RELATIVE PERFORMANCE OF SEMI-PARAMETRIC NONLINEAR MODELS IN FORECASTING BASIS

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Relative Performance of Semi-Parametric Nonlinear Models in Forecasting Basis

Abstract

Many risk management strategies, including hedging the price risk using forward or futures contracts require accurate forecasts of basis, i.e., spot price minus the futures price. Recent literature in this area has applied nonlinear time-series models, which are refinements of the linear autoregressive models that allow the parameters to transition from one regime to another. These parametric nonlinear models, however, involve complex estimation problems, and may diminish forecasting accuracy, especially in longer horizons. We propose using a semi-parametric, generalized additive model (GAM) that may improve the forecasting performance with its simplicity and flexibility while still accounting for nonlinearities in local prices and basis. Empirical results based on weekly futures and spot prices for North Carolina soybean and corn markets support evidence of nonlinear effects in basis. In general, generalized additive models seem to yield better forecasts of basis.

Key words: basis, futures markets, forecasting, generalized additive models, nonlinear models

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Introduction

Many risk management strategies, including hedging the price risk using forward or futures contracts, involve forming price expectations and making storage decisions, which in turn require accurate forecasts of basis, i.e., spot price minus the futures price (CBOT, 1990; Hatchett, Brorsen, and Anderson, 2010). Market frictions and uncertainties involved in production may cause nonlinearities in prices over time, increasing the basis unpredictability. As a result, level of confidence in forming expectations of future basis may diminish, obstructing producers' ability to effectively manage their price risk.

Basis forecasting has been an area of continued research interest. The most typical approach to forecasting basis in the literature has been averaging historical basis levels across years. This approach is widely used because of its simplicity and usually referred as the naïve forecast. One disadvantage of using historical averages is that they do not incorporate current market information. Earlier studies have used different number of years in their historical moving averages. Dhuyvetter and Kastens (1998), for example, conclude that longer averages ranging from three to seven years is optimal, while Taylor, Dhuyvetter, and Kastens (2006) find that the one-year average is optimal for harvest basis forecasts for several crops. More recently, Hatchett, Brorsen, and Anderson (2010) revisit this commonly-used historical moving average approach to forecast basis and conclude the optimal length of moving average is not constant over time and suggest using longer moving averages in the absence of structural breaks and last year's basis in the presence of structural changes.

Another common approach to forecasting basis has been using mean-reverting time-series models such as linear autoregressive integrated moving average (ARIMA) models (Jiang and Hayenga, 1997; Sanders and Manfredo, 2006). However, it has been generally found that naïve forecast models perform better in the long horizon, while some linear time-series models perform well only in the short horizon. The more recent literature in this area has applied smooth-transitioning autoregressive (STAR) model, which is a refinement of the autoregressive model that allows the parameters to smoothly transition from one regime to another (Sanders and Baker, 2012). In a similar area, Goodwin and Piggott (2001), in their study of spatial linkages between crop markets, use a threshold autoregressive (TAR) model, which allows for discrete regime switches. Although these approaches follow a long progression of the development of time-series methods for identifying nonlinear effects in empirical models, several issues remain in practice. First, the variable causing the "regime shift" is assumed to be known even though economic theory rarely dictates a likely candidate. Second, there is typically little or no guidance on what the most appropriate functional form or transition function for a given application might be. Finally, the specifications applied in recent work typically involve a significant number of additional parameters to be estimated, and thus add significantly to the complexity of estimation and hypothesis testing.

It is acknowledged in the forecasting literature that the more complicated and sophisticated the forecasting model is, the poorer forecasting performance that particular model yields. Furthermore, simple models are difficult to improve upon for more distant-horizon forecasting. To this extend, we propose using a nonparametric, generalized additive model (GAM) that may improve the forecasting performance with its simplicity and flexibility, and at the same time, will still account for nonlinearities in the local prices and basis.

In particular, we aim to compare the forecasting accuracy of GAM to those of other nonlinear but parametric time-series models, such as TAR and STAR. In addition, we perform forecasting using standard naïve and linear AR models for comparison of the forecasting performance between linear and nonlinear models. Our application is to corn and soybean markets in North Carolina. Preliminary results support evidence for nonlinearities in basis in these markets. Forecasts based on semi-parametric, nonlinear generalized additive models seem more accurate than their linear counterparts.

Earlier Work on Basis Forecasting

There is extensive literature analyzing the determinants of basis using structural models. Factors used as explanatory variables include seasonality, demand (consumption), supply (production), inventories, storage costs (carrying charge), transportation costs (for the non-delivery points), insurance, and interest rate (Martin, Groenewegen, and Pidgeon, 1980; Garcia and Good, 1983; Bailey and Chan, 1993).

