Should we internalize intertemporal production externalities in the case of pest resistance?

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Abstract

Pesticides efficiency decreases with their global application by farmers. Within a strategic dynamic framework, this results in a classic intertemporal production externality. We analyze tax and subsidy schemes that can be used in order to internalize this externality. We show that they are able to restore socially optimal paths but that final time of pesticide use differs. With these schemes, farmers have a tendency to switch to alternative pest-control technology, as integrated pest management, earlier than is optimal. A lump-sum transfer is shown to be necessary to obtain a switching time equal to the socially optimal one, for the subsidy case only. Furthermore, the socially optimal switching time can be later than the one obtained under a situation without control.

Keywords stock externality, pest resistance, technology change.

JEL code Q10, Q3, H23, C73.

1 Introduction

Pests are well-known for being difficult to manage because they often develop resistance to pesticides. Resistance has been documented for a long time in several ways. For instance, Georghiou (1986) enumerated pest species found to be resistant at one or more locations for at least one crop season for various pesticides. Carlson (1977) aggregated resistant pests for major crop areas at various points over time and measured farmer pesticide use choices over time for various pesticide classes. The design of resistance management programs has consequently been a question of huge interest. For instance, Roush (1989) critically reviewed a list of resistance management tactics to bring to the fore the most promising tactics for general use in resistance management.

In a dynamic framework, effectiveness of a pesticide can be considered as a stock that is declining over time because of pest resistance (like a non-renewable resource). Regev *et al.* (1983), in an extension of Regev *et al.* (1976), developed a theoretical model in order to determine optimal pesticide use in this framework. They compared the optimal solution with the competitive solution in order to bring to the fore intertemporal production externalities at work and to derive some policy recommendations. These recommendations were quite basic since they consisted in tax per unit of pesticide use or in restrictions on pesticide use. The major purpose of our work is to refine the policy recommendations that can be formulated with respect to the internalization of such intertemporal production externalities, especially by explicitly taking into account the possibility of farmers to switch to other technologies than pesticide use.

The quite recent development of transgenic agriculture has induced a wide literature that focus on refuge policies. Within the framework of refuge policies, farmers can cultivate trangenic crops resistant to pest upon the condition that some refuge areas are cultivated with conventional crops. The main idea is to avoid the development of pests that are resistant to transgenic crops and to biological control (specially for Bt crops). Some authors followed a quite similar dynamical theoretical framework as Regev *et al.* (1983) in order to study the optimal management of pest resistance with refuge policies. For instance, Laxminarayan and Simspon (2002) derived the optimality conditions for the refuge size in a stylized dynamic model adapted from epidemiology; Ambec and Desquilbet (2012) added a spatial setting into the basic dynamic setting and compared the performance of refuge areas and taxes on pesticide varieties. Other authors proposed to assess economically the effets of refuge policies. For instance, Hurley *et al.* (2001) assessed their impact on agricultural productivity, on conventional pesticide use, and on pest resistance; Frisvold and Reeves (2008) focused on their welfare impacts by taking into account producer surplus, consumer surplus, seed supplier profits, and commodity program costs; Qiao *et al.* (2009) proposed an assessment applied to a developing country framework. All these papers mainly focus on a single technology that allows farmers to struggle against pests: transgenic crops. The purpose of our work is to consider the possibility of farmers to switch to technologies that constitute an alternative to a single pesticide use, without specifying this technology.

Another part of the recent literature concerned with pest resistance focus on R&D targeted towards technologies that can struggle against pests. Some authors focused on innovation process of the biotechnology sector. For instance, O'Shea and Ulph (2008) studied the role of pest resistance in biotechnology R&D investment strategy; Yerokhin and Moschini (2008) studied the impact of intellectual property rights on the investments in biological innovations which value can be reduced to zero because of pest resistance. Here the new technology is mainly understood as an alternative molecule of pesticide or as transgenic crops. Other authors focused on new technologies that rely on farmers' strategies. For instance, Mullen *et al.* (2005) described the role of public investments in pest-management R&D and emphasized the development of integrated pest management (IPM) strategies. The IPM concept was first developped by Stern *et al.* (1959). It is a multidisciplinary concept which main principle is to combine chemical, mecanical and biological control strategies against pests. Mullen *et al.* (2005) summed up that *"the antithesis of IPM is applying broad-spectrum pesticides on a fixed schedule related to the physiological development of the crop, irrespective of pest populations"*. Knight and Norton (1989) explained that one of the motivations for the development of IPM strategies had been the problem of pest resistance. IPM is thus an alternative technology that is at farmers' disposal in order to struggle against pests. For instance, Llewellyn *et al.* (2007) studied the case of integrated weed management strategies. They investigated the determinants of the adoption of these strategies by Western Australian grain growers. We seek to design a framework that is able to consider alternative pest-control technologies as IPM in order to study into more details the policy that can be implemented in order to internalize intertemporal production externalities linked with pest resistance.

