities, either due to weather events or machinery breakdown, were common then investment in larger gear would be warranted.¹

Consider the numerical example given earlier where $S = 1000$ hectares, $\bar{p} = $125 per tonne, $b = $250 ha per day and $E(d) = 15$ kg per ha per day; then $R_{\text{opt}}$ was 61 ha per day. Now introduce $E(e) = 20$ kg per ha per day which causes $R_{\text{opt}}$ to increase to 71 ha per day. This difference between the values for $R_{\text{opt}}$ illustrates the importance of discontinuities in sowing opportunities.

Sometimes simple calculations are made to indicate appropriate technical sizes of sowing machinery. For example, if the farmer’s cropping program is $S$ hectares and the crop needs to be sown in $N$ calendar days then the work rate ($R$) of the machinery, it is often suggested, needs to be $S/N$. However, this approach underestimates machinery requirements when discontinuities in sowing opportunities are present because the number of effective sowing days in the period of $N$ calendar days will be less than $N$. Hence to achieve the sowing of $S$ hectares within $N$ calendar days will actually require a work rate ($R$) of the machinery greater than $S/N$.

**Varetal Portfolio**

Thus far, we have implicitly assumed that the farmer only grows a single variety. Now consider the case where a farmer has access to two varieties. If we assume the farmer is rational, preferring more yield to less, then varietal selection on each effective day of sowing will be according to whichever variety offers the higher expected yield on that day, assuming there are sufficient seed stocks of each variety and that changeover costs are negligible. It is known that the yields of varieties differ throughout the sowing period. For example, the 1993 Crop Variety Sowing Guide for Western Australia (WADA, 1992) comments that:

*The semi-dwarf wheat varieties such as Aroona, Eradu and Kulin lose yield at the rate of 25 to 30 kg/ha/day for sowings after mid-May, whereas taller*

¹ This assumes the frequency of breakdown is constant across size of gear.
varieties like Gamunya and Guha lose yield at 15 to 20 kg/ha/day for later sowings.

Crop production in year \( t \) can be represented by:

\[
Q_t = \sum_{x} Y_{t,x} \cdot R \quad \text{where} \quad Y_{t,x} = \text{Max}[Y_{t,1}, Y_{t,2}] \quad \text{and where}
\]

\( Y_{t,1} \) is the yield of variety 1 on day \( x \) in the set of effective working days \( \{s, \ldots, l\} \) in weather-year \( t \) where \( s \) is the starting day for sowing, \( l \) is the last effective day of sowing and where \( Y_{t,2} \) is the yield of the new variety, variety 2, on day \( x \) in the set of effective working days \( \{s, \ldots, l\} \) in weather-year \( t \). The number of elements (\( N \)) in the set \( \{s, \ldots, l\} \) is determined by \( R \) and \( S' \), where \( N = \frac{S}{R} \).

For the case of two varieties \( E(Q) \) can be represented as a weighted sum where each variety’s contribution to \( E(Q) \) depends on the size and duration of its yield superiority within the sowing programme. The expression for \( E(Q) \) becomes:

\[
E(Q) = R((\frac{1}{2}\phi S \cdot R^{-1}(2E(Y_{t,1}) + (\phi S \cdot R^{-1}1).E(e))) + R((\frac{1}{2}(1-\phi) S \cdot R^{-1}1(2E(Y_{t,2}) + (1-\phi) S \cdot R^{-1}1).E(f)))
\]

where \( \phi \) is the proportion of effective days in the set \( \{s, \ldots, l\} \) in which \( E(Y_{t,1}) > E(Y_{t,2}) \) and \( 0 < \phi < 1 \).

\( E(Y_{t,1}) \) is the expected yield of variety 1 on the first day of sowing on which \( E(Y_{t,1}) > E(Y_{t,2}) \),

\( E(Y_{t,2}) \) is the expected yield of variety 2 on the first day of sowing on which \( E(Y_{t,2}) > E(Y_{t,1}) \),

\( E(e) \) is the expected rate of yield decline per effective day of crop sowing in which \( E(Y_{t,1}) > E(Y_{t,2}) \) and

\( E(f) \) is the expected rate of yield decline per effective day of crop sowing in which \( E(Y_{t,2}) > E(Y_{t,1}) \).

For ease of exposition we assume that \( \phi \) is independent of \( R \). In practice, \( \phi \) is likely to be influenced by the size of \( R \), \( S' \) and the nature of the yield relativities of varieties 1 and 2.

