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# Measurement Error in the Schaefer Production Model 

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# Measurement Error in the Schaefer Production Model 

Christopher Burns


#### Abstract

This paper contributes to the existing literature on the Schaefer Production Model by applying a relatively new method for reducing the bias induced by measurement error, known as Simulation Extrapolation (SIMEX). We test this new method on logbook data from the MidAtlantic surfclam fishery from 2001-2009. We show that the SIMEX estimator reduces bias in the parameter estimates for the biological parameters and biomass estimates. This contribution is important because fishery managers may only have logbook catch and effort data on a few time periods, which could lead to poor biomass estimates, and potentially leading to a state of overfishing.


Key words: commercial fisheries, measurement error

## 1 Introduction

Since the reauthorization of the Magnuson-Stevens Fishery Conservation Act in 2007 (M.S. Act), fishery managers have been given high standards to meet when determining future management plans. National Standards One and Two state that management plans must prevent overfishing while achieving the optimum yield for the fishery, and that new management measures be based on the best scientific information available (U.S. Department of Commerce, 2007). In order to achieve the objectives outlined in the M.S. Act, fishery managers must know what constitutes overfishing. This requires information on the biological parameters of fishery. Because biological data are often not available, fishery managers must make use of catch and effort data to make inference about the status of the biomass (stock). The most commonly used model is the Schaefer production model (Schaefer, 1954). This paper ${ }^{1}$ extends recent work on the Schaefer Production model (Zhang and Smith, 2011), attempting to deal with the estimation issues, such as measurement error, by using a Monte Carlo method known as SIMEX.

[^0]The Schaefer Production model is a bioeconomic model that links an economic model of production to a biological growth model for a fishery. The advantage to this model is that fishery managers can use logbook data obtained from active vessels in the fishery to estimate the biological parameters. The Schaefer production model is estimated as a two-stage model. In the first stage production function, harvest is modeled as a function of effort and the biomass. Typically a Cobb-Douglas functional form is assumed, though more recent work has looked at more flexible functional forms (Zhang and Smith, 2011). This first stage model cannot be directly estimated with the available data, since biomass is not observed. Instead a proxy variable, such as catch-per-unit effort (CPUE), is used in its place. This proxy variable is then substituted into the second stage growth model, and biological parameters for the fishery can be estimated. The problem with this method is that using the CPUE proxy contains measurement error, leading to inconsistent estimates of biological parameters in the second stage of the model. This paper examines how to mitigate the impact of measurement error, so that fishery managers can have more precise estimates of biological parameters.

Previous work on this model has highligted important considerations in the methods used to obtain the biological parameters. Punt (1992) shows that when estimating the biomass using catch and effort data, the method used to fit the model is much more important than the actual parametric form. Polacheck et al. (1993) note that there are three models which are widely used for fitting a dynamic biomass to observed data; the effort-averaging estimation method (Gulland, 1961; Fox, 1975), process-error estimators (Walters and Hilborn, 1976; Schnute, 1977) and observation-error estimators (Pella and Tomlinson, 1969: Butterworth and Andrew, 1984). Observation error estimators refer to appending error in the first stage production model, while process-error estimators refer to appending error to the second stage growth model. The effort-averaging estimation method refers to averaging effort from the vessel data. Each of these methods has potential drawbacks. Polacheck et al. (1993) examine each of these estimators, concluding that the effort-averaging estimator is biased and process-error estimators have high variability. They further conclude that the observation error method should provide the most precise parameter estimates, but that any method should be assessed with a Monte Carlo study before implementation.

More recently, Zhang and Smith (2011) proposed a more generalized production function, and use panel-data methods to obtain a consistent estimator for biological parameters. They note three
empirical problems with estimating the classic Schaefer model; 1) the biological dynamics have natural variation or process error; 2) the production function has stochastic shocks, which makes the inferred stock noisy; and 3) the fishing production function, usually Cobb-Douglas, has an extremely restrictive form. Without taking these three factors into account simultaneously, the resulting estimation of the classic Schaefer production model will lead to biased and inconsistent parameter estimates. This means that estimates of the growth parameters and the biomass estimates are not reliable. Given that the growth model is non-linear in the parameters, there is great potential for biased results. Uhler (1980) showed that parameter estimates from the Schaefer production model could be biased by up to $40 \%$.

This paper further extends the existing literature on the Schaefer production model by applying a relatively new method for reducing the bias induced by measurement error. This method, known as simulation extrapolation (SIMEX), is a Monte Carlo method for reducing bias first developed by Cook and Stefanski (1994). After estimating a more generalized Schaefer production model, we apply the SIMEX estimator to the second stage growth model. We show that this method can further improve the generalized Schaefer model, which can be particularly useful when a fishery manager has limited data. To test our method we use data from the Mid-Atlantic surfclam fishery for 2001-2009.

Using logbook data from the Mid-Atlantic surfclam fishery, we estimate three different models; the classic Schaefer model, the generalized Schaefer Production model and a SIMEX model. In order to apply the measurement error correction method known as SIMEX, we must also estimate the measurement error variance. To assess the bias in the estimates of each model, we use data on the biomass estimates for the Mid-Atlantic surfclam fishery, obtained from the Northeast Fisheries Science Center (Northeast Fisheries Science Center, 2010). Although the scientific estimates are known to contain some measurement error, we use these estimates as the truth when evaluating each model.