Earlier studies on forecasting basis generally test the forecasting accuracy of using moving averages of various lag lengths to form basis expectations. Hauser, Garcia, and Tumblin (1990), for example, compare several naïve models with one- or three-year historical averages to forecast soybean basis for 10 Illinois elevators and find that historical average models perform comparably to models incorporating current market information. Kastens, Jones, and Schroeder (1998) compare relative performance across various competing naïve and futures-based localized basis forecasts for corn, soybean, wheat, and livestock in Kansas and Missouri using regression models of forecast errors. They show that complex regression models capturing nonlinearity do not improve forecasting accuracy, and suggest that historical localized basis to deferred futures contract should be used as a forecast. Tonsor, Dhuyvetter, and Minert (2004) study weekly basis on live cattle and feeder cattle in Kansas. They find that using time-to-futures-contract-estimation technique does not improve forecasting efficiency and the optimal number of years to include in a historical average depends on the particular time period. They also show that incorporating current basis information increases forecast accuracy.

Taylor, Dhuyvetter, and Kastens (2006) compare basis forecasting methods for wheat, soybeans, corn, and milo basis in Kansas. They distinguish between harvest and post-harvest basis forecasts. They find that while the optimal harvest and post-harvest basis forecast for corn, milo, soybeans basis is the historical one-year average, the optimal harvest and post-harvest basis forecasts for wheat are the historical five-year and one-year averages, respectively. They show that incorporating current market information (i.e. basis deviation from historical average) improves forecast accuracy for post-harvest basis. Sanders and Manfredo (2006) also compare basis forecasting methods for soybean, soybean oil, and soybean meal in Central Illinois. They consider three naïve models (the historical five-year average for the month being forecasted, basis in the same month a year ago, the most recent observed basis) and two time-series models (and ARMA and VAR models). They show that the historical five-year average (as argued as the best in the literature) is not the best method for all commodities. They further show that time-series models do better for short-horizon forecasts but the gain from using them is much smaller in long-horizon forecasts.

More recently Hatchett, Brorsen, and Anderson (2010) reassess the earlier studies on the optimal length of moving historical averages to be used in basis forecasts of hard wheat, soft wheat, corn, and soybeans. They find that the optimal forecast length is generally shorter than what previous studies suggested and argue that this is due to structural changes. They recommend using longer moving averages for locations or time periods when there were no structural changes and the previous year's basis when a structural change occurred.

Our paper takes somewhat a similar approach and revisits methods to forecasting basis. However, our main focus is on the performance of nonlinear (and non-parametric) time-series models as recent developments in commodity markets signal nonlinearities in prices over time.

Methodology

We propose using potentially fully-nonparametric and nonlinear models to forecast basis in grain markets. Specifically, we use the Generalized Additive Models (GAM) proposed by Hastie and Tibshirani (1986, 1990). These models assume that the mean of the response variable depends on an additive predictor through a link function. The appealing feature of GAMs is their ability to deal with highly nonlinear and non-monotonic relationships between the response and the set of explanatory variables without imposing strict parametric restrictions in the model.

Consider a standard linear regression model given by:

(1)
$$y_t = \beta_0 + \sum_{j=1}^{k} \beta_j X_{jt} + \varepsilon_t,$$

where y_t, the response variable, is assumed to be a linear additive function of k independent variables, $X_{jt}, j = 1, \cdots k$. Additive models generalize this linear model by modelling the dependent variable as:

(2) $$y_t = \beta_0 + \sum_{j=1}^{k} f_j(X_{jt}) + \varepsilon_t \ ,$$

where $f_j(\cdot)$ is an unspecified smooth nonparametric function. Estimation of this model can be accomplished by fitting a weighted additive model through a backfitting algorithm. The estimated nonparametric components, $\hat{f}_j(\cdot)$, can be thought of as estimates of the functions transforming each explanatory variable so as to maximize the fit of their additive combination to the dependent variable, subject to constraints about the smoothness of the link function. We focus on two types of link functions; locally weighted regression smoothers (LOESS) and cubic smoothing splines (SPLINE), which have well-understood properties.

Specifically, we model the basis as an additive function consisting of parametric and nonparametric terms. We argue that previous period's basis has linear and nonlinear effects on the current period's basis, while time trend has linear effects:

(3) $$basis_t = \beta_0 + \beta_1 trend_t + \beta_2 basis_{t-1} + f_1(basis_{t-1}) + \varepsilon_t.$$

The actual values of $\hat{f}_1(\cdot)$ are not meaningful per se, but the shape of the fitted function reveals the nature of any estimated nonlinearities in the basis. We consider various metrics of the goodness of fit, providing a guide as to whether the fitted nonlinear function is distinguishable from and favorable to a linear fit. We, then, compare the forecasting accuracy of different models using various forecast error criterions.