For example, if the yield relativities are as depicted in Figure 3a then for a given \( S' \) increases in \( R \) could diminish \( \phi \). Conversely, if the yield relativities are as in Figure 3b then for a given \( S' \) increases in \( R \) could increase \( \phi \). Hence, strictly speaking \( \phi \) should be specified as a function of \( R \), \( S' \) and the nature of the yield relativities.

For a risk neutral decision-maker with a fixed size of cropping programme \( (S') \), the decision problem remains as equation (5). That is, to discover the optimal investment in \( R \) such that expected profit \( (\pi) \) is maximized.

From equation (9) \( \frac{\partial E(Q)}{\partial R} \) is derived and included in the first-order conditions of equation (5) to yield the following expression:

\[
\frac{\partial E(\pi)}{\partial R} = -\frac{1}{2} \bar{p} \cdot \phi^2 \cdot E(e) \cdot S'^2 \cdot R^{-2} - \frac{1}{2} \bar{p} \cdot (1-\phi)^2 \cdot E(f) \cdot S'^2 \cdot R^{-2} - b = 0
\]

Re-arranging gives.
\[-\frac{1}{2} \bar{p} \tilde{S}^2 (\phi^2 E(e) + (1-\phi)^2 E(f)) = b R^2 \]

\[-\bar{p} \tilde{S}^2 (\phi^2 E(e) + (1-\phi)^2 E(f))/2b = R^2 \]

\[R_{\text{opt}} = \sqrt{\frac{\bar{p}(\phi^2 E(e) + (1-\phi)^2 E(f))}{2b}} \quad \ldots (10)\]

Comparing equations (8) and (10) reveals that \(R_{\text{opt}}\) in the two variety case will be less than \(R_{\text{opt}}\) in the single variety case if \((\phi^2 E(e) + (1-\phi)^2 E(f)) > E(e)\). Conversely, if \((\phi^2 E(e) + (1-\phi)^2 E(f)) < E(e)\) then \(R_{\text{opt}}\) in the two variety case will be more than \(R_{\text{opt}}\) in the single variety case. Whether \(R_{\text{opt}}\) in the two variety case will be more or less than \(R_{\text{opt}}\) in the single variety case depends on the relativities of \(\phi, E(e)\) and \(E(f)\). Mostly \(R_{\text{opt}}\) in the two variety case would be less than \(R_{\text{opt}}\) in the single variety case. Only where low values of \(\phi\) are recorded and where \(E(e)\) is much greater than \(E(f)\) is it likely for \(R_{\text{opt}}\) in the two variety case to be more than \(R_{\text{opt}}\) in the single variety case. The outcomes of an increase or a decrease in \(R_{\text{opt}}\) are illustrated in Figures 3a and 3b.

**Figure 3a:** An increase in crop machinery investment due to a varietal portfolio

In Figure 3a, the proportion of the 15 day sowing period in which variety 2 has a higher expected yield \((E(Y_{x2}) > E(Y_{x1}))\) is \((1-\phi)\) or two-thirds, corresponding to the first 10 effective days of sowing. Thereafter variety 1 has the higher expected yield. So, in this example:

\[(\phi^2 E(e) + (1-\phi)^2 E(f)) < E(e) \quad \ldots (11)\]

In words, equation (11) says the rate of yield decline associated with crop sowing using the two varieties is more due to selection of the higher yielding variety 2, with its greater rate of yield decline in proportion \((1-\phi)\) of the crop sowing period. This leads to an increase in \(R_{\text{opt}}\).

In Figure 3b, the proportion of the 15 day sowing period in which variety 1 has a higher expected yield \((E(Y_{x1}) > E(Y_{x2}))\) is a third, or in other words \(\phi\) is a third. So, in this example:

\[(\phi^2 E(e) + (1-\phi)^2 E(f)) > E(e) \quad \ldots (12)\]
In words, equation (12) says the rate of yield decline associated with crop sowing using the two varieties is less due to $E(f) > E(e)$ and selection of variety 2 in proportion $(1-\phi)$ of the crop sowing period. This leads to a decrease in $R_{opt}$.