This has important policy implications for fishery managers. If fishery managers make decisions about setting the TAC based on biased estimates of the biomass, it can lead to reduced economic rent for harvesters and possible collapse of the fishery. Additionally, the efficiency gains from moving to an individual tradable quota fishery (ITQ), such as the Mid-Atlantic surfclam fishery, depend on correct setting of the TAC. Thus if fishery managers use biased estimates of the
biomass, the goals set in the reauthorized M.S. Act will not be met (U.S. Department of Commerce, 2007) and gains in economic efficiency will not be realized.

The rest of the paper will proceed as follows: Section 2 explains the methodology of the classic Schaefer and generalized Schaefer Production Model. In Section 3 we build a measurement error model and describe the SIMEX estimator. Section 4 describes the Mid-Atlantic surfclam logbook data. Section 5 reports results from all three models and compares the biomass estimates to the scientific estimates. Section 6 summarizes the findings of this paper and discusses future research.

## 2 Methodology

We begin by describing the two-stage classic Schaefer production model. In the first stage a production function is estimated, followed by the second stage estimation of a logistic growth model for the biomass.

### 2.1 Classic Schaefer Production Model

The classic Schaefer production model is

$$
\begin{equation*}
H_{t}=q E_{t} X_{t} \tag{1}
\end{equation*}
$$

where $X_{t}$ is the biomass in time $\mathrm{t}, H_{t}$ is harvest in time t , and $E_{t}$ is fishing effort in time t . The state equation or growth model is

$$
\begin{equation*}
X_{t+1}=X_{t}+r X_{t}\left(1-\frac{X_{t}}{K}\right)-H_{t} \tag{2}
\end{equation*}
$$

where the intrinsic growth rate $r$, carrying capacity $K$, and catchability coefficient $q$ are the biological parameters to be estimated. The second stage growth model expresses the biomass in the next period as the sum of the current period biomass and growth, minus the current period harvest. If biomass, harvest and effort information are all observed, this model can be estimated under regularity conditions Zhang and Smith, 2011. However, the fishery biomass is typically not observed, and is therefore a latent variable to the econometrician.

To estimate the above model when biomass is unknown a proxy such as catch-per-unit-effort (CPUE) is often used. Let $y_{t}$ denote CPUE, defined as $y_{t}=\frac{H_{t}}{E_{t}}$. From equation 2. CPUE is pro-
portional to the unobserved biomass, such that $X_{t}=\frac{y_{t}}{q}$. The standard approach to the Schaefer production model substitutes this proxy into equation 2 and appends an additive error term. The resulting second stage growth model is

$$
\begin{equation*}
y_{t+1}=(1+r) y_{t}-\frac{r}{q K} y_{t}^{2}-q H_{t}+\epsilon_{t} \tag{3}
\end{equation*}
$$

Under exogeneity conditions, $E\left(\epsilon_{t} \mid y_{t}\right)=0$, this equation can be estimated by ordinary least squares.

There are several drawbacks to this particular estimator. First, the production function assumes a very restrictive form, with constant returns to scale imposed. Second, the production function is usually assumed to have an error term that captures variability in the harvest from random events, such as weather. Thus when $y_{t}$ is substituted in equation 3 , the right-hand side of the equation has $y_{t}$ and $y_{t}^{2}$, both of which are measured with error. Uhler (1980) points out that if the stock is measured with significant noise due to the error in the production function, then the CPUE proxy for stock will lead to biased estimators. Additionally, the functional form of equation 3 assumes a logistic growth model. Many other functional forms for the growth model could be fitted to the data, such as exponential or quadratic form.

### 2.2 Generalized Schaefer Production Model

Following previous work (Burns, 2013) we specify a more flexible Cobb-Douglas production function in the first stage.

$$
\begin{equation*}
H_{i t}=\left(Z_{i t 1}^{\boldsymbol{\beta}_{1}} Z_{i t 2}^{\boldsymbol{\beta}_{2}}\right) q X_{t} \exp ^{\left(b_{i}+\epsilon_{i t}\right)} \tag{4}
\end{equation*}
$$

with $i=1, \ldots, 70$ vessels, $t=1, \ldots, 9$ time periods, $b_{i} \stackrel{\text { ind. }}{\sim} N\left(0, \sigma_{b}^{2}\right)$, and $\epsilon_{i t} \stackrel{\text { i.i.d. }}{\sim} N\left(0, \sigma^{2}\right)$. Let $H_{i t}=$ total bushels harvested by vessel $i$ in year $t, Z_{i t 1}=$ total hours fished by vessel $i$ in year $t$, and $Z_{i t 2}$ $=$ length of vessel $i$ in year $t$. This functional form does not impose constant returns to scale and also captures unobserved heterogeneity in the fishing fleet through the random effect term $b_{i}$.

