On the Relationship between Financial Instability and Economic Performance: Stressing the Business of Nonlinear Modelling

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Abstract

The recent global financial crisis and the subsequent economic recession have revitalized the discussion on the causal relationship between financial and economic sectors. This study examines financial and economic indices developed by the Federal Reserve Banks of Kansas City and Chicago, respectively, to identify the impact of financial uncertainty on the overall economic performance. Using smooth transition and vector smooth transition autoregressions, this research assesses nonlinear dynamics of these indices, and tests the Granger non-causality hypothesis between the financial stress and economic activity in an out-of-sample setting. Results of this study confirm the causal relationship between financial and economic indices. Moreover, the improved in-sample fit of nonlinear models translates into better forecast performance, in comparison with the linear models, in an out-of-sample setting as well.

Keywords: business cycles; financial stress; forecast evaluation; Granger causality; smooth transition modelling.

1 Introduction

The recent global financial crisis and the subsequent economic recession have fortified the discussion regarding the scope and extent of the interrelationship between financial and economic sectors (e.g. Gertler and Kiyotaki, 2010; Jermann and Quadrini, 2012; Brunnermeier and Sannikov, 2014). The topic is not new, however. Almost a century ago, Schumpeter (1934) has argued about the benefits of financial services on the economic growth, while Fisher (1933) and Keynes (1936) have attributed economic recessions to the financial market failures (e.g. King and Levine, 1993; Brunnermeier and Sannikov, 2014). Since then, questions of whether financial development results in economic growth, or, by corollary, financial crises cause economic recessions, have been broadly discussed and analyzed in the economics literature (e.g. King and Levine, 1993; De Gregorio and Guidotti, 1995; Levine, 1997; Arestis et al., 2001; Calderón and Liu, 2003). The economic rationale behind the considered relationship has hardly been a reason of skepticism. The general ambiguity around the predictability, or out-of-sample causality in Granger (1969) sense, remains to be an issue, however. This was famously summarized by Samuelson (1966) in his quote, stating: "Wall Street indexes predicted nine out of the last five recessions!"

Business cycles, as defined by Burns and Mitchell (1946), reflect the co-movement of multiple individual economic series (Diebold and Rudebusch, 1996). Thus, a logical way to assess business cycles would be examining the dynamics of a composite index of economic variables (e.g. Stock and Watson, 1989, 1999a). Similarly, factor methods can be applied to financial variables to obtain the so called financial stress index (e.g. Hakkio and Keeton, 2009). Composite indices have long been used as leading indicators for economic growth and recessions (e.g. Stock and Watson, 1993; Granger et al., 1993; Lahiri and Wang, 1994; Stock and Watson, 2003; Camacho, 2004; Marcellino, 2006; Anderson et al., 2007), with a varying degree of success (e.g. Diebold and Rudebusch, 1991; Giacomini and Rossi, 2013). The main focus of this study, however, is to investigate if the composite financial index leads the composite economic index. Put differently, this research examines dynamics of the financial and the economic indicators, and assesses the out-of-sample Granger causality between the two variables.

This study, moreover, argues that regime-dependent nonlinear models, designed to identify multiple regimes in time series dynamics, may be better suited to analyze causal relationship between financial stress and economic health, and possibly contribute to improved predictability of the composite index of economic variables. It has been well documented that contractions and expansions of business cycles do not happen at the same rate (e.g. Diebold and Rudebusch, 1996). More recently, studies have also emphasized the nonlinear nature of the relationship between the financial and economic sectors (e.g. Davig and Hakkio, 2010; Liu et al., 2011; Hubrich and Tetlow, 2012; Mittnik and Semmler, 2013; Brunnermeier and Sannikov, 2014), implying that linear models may not be ideal in identifying the causal links between financial instability and economic recessions, and emphasizing potential advantages of regime-dependent nonlinear modelling. For example, Hubrich and Tetlow (2012) argue that the "episodic" nature of the relationship is one reason researchers have failed to find sufficient evidence of financial frictions affecting economic activity. In particular, economic agents tend to behave differently in times of higher financial uncertainty, as compared to a relatively stable environment. As such, the effect of financial stress on economic activity can vary, depending on the state of nature of the financial sector. Davig and Hakkio (2010) rationalize such discrepancy using the real options and the financial accelerator theories.

This research answers the questions whether there are causal links between financial and economic indices, and if nonlinear models help to better identify such a relationship. In so doing, this adds to the growing literature examining intricacies in the system of financial and economic indicators. In what follows, we first introduce the smooth transition modelling and testing frameworks. We then present and interpret the estimated models, and assess their dynamics, using generalized impulse–response functions. We then turn to the forecasting exercise, and evaluate forecast performance of linear and nonlinear models, with a particular emphasis on an out-of-sample test of the null hypothesis of Granger non-causality. In conclusion, we summarize the main findings of this research.

2 Econometric Modelling and Forecasting

This research adopts the smooth transition modelling framework to examine nonlinearities in the financial stress and the economic activity indicators. Smooth transition regressions were first proposed by Bacon and Watts (1971), and subsequently advocated for the time series analysis by Chan and Tong (1986). More formally Luukkonen et al. (1988); Teräsvirta (1994) introduced and developed smooth transition autoregression (STAR) modelling and testing frameworks. The family of smooth transition models was later extended to the multivariate framework (e.g. Anderson and Vahid, 1998; Rothman et al., 2001), resulting in a vector smooth transition autoregression (VSTAR). The regime-dependent (and piecewise linear) modelling facilitates the analysis of the relationship between the variables of interest, where causal inferences may vary across the regimes depending on the transition variable's state of nature. Moreover, the so-called "smooth" transition supports the continuum of intermediate steps between the two extreme regimes, which is more intuitive from an economic standpoint. Since the introduction, the smooth transition models have gained popularity and have been given consideration in studies modelling asymmetric cyclical variations of business cycles, and other related economic indicators (e.g. Teräsvirta, 1995; Skalin and Teräsvirta, 2002; Franses and van Dijk, 2005).

2.1 The Smooth Transition Models

Consider a basic autoregressive model with an additive error term:

$$y_t = f\left(y_{t-1}, y_{t-2}, \ldots; \boldsymbol{\theta}\right) + \varepsilon_t \tag{1}$$

where y_t is the dependent variable, $\boldsymbol{\theta}$ is the parameter set defining the autoregressive structure of the model, and $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$ is a white noise process.