**Figure 3b: A decrease in crop machinery investment due to a varietal portfolio**

![Graph showing yield decrease with effective day of sowing](image)

We can further illustrate the importance of varietal portfolios with our numerical example given earlier where $\bar{S} = 1000$ hectares, $\bar{p} = 125$ per tonne, $b = 250$ ha per day and $E(1|f|) = 20$ kg per ha per day and $R_{opt}$ is 71 ha per day. Now introduce another variety with $E(1|f|) = 15$ kg per ha per day and $(1-\phi)$ equal to 0.25 which causes $R_{opt}$ to decrease to 55 ha per day. Conversely suppose the additional variety had its $E(1|f|) = 30$ kg per ha per day and $(1-\phi)$ equal to 0.9 which causes $R_{opt}$ to increase to 78 ha per day. This difference between the values for $R_{opt}$ illustrates the importance of yield relativities between varieties and their respective durations of yield superiority within the sowing programme.

**Soil Types**

Commonly on larger farms cropping occurs on more than one soil class. Physical differences between soils can affect the power requirements of crop sowing machinery. Hence, the nature and mix of soils to be cropped can influence investment in crop sowing machinery. To illustrate this the original simple model of profit from crop production in weather-year $t$ (see equation (1)) can be modified to represent two soil classes as follows:

$$\pi = Q_i \bar{p} - \bar{r} \bar{S} - \bar{r} \bar{S} - \bar{r} - g$$

where $\bar{p}$ is the fixed or certain price of the crop,

$\bar{r}$ and $\bar{r}$ are the certain production costs per hectare on soil classes 1 and 2,

$\bar{S}$ and $\bar{S}$ are the fixed areas in hectares to be sown to crop on soil classes 1 and 2

and $\bar{S} + \bar{S} = \bar{S}$ of equation (1).

$\bar{r}$ is the fixed overhead costs associated with the cropping programme

$g$ is the opportunity cost of investment in seeding gear.

$Q_i$ is the production of grain (tonnes) from $\bar{S}$ and $\bar{S}$ hectares of crop sown with machinery of work rates $R_1$ and $R_2$ on soil classes 1 and 2 in weather year $t$. 
The nature of the soil classes will be such that \( R_2 = h \cdot R_1 \), where \( h = 1 \) or \( h < 1 \) or \( h > 1 \). In words, the work rate of purchased machinery will either be the same or different across the soil classes. As in preceding analyses we assume opportunity cost of investment in seeding gear \( (g) \) is a simple linear function of work rate \((R)\) whereby \( g = a + bR_1 \) or \( g = a + bh \cdot R_2 \). We further assume that the farmer has access to a single variety.

Regarding the soil classes two cases can be considered. Firstly, imagine that the water-holding properties of the two soil classes differ (e.g. one of the soils is sandier) which enables, in some weather-years, the variety to be sown earlier on one of the soil classes. This situation is portrayed in Figure 4. Secondly, consider the case where the soil properties do not allow earlier sowing opportunities on one of the soils. In both cases there is a yield response to time of sowing that differs according to soil type. In the second case the yield responses according to time of sowing would similar to those in Figures 3a and 3b except that the responses would be for soil classes rather than varieties. The following exposition encompasses both cases.

![Figure 4: An example of yield response to time of sowing on two soil classes](image)

Whether or not an opportunity exists for earlier sowing on only one of the soil classes the decision problem for the risk-neutral manager remains as:

\[
\text{Max } E(\pi) = E[Q \cdot \bar{p} - \bar{r}_1 \bar{S}_1 - \bar{r}_2 \bar{S}_2 - \bar{r} - a - h \cdot R_1] \quad \text{(13a)}
\]

or

\[
\text{Max } E(\pi) = E[Q \cdot \bar{p} - \bar{r}_1 \bar{S}_1 - \bar{r}_2 \bar{S}_2 - \bar{r} - a - bh \cdot R_2] \quad \text{(13b)}
\]

s.t.

\[
\bar{S}_1 + \bar{S}_2 = \bar{S}
\]

The first-order conditions for equation (13a) are:

\[
\frac{\partial E(\pi)}{\partial R_1} = \bar{p} \cdot \frac{\partial E(Q)}{\partial R_1} - h = 0 \quad \text{(14a)}
\]

and

for equation (13b) they are:

\[
\frac{\partial E(\pi)}{\partial R_1} = \bar{p} \cdot \frac{\partial E(Q)}{\partial R_1} - bh = 0 \quad \text{(14b)}
\]
\[ \frac{\delta E(Q)}{\delta R_t} = p \quad \frac{\delta E(Q)}{\delta R_s} = -h = 0 \]  

\hspace{0.5cm} \ldots \ldots (14b)

We can derive expressions for either first-order condition by maintaining the assumption that the farmer is rational, preferring more yield to less. Hence, the selection of the soil class on which to sow on each effective day of sowing will be according to whichever soil class offers the higher expected yield on that day, assuming the travel costs between the soil classes are not greater than the net revenue benefits of switching soil classes.