Taking natural logs we can rewrite equation 4 as linear in the parameters. Let lowercase letters denote natural log, e.g $\ln \left(X_{t}\right)=x_{t}$, then production function becomes

$$
\begin{equation*}
h_{i t}=\boldsymbol{\beta}_{\mathbf{1}} z_{i t 1}+\boldsymbol{\beta}_{\mathbf{2}} z_{i t 2}+b_{i}+\epsilon_{i t}+\ln q+x_{t} \tag{5}
\end{equation*}
$$

Following (Zhang and Smith, 2011) the unobserved stock in year $t$ can be estimated by a timevarying fixed effect. We create a series of indicator variables $\left(D_{t}\right)$ and then estimate a stock index for each year $\left(\delta_{t}\right)$. Let $D_{t}=1$ if year $t$ and 0 otherwise. Then equation 5 can be written

$$
\begin{equation*}
h_{i t}=\boldsymbol{\beta}_{\mathbf{1}} z_{i t 1}+\boldsymbol{\beta}_{2} z_{i t 2}+b_{i}+\epsilon_{i t}+\sum_{t=1}^{9} \delta_{t} D_{t} \tag{6}
\end{equation*}
$$

From equation 6 the stock index is written as $\delta_{t}=\ln q+x_{t}$. The stock index can be estimated by

$$
\begin{equation*}
\hat{\delta}_{t}=\left(\bar{h}_{t}-\bar{z}_{t 1} \hat{\boldsymbol{\beta}}_{1}-\bar{z}_{t 2} \hat{\boldsymbol{\beta}}_{2}\right)+\hat{\epsilon}_{t} \tag{7}
\end{equation*}
$$

where $\bar{h}_{t}=\frac{\sum_{i=1}^{n_{t}} h_{i t}}{n_{t}}, \bar{z}_{t j}=\frac{\sum_{i=1}^{n_{t}} z_{i t j}}{n_{t}}$ for $j=1,2$ and $\hat{\epsilon}_{t}=\frac{\sum_{i=1}^{n_{t}} e_{i t}}{n_{t}}$. Rearranging the stock index, the biomass in time period $t$ can be estimated as

$$
\begin{equation*}
\hat{X}_{t}=\frac{\exp \left(\hat{\delta}_{t}\right)}{\hat{q}} \tag{8}
\end{equation*}
$$

In order to estimate this quantity the catchability coefficient, $q$, must be estimated by the secondstage growth model.

$$
\begin{equation*}
\Delta Y_{t+1}=\hat{Y}_{t+1}-\hat{Y}_{t}=\alpha_{1} \hat{Y}_{t}-\alpha_{2} \hat{Y}_{t}^{2}-\alpha_{3} H_{t}+\epsilon_{t} \tag{9}
\end{equation*}
$$

where $\hat{Y}_{t}=\exp \left(\hat{\delta}_{t}\right), \alpha_{1}=r, \alpha_{2}=\frac{r}{q K}$ and $\alpha_{3}=q$. Then $\exp \left(\hat{\delta}_{t}\right)=q \hat{X}_{t}$, and stock is estimated as $\hat{X}_{t}=\frac{\exp \left(\hat{\delta}_{t}\right)}{\hat{q}}$

The measurement error in equation 9 comes from the estimation of $\hat{Y}_{t+1}, \hat{Y}_{t}$, and $\hat{Y}_{t}^{2}$, which are a function of the estimated stock index, $\hat{\delta}_{t}$. The remaining error in the estimated stock index will be correlated with the disturbance in the model, which violates an important assumption of the classical regression model. The consequences are that the estimated biological parameters in the second-stage growth model will be biased an inconsistent. We propose another way to correct for this measurement error, using a Monte Carlo method known as Simulation Extrapolation (SIMEX).

## 3 Measurement Error Model

In this section we specify a measurement error model for the estimated stock index, $\hat{\delta}_{t}$. With variance for $\hat{\delta}_{t}$ known, or at least approximately known, it will then be possible to reduce the bias by implementing SIMEX. The error is assumed to be additive, with the estimated stock index equal to the true index plus random error. The measurement error for the stock index is specified as

$$
\begin{equation*}
\hat{\delta}_{t}-\delta_{t}=u_{t}=\bar{z}_{t}^{\prime} \beta-\bar{z}_{t}^{\prime} \hat{\beta}+\bar{\eta}_{t} \tag{10}
\end{equation*}
$$

where $u_{t} \mid \bar{z}_{t} \sim N\left(0, \sigma_{u t}^{2}\right)$. The measurement error model requires $E\left[u_{t} \mid \bar{z}_{t}\right]=0$ but does not require the error to be independent of the variables conditioned on. The assumption that $u_{t}$ is independent of $\bar{z}_{t}$ is a much stronger assumption than required here.

From the measurement error model above, we can write the measurement error variance as

$$
\begin{equation*}
\hat{\sigma}_{u t}^{2}=\bar{z}_{t}^{\prime}\left[\frac{\hat{\operatorname{Cov}(\hat{\beta})}}{n_{t}}\right] \bar{z}_{t}+\frac{\hat{\sigma}_{\eta}^{2}}{n_{t}} \tag{11}
\end{equation*}
$$

where $\operatorname{Cov}(\beta)$ is the variance-covariance matrix for the estimated parameters in the first stage production function. Equation 11 defines the measurement error variance for the stock index. The variance is allowed to vary over time. We note that because of the off-diagonal elements in the variance-covariance matrix, the measurement error is not independent between years. We also allow the measurement error variance to be heteroscedastic over the nine years of our observed data. Given this estimated of the measurement error variance, we can apply SIMEX as a method to reduce bias in the parameter estimates. Before addressing the SIMEX algorithm, we first describe the measurement error problem in the second stage growth model in more detail.