We propose to develop a single pest and single crop management model. We assume, like in the non-renewable resources literature[3], that there is a backstop technology with respect to the pesticide, i.e. a technology that does not increase the pesticide resistance of the pest population. This technology can be new molecule of pesticide, transgenic crop resistant to pests or IPM strategies. In order to do so, we mainly rely on Regev *et al.* (1983) work. Their competitive solution relied on the assumption that farmers were non-strategic: they were static maximizers that did not take into account effects of their actions on the dynamics of the system. This assumption was a plausible one in the 1980s. Nowadays, progress of knowledge allowed farmers to be perfectly aware of the phenomena of pest resistance (see for instance Bentley and Thiele, 1999, for a literature review applied to developing countries). Furthermore, pest resistance is a spatial phenomena (see for instance Georghiou and Mellon, 1983 or Peck *et al.*, 1999 or Peck and Ellner, 1997) that can be at work at a local level. As a consequence, the number of farmers concerned with the phenomena of pest resistance can be small. Within such a framework, the farmers perfectly know the effects of their actions on the dynamic of the system.

One question of interest is thus to think of farmers that consider the impact of their choices on the stock of the pesticide effectiveness. In this work, we propose to extend the question of Regev *et al.* (1983) to strategic farmers. For this purpose, we have to resort to differential games theory. Within the framework of differential games, it is necessary to make strict assumptions on the specifications of the functions in order to obtain closed form solutions both for the steady states and for the paths. Closed form solutions are essential to bring to the fore intertemporal production externalities in a quite general case. Indeed, to do so, solutions obtained in the MPNE case have to be compared to the solutions obtained in the socially optimal case. In most works concerned with differential games, functions are assumed linear quadratic. Here, we explore the results of Long and Shimomura (1998) by choosing homogeneous functions that are more general than linear quadratic functions. In a companion paper of Long and Shimomura (1998), Cornes *et al.* (2001) developed non-cooperative differential games based on concepts of

[3] See for instance Amit (1986).

open-loop Nash equilibrium and Markov perfect Nash equilibrium in order to investigate the question of pest resistance in a differential game setting. These concepts of equilibrium differ by the fact that players commit themselves at the beginning of the game for the entire horizon of time in the open-loop case. In the Markov perfect case, they revise their decision at each period of time according to the state of the stock. Applied to the pesticide case, an open-loop structure means that farmers are able to commit themselves with respect to their pesticide application for an entire time horizon. Since farmers can observe the state of the pesticide effectiveness at each period of time, we focus on the Markov perfect equilibrium concept in which farmers are able to revise their decision at each period of time according to the state of the pesticide effectiveness. Cornes *et al.* (2001) developed a discrete time model with three periods and a continuous time one with an infinite number of periods. For clarity of presentation of results, we concentrate on a continuous time model.

Cornes *et al.* (2001) assumed that the terminal time was fixed (to three in the discrete time model and to infinite in the continuous time one). As Example 1 of Long and Shimomura (1998), we assume a free finite terminal time that is endogenous: the planners (farmers and central planner) choose the final time of the pesticide use. Furthermore and contrary to Cornes *et al.* (2001) and Example 1 of Long and Shimomura (1998), we assume that there is a backstop technology with respect to the pesticide, i.e. a technology that does not increase pesticide resistance of the pest population. In order to consider this possibility of an alternative pest-control technology without a priori knowledge on the exact form of this technology, we introduce a scrap value function that reflects the value attached by the decision-maker to the pesticide effectiveness that is left over at the terminal time. This assumption is linked with technologies evocated previously: new molecule of pesticide, transgenic crops and IPM. A new molecule of pesticide is generally used in combination with old one that must still be effective. In order to avoid that pests develop resistance to transgenic crops, it is generally imposed by government to keep traditional crops on which pesticide could still be effective. IPM strategies consist in combination of mechanical, chemical and biological control that can be effective only if chemical control works.