Data and Empirical Results

There have been several important developments in corn and soybean markets since 1990s. For instance, soybean production in Brazil and Argentina has increased dramatically between 1990 and 2002. This put downward pressure on the U.S. prices (Plato and Chambers, 2004). Corn, soybean, and wheat futures contracts experienced poor convergence between late 2005 and 2008. Low contract storage rates at the Chicago Board of Trade (CBOT), structural problems in the delivery mechanism, and changes in storage market conditions caused cash prices to be delinked from futures prices (Irwin et al., 2011). Irwin and Good (2009) conclude that a new era with a permanent upward shift in the level of price and volatility started for corn, soybean, and wheat prices after 2006. Bekkerman, Goodwin, and Piggott (2008) also show that soybean markets have experienced instability between 2007 and 2008 due to lower supply and stable demand and the increase in the probability of soybean rust infections. These developments in corn and soybean markets can be observed in Figure 1, in which North Carolina spot prices are presented. In particular, we observe a permanent upward trend in prices toward the end of the sample, for both corn and soybean markets.

Figure 2 presents plots of basis in North Carolina grain markets. Similar to Figure 1, we can observe increased volatility in basis after 2006. The large increase in the soybean basis toward the end of the sample period can be attributed to recent developments in these markets. In April 2012, soybean futures prices hit a seven-month high due to strong export demand from China (16% increase from previous month, 44% increase from previous year), the announcement by USDA about increased sales of soybeans to China and other destinations, and decreased supply in South America due to adverse weather (DJN, 2012).

All these events in corn and soybean markets suggest possible structural changes and nonlinearities in the basis behavior. Thus, these two commodities provide a good avenue to test forecasting performance of generalized additive models.

Weekly local spot prices of corn and soybean are provided by North Carolina State University, Grain Marketing Extension Program.[1] Specifically, corn prices are for the cities Candor, Candor, Cofield, and Roaring River for the sample period of January 1988-July 2013; and soybean prices are for the cities Elizabeth City, Fayetteville, and Raleigh for the sample period of January 1980-July 2013.

For the futures price data we use the settlement prices of corn and soybean contracts traded at the Chicago Mercantile Exchange (CME) Group. The data are obtained from the Commodity Research Bureau (CRB). We construct a continuous futures price series by rolling over the nearby contracts at the end of the month preceding the delivery month. All price series are recorded weekly on Wednesdays.

We start with splitting the whole sample at the cutoff dates to "holdout" the last 24 observations for forecast evaluations. Then with the first part of the sample, we estimate a simple linear AR(p) and two GAMs, with LOESS and SPLINE smoothers, respectively. From each of these models, we compute multi-period forecasts for horizon 1 to 24 (weeks). Using the holdout data, we compute the Mean Absolute Percent Error (MAPE) of forecasts. Finally, we compare the forecasting performance of different models based on these statistics.

Table 1 presents the estimation results of autoregressive GAM for NC soybean markets. The model allows for a linear trend term, however the previous period's basis has a linear

[1] Weekly data are proprietary; however, monthly and annual summaries can be downloaded from http://www.ces.ncsu.edu/depts/agecon/piggott/grainmarket/datasets.html

parametric and a nonlinear nonparametric component. We use LOESS smoother in the GAM.[2] Linear trend estimates are practically zero. Both linear and smooth components of last period's basis are significant in all three soybean markets. Figure 3 shows the nonlinear effects of $basis_{t-1}$ on $basis_t$. We observe that when the last period's basis is small, its effect on this period's basis is closer to a linear effect. However, a large and negative $basis_{t-1}$ has a larger and positive effect on the current period's basis. This holds in all three soybean markets, Raleigh, Fayetteville and Elizabeth City. A large and positive basis in the previous period, on the other hand, triggers a large and negative response in the current period's basis in Raleigh and Fayetteville markets. The same nonlinear response, however, is positive in Elizabeth City markets (though, more subtle in size).

Table 2 presents the estimation results of GAM for North Carolina corn markets. The results are very similar to those reported for soybean markets. In particular, both linear and smooth components of last period's basis are significant in all corn markets. Again, linear trend term seems to be practically zero. Figure 4 shows the nonlinear effects of $basis_{t-1}$ on current period's basis. A large and negative $basis_{t-1}$ has a larger and positive effect on the current period's basis. Similar to soybean markets, we observe asymmetric response to last period's basis. In particular, a large positive basis in the last period leads to a much smaller change in the current period's basis compared to a negative $basis_{t-1}$ of the same size.

Table 3 and 4 report the forecasting performance of semi-parametric GAMs and the linear AR model. In soybean markets (Table 3), in general, forecast errors measured by MAPE from GAMs are smaller than those from a linear AR model. In addition, as the forecast horizon

[2] Results with SPLINE smoother are similar, and therefore they are not reported for the sake of brevity.

increases, forecast accuracy decreases somewhat quickly. In general, SPLINE smoother leads to better forecasts compared to LOESS smoother in GAMs. Elizabeth City has the smallest one-step-ahead forecast errors. Forecasting performance for Raleigh and Fayetteville are very similar. This is not surprising as these two markets are geographically very close to each other.