### 3.1 Linear Models with Nonadditive Measurement Error

In this section we discuss the implications of nonadditive error in the estimation of the generalized Schaefer Production Model. We start by examining how the second stage growth model is estimated. This model estimates the biological parameters using the three covariates; $\hat{Y}_{t}=\exp \left(\hat{\delta}_{t}\right)$, $\hat{Y}_{t}^{2}=\exp \left(\hat{\delta}_{t}\right)^{2}$, and $H_{t}$, where $\hat{\delta}_{t}$ is the stock index in time period t and $H_{t}$ is the harvest or catch in time period t. $\hat{\delta}_{t}$ has approximately known additive measurement error. While the measurement
error for $\hat{c}_{t}$ is additive, the model is estimated by using $\hat{Y}_{t}, \hat{Y}_{t}^{2}$ and $H_{t}$. Since the first two covariates are nonlinear function of the mismeasured $\hat{\delta}_{t}$, the second stage growth model parameter estimates will not be unbiased.

We motivate the measurement error problem by describing the properties of a naive estimator for the model. Let $\tilde{\mathbf{x}}_{t}$ be a $(1 \times 3)$ vector containing the true variables $\left[\delta_{t} \delta_{t} H_{t}\right.$ ]. Then let $\tilde{\mathbf{w}}_{t}$ be a ( 1 $x 3)$ vector containing the estimated or mismeasured variables $\left[\hat{\delta}_{t} \hat{\delta}_{t} H_{t}\right]$, such that

$$
\begin{equation*}
\tilde{\mathbf{w}}_{t}=\tilde{\mathbf{x}}_{t}+\tilde{\mathbf{u}}_{t} \tag{12}
\end{equation*}
$$

Defining the vector $\tilde{\mathbf{u}}_{\mathrm{t}}$ as a $(1 \times 3)$ vector, $\tilde{\mathbf{u}}_{t}=\left[u_{t} u_{t} 0\right]$, then $E\left[\tilde{\mathbf{u}}_{t} \mid \tilde{\mathbf{x}}\right]=0$. However, because $\tilde{\mathbf{x}}_{t}$ enter into the model through a nonlinear function, call it $g\left(\tilde{\mathbf{x}}_{t}\right)$, this means $E\left[g\left(\tilde{\mathbf{w}}_{t}\right)\right] \neq g\left(\tilde{\mathbf{x}}_{t}\right)$.

To show how this will impact estimation of the model, we specify the design matrix by stacking the vector of $\tilde{\mathbf{w}}_{t}$ for all nine time periods, $t=1, \ldots, 9$

$$
\tilde{\mathbf{W}}=\left(\begin{array}{ccc}
\hat{Y}_{1} & \hat{Y}_{1}^{2} & H_{1}  \tag{13}\\
\vdots & \vdots & \vdots \\
\hat{Y}_{9} & \hat{Y}_{9}^{2} & H_{9}
\end{array}\right)
$$

In the traditional setup of the generalized Schaefer production model, the second stage growth model is specified as linear in the parameters. Under the classical regression model assumptions, the biological parameters can be estimated using ordinary least squares. Let y be the response vector of the difference $\hat{Y}_{t+1}-\hat{Y}_{t}$ for $t=1, \ldots, 9$, such that

$$
\mathrm{y}=\left(\begin{array}{c}
\mathrm{y}_{1}  \tag{14}\\
\vdots \\
\mathrm{y}_{9}
\end{array}\right)
$$

The naive growth model can be written

$$
\begin{equation*}
\mathbf{y}=\tilde{\mathbf{W}} \beta+\epsilon \tag{15}
\end{equation*}
$$

The ordinary least squares estimator for the naive growth model parameters is

$$
\begin{equation*}
\hat{\beta}=\left(\tilde{\mathbf{W}}^{\prime} \tilde{\mathbf{W}}\right)^{-1}\left(\tilde{\mathbf{W}}^{\prime} \mathbf{y}\right) \tag{16}
\end{equation*}
$$

The classical regression model assumptions define this estimator to be unbiased and consistent when $\tilde{\mathbf{W}}$ is not measured with error. Using our naive estimator above, the asymptotic properties can be evaluated by making a few more assumptions. Following, Greene (2003) let

$$
\begin{equation*}
\operatorname{plim}\left(\frac{\tilde{\mathbf{w}}^{\prime} \tilde{\mathbf{W}}}{n}\right)=\mathbf{Q}^{*}+\boldsymbol{\Sigma}_{u} \tag{17}
\end{equation*}
$$

where $\mathbf{Q}^{*}=\operatorname{plim} \frac{\tilde{\mathbf{X}}^{\prime} \tilde{\mathbf{x}}}{n}$, and $\boldsymbol{\Sigma}_{u}$ is the Variance-Covariance matrix of $\tilde{\mathbf{u}}$, i.e. the measurement error variance-covariance matrix. The probability limit of $\hat{\beta}$ is then

$$
\begin{equation*}
\left[\mathbf{Q}^{*}+\boldsymbol{\Sigma}_{u}\right]^{-1} \mathbf{Q}^{*} \beta \neq \beta \tag{18}
\end{equation*}
$$

Thus, the estimator is not consistent for the true parameters. The result of nonadditive measurement error in the linear model is that the estimates of biomass, which are a function of the biased parameter estimates, will be biased and inconsistent. In a situation where measurement error is present, identification of the parameters in the model is often an associated issue. One option is to bring in outside information to help identify the model (Greene, 2003). We identify the parameter estimates through the estimated measurement error variance. In the next section we show how the estimated measurement error variance to reduce bias in the naive estimates using SIMEX.