Assuming linearity, equation (1) can be expressed as the AR(p) process, where p is the autoregressive order:

$$y_t = \alpha + \sum_{i=1}^{p} \beta_i y_{t-i} + \varepsilon_t \tag{2}$$

One can relax the linearity assumption by introducing the regime-dependency in the autoregressive process, resulting in:

$$y_t = \alpha_0 + \sum_{i=1}^{p} \beta_{0i} y_{t-i} + \left(\alpha_1 + \sum_{i=1}^{p} \beta_{1i} y_{t-i}\right) \mathcal{I}\left(s_t; c\right) + \varepsilon_t \tag{3}$$

where $\mathcal{I}(s_t; c)$ is an indicator function, which can take values of zero or one, depending on the location of the transition variable, s_t, in relation to the threshold parameter, c. Put differently, equation (3) is piecewise linear, conditional on $\mathcal{I}(\cdot)$, and is better known as the threshold autoregression (TAR) pioneered by Tong and Lim (1980). A somewhat more convoluted version of this model is the smooth transition autoregression (STAR) of Teräsvirta (1994):

$$y_t = \alpha_0 + \sum_{i=1}^{p} \beta_{0i} y_{t-i} + \left(\alpha_1 + \sum_{i=1}^{p} \beta_{1i} y_{t-i} \right) \mathcal{G}\left(s_t; \gamma, c\right) + \varepsilon_t \tag{4}$$

where $\mathcal{G}(s_t; \gamma, c)$ is the so called transition function that is also bounded between zero and one, but can take any value in between. The transition function varies with s_t, the transition variable. The other factors of the transition function are the parameters γ and c, where γ, the speed–of–adjustment parameter, determines how rapidly shifts from one regime to another occur, and c, the centrality parameter, determines the value of s_t around which regime changes are centered.

There are several choices for the transition functions. The most popular is the logistic function, a general form of which is:

$$\mathcal{G}\left(s_t; \gamma, \boldsymbol{c}\right) = \left\{ 1 + \exp\left[-\gamma^* \prod_{j=1}^{k} \left(s_t - c_j\right) \right] \right\}^{-1} \qquad \gamma^* > 0,\ c_1 < \ldots < c_k \tag{5}$$

where $\boldsymbol{c} = (c_1, \ldots, c_k)$ is a vector of centrality parameters. In practice most analysts choose either $k = 1$ or $k = 2$, respectively resulting in logistic STAR (LSTAR) and quadratic STAR (QSTAR) models. The former is useful in situations with asymmetries in autoregressive dynamic in relation to s_t are suspected; the latter is useful for situations where nonlinearity in dynamics is linked to the absolute value of s_t. Finally, in equation (5) the parameter γ is normalized by $\sigma_{s_t}^k$, $\gamma^* = (\gamma / \sigma_{s_t}^k)$, where σ_{s_t} is the standard deviation of the transition variable. Doing so effectively makes the smoothness parameter unit free.

The smooth transition autoregressive framework can be extended to a multivariate framework (e.g. Rothman et al., 2001; Camacho, 2004). First, consider a linear vector autoregression of order p, VAR(p):

$$\boldsymbol{x}_t = \boldsymbol{\mu} + \sum_{i=1}^{p} B_i \boldsymbol{x}_{t-i} + \boldsymbol{\upsilon}_t \tag{6}$$

where $\boldsymbol{x}_t = (x_{1t}, \ldots, x_{nt})'$ is a n-dimensional vector of dependent variables; and $\boldsymbol{\upsilon}_t \sim iid\left(\mathbf{0}, \Sigma_\upsilon\right)$, where Σ_υ is a positive definite covariance matrix.

A nonlinear version of equation (6), the vector smooth transition autoregression of order p, VSTAR(p) is specified as follows:

$$\boldsymbol{x}_t = \boldsymbol{\mu_0} + \sum_{i=1}^{p} B_{0i} \boldsymbol{x}_{t-i} + \left(\boldsymbol{\mu_1} + \sum_{i=1}^{p} B_{1i} \boldsymbol{x}_{t-i} \right) \boldsymbol{\iota} \mathcal{G}(s_t, \gamma, c) + \boldsymbol{\upsilon}_t \tag{7}$$

where $\boldsymbol{\iota}\mathcal{G}(s_t, \gamma, c)$ is a vector of transition functions, where $\boldsymbol{\iota}$ is an unit vector restricting the transition function to be common across the equations.

2.2 Testing Linearity in the Auxiliary Regression Setting

The question of whether STAR-type nonlinearity is truly an underlying feature of the data-generating process is a testable hypothesis. However, one cannot directly test the linearity hypothesis, i.e. $H_0 : \gamma = 0$, in a STAR model due to Davies' problem associated with unidentified nuisance parameters (Davies, 1977, 1987). For example, consider equation (4) in conjunction with equation (5), where $K = 1$: the nonlinear model will reduce to the linear AR(p) model either by imposing $\gamma = 0$ or by imposing $\alpha_1 = \beta_{11} = \ldots = \beta_{1p} = 0$. Therefore, the standard test statistics are no longer applicable. Luukkonen et al. (1988) proposed a solution to the problem by approximating the transition function using Taylor series expansion. This results in a testable auxiliary regression:

$$y_t = \varphi + \sum_{j=0}^{3}\sum_{i=1}^{p} \phi_{ji} y_{t-i} s_t^j + \xi_t \tag{8}$$

where ξ_t combines the original error term, ε_t, and the approximation error resulted from the Taylor series expansion. Conventional testing methods can now be applied to equation (8) to test for linearity against the STAR specification. The test is equivalent to testing the null hypothesis of H_0': $\phi_{11} = \ldots = \phi_{1p} = \phi_{21} = \ldots = \phi_{3p} = 0$.

Similar approach can be applied to test linearity in a multivariate setting (Camacho, 2004). In particular, the system of auxiliary regressions can be defined as follows:

$$\boldsymbol{x}_t = \boldsymbol{\pi} + \sum_{j=0}^{3}\sum_{i=1}^{p} \Phi_{ji} \boldsymbol{x}_{t-i} \boldsymbol{s}_t^j + \boldsymbol{\zeta}_t \tag{9}$$

The test is equivalent to a system-wide test of all elements of the Φ_{ji}, $i = 1, \ldots, p$, $j = 1, 2, 3$, being equal to zero. In small samples the higher order polynomial terms can be ignored (Weise, 1999), yielding somewhat more "economical" version of the auxiliary regression:

$$\boldsymbol{x}_t = \boldsymbol{\pi} + \sum_{i=1}^{p} \Phi_{0i} \boldsymbol{x}_{t-i} + \sum_{i=1}^{p} \Phi_{1i} \boldsymbol{x}_{t-i} \boldsymbol{s}_t + \boldsymbol{\zeta}_t \tag{10}$$

The test being equivalent to a system-wide test of $\Phi_{1i} = \mathbf{0}$, $i = 1, \ldots, p$.

