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# Agricultural productivity and soil carbon dynamics: a bio-economic model

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## Abstract

The strong link between poverty, natural resources and the environment is apparent in smallholder agriculture: farmers are making repeated land use and management decisions while facing diverse resource endowments and significant environmental constraints on production. To investigate the likely effects of changes in agricultural practices on the natural resource base and on farmer welfare, we develop a bio-economic dynamic model of agricultural households in the western Kenya highlands. Our modeling framework extends economic farm household models to incorporate the dynamic nature of natural resource management and its implications for household welfare, and to permit a meaningful interface with biophysical processes through soil carbon management. Using an eight-year panel data set, the model combines econometrically estimated production and soil carbon flow equations in a dynamic programming framework. We use the model to determine the optimal management of the farming system over time in terms of the quantity of mineral fertilizer and crop residues to apply, taking into consideration initial resource endowments and prices. Understanding how soil resources respond to the combined applications of mineral and organic resources is important for improved resource allocation at the farm level and for national agricultural policy decisions.

**Keywords:** natural resource management, agricultural productivity, bio-economic model, soil carbon dynamics, western Kenya.

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# Agricultural productivity and soil carbon dynamics: a bio-economic model

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## Motivation

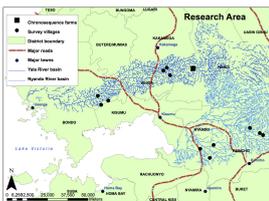
- Extreme poverty is often associated with environmental degradation (Dasgupta 2010).
- Cereal yields in Sub-Saharan Africa have remained stagnant over past 40 years, largely due to depletion of soil fertility (Sanchez 2002).
- Soil fertility constraints require combined applications of chemical fertilizer and organic resources to address crop nutrient demands and agronomic productivity and sustainability goals.

## Research goals

A bio-economic model of agricultural households in the Western Kenya Highlands to:

- study the link between poverty, agricultural production and natural resources,
- analyze soil carbon in a dynamic setting,
- consider initial resource endowment,
- determine optimal application rates of mineral fertilizer and crop residues, and
- evaluate the value of soil carbon.

## Western Kenya Highlands



- Densely populated and poor: 55% of population living below national rural poverty line.
- Current practices: maize monoculture, removal of maize residues for fodder and fuel, limited use of hybrid seeds and mineral fertilizer, no following.
- **Data:** agronomic experiment in 2005-2012, household and market surveys in 2011-2013.

## Focus on soil carbon

- Strong relation between soil carbon, soil quality, and crop productivity (Lal 2006).
- Land use and management decisions influence the stock of soil carbon (e.g., land conversion to agriculture, residue retention).
- Potential to sequester carbon and offset emissions from fossil fuels.

## Economic model

- A farmer cultivates a hectare of land with maize.
- $c_t$  stock of soil carbon in year  $t$ .
- $f_t$  quantity of mineral nitrogen applied in year  $t$ .
- $\alpha_t$  share of maize residues left on the field for soil fertility management at the end of year  $t$ .
- Maize production  $y_t = y(c_t, f_t, \alpha_t)$ .
- $c_{t+1} - c_t = g(c_t, f_t, \alpha_t)$ , where  $g(\cdot)$  is a function describing carbon dynamics.
- $c_0 = a$  initial level of soil carbon (given).
- $\pi_t = \pi(c_t, f_t, \alpha_t) = py(c_t, f_t, \alpha_t) - nf_t - q\alpha_t - m$  is annual profit, where  $p$ ,  $n$ ,  $q$  and  $m$  are prices of maize, nitrogen, crop residues, and per-hectare cost of planting and harvesting maize.
- $\rho$  discount factor =  $1/(1 + \delta)$  for discount rate  $\delta$ .

$$\max_{\{f_t, \alpha_t\}} \pi = \sum_{t=0}^{\infty} \rho^t [py(c_t, f_t, \alpha_t) - nf_t - q\alpha_t - m]$$

subject to

$$c_{t+1} - c_t = g(c_t, f_t, \alpha_t),$$

$$c_0 = a > 0, \text{ given.}$$

## Maize yield function

Generalized quadratic function of carbon stocks and nitrogen applications:

$$y_{kit} = \gamma_0 + \gamma_c c_{kit} + \gamma_{cc} c_{kit}^2 + \gamma_f f_{kit} + \gamma_{ff} f_{kit}^2 + \gamma_{cf} c_{kit} f_{kit} + \eta_k + \zeta_i + \theta_t + \xi_{kit} + \epsilon_{kit}$$