### 3.2 SIMEX Parameter Estimates

SIMEX is a two-step simulation-based method of estimating and reducing bias due to measurement error. In the first step, simulated data are obtained by adding additional measurement error to the data in a resampling-like process. This establishes a trend of measurement error-induced bias versus the variance of the added measurement error. In the second step, an extrapolation method fits a trend line back to a point where the measurement error variance is zero. The key underlying SIMEX is the fact that the effect of measurement error on an estimator can be determined experimentally through simulation (Carroll et al., 2012). It can be shown that under a number of
different measurement error specifications that SIMEX provides approximately consistent parameter estimates. SIMEX is very general in the sense that the bias due to measurement error in almost any estimator of almost any parameter can be estimated and corrected, at least approximately.

In the simulation step we generate measurement error with variance $(1+\lambda) \hat{\sigma}_{u t}^{2}$ for the stock index. The additional measurement error variance $\lambda \hat{\sigma}_{u t}^{2}$, is generated at different values of $\lambda$, where $\lambda=(0,0.375,0.75,1.125,1.50)$. Using the simulated data, the growth model is estimated 10,000 times for each value of $\lambda$. Finally, at each value of $\lambda$, we calculate the mean for each parameter estimate. Let the mean for each of the $k$ parameter estimates be $\bar{\alpha}_{k}$, shown as dots in the figure 1 .


Figure 1: An example of the SIMEX Simulation step for $\hat{\alpha}$

The extrapolation step fits a model by regressing the estimated $\bar{\alpha}_{k}$ on $\lambda$, and then extrapolating back to $\lambda=-1$. Since the generated measurement error variance is now $(1+\lambda) \hat{\sigma}_{u t}^{2}$, at $\lambda=-1$ the measurement error collapses to zero.

The last step of the SIMEX method is the extrapolation step. There are several functional forms which can be chosen, including the linear, quadratic and rational extrapolant functions. Figure 2 shows an example of both the linear and quadratic extrapolation functional forms. We choose a quadratic extrapolation function for this model, reporting results for both the naive estimates and quadratic extrapolation function. It should be noted that the choice of the extrapolation function will affect the SIMEX estimates.


Figure 2: An example of the SIMEX Extrapolation Step for $\hat{\alpha}$

### 3.3 Standard Errors of SIMEX Estimates

To obtain standard errors for the SIMEX parameter estimates we use a two-stage bootstrap. The two-stage bootstrap is different from a one-stage bootstrap because it generates both a response from a regression model, and the mismeasured covariates, similar to a parametric bootstrap procedure. One advantage of the two-stage bootstrap is that it gives an estimate of bias.

Below we describe the steps for the two-stage bootstrap from (Buonaccorsi, 2010). This procedure is used to obtain the standard errors for the estimated parameters in the second stage growth model. Two of the covariates in the model, $\hat{Y}_{t}$ and $\hat{Y}_{t}{ }^{2}$, are measured with error. In the two-stage bootstrap we generate these variables in $b$ repeated simulations and denote them, $w_{b t 1}$ and $w_{b t 2}$ respectively. The third covariate is the harvest, denoted $x_{b t 3}$, is assumed not measured with error. For the $b^{t h}$ bootstrap sample, generate $\left[y_{b t}, w_{b t 1}, w_{b t 2}, x_{b t 3}\right]$, where

$$
\begin{equation*}
y_{b t}=\beta_{1} w_{b t 1}+\beta_{2} w_{b t 2}+\beta_{3} x_{b t 3}+e_{b t} \tag{19}
\end{equation*}
$$

and the two mismeasured covariates are

$$
\begin{equation*}
w_{b t 1}=\exp \left(\hat{\delta}_{t 1}+u_{b t}\right) \tag{20}
\end{equation*}
$$

and

$$
\begin{equation*}
w_{b t 2}=\left(w_{b t 1}\right)^{2} \tag{21}
\end{equation*}
$$

where $u_{b t} \mid w_{b t 1}, w_{b t 2}, x_{b t 3} \stackrel{\text { ind. }}{\sim} N\left(0, \sigma_{u_{t}}^{2}\right)$ and $e_{b t} \stackrel{\text { i.i.d. }}{\sim} N\left(0, \sigma_{\epsilon}^{2}\right)$.
The standard errors reported for the SIMEX estimates are from 10,000 simulations using this two-step data generating process. Estimates of bias are calculated as

$$
\begin{equation*}
\operatorname{Bias}(b o o t)=\frac{\sum_{b=1}^{B} \hat{\beta}_{b}}{B}-\hat{\beta} \tag{22}
\end{equation*}
$$