2.3 Out-of-Sample Forecasting, Forecast Evaluation, and Granger Causality

Continuing with the notation from equation (1), the expected one-step-ahead forecast can then be written as:

$$\hat{y}_{t+1|t} = \hat{f}\left(y_t, y_{t-1}, \ldots; \hat{\boldsymbol{\theta}}_t\right) \qquad t = R, \ldots, T-1 \tag{11}$$

where $\hat{f}(\cdot)$ is the estimated functional form of the model, and $\hat{\boldsymbol{\theta}}_t$ is a set of parameter estimates. R denotes the estimation window size, i.e. the subset of observations used to obtain a set of parameter estimates. $P-1$ such windows are estimated, where P denotes the subset of observations used for forecast evaluation, so that $R+P=T$. The associated forecast error, then, is:

$$\hat{e}_{t+1} = y_{t+1} - \hat{y}_{t+1|t} \tag{12}$$

where y_{t+1} is the actual one-step-ahead realization of the variable of interest. One can, in principle, define a loss function, $\mathcal{L}(\hat{e}_{t+1})$, which can take various different forms. Assuming the most popular (for good reasons) quadratic loss, i.e. $\mathcal{L}(\hat{e}_{t+1}) = \hat{e}_{t+1}^2$, an appropriate measure for the out-of-sample forecast evaluation is the mean squared forecast error (MSFE):

$$\hat{\sigma}^2 = \frac{1}{P-1}\sum_{t=R}^{T-1}\hat{e}_{t+1}^2 \tag{13}$$

where P is the total number of out-of-sample forecasts. The smaller is the MSFE measure, the better is a particular forecast. So, in order to compare the forecasts from two competing models, one should assess a relative difference between two MSFE measures.

Consider two competing forecasts. One as defined in equation (11), and the other as follows:

$$\hat{y}_{c,t+1|t} = f\left(y_t, y_{t-1}, \ldots, z_t, z_{t-1}, \ldots; \hat{\boldsymbol{\theta}}_{c,t}\right) \tag{14}$$

with the associated forecast error:

$$\hat{e}_{c,t+1} = y_{t+1} - \hat{y}_{c,t+1|t} \tag{15}$$

and the quadratic loss function, $\mathcal{L}(\hat{e}_{c,t+1})$, and the associated MSFE:

$$\hat{\sigma}_c^2 = \frac{1}{P-1}\sum_{t=R}^{T-1}\hat{e}_{c,t+1}^2 \tag{16}$$

Causality in Granger sense (and its original spirit) implies improved out-of-sample predictive ability of the model that incorporates the potentially causal variable, as compared to the model that omits such variable (e.g. Ashley et al., 1980). In the current context, $\hat{\sigma}_c^2 < \hat{\sigma}^2$ would imply that $\{z_t\}$ Granger causes $\{y_t\}$.

A number of methods have been proposed to assess forecast accuracies. One of the most frequently applied methods is the Diebold and Mariano (1995) (DM) measure, which tests predictive accuracy of two (non-nested) competing models. However, when one of the competing models nests another (which is certainly the case in Granger causality testing), then the original DM measure is no longer applicable (e.g. Clark and McCracken, 2001). For such cases, alternative measures have been proposed in lieu of the DM measure. In this study, we will use one such measure proposed by Clark and West (2007). Consider forecast errors from two competing models: $\hat{e}_{t+1}$ and $\hat{e}_{c,t+1}$, where the latter model nests the former. Further, define an "adjustment error", $e_{a,t+1} = (\hat{y}_{t+1|t} - \hat{y}^c_{t+1|t})$. Then, the null hypothesis of equal mean squared (forecast) errors, or currently Granger non-causality, is rejected if the difference given by $\hat{\sigma}^2 - (\hat{\sigma}^2_c - \hat{\sigma}^2_a)$ is sufficiently positive (for more details see Clark and West, 2007), where:

$$\hat{\sigma}^2_a = \frac{1}{P-1}\sum_{t=R}^{T-1} \hat{e}^2_{a,t+1} \tag{17}$$

Then the test of equal forecast ability is equivalent of the null hypothesis $H_0 : \mu = 0$ in the following regression:

$$\mathcal{L}' = \mu + \upsilon_{t+1} \tag{18}$$

where, in the case of non-nested models, $\mathcal{L}' = \hat{e}^2_{t+1} - \hat{e}^2_{c,t+1}$, and, in the case of nested models, $\mathcal{L}' = \hat{e}^2_{t+1} - [\hat{e}^2_{c,t+1} - (\hat{y}_{t+1|t} - \hat{y}_{c,t+1|t})^2]$. Heteroskedasticity and autocorrelation consistent standard errors are used to test the null hypotheses.

3 Empirical Framework

This research uses financial and economic indicators, obtained from the Federal Reserve Banks of Chicago and Kansas City. In particular, the study applies the monthly series of the financial stress index (FSI) of the Federal Reserve Bank of Kansas City, and the national activity index (NAI) of the Federal Reserve Bank of Chicago as proxies for financial instability and economic performance, respectively. The indices (see Figure 1) are constructed based on principal components methodology, and incorporate an array of major financial and economic indicators. In particular, the FSI is based on 11 financial variables (Hakkio and Keeton, 2009); and the NAI is based on 85 economic variables, and corresponds to the economic activity index of Stock and Watson (1999b). Figure 1 reveals several features of interest. Firstly, the two indices are highly correlated – the financial stress episodes coincide with the decreased economic activity. Moreover, both indices well depict the most recent financial crisis and the economic recession.

3.1 Model Selection and Estimation

Initially, we assessed the null hypothesis of a unit root in the considered series. Based on augmented Dickey-Fuller test results, both indices appear to be integrated of order zero. We therefore proceed by modelling the variables in levels. We set the lag length, p, equal to four, both in univariate and multivariate cases, and thus, estimated linear AR(4) and VAR(4) models.[1] The Akaike Information Criteria (AIC) of the estimated models are -1.27 and -3.92, respectively.

Next step is to test for STAR-type nonlinearities using the procedure described in the previous section. We consider lagged levels of both indices (i.e. y_{t-d} and z_{t-d}, where $1 \leq d \leq p$) as candidate transition variables. Results present overwhelming evidence against the linearity assumption (see Table 1). We therefore proceeded to estimate STAR models using each of the two indices as transition variables. Parameter estimates, and estimated transition functions of each of the two models are as follows:

$$\begin{aligned} y_t = & -\underset{(0.16)}{0.14} + \underset{(0.12)}{0.64} y_{t-1} + \underset{(0.12)}{0.40} y_{t-2} - \underset{(0.15)}{0.11} y_{t-3} - \underset{(0.12)}{0.08} y_{t-4} \\ & + \left(\underset{(0.17)}{0.14} - \underset{(0.14)}{0.63} y_{t-1} - \underset{(0.15)}{0.07} y_{t-2} + \underset{(0.17)}{0.44} y_{t-3} + \underset{(0.14)}{0.13} y_{t-4} \right) \mathcal{G}\left(s_t; \hat{\gamma}, \hat{c}\right) + \hat{\varepsilon}_t \end{aligned} \tag{19}$$

where

$$\mathcal{G}\left(s_t; \hat{\gamma}, \hat{c}\right) = \left\{ 1 + \exp\left[-\underset{(11.94)}{9.41} / \sigma_{s_t} \left(y_{t-3} + \underset{(0.13)}{0.72} \right) \right] \right\}^{-1} \tag{20}$$

and where σ_{s_t} is the standard deviation of the transition variable, $s_t = y_{t-3}$. The AIC of the estimated model is -1.32.