- $y_{kit}$  yield for treatment  $k$  on farm  $i$  at time  $t$ ,
- $c_{kit}$  carbon stock,
- $f_{kit}$  nitrogen fertilizer input,
- $\gamma_0, \gamma_c, \gamma_{cc}, \gamma_f, \gamma_{ff}, \gamma_{cf}$  estimated coefficients,
- $\eta_k, \zeta_i, \theta_t, \xi_{kit}$  fixed effects,
- $\epsilon_{kit}$  i.i.d., mean zero, normally distributed error.

## Soil carbon function

Addition from maize residues left on the field to replenish carbon and carbon loss from mineralization:

$$c_{t+1} - c_t = -Dc_t + A(\alpha_t M)^\beta,$$

- $D$  rate of annual soil carbon loss,
- $A, \beta$  and  $M$  parameters calibrated with **Rothamsted Carbon Model** for turnover of carbon in soil (Coleman and Jenkinson 1999).

Variable	Description	Value	Unit
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Maize yield function			
$\gamma_c$	Coefficient on $c_t^2$	0.113	—
$\gamma_{cc}$	Coefficient on $c_t^2$	-0.0004	—
$\gamma_f$	Coefficient on $f_t$	27.038	—
$\gamma_{ff}$	Coefficient on $f_t^2$	-41.295	—
$\gamma_{cf}$	Coefficient on $c_t f_t$	-0.218	—
$\gamma_0$	Constant	-0.810	—

Soil carbon equation			
$D$	Rate of soil carbon loss	0.11	—
$A$	Carbon plant input parameter	4.45	—
$\beta$	Carbon plant input parameter	0.79	—
$M$	Total residues	1	—

Prices			
$p$	Price of maize	356	\$ Mg <sup>-1</sup>
$n$	Price of nitrogen fertilizer	4,337	\$ Mg <sup>-1</sup>
$q$	Opportunity cost of crop residues	397	\$ share <sup>-1</sup>
$m$	Maize production cost	448	\$ ha <sup>-1</sup>
$\delta$	Discount rate	10	%

Initial conditions			
$c_0$	Depleted soils	20	Mg ha <sup>-1</sup>
	Medium-fertility soils	33	Mg ha <sup>-1</sup>
	Fertile soils	55	Mg ha <sup>-1</sup>

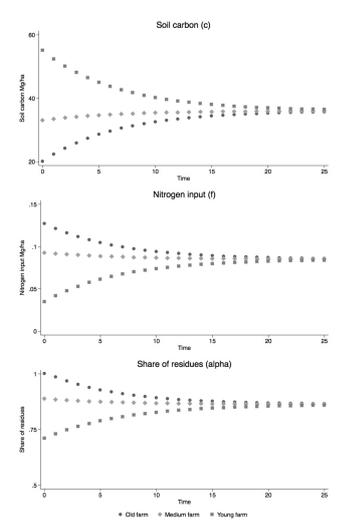
## Steady-state analysis

Variable	$\delta = 5\%$	$\delta = 10\%$	$\delta = 15\%$
Carbon stock, $c_{ss}$	40.45	35.97	20.15
Maize yield, $y_{ss}$	4.19	4.04	3.50
Nitrogen input, $f_{ss}$	0.07	0.08	0.13
Share of residues, $\alpha_{ss}$	1	0.86	0.41
Value of carbon, $\lambda_{ss}$	148.23	120.40	107.90

Discounted present value of annual profits,  $\delta = 10\%$ :

- Depleted soils:  $\pi = \$1,133$ ,
- Medium-fertility soils:  $\pi = \$2,735$ ,
- Fertile soils:  $\pi = \$5,332$ .

## Simulations



## Policy implications

- Value of soil carbon is high: 108-148 \$/Mg.
- Both mineral fertilizer and organic resources are needed to replenish carbon and sustain yields.
- Initial soil fertility level matters.
- Over 25 years: 58 and 11 Mg CO<sub>2</sub>/ha sequestered on depleted and medium-fertility soils, 69 Mg CO<sub>2</sub>/ha lost from fertile soils.

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