## 4 Data

The data for the empirical analysis come from the National Marine Fishery Service logbook reporting system, which documents every harvesting trip taken by every vessel in the Mid-Atlantic surfclam fishery in the U.S. EEZ (3-200miles offshore). The logbook data are a panel data set containing approximately 24,000 vessel-trip observations, for years 2001-2009. The trip-level data set includes variables such as bushels harvested, time fishing, time-at-sea, and vessel characteristics such as vessel length, gross-tons and horsepower. There are a total of eighty-eight different vessels observed over the nine year period. To simplify the correlation structure within each vessel and because biomass is observed annually, data are aggregated by vessel-year. The resulting data are reduced to 70 vessels and 285 vessel-year observations. The biological data on the biomass comes from the Northeast Fisheries Science Center. Summary statistics for the data are presented in Table 1

|  | Obs | Mean | Std.Dev | Min | Max |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Harvest (bushels) | 285 | 93749 | 82007.6 | 864 | 442496 |
| Time Fishing (hours) | 285 | 1209.2 | 951.9 | 58 | 3959.4 |
| Fuel (gallons) | 285 | 65896 | 65979.3 | 876 | 388204 |
| Length (feet) | 70 | 85.7 | 18.4 | 28 | 162 |
| Biomass (1000 metric tons) | 9 | 1037 | 171.9 | 750 | 1294 |

Table 1: Summary statistics 2001-2009

A closer inspection of the data reveals that CPUE has been declining over the nine year time period, seen in Figure 3. This downward trend reflects declining resource abundance due to repeated harvesting from a particular densely populated area of surfclams, located off the northern New Jersey coast. The result of this fishing behavior means that harvesters are spending more time-at-sea as the resource is becoming depleted.


Figure 3: CPUE for Mid-Atlantic Surfclam Fishery (2001-2009)

The resulting scenario in which effort is increasing while stock abundance decreases can potentially affect the ability of the generalized Schaefer model to estimate the biomass with any precision. Hilborn (1979) notes that this "one-way trip" of increasing effort and decreasing index of abundance can sometimes lead to uninformative results. Unfortunately, there are many regulated fisheries which share this fate. We proceed with estimation in with the knowledge that the direction of bias in our estimates may be specific to the Mid-Atlantic surfclam fishery.

## 5 Results

In this section we present and discuss the results from three different models; the classic Schaefer Model, the generalized Schaefer production model, and the adapted Schaefer production model with SIMEX. We compare and contrast the various estimates of the biomass from each model, using the scientific estimates from the Northeast Fisheries Science Center (NEFSC) as estimates of the truth. To estimates the biomass, we convert surfclam bushels to pounds of surfclams using the standard conversion, 1 bushel = 17 pounds (Northeast Fisheries Science Center, 2010).

### 5.1 Classic Schaefer Production Model

In table 2 we present estimation results for the second-stage growth model from classic Schaefer Production Model. This model uses CPUE as a proxy for the stock. In the table below $r$ represents intrinsic growth rate, $K$ is carrying capacity and $q$ is the catchability coefficient.

| Variable | Estimate |
| :--- | :--- |
| $r$ | 0.532 |
|  | $(0.379)$ |
| $r / q K$ | $-5.4 \mathrm{E}-04$ |
|  | $(0.002)$ |
| $q$ | $2.72 \mathrm{E}-06$ |
|  | $(5.1 \mathrm{E}-06)$ |
|  |  |
| $\hat{\sigma}_{e}^{2}$ | 7.416 |
| ${ }^{*} \mathrm{p}<0.1$, | ${ }^{* *} \mathrm{p}<0.05,{ }^{* * *} \mathrm{p}<0.01$ |

Table 2: Classic Schaefer Model Estimates for Growth Equation

Using the CPUE estimator, we can estimate the biomass as $\hat{X}_{t}=\frac{\hat{y}_{t}}{\tilde{q}}$. Biomass estimates from classic Schaefer Production Model (Sch est.) are shown below in Table 3. The NEFSC estimates of biomass are treated as true measures in the table below. All biomass estimates are reported in thousands of metric tons. Variance and standard error estimates for the biomass are obtained using a bootstrap method. Confidence intervals are reported as $95 \%$ approximate Wald intervals.

| Year | NEFSC est. | Sch est. | $95 \%$ lower | $95 \%$ upper | \% Bias |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2001 | 1294 | 520 | 312 | 728 | $-60 \%$ |
| 2002 | 1207 | 476 | 290 | 662 | $-61 \%$ |
| 2003 | 1128 | 438 | 265 | 611 | $-61 \%$ |
| 2004 | 1104 | 402 | 239 | 565 | $-64 \%$ |
| 2005 | 1079 | 375 | 209 | 541 | $-65 \%$ |
| 2006 | 1013 | 350 | 207 | 493 | $-65 \%$ |
| 2007 | 912 | 294 | 177 | 411 | $-68 \%$ |
| 2008 | 827 | 257 | 152 | 362 | $-69 \%$ |
| 2009 | 750 | 235 | 140 | 330 | $-69 \%$ |

Table 3: Classic Schaefer Biomass Estimates (1000MT)

Using the NEFSC estimates as the true value, the average bias for the classic Schaefer Model is $-65 \%$. Clearly the model does a poor job of estimating the biomass for the fishery, with significant downward bias in each year. Although a fishery manager might not see this as a problem, given that estimates of maximum sustainable yield (MSY) would be conservative, we cannot be sure this
would be the case in another fishery. As Uhler (1980) points out, the biases in the model are too complex to estimate beforehand, and so a different fishery could certainly have significant bias in the opposite direction. It is certainly possible this result is limited to the Mid-Atlantic surfclam fishery. At the very least, these estimates of the biomass are not very informative because they are biased downward by a substantial amount.