We also estimate the STAR model with the lagged level of the financial stress index as a transition variable:

$$\begin{aligned} y_t = & \underset{(0.04)}{0.08} + \underset{(0.08)}{0.01} y_{t-1} + \underset{(0.08)}{0.30} y_{t-2} + \underset{(0.07)}{0.23} y_{t-3} + \underset{(0.07)}{0.01} y_{t-4} \\ & + \left(-\underset{(0.24)}{0.59} + \underset{(0.15)}{0.49} y_{t-1} + \underset{(0.16)}{0.08} y_{t-2} - \underset{(0.15)}{0.18} y_{t-3} - \underset{(0.15)}{0.19} y_{t-4} \right) \mathcal{G}\left(s_t; \hat{\gamma}, \hat{c}\right) + \hat{\varepsilon}_t \end{aligned} \tag{21}$$

where

$$\mathcal{G}\left(s_t; \hat{\gamma}, \hat{c}\right) = \left\{ 1 + \exp\left[-\underset{(2.68)}{5.02} / \sigma_{s_t} \left(z_{t-1} - \underset{(0.16)}{0.75} \right) \right] \right\}^{-1} \tag{22}$$

The AIC of the estimated model is -1.37.

Interestingly, there was evidence of remaining nonlinearities in each of the two estimated STAR models. So, we also estimated the multiple-regime additive STAR model (MRSTAR), i.e. the

[1] Results are not reported here, but are available upon request.

STAR with two distinct transition functions, using both indices together as transition variables:

$$y_t = -\underset{(0.48)}{1.11} + \underset{(0.34)}{0.70}\, y_{t-1} + \underset{(0.28)}{0.22}\, y_{t-2} - \underset{(0.46)}{1.32}\, y_{t-3} - \underset{(0.38)}{0.53}\, y_{t-4} \tag{23}$$
$$+ \left(\underset{(0.37)}{1.46} - \underset{(0.23)}{0.57}\, y_{t-1} + \underset{(0.22)}{0.10}\, y_{t-2} + \underset{(0.31)}{1.26}\, y_{t-3} + \underset{(0.24)}{0.38}\, y_{t-4} \right) \mathcal{G}\left(s_t; \hat{\gamma}_1, \hat{c}_1\right)$$
$$+ \left(-\underset{(0.40)}{1.24} - \underset{(0.40)}{0.50}\, y_{t-1} - \underset{(0.34)}{0.08}\, y_{t-2} + \underset{(0.50)}{1.19}\, y_{t-3} + \underset{(0.44)}{0.63}\, y_{t-4} \right) \mathcal{G}\left(s_t; \hat{\gamma}_2, \hat{c}_2\right) + \hat{\varepsilon}_t$$

where

$$\mathcal{G}\left(s_t; \hat{\gamma}_1, \hat{c}_1\right) = \left\{ 1 + \exp\left[-\underset{(—)}{100}\sigma_{s_{1t}} \left(y_{t-3} + \underset{(0.39)}{0.99} \right) \right] \right\}^{-1} \tag{24}$$

and

$$\mathcal{G}\left(s_t; \hat{\gamma}_2, \hat{c}_2\right) = \left\{ 1 + \exp\left[-\underset{(0.41)}{1.00}\sigma_{s_{2t}} \left(z_{t-1} - \underset{(0.66)}{0.87} \right) \right] \right\}^{-1} \tag{25}$$

The AIC of the estimated model is -1.42.

We next moved on to estimate the vector smooth transition autoregression with z_{t-1} as the transition variable. Below we only present the first estimated equation of the system:

$$y_t = \underset{(0.05)}{0.03} + \underset{(0.07)}{0.02}\, y_{t-1} + \underset{(0.07)}{0.27}\, y_{t-2} + \underset{(0.07)}{0.20}\, y_{t-3} - \underset{(0.07)}{0.01}\, y_{t-4} \tag{26}$$
$$- \underset{(0.19)}{0.27}\, z_{t-1} + \underset{(0.25)}{0.31}\, z_{t-2} - \underset{(0.24)}{0.10}\, z_{t-3} - \underset{(0.18)}{0.07}\, z_{t-4}$$
$$+ \left(-\underset{(0.24)}{0.63} + \underset{(0.17)}{0.13}\, y_{t-1} + \underset{(0.20)}{0.44}\, y_{t-2} + \underset{(0.18)}{0.19}\, y_{t-3} + \underset{(0.16)}{0.05}\, y_{t-4} \right.$$
$$\left. + \underset{(0.25)}{0.31}\, z_{t-1} - \underset{(0.34)}{0.63}\, z_{t-2} + \underset{(0.35)}{0.02}\, z_{t-3} + \underset{(0.27)}{0.91}\, z_{t-4} \right) \mathcal{G}\left(s_t; \hat{\gamma}, \hat{c}\right) + \hat{\nu}_t$$

where

$$\mathcal{G}\left(s_t; \hat{\gamma}, \hat{c}\right) = \left\{ 1 + \exp\left[-\underset{(2.10)}{5.21}/\sigma_{s_t} \left(z_{t-1} - \underset{(—)}{0.87} \right) \right] \right\}^{-1} \tag{27}$$

The AIC of the estimated model is -4.08.

These functions are better illustrated in figures 2 – 5. As seen, the time series dynamics appear to switch between the regimes when the economy and the financial sector move from the less volatile to the more volatile conditions. While the regime shifts tend to coincide whether the financial stress index or the national activity index is used as a transition variable, notably, there is considerable additional information, when both variables are used, as suggested by the AIC measures.

3.2 Interpretation: Generalized Impulse–Response Analysis

To better illustrate the model dynamics, figures 2 – 5 also feature the generalized impulse–response (GIR) functions. The approach, proposed by Koop et al. (1996), is imperative when analyzing the nonlinear model dynamics, which, unlike linear models, are not invariant to the information set prior the shocks, the sign and magnitude of the shocks, as well as, the idiosyncratic disturbances that occur throughout the forecast horizon. The method implements bootstrap resampling, and thus generates an empirical distribution around the expected GIRs at each point in the horizon, which allows for additional information about the impulse response functions beyond their first moments.