Crop production in weather-year \( t \) can be represented by:

\[ Q_t = \sum_{s=1}^{t} Y_s * R \]  

\hspace{0.5cm} \text{where} \ Y_{s,t} = \text{Max}[Y_{s,t1}, Y_{s,t2}] \text{ and where}

\( Y_{s,t1} \) and \( Y_{s,t2} \) are the yields of the variety on soil class \( 1 \) and soil class \( 2 \) respectively on day \( v \) in the set of effective working days \{s, \ldots , l\} in weather-year \( t \) where \( s \) is the starting day for sowing, \( l \) is the last effective day of sowing. The number of elements \((N)\) in the set

\{v, \ldots , l\} is determined by \( R \) and \( s \), where \( N = (\frac{\bar{S}_1}{R_1} + \frac{\bar{S}_2}{hR_1}) \).

\[ E(Q_t) = (r_2 \gamma \bar{S}_1 h + r_2 \gamma \bar{S}_2)^{(2E(Y_{s,t1}) + (\frac{\phi h \bar{S}_1 - \bar{S}_2}{hR_1} y)} E(0) + \]

\[ (r_2 \phi \bar{S}_1 + r_2 \phi \bar{S}_2 h) (2E(Y_{s,t}) + (\frac{\phi h \bar{S}_1 - \bar{S}_2}{hR_1} y)} E(0) + \]

\[ (r_2 \psi \bar{S}_1 h + r_2 \psi \bar{S}_2) (2E(Y_{s,t2}) + (\frac{\phi h \bar{S}_1 - \bar{S}_2}{hR_1} y)} E(0) + \]

\[ (r_2 \gamma \bar{S}_1 h + r_2 \gamma \bar{S}_2 h) (2E(Y_{s,t}) + (\frac{\phi h \bar{S}_1 - \bar{S}_2}{hR_1} y)} E(0) + \]

\[ E(Y_{s,t}) = \text{the expected yield of the variety on soil class } 2 \text{ on the first day of sowing in the } \gamma \text{ proportion of effective days in the set } \{s, \ldots , l\} \text{ when sowing only on soil class } 2 \text{ is possible.} \]

\[ E(Y_{s,t1}) = \text{the expected yield of the variety on soil class } 1 \text{ on the first day of sowing on which either } E(Y_{s,t1}) > E(Y_{s,t2}) \text{ in the } \phi \text{ proportion of effective days in the set } \{s, \ldots , l\} \text{ or sowing area } \bar{S}_2 \text{ has been completed.} \]

\[ E(Y_{s,t2}) = \text{the expected yield of the variety on soil class } 2 \text{ on either the first day of sowing in the remaining proportion of effective days when } E(Y_{s,t2}) > E(Y_{s,t1}), \text{ and this proportion equals } \psi, \text{ or sowing area } \bar{S}_1 \text{ has been completed.} \]
$E(Y_{st1})$ is the expected yield of the variety on soil class 1 on the first day of sowing in the remaining proportion of effective days when $E(Y_{st2}) > E(Y_{st1})$, and this proportion equals $\bar{w}$, and when sowing area $\bar{S}_2$ has been completed.

$E(Y_{st2})$ is the expected yield of the variety on soil class 2 on the first day of sowing in the remaining proportion of effective days when $E(Y_{st1}) > E(Y_{st2})$, and this proportion equals $(1-\gamma-\psi-\phi-\bar{w})$, and when sowing area $\bar{S}_1$ has been completed.

$E(e)$ is the expected rate of yield decline per effective day of crop sowing in which $E(Y_{st1}) > E(Y_{st2})$ or sowing area $\bar{S}_2$ has been completed.

$E(f)$ is the expected rate of yield decline per effective day of crop sowing in which $E(Y_{st2}) > E(Y_{st1})$ or sowing area $\bar{S}_1$ has been completed.

and where $0 \leq \phi \leq 1$, $0 \leq \psi \leq 1$, $0 \leq \bar{w} \leq 1$ and $0 \leq \gamma \leq 1$. Again, for simplicity we assume that $\phi, \psi, \bar{w}$ and $\gamma$ are independent of $R$.