In section 2, we will present the model and assumptions. Section 3 will be devoted to the derivation of the Markovian perfect Nash equilibrium. This will help us to bring to the fore in section 4 the inefficiencies that can occur in such a framework. Section 5 will be devoted to the schemes that can be used in order to obtain a socially optimal switch to an alternative pest-control technology. Finally, we will conclude. All technical proof will be relegated to an appendix.

2 The model

We consider n symmetrical farmers that use pesticides in order to produce a homogenous good. Each farmer i applies a quantity $a_{i,t}$ of pesticide at time t. The effectiveness of this application is given by $a_{i,t}.e_t$ where e_t denotes the effectiveness of the pesticide under consideration. We know that pests develop resistance to pesticide over time. We assume that resistance is a negative linear function of the pesticide application. Therefore, the motion of the pesticide effectiveness over time is given by:

$$\dot{e}_t = -b\sum_{i=1}^{n} a_{i,t} \qquad (1)$$

where b measures the sensitivity of the pesticide efficiency to the amount of pesticide applied; it is a constant that is exogenously given. We consider that the pesticide effectiveness is like a non-renewable stock that declines over time. The pesticide effectiveness before pests develop resistance is exogenously given: $e_0 = e_{\max}$.

We disregard other inputs than pesticide for simplicity and assume that each farmer's profit increases with the effective amount of the pesticide that he applies, at a decreasing rate:

$$\pi_{i,t} = \left[a_{i,t} e_t\right]^{\alpha/2}, \quad 0 < \alpha < 1 \tag{2}$$

where $\alpha / 2$ is the elasticity of the profit to the effective amount of the pesticide applied. This assumption does not seem excessive in view of the high use of pesticide by farmers despite the effects on their health for instance (see Mullen *et al.*, 2005 for instance).

To sum up, the current pest-control technology is used by farmer i until some future time T_i when he will use an alternative pest-control technology. At time T_i, since we do not make any assumption on the form of the alternative pest-control technology, we assume that the alternative pest-control technology brings in a net return of:

$$E_{T_i} = e_{T_i} \int_{T_i}^{\infty} \exp(-\rho(t - T_i))dt = \frac{e_{T_i}}{\rho}$$

where the effectiveness of the pesticide at time T_i, e_{T_i}, is a proxy of the net benefit from the stock of the pesticide effectiveness left over at the terminal time T_i and ρ is a fixed discount rate. E_{T_i} is also named a scrap current value function. It represents the maximum current value of an integral of future utility flow starting from time T_i with an initial stock of the pesticide effectiveness e_{T_i}.

The economic problem is defined as the choice of the amount of pesticide to apply, $a_{i,t}$, and the switching time to the new technology, T_i which maximize:

$$\int_0^{T_i} \pi_{i,t} \exp(-\rho t)dt + E_{T_i} \exp(-\rho T_i) \tag{3}$$

subject to (1) and $e_0 = e_{\max}$ and $a_{i,t} \geq 0$. For the sake of clarity the last constraint will be omitted in the continuation.

Finally, our model is mainly a development of Example 1 of Long and Shimomura (1998) in which we add a scrap value function in the same way as Regev *et al.* (1983).

3 The Markovian Perfect Nash Equilibrium (MPNE)

We look first at the situation in which the n farmers play a differential game. Farmer i looks for a strategy of pesticide application ignoring the strategy chosen by the other farmers. Each farmer strategy is a function of the pesticide effectiveness, e_t, that is easily observable: $\phi(e_t)$. Therefore, e_t sums up the influence of the past at time t. Each farmer is perfectly aware of the fact that the others select their strategy on the basis of the current pesticide effectiveness. This is why we assume that the farmers' strategy are Markovian.[4]

[4] See Chapter 4 of Dockner *et al.* (2001) for more details on Markovian equilibriums.