In corn markets (Table 4), we observe that forecast errors and MAPEs are much smaller compared to those reported for the soybean markets. Similar to soybean markets, nonlinear GAMs lead to better forecasts for corn basis almost at all horizons than the linear AR model does. Except for Cofield, a SPLINE smoother in GAM results in better forecast accuracy than the LOESS smoother. As usual, forecast errors increase with the forecast horizon.

Concluding Remarks

In this paper, we suggest using semi-parametric Generalized Additive Models (GAMs) as an alternative to traditional parametric time-series models in forecasting basis. Using weekly North Carolina corn and soybean basis data, we show that nonlinear effects obtained from GAMs were significant. Semi-parametric GAMs in almost all cases yield better forecasts than a linear model. What remains is adding forecasts from various parametric, nonlinear models, such as TAR and STAR. Moreover, bootstrapping of forecast standard errors needs to be performed to determine confidence intervals of forecasts based on GAMs.

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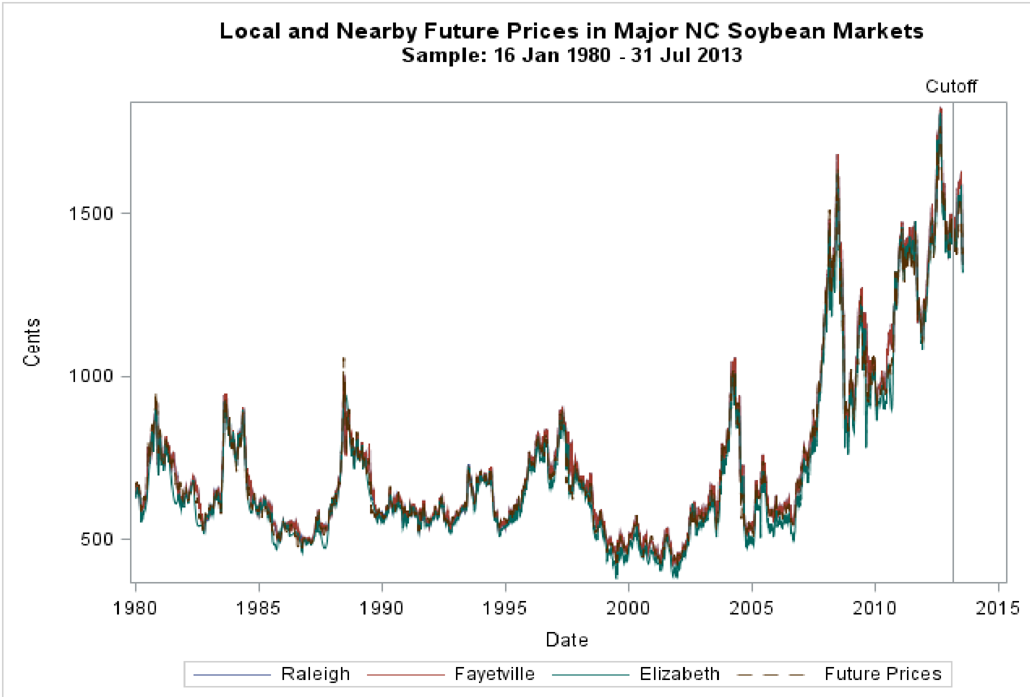
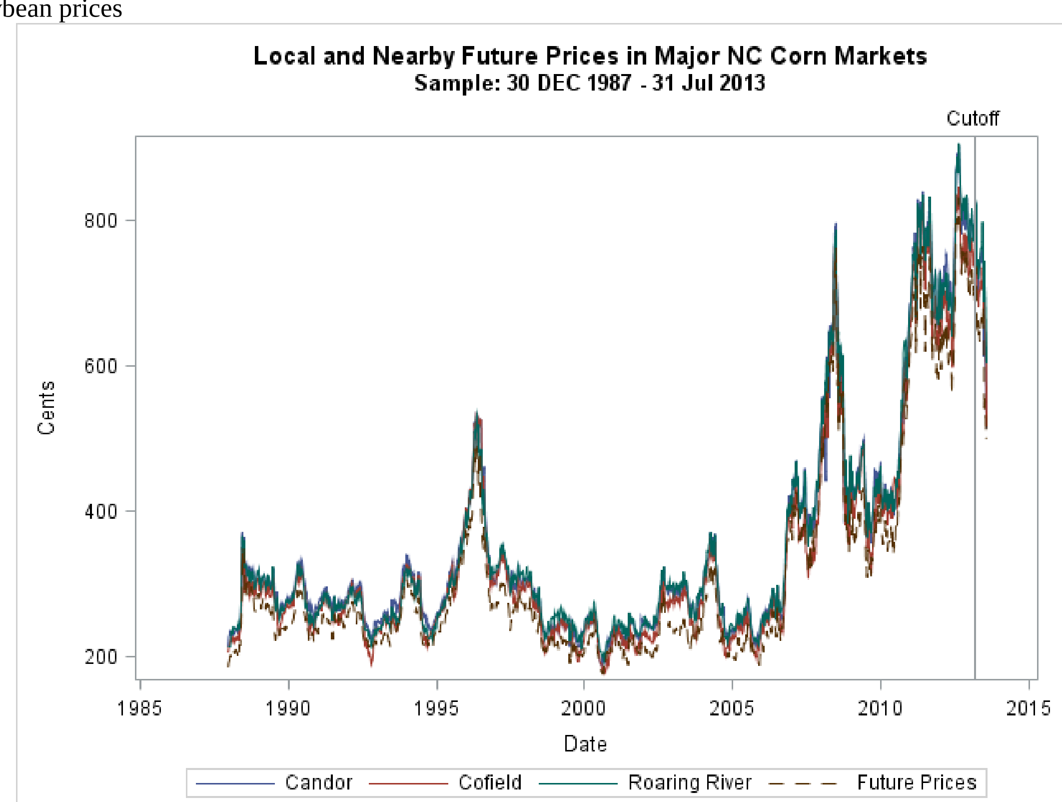
FIGURES