A GIR at a horizon h, for a given shock ν, and a subset of histories $\mathcal{H}$, is defined as:

$$\text{GIR}\left(h, \nu, \mathcal{H}\right) = E\left(y_{t+h} | \nu, \omega_t \in \mathcal{H}\right) - E\left(y_{t+h} | \omega_t \in \mathcal{H}\right) \tag{28}$$

where ω_t denotes a point in time with initial conditions from the considered subset of histories. The method is executed in a manner similar to Skalin and Teräsvirta (2002), wherein all available starting points are used as histories. Within each history, a total of 100 bootstrap vectors equal in length to 24 months are computed. This way, a sufficiently large number of forecast sequences are obtained to approximate the densities at each horizon. The bootstrap vectors are extrapolated both with and without the initial shock, by incorporating randomly sampled innovations from the pool of residuals of the estimated model. Additionally, GIRs are computed for the initial shocks measured as $\pm 2\sigma_\epsilon$, and are referred as “statistically significant” shocks, where σ_ϵ is the standard deviation of the residuals in the estimated model. The aforementioned steps are reiterated for all combinations of histories and initial shocks. Mean responses are obtained by averaging the realized GIRs across the bootstrap iterations and selected histories.

Additionally, we obtain regime-specific GIRs. In particular, GIRs are calculated for histories when the financial stress index is greater than one and the national activity index is less than one. This referred as the “volatile” regime. A total of 16 such histories are identified (see Figure 6). On the contrary, the “normal” regime is defined when the absolute values of the indices are less than 0.5, and 16 histories are randomly selected from this pool. A total of 500 bootstrap vectors are obtained per each history. The rest of the procedure is similar to the one described above. The results of this exercise are presented in Figure 7. Regime-specific asymmetries are apparent. Moreover, the graphs also illustrate sign-specific asymmetries, i.e. responses to positive shocks are not mirror images of those to negative shocks.

3.3 Out-of-Sample Forecast Evaluation

This paper uses a rolling window approach to obtain $P-1$ one-step-ahead forecasts. The first window starts in January 1991 and ends in December of 2009. Every successive window is rolled over by one month. As such, 47 out-of-sample forecasts are obtained from each considered model. The forecasting exercise, thus, is designed so that every estimation window contains the high-financial-stress/low-economic-activity period of 2008-2009. The selected transition variables and the functional form of the transition functions are maintained across the estimation windows. However, the autoregressive and the transition parameters are re-estimated in each window. The out-of-sample forecasts are then based on the estimated parameters.

Results of this forecasting exercise are presented in Table 2. The table reports AIC measures averaged across all rolling windows. As seen, the nonlinear models perform better than their linear counterparts, and the multivariate models perform better than their univariate counterparts, thus, the VSTAR model offering the best data fit.

Forecast errors are evaluated for possible bias and inefficiency. The error unbiasedness is equivalent of testing the null hypothesis of $H_0 : \alpha = 0$ in: $e_{i,t+1} = \alpha + \upsilon_{t+1}$, and the error efficiency is equivalent of testing $H_0 : \beta = 0$ in: $e_{i,t+1} = \alpha + \beta\hat{y}_{i,t+1|t} + \upsilon_{t+1}$. Results of these tests are presented in Table 2. Forecast errors across all models appear to be unbiased, but we failed to reject the efficiency hypothesis of forecast errors from all models except for the VSTAR model.

The root mean squared forecast errors (RMSFE) and the forecast accuracy measures suggest improved predictability of the national activity index, when the financial stress index is incorporated, i.e. financial stress does Granger cause economic activity. Moreover, nonlinear models, and VSTAR in particular, tend to improve forecast accuracy. So, nonlinearity matters.

4 Conclusion

Findings of this study support the hypothesis that financial stress Granger causes economic performance, in particular when nonlinearity is accounted for. The vector smooth transition autoregression outperforms its linear and univariate counterparts from the standpoints of both in-sample fit as well as out-of-sample forecast performance. Moreover, the dynamics vary considerably between the normal and the volatile regimes.

The out-of-sample forecast comparison is performed for the period after the most recent recession. This was done for the identification purposes, i.e. to capture the sufficiently large amount of observations in the volatile regime. The downside of it is that much of the forecast accuracy measures are assessed in the normal regime. Even so, the advantage of nonlinear modelling is apparent. That is, a more proper accounting of the underlying nonlinear dynamics has led to more accurate forecasts.

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Figures

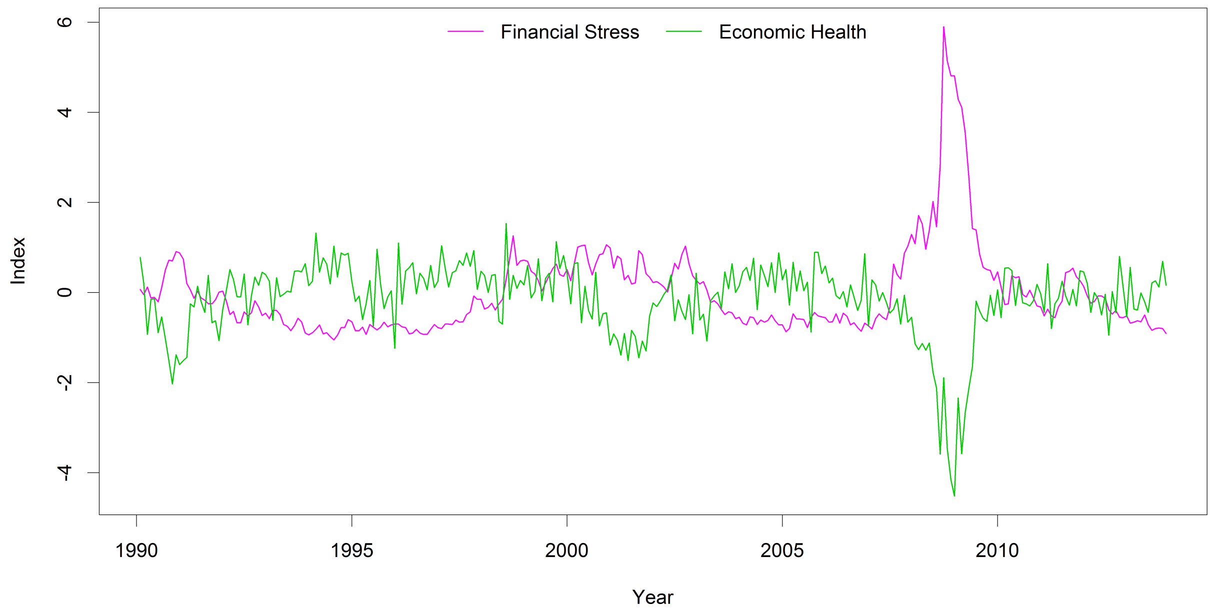


Figure 1: Time series of the FSI (Financial Stress) and the NAI (Economic Health).