### 5.2 Generalized Schaefer Production Model

The generalized Schaefer model uses a two-stage panel data (Zhang and Smith, 2011)estimator. Table 4 presents estimation results from first stage generalized Schaefer production model. The stock index is estimated as a year fixed-effect.

| Variable | Estimate |
| :--- | :--- |
| log(timefish) | 1.051 |
|  | $(0.022)^{* * *}$ |
| log(length) | 0.149 |
|  | $(0.132)$ |
| Year 2001 | $3.770^{* * *}$ |
|  | $(0.593)$ |
| Year 2002 | $3.602^{* * *}$ |
|  | $(0.593)$ |
| Year 2003 | $3.480^{* * *}$ |
|  | $(0.595)$ |
| Year 2004 | $3.388^{* * *}$ |
|  | $(0.597)$ |
| Year 2005 | $3.283^{* * *}$ |
|  | $(0.580)$ |
| Year 2006 | $3.155^{* * *}$ |
|  | $(0.599)$ |
| Year 2007 | $2.965^{* * *}$ |
|  | $(0.597)$ |
| Year 2008 | $2.883^{* * *}$ |
|  | $(0.597)$ |
| Year 2009 | $2.801^{* * *}$ |
|  | $(0.596)$ |
|  |  |
| $\hat{\sigma}_{b}^{2}$ | 0.070 |
| $\hat{\sigma}_{e}^{2}$ | 0.061 |
| ${ }^{*} \mathrm{p}<0.05$, | ${ }^{* *} \mathrm{p}<0.01, * * \mathrm{p}<0.001$ |

Table 4: Generalized Schaefer Model Estimates

Using the estimated stock index from table 4 , we can estimate the second stage growth model.

Table 5 presents results from second stage estimation of the growth model for the generalized Schaefer Production Model. Just as in the classic Schaefer Production Model, the parameter estimates are not significant due to a high degree of collinearity among the right-hand side variables.

| Variable | Estimate |
| :--- | :--- |
| $r$ | 0.181 |
|  | $(0.318)$ |
| $r / q K$ | $1.38 \mathrm{E}-04$ |
|  | $(0.005)$ |
| $q$ | $5.69 \mathrm{E}-07$ |
|  | $(1.46 \mathrm{E}-06)$ |
|  |  |
| $\hat{\sigma}_{e}^{2}$ | 0.940 |
| ${ }^{*} \mathrm{p}<0.1$, | ${ }^{* *} \mathrm{p}<0.05,{ }^{* * *} \mathrm{p}<0.01$ |

Table 5: Generalized Schaefer Model Estimates for Growth Equation
From table 5, the catchability coefficient $q$ is used to estimate the biomass for the fishery $\hat{X}_{t}$, where $\hat{X}_{t}=\frac{\hat{y}_{t}}{\hat{q}}$. Table 6 below shows estimates of biomass using the Generalized Schaefer Model (Gen. Schaefer). All estimates are reported in thousands of metric tons (1000 MT). Variance estimates for the biomass were obtained using a stratified bootstrap method.

| Year | NEFSC | Gen. Schaefer | 95\% Lower | 95\% Upper | \% Bias |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2001 | 1294 | 996 | 671 | 1321 | $-23 \%$ |
| 2002 | 1207 | 842 | 564 | 1120 | $-30 \%$ |
| 2003 | 1128 | 745 | 504 | 986 | $-34 \%$ |
| 2004 | 1104 | 681 | 461 | 901 | $-38 \%$ |
| 2005 | 1079 | 612 | 419 | 804 | $-43 \%$ |
| 2006 | 1013 | 539 | 366 | 712 | $-47 \%$ |
| 2007 | 912 | 446 | 300 | 592 | $-51 \%$ |
| 2008 | 827 | 411 | 278 | 544 | $-50 \%$ |
| 2009 | 750 | 368 | 242 | 494 | $-50 \%$ |

Table 6: Generalized Schaefer Model Biomass Estimates (1000MT)

The average bias for the Generalized Schaefer Model is $-41 \%$. While this is much smaller than the classic Schaefer Production Model, it does consistently underestimate the biomass in each year except 2001. The $95 \%$ confidence intervals for the generalized Schaefer model contain the NEFSC biomass estimate for 2001 only. If we are to believe that the NEFSC biomass estimates represent an unbiased picture of the fishery stock, then this would seem to be a significant problem for fishery managers. A fishery manager using this method with this particular data would likely set the
total allowable catch too low or simply not be able to use this model for estimation of the biomass. The results from table 6 show that the confidence intervals from the generalized Schaefer Model do not contain the true biomass in any year, a disappointing result.