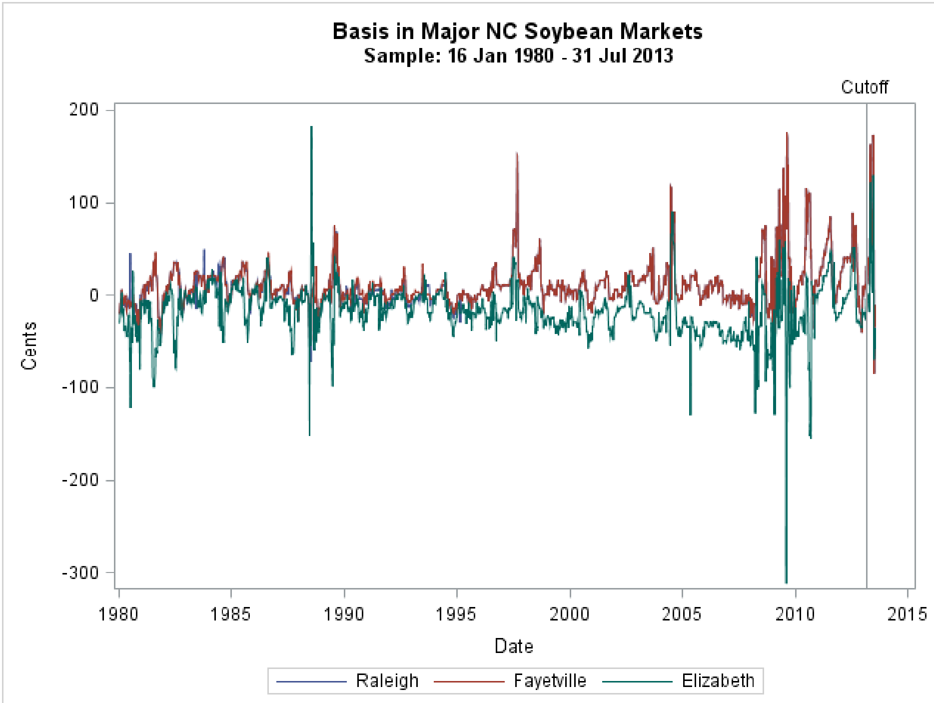


1a) Soybean prices

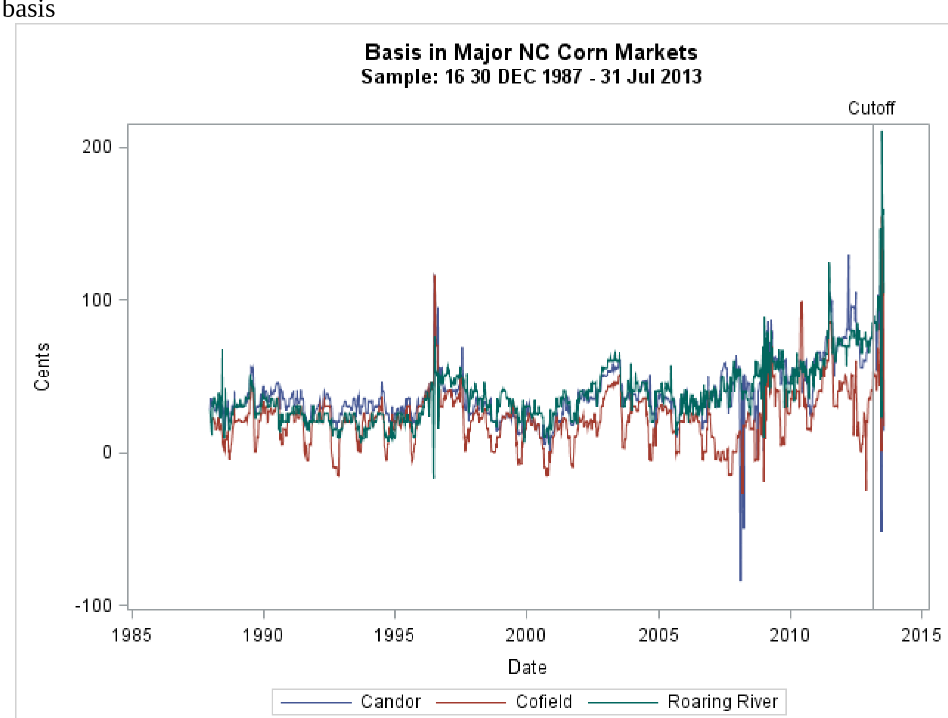


1b) Corn Prices

Figure 1. Soybean and corn prices in North Carolina markets.



1a) Soybean basis