From equation (15) $\frac{\partial E(Q)}{\partial R_1}$ is derived and the following first-order expression is derived:

$$\frac{\partial E(Q)}{\partial R_1} = -\bar{p}(\bar{S}_1 + \bar{S}_2)h^{-1})Z^2E(f) + X^2E(e) 2bR_1^2 = 0$$

where $Z^2 = (\psi^2 + \gamma^2 + (1-\phi-\psi-\bar{w}-\gamma)^2$ and $X^2 = \phi^2 + \bar{w}^2$.

Re-arranging gives:

$$R_1^2 = \frac{-p(S_1 + S_2)h^{-1})Z^2E(f) + X^2E(e)}{2b}$$

$$R_{1\text{opt}} = (\bar{S}_1 + \bar{S}_2)h^{-1})\sqrt{\frac{-p(Z^2E(f) + X^2E(e))}{2b}} \quad \ldots \ldots (16)$$

As might be expected the expression for $R_{1\text{opt}}$ is more complicated for this case of two soils, with $R_2 = hR_1$ and where $h = 1$ or $h < 1$ or $h > 1$. If we assume the soil class in equation (8) is $S_1$ then $R_{1\text{opt}}$ in equation (16) can be more or less than the $R_{1\text{opt}}$ in equation (6) depending on particular values of parameters in equation (16). For example, the tendency of effects is for $R_{1\text{opt}}$ in equation (16) to be less (more) than $R_{1\text{opt}}$ in equation (6) if $h$ is greater (less) than 1. In other words, if the characteristics of the other soil ($S_2$) is such that it allows a greater (lesser) work rate for a given complement of seeding machinery then a lesser (greater) investment in work rate is required. Similar to the findings in the case of two varieties, an increase or decrease in investment in work rate is possible depending on the yield decline relativities on each soil class ($E(e)$ and $E(f)$) and the duration of the sowing period on each soil class. If for example, the rate of yield decline is less on $S_2$ ($E(f) > E(e)$) and the period when sowing only on soil class 2 is possible forms a large proportion of the overall period of crop sowing (i.e. $\gamma$ is large) then a decrease in investment in work rate is likely. Conversely, if $\gamma$ is very small and ($E(f) < E(e)$) then an increase in $R_{1\text{opt}}$ is possible. These examples illustrate how in practice numeric solutions would be necessary to determine the change in $R_{1\text{opt}}$.

As a further example suppose $S = 1000$ hectares, $\bar{p} = $125 per tonne, $b = $250 ha per day and $E(e) = 20$ kg per ha per day and $R_{1\text{opt}}$ is 71 ha per day. Then introduce $\bar{S}_1$ equal to 750 hectares and $\bar{S}_2$ equal to 250 hectares and with $E(e) = 20$ kg per ha per day and
E(t|f; \eta) = 15 \text{ kg per ha per day. Further suppose } (\gamma = 0.2, \phi = 0.5, \psi = 0.3 \text{ and } \omega = 0) \text{ and } h = 1.5 \text{ in } R_2 = hR_1. \text{ These parameter values result in a decrease in } R_{1\text{opt}} \text{ to } 47 \text{ ha per day. This difference between the values for } R_{\text{opt}} \text{ illustrates how the nature and mix of soils to be cropped can influence investment in crop sowing machinery.}

**Tillage Technologies**

Traditional crop sowing often involves tilling the soil, usually twice, to ensure adequate weed kill and tilth for the sowing of crops. However, herbicide technology has facilitated weed control and reduced the need for the repetitious working of the soil. Adopting herbicides and reduced tillage technology often enables a crop to be sown sooner (see Figure 5) and, for a given size of tillage gear, raises the effective work rate because fewer workings of the soil are required. The effect of tillage technologies on the optimal investment in crop sowing machinery can be illustrated by examining the case of a single variety and single soil class yet where areas \(S_1\) and \(S_2\) are sown using traditional and reduced tillage technology respectively.

![Figure 5: An example of yield response to type of tillage](image)

The derivation for establishing \(R_{1\text{opt}}\) in this case is similar to that as outlined in the earlier section dealing with two soil classes, except that two tillage technologies are considered; reduced and conventional tillage, \(E(e)\) equals \(E(f)\) and \(h \) is greater than 1 (i.e. \(R_2 > R_1\) meaning the work rate for reduced tillage is greater than that of conventional tillage). The expression for \(R_{1\text{opt}}\) in equation (16) can be re-expressed as:

\[
R_{1\text{opt}} = (S_1 + \bar{S}_2 h^{-1}) \sqrt{\frac{pY^2|e|}{2b}} \quad \text{(17)}
\]

where \(Y^2 = \gamma^2 + (1-\gamma)^2 = 1-2\gamma + 2\gamma^2\) and where \(\gamma\) is the proportion of effective days in the set \(\{s,\ldots,l\}\) when sowing only using reduced tillage is possible followed by continued sowing until area \(\bar{S}_2\) is completed and.