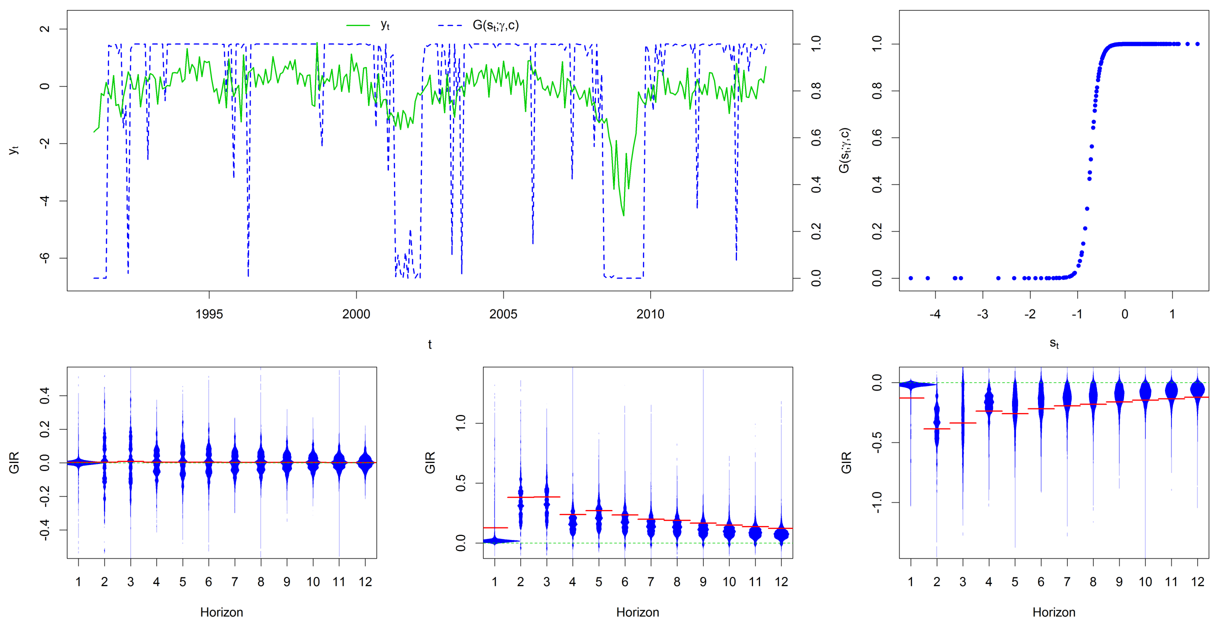


Figure 2: Transition function of the estimated STAR model with y_{t-3} as the transition variable.

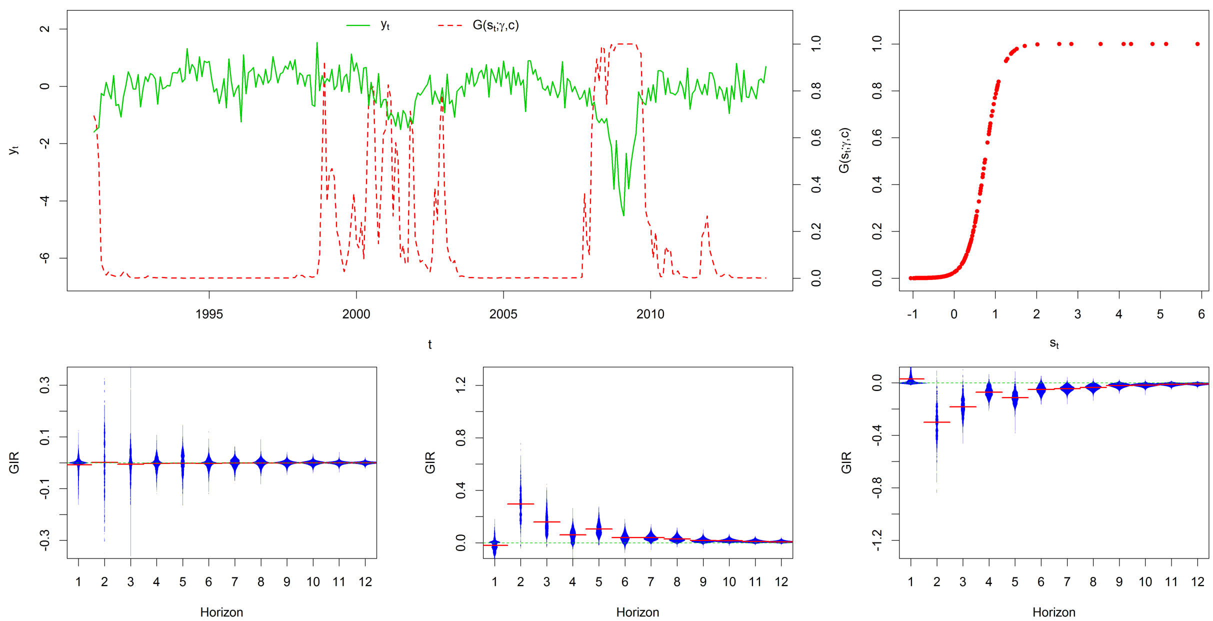


Figure 3: Transition function of the estimated STAR model with z_{t-1} as the transition variable.

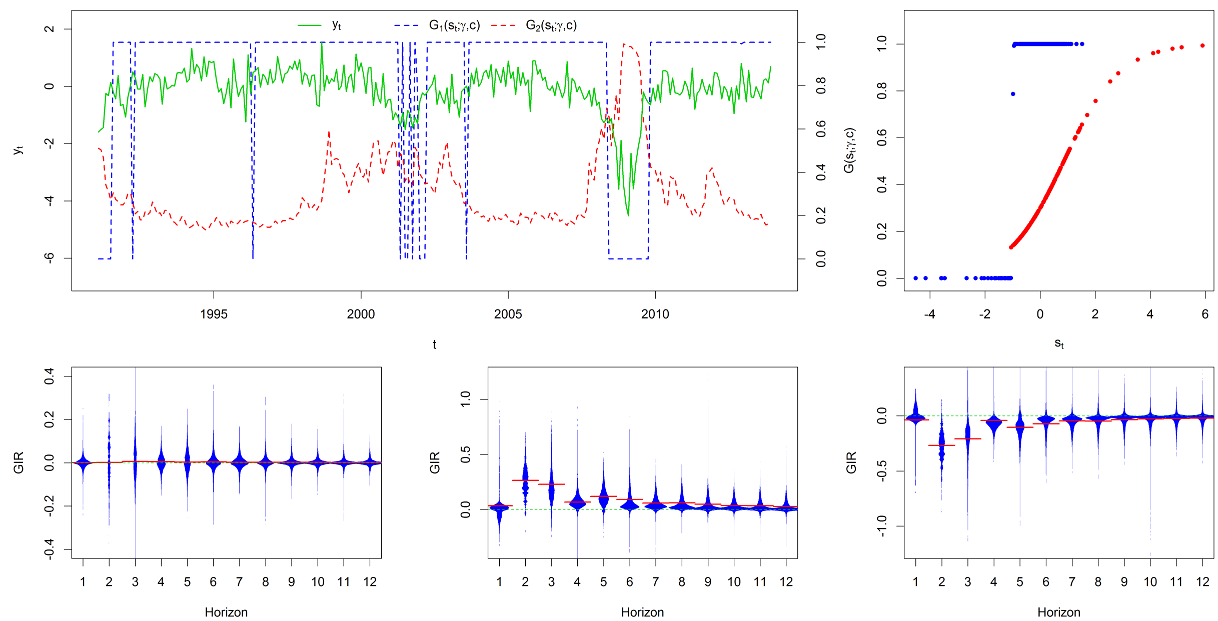


Figure 4: Transition function of the estimated STAR model with y_{t-3} and z_{t-1} as the transition variables.