1b) Corn basis

Figure 2. Soybean and corn basis in North Carolina markets.

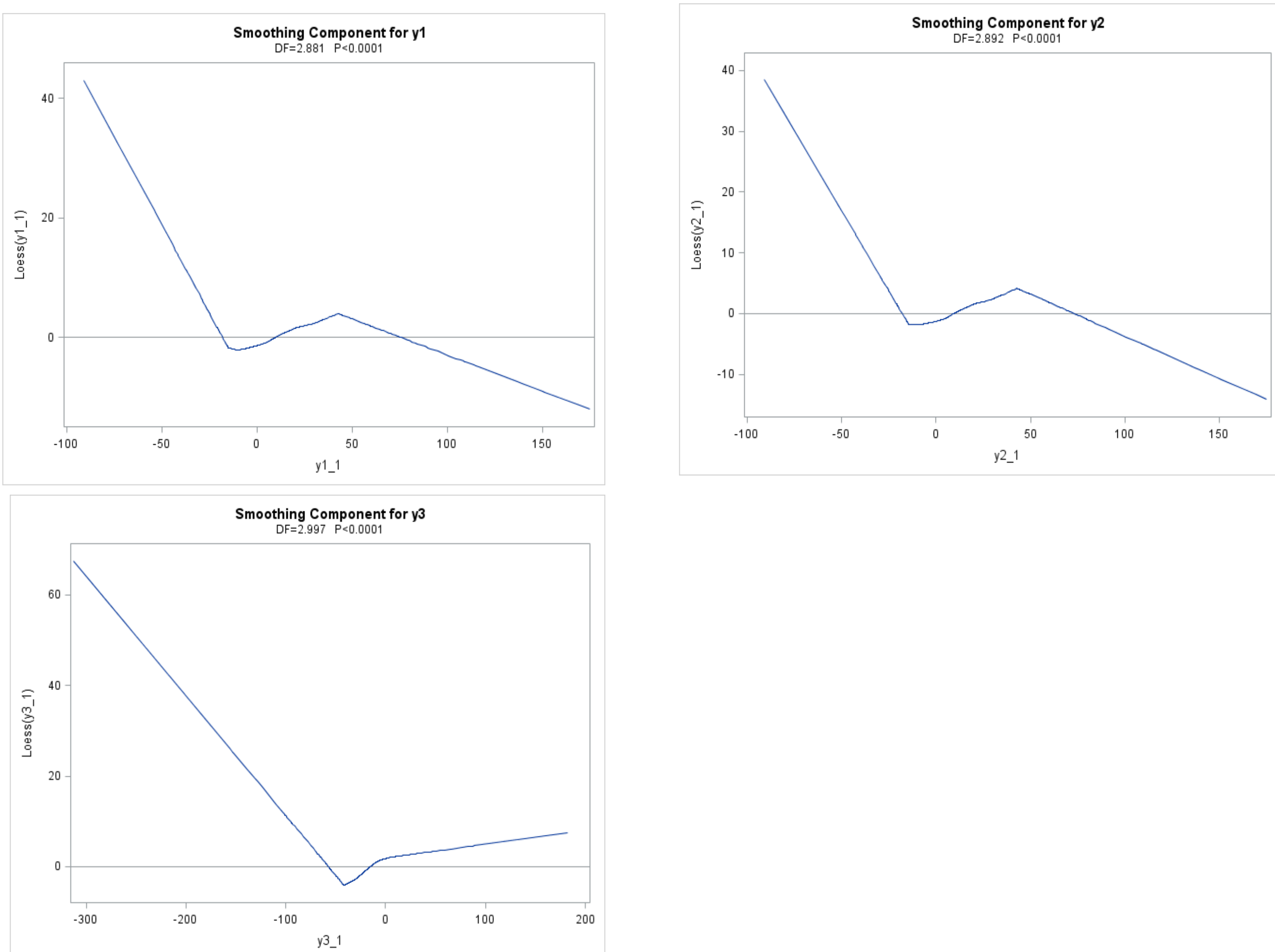


Figure 3. Nonlinear effects of $Basis_{t-1}$ on $Basis_t$ in North Carolina soybean markets.