---

2 This assumes that among weed populations resistance to herbicides has not developed to a stage where there is little incentive for reduced tillage.
(1-\gamma) is the proportion of effective days in the set \{s, \ldots, l\} when area \overline{S}_1 \ is sown using conventional tillage.

R_{1\text{opt}} in equation (17) will be less than the R_{1\text{opt}} in equation (8) due to \gamma > 1 and \gamma > 0. The possibility of earlier sowing using reduced tillage technology and the greater work rate of machinery undertaking reduced tillage enable the optimal investment in work rate to diminish. A numerical example of equation (8) gave \text{R}_{\text{opt}} as 71 ha per day. In equation (17) let \overline{S}_1 equal to 750 hectares and \overline{S}_2 equal to 250 hectares and with \text{E(}\epsilon\text{)} = 20 kg per ha per day. Further suppose \gamma = 0.2 and \ h = 1.5 in \text{R}_2 = h.\text{R}_1. These parameter values result in a decrease in R_{1\text{opt}} to 52 ha per day. This difference between the values for \text{R}_{\text{opt}} illustrates how reduced tillage enables a lesser investment in work rate.

An application of the model: work in progress

To illustrate the practical importance of some of these technical factors on investment in work rate of sowing machinery, some of the models outlined in previous sections of this paper are being applied to a dryland cropping region of Western Australia. An indication of the relationship between investment in cropping gear (g), work rate (R) and cropping programme \overline{S} is being gained from a survey of farm machinery utilisation and cropping programmes in the region. In late December 1994 a mail questionnaire was sent to 300 farmers. Questionnaire returns are still being received and collated.

Data for the relationships \text{E(}\text{Y}_{\text{e}}|\text{cont}) = f(x) = \text{E(}\text{Y}_{\text{s}})+ \text{E(}\epsilon\text{)} x \text{ and E(}\text{Y}_{\text{d}}|\text{discont}) = f(x) = \text{E(}\text{Y}_{\text{s}})+ \text{E(}\epsilon\text{)} x \text{ are being derived from a wheat growth simulation model (Robinson, 1993) validated for the eastern wheatbelt of Western Australia. Typical results from the simulation model are shown in Figure 6. The simulation model is a daily time step water balance model of crop growth. Using daily weather data from the Merredin Research Station for the years 1912 to 1991 and assuming "varietal selection and crop management typical of current practice, yield estimates over the period have been generated.

The yields in Figure 6 are estimates of \text{Y}_{\text{e}} in each year over the period 1912 to 1991. The expected yield for the first feasible day of crop sowing (\text{E(}\text{Y}_{\text{s}})) in Figure 6 is 1314 kg/ha and \text{Var(}\text{Y}_{\text{s}}) is 625758. The wheat growth simulation model is also being used to derive estimates for \text{E(}\epsilon\text{)} and \text{E(}\epsilon\text{)}. Soil water balance conditions as described by the crop growth simulation model are being used to define when sowing opportunities are continuous or discontinuous. Weather-years can thus be classed as displaying conditions for continuous or discontinuous crop sowing. The estimates for \text{E(}\epsilon\text{)} are derived from weather-years classed as having no discontinuities in crop sowing opportunities whereas estimates for \text{E(}\epsilon\text{)} are derived from weather-years classed as having discontinuities in crop sowing opportunities. These estimates for \ g = f(R), \overline{S}, \text{E(}\epsilon\text{)}, \text{E(}\epsilon\text{)} and \text{E(}\epsilon\text{)} can be used to further and more realistically illustrate the model and help identify the relative potential importance of some technical factors that influence machinery investment.
Figure 6: Simulated Wheat Yields (kg/ha) at Merredin from 1912 to 1991
References


Malcolm, B. (1994) Managing farm risk: there may be less to it than is made of it. Paper presented to the conference on Risk Management in Australian Agriculture, June 15-16, University of New England, Armidale.


Ripley, J. and Kingwell, R. (1984) Farm indebtedness in wheat-growing areas of Western Australia: survey results. Western Australian Department of Agriculture, Baron-Hay Court, South Perth.