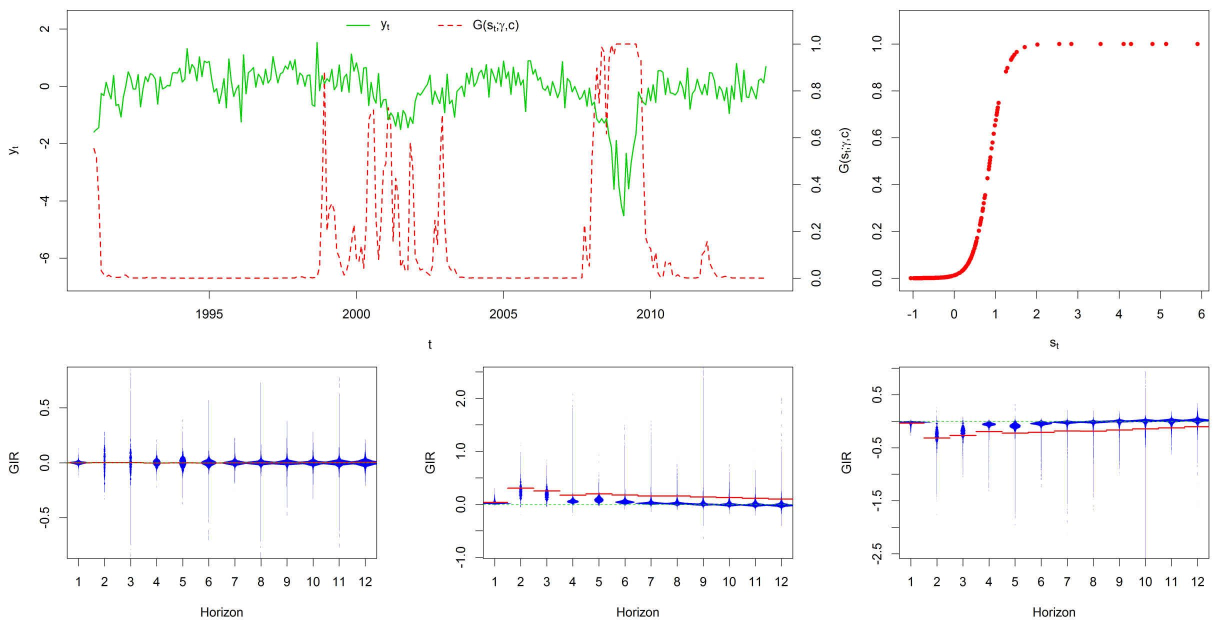


Figure 5: Transition function of the estimated VSTAR model with z_{t-1} as the transition variable.