### 5.3 Generalized Schaefer Production Model with SIMEX

Table 7 below shows estimates from the second stage growth model using the SIMEX with a linear extrapolation functional form. All SIMEX estimates are 10,000 simulations at each $\lambda$, resulting in a total of 40,001 simulated datasets. Standard errors reported are from using a two-stage bootstrap method. It can be seen that none of the parameter estimates are statistically significant, due to high multicollinearity in the variables. The important difference is the point estimates are different than the generalized Schaefer Production Model. The difference in the estimate of $q$ is particularly important because it is used to estimate the biomass. These estimates can be seen in table 8

| Variable | Estimate |
| :--- | :--- |
| $r$ | 0.103 |
|  | $(0.620)$ |
| $r / q K$ | $1.36 \mathrm{E}-03$ |
|  | $(0.130)$ |
| $q$ | $2.67 \mathrm{E}-07$ |
|  | $(5.354 \mathrm{E}-06)$ |
|  |  |
| $\hat{\sigma}_{e}^{2}$ | 0.778 |
| ${ }^{*} \mathrm{p}<0.1$, | ${ }^{* *} \mathrm{p}<0.05,{ }^{* * *} \mathrm{p}<0.01$ |

Table 7: Generalized Schaefer Model with SIMEX, Estimates for Growth Model

The biomass estimates, calculated in thousands of metric tons (1000MT), are shown in table 8 . Variance and standard error estimates of the biomass were obtained using the two-stage bootstrap method. The average bias for the SIMEX estimates of biomass, using the NEFSC biomass estimates as truth, is $-17 \%$. This is less than half the amount of bias in the generalized Schaefer model estimates. Additionally, the $95 \%$ confidence interval in each year covers the NEFSC estimate. Note that these intervals are wider than the generalized Schaefer model, because they account for measurement error.

| Year | NEFSC est. | SIMEX est. | 95\% Lower | 95\% Upper | \% Bias |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2001 | 1294 | 1399 | 937 | 1861 | $8 \%$ |
| 2002 | 1207 | 1183 | 781 | 1585 | $-2 \%$ |
| 2003 | 1128 | 1047 | 656 | 1438 | $-7 \%$ |
| 2004 | 1104 | 956 | 603 | 1309 | $-13 \%$ |
| 2005 | 1079 | 860 | 549 | 1171 | $-20 \%$ |
| 2006 | 1013 | 757 | 481 | 1033 | $-25 \%$ |
| 2007 | 912 | 626 | 400 | 852 | $-31 \%$ |
| 2008 | 827 | 577 | 378 | 776 | $-30 \%$ |
| 2009 | 750 | 531 | 336 | 726 | $-29 \%$ |

Table 8: SIMEX Biomass Estimates (1000MT)

## 6 Discussion

This paper explores the impact of measurement error in the Schaefer Production Model and describes a Monte Carlo method for reducing bias in the parameters. The Schaefer production model is a two-stage model that estimates the biomass of a fishery using only information on catch and effort, usually obtained through vessel logbook data. We build on previous work by Zhang and Smith (2011) to reduce bias in the parameter estimates of a more generalized version of the Schaefer Production Model by using a Monte Carlo method called simulation extrapolation (SIMEX).

Using data from the Mid-Atlantic surfclam fishery from 2001-2009, along with scientific estimates of the biomass from NEFSC, we show that the remaining measurement error in a generalized Schaefer model can still contribute to significant bias in the estimates of biomass, up to $-41 \%$ in this case. A fishery manager using this method would not have much reason to be confident in setting a total allowable catch with such bias. Obtaining unbiased estimates of the biomass is crucial to effective fishery management. A total allowable catch that is set too high could put a fishery on a trajectory towards collapse, while a total allowable catch that is set too low can lead to economic loss of rents. We show that fishery managers can obtain more truthful estimates of the biomass after correcting for the measurement error with SIMEX. The results show that the bias in the biological parameters, and biomass estimates, is dramatically less than either the classic Schaefer model or the generalized Schaefer model.

The generalized Schaefer production model attempts to simultaneously deal with three problems inherent in the Schaefer production model, including; 1) the error in the production function, 2) the error in the second stage growth model, and 3) restrictive functional form in the production
function. The idea behind the generalized Schaefer production model is that a stock index can be created using a fixed effects estimator for the production function. We further reduce bias using the SIMEX algorithm. The average remaining bias in the SIMEX estimates is $-17 \%$. This is less than half the average bias in the generalized Schaefer production model, at -41\%. Additionally, all 95\% confidence intervals from the SIMEX estimates cover the NEFSC estimate. Only one confidence interval from the generalized Schaefer production model covers the NEFSC estimate, while none of the intervals cover the truth using the classic Schaefer production model.

While these findings would suggest this additional measurement error correction method improves on previous models, it should be noted that these findings may only apply to the MidAtlantic surfclam fishery. Because the surfclam is a slow-growing mollusk, the dynamics of the growth model may be very different from a pelagic fishery. We intend to apply this method in several different kinds of fisheries, including pelagic and other fisheries with significantly more variation in the biomass, before making any definitive conclusions about the model. We think this model is a good first step towards giving fishery managers a more reliable estimate of the biomass when they only have catch and effort data obtained through vessel logbooks.

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