Note. y1, y2, y3 denote Raleigh, Fayetteville, and Elizabeth City soybean markets, respectively.

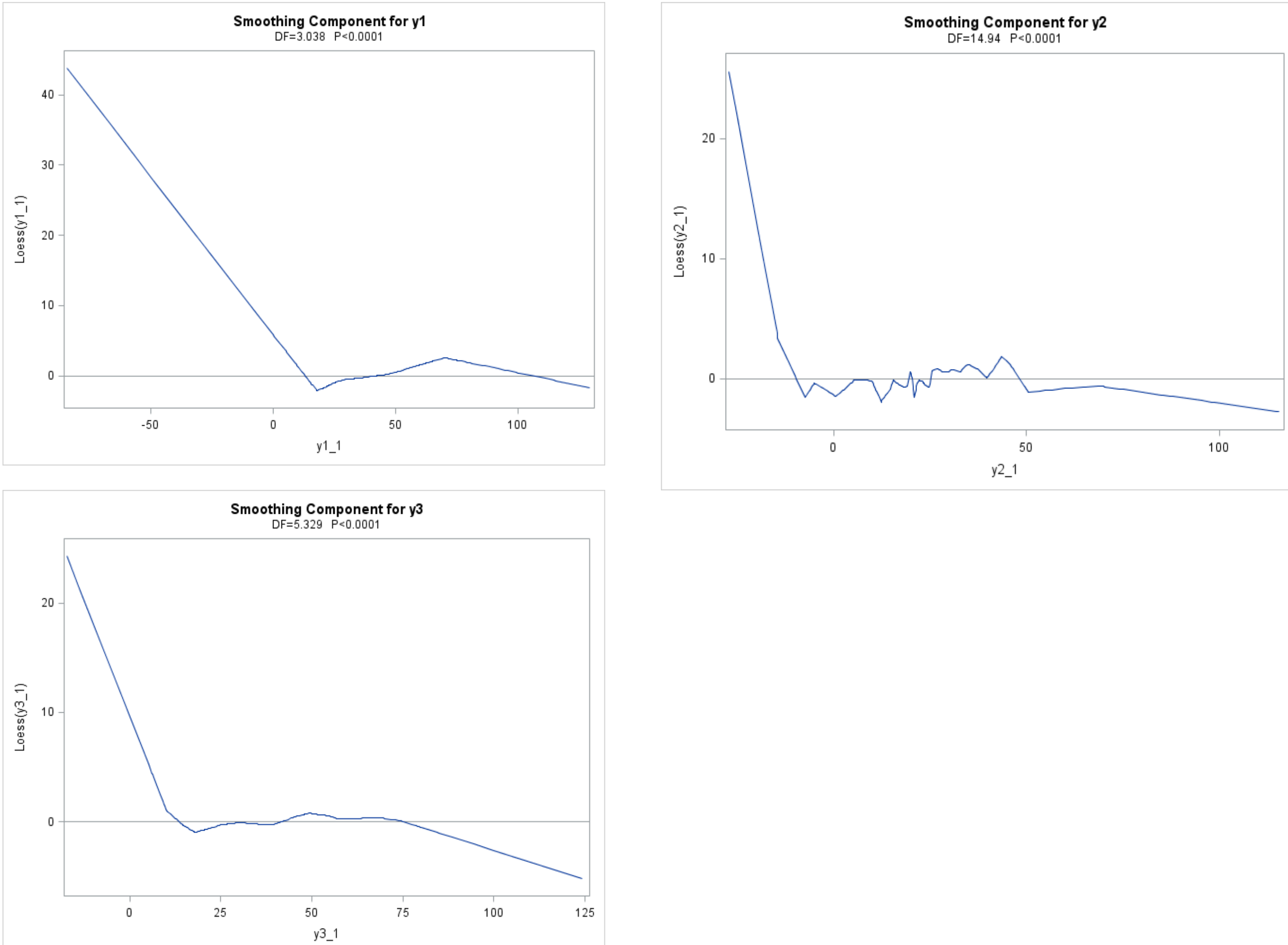


Figure 4. Nonlinear effects of $Basis_{t-1}$ on $Basis_t$ in North Carolina corn markets.

Note. y1, y2, y3 denote Candor, Cofield, and Roaring River corn markets, respectively.