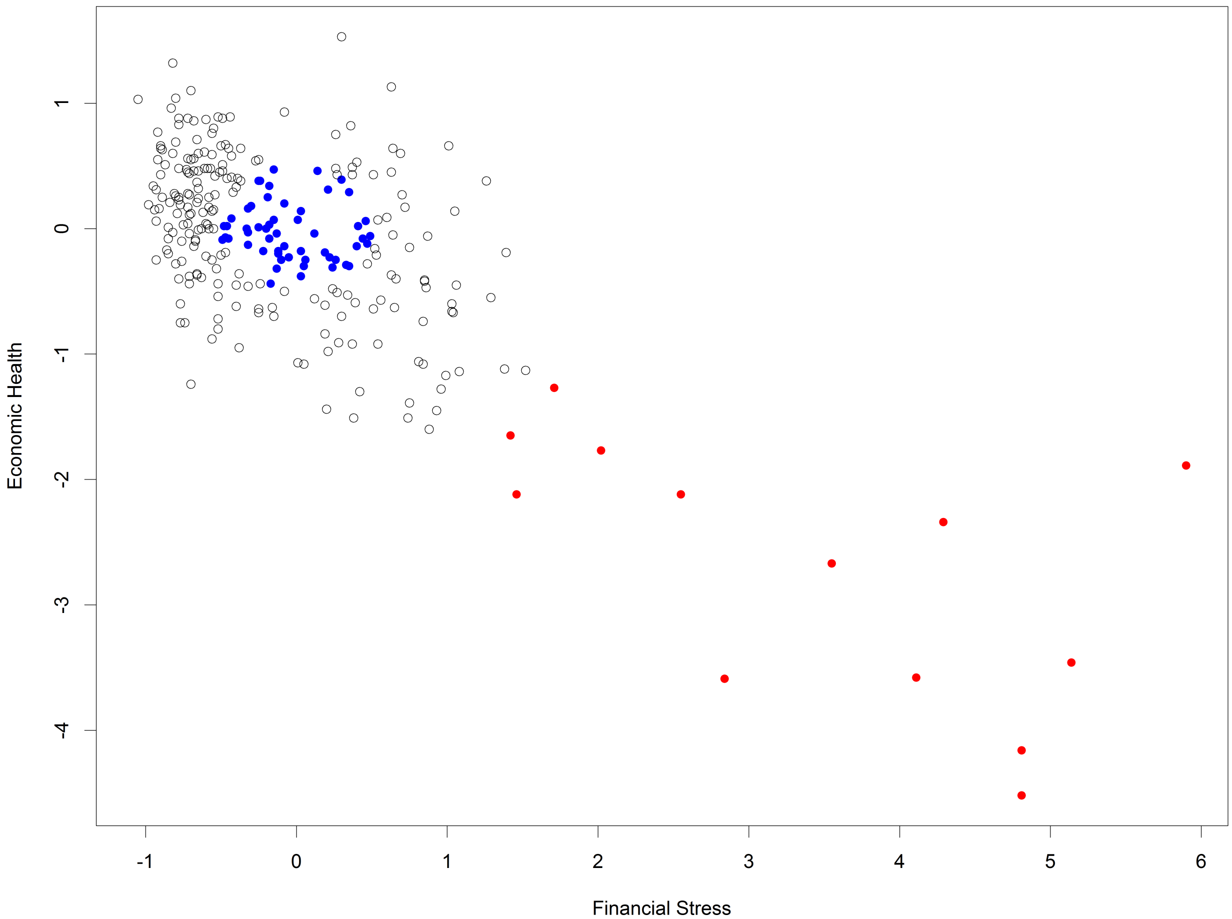
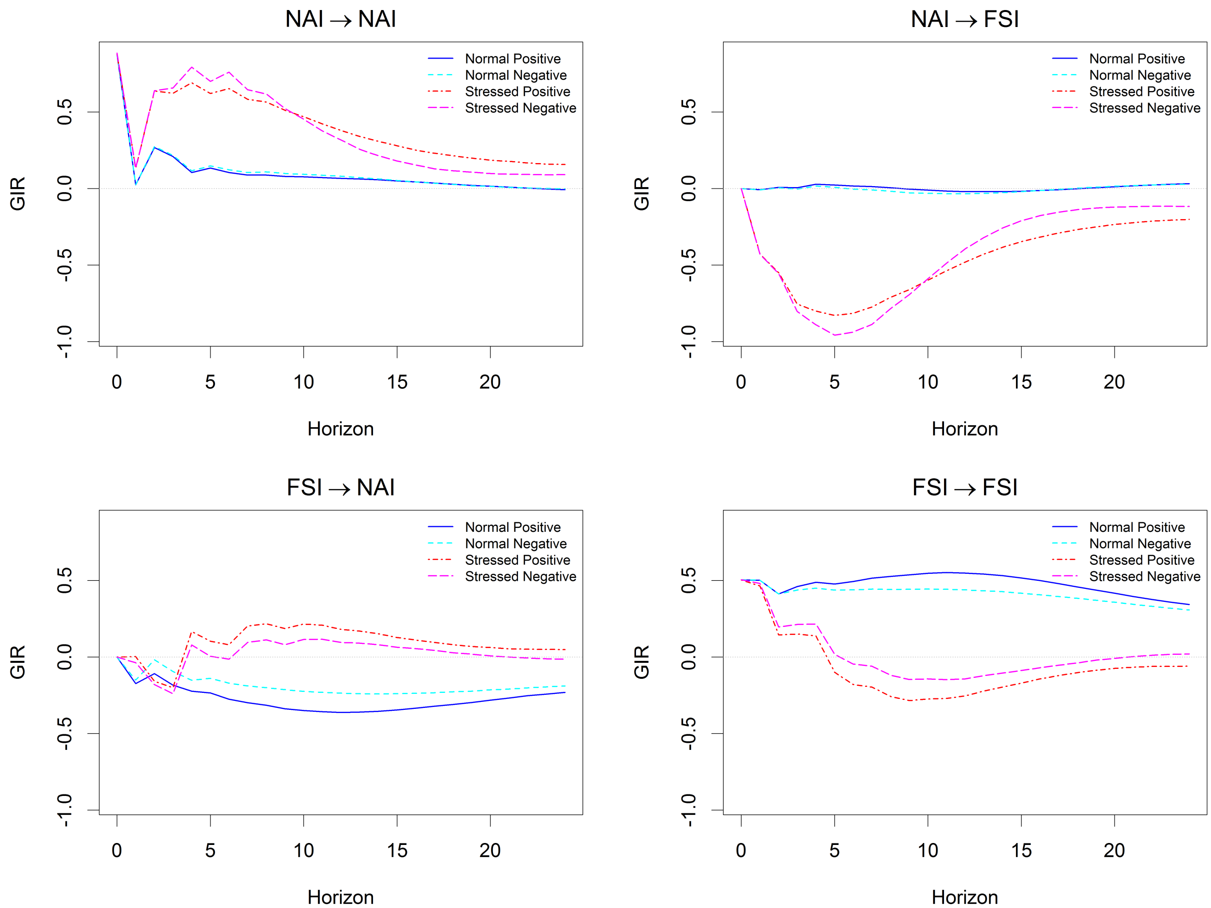


Figure 6: Scatter-plot of the financial stress and the national activity indices.

Note: the red dots identify histories of the relatively volatile (i.e. "financially stressed" or "economically unhealthy") regime; while the blue dots identify histories of the relatively normal regime.



Note: the GIRs of negative shocks are reversed (i.e. multiplied by -1) to ease the visualization of asymmetries in comparison with the GIRs of positive shocks.

Figure 7: Generalized Impulse-Responses during the normal and the stressed regimes.

Tables

Table 1: Nonlinearity Test Results

s_t	H'_{AR}	H'_{STAR}	H'_{MRSTAR}	H'_{VAR}	H'_{VSTAR}
y_{t-1}	<0.001	<0.001	0.039	<0.001	<0.001
y_{t-2}	<0.001	0.006	0.240	<0.001	0.023
y_{t-3}	<**0.001**	0.036	0.010	0.140	0.038
y_{t-4}	<0.001	0.200	0.075	0.032	<0.001
z_{t-1}	<**0.001**	<0.001	0.003	<**0.001**	0.030
z_{t-2}	<0.001	<0.001	0.015	<0.001	<0.001
z_{t-3}	<0.001	<0.001	0.005	<0.001	<0.001
z_{t-4}	<0.001	<0.001	0.004	0.005	<0.001

Note: the table entries are probability values associated with the null hypotheses denoted in the column headings. The values in bold identify the selected transition variable.

Table 2: Forecast Evaluation

	AR	STAR(y_{t-d})	STAR(z_{t-d})	MRSTAR	VAR	VSTAR
$\overline{AIC}$	-1.220	-1.270	-1.310	-1.360	-1.420	-1.450
H_u	0.540	0.780	0.449	0.578	0.910	0.561
H_e	0.005	0.003	0.043	0.008	0.018	0.100
$\hat{\sigma}$	0.447	0.455	0.426	0.454	0.423	0.416
$\hat{\sigma}_i/\hat{\sigma}_{\text{AR}}$		1.018	*0.953*	1.016	**0.946**	**0.931**
$\hat{\sigma}_i/\hat{\sigma}_{\text{STAR}(y_{t-d})}$			0.936	0.998	0.930	*0.914*
$\hat{\sigma}_i/\hat{\sigma}_{\text{STAR}(z_{t-d})}$				1.066	0.993	*0.977*
$\hat{\sigma}_i/\hat{\sigma}_{\text{MRSTAR}}$					0.932	*0.916*
$\hat{\sigma}_i/\hat{\sigma}_{\text{VAR}}$						*0.983*

Note: $\overline{AIC}$ is the average AIC across all estimated windows; H_u is the null of forecast error unbiasedness, and H_e is the null of forecast error efficiency. $\hat{\sigma}$ is root mean squared forecast error; $\hat{\sigma}_i/\hat{\sigma}_j$, $j = \text{AR}, \text{STAR}(y_{t-d}), \text{STAR}(z_{t-d}), \text{MRSTAR}, \text{VAR}$, are relative RMSFE measures. Values in italic denote statistical significance at $\alpha = 0.1$ level, while values in bold denote statistical significance at $\alpha = 0.01$ level.