TABLES

Table 1. Autoregressive GAM Estimation Results for North Carolina Soybean Markets.

	Raleigh		Fayetteville		Elizabeth	
	Linear Parameters					
	Estimate	**Pr > |t|**	**Estimate**	**Pr > |t|**	**Estimate**	**Pr > |t|**
Intercept	1.25	0.12	-1.43	0.23	1.58	0.24
trend	0.00	0.00	0.01	0.00	0.00	0.12
Linear(basis_1)	0.80	<.0001	0.81	<.0001	0.77	<.0001
	Smoothing Parameter					
	Estimate	**Pr > ChiSq**	**Estimate**	**Pr > ChiSq**	**Estimate**	**Pr > ChiSq**
Loess(basis_1)	0.62	<.0001	0.62	0.00	0.71	<.0001

Note. LOESS smoother is used.

Table 2. Autoregressive GAM Estimation Results for North Carolina Corn Markets.

	Candor		Cofield		Roaring River	
	Linear Parameters					
	Estimate	**Pr > |t|**	**Estimate**	**Pr > |t|**	**Estimate**	**Pr > |t|**
Intercept	2.19	0.10	0.62	0.56	-3.71	0.00
trend	0.00	0.04	0.00	0.20	0.00	<.0001
Linear(basis_1)	0.86	<.0001	0.91	<.0001	0.85	<.0001
	Smoothing Parameter					
	Estimate	**Pr > ChiSq**	**Estimate**	**Pr > ChiSq**	**Estimate**	**Pr > ChiSq**
Loess(basis_1)	0.57	<.0001	0.11	<.0001	0.31	<.0001

Note. LOESS smoother is used.

Table 3. Relative Forecasting Performance of GAM in North Carolina Soybeans Markets.

		Horizon:								
Location:	**Model:**	1	2	3	4	6	8	12	18	24
Raleigh	AR	29.16	79.90	67.12	57.95	50.30	48.17	55.21	58.36	77.01
	GAM_LOESS	26.60	47.53	55.85	41.94	30.10	24.41	42.30	44.92	56.42
	GAM_SPLINE	12.09	29.80	41.84	32.09	26.51	21.45	38.66	42.29	54.56
Fayetteville	AR	23.98	63.59	57.70	50.57	44.76	44.06	52.45	56.23	75.52
	GAM_LOESS	25.67	46.54	55.31	41.67	29.75	24.11	42.26	44.78	58.20
	GAM_SPLINE	14.56	37.51	45.79	36.25	30.85	25.49	40.36	43.36	50.85
Elizabeth City	AR	9.60	13.46	132.56	195.14	268.05	2630.50	1789.16	1335.60	1031.09
	GAM_LOESS	8.93	15.78	168.28	139.78	113.24	648.63	457.72	333.89	273.18
	GAM_SPLINE	6.36	15.40	162.17	133.17	106.02	628.51	444.48	325.74	267.76

Note. Comparison criteria reported in the table is MAPE (Mean Absolute Percent Error). GAM_LOESS is an autoregressive GAM model with aLOESS smoother; and GAM_SPLINE represents an autoregressive GAM model with a SPLINE smoother. AR is the linear autoregressive model.

Table 4. Relative Forecasting Performance of GAM in North Carolina Corn Markets.

		Horizon:								
Location:	**Model:**	1	2	3	4	6	8	12	18	24
Candor	AR	3.8288	5.2238	9.5958	11.8347	14.4313	16.3996	17.0985	22.4286	44.3645
	GAM_LOESS	2.7693	2.7683	6.5582	5.9302	5.5573	5.4053	12.186	18.4007	47.2713
	GAM_SPLINE	0.5823	0.5828	4.1151	3.8813	3.8512	3.8713	11.1096	18.2775	45.2877
Cofield	AR	2.254	2.998	12.766	15.916	20.14	23.04	25.872	30.578	279.456
	GAM_LOESS	2.233	2.232	11.039	9.354	8.284	7.808	11.052	18.343	334.607
	GAM_SPLINE	4.29	4.29	12.912	10.805	9.8	9.353	12.533	19.454	321.039
Roaring River	AR	14.1478	14.2608	14.5546	16.2424	16.3691	16.627	18.0582	23.616	37.391
	GAM_LOESS	15.0965	10.7115	9.248	9.885	7.9719	7.5948	8.3278	9.5603	22.2997
	GAM_SPLINE	11.8719	10.5247	10.0739	11.1757	12.5875	11.8464	11.9767	12.8971	22.4454

Note. Comparison criteria reported in the table is MAPE (Mean Absolute Percent Error). GAM_LOESS is an autoregressive GAM model with a LOESS smoother; and GAM_SPLINE represents an autoregressive GAM model with a SPLINE smoother. AR is the linear autoregressive model.