For the purpose of the resolution, it is convenient to transform variables. We define a new control variable $c_{i,t} = \left[a_{i,t} e_t \right]^{1/2}$ that induces modification of equations (1) and (2) as, respectively, $\dot{e}_t = -b \sum_{i=1}^{n} \frac{c_{i,t}^2}{e_t}$ and $\pi_{i,t} = \left[c_{i,t} \right]^{\alpha}$. More precisely, the MPNE of the game modified is the solution of problem:

$$\max \int_0^{T_i} \left[c_{i,t} \right]^{\alpha} \exp(-\rho t) dt + E_{T_i} \exp(-\rho T_i)$$
$$s.t.: \dot{e}_t = -b \left(\frac{c_{i,t}^2}{e_t} + \sum_{j \neq i} \frac{\phi_j(e_t)^2}{e_t} \right)$$
$$e_0 = e_{\max}$$

We use the Pontryagin's maximum principle in order to solve this problem. We focus on a symmetrical solution. We define the Hamiltonian of this problem in current value:

$$H(c_{i,t}, e_t, \lambda_t) = \left[c_{i,t} \right]^{\alpha} - \lambda_{i,t} b \left(\frac{c_{i,t}^2}{e_t} + \sum_{j \neq i} \frac{\phi_j(e_t)^2}{e_t} \right)$$

If $\phi_i(e_t)$ is a solution to this problem and T_i the optimal switching time (from farmer's i point of view), then there exists a continuous function $\lambda_{i,t}$ that satisfies the conditions:

$$\alpha \phi_i(e_t)^{\alpha - 1} = \frac{2 \lambda_{i,t} b \phi_i(e_t)}{e_t} \tag{4}$$

$$\dot{\lambda}_{i,t} = \rho \lambda_{i,t} - \partial_{e_t} H(\phi_i(e_t), e_t, \lambda_{i,t}) \tag{5}$$

$$\dot{e}_t = -\frac{b n \phi_i(e_t)^2}{e_t} \tag{6}$$

$$\left(\phi_i(e_t)^{\alpha} - \frac{\lambda_{i,t} b n \phi_i(e_t)^2}{e_t} + \frac{1}{\rho} \partial_t e_t - e_t \right)_{t = T_i} = 0 \tag{7}$$

Based on Long and Shimomura's (1998) result, we assume that farmer i's strategy is of the form $\phi_i(e_t) = \beta_i . e_t$ if all other farmers use strategies that are homogeneous of degree one: $\phi_j(e_t) = \beta_j . e_t$, $j \neq i$.

Equation (4) is the usual marginal condition of long-run profit maximization. It states that, for each period of time, the marginal profit of pesticide application is equal to its marginal cost. The cost is a measurement of future losses implied by increasing resistance. It is given by farmer i's private shadow price $\lambda_{i,t}$. Equation (5) defines the dynamics of the co-state variable $\lambda_{i,t}$. When equation (6) is also valid, we can conclude, as in Example 1 of Long and Shimomura (1998), that:

Remark 1 *If* $2 - n\alpha > 0$ *and farmers' strategies are of the form* $\phi_i(e_t) = \beta_i . e_t$*, then there* *exists a unique MPNE for which the equilibrium strategy of farmer* i *is* $a_{i,t}^N = \beta_i^{N^2} e_t^N$ *where* $\beta_i^N = \sqrt{\frac{\rho}{b(2-n\alpha)}}$. *The resulting time path of the pesticide effectiveness is* $e_t^N = e_{\max}\exp(-nb\beta_i^{N^2} t)$.

We see that when n approaches $\frac{2}{\alpha}$, the amount of pesticide applied tends to infinity because farmers know that they will not be able to apply pesticide the time after: their effectiveness will be too low. Further computations state that:

$$\partial_t a_{i,t}^N = -\frac{n\rho^2 e_{\max}}{b(-2+n\alpha)^2}\exp\left(\frac{n\rho t}{-2+n\alpha}\right) \le 0$$

$$\partial_t e_t^N = \frac{n\rho e_{\max}}{-2+n\alpha}\exp\left(\frac{n\rho t}{-2+n\alpha}\right) \le 0$$

This means that the effectiveness and the application of pesticide decline over time but until when? To answer this question, we need to achieve the resolution of the problem by determining the optimal switching time (from farmer's i point of view) T_i^N. To do so, we can directly apply Theorem 7.6.1 of Leonard and Long (1992). The condition is valid for an Hamiltonian function in present value. We work with an Hamiltonian in current value. Standard computations lead to equation (7) that is the necessary transversality condition. Equation (7) controls the switch to a new pest-control technology and it states that the optimal switching time (from farmer's i point of view) is such that the value of the optimal Hamiltonian evaluated at T equals the marginal scrap value. We obtain:

Proposition 1 *The optimal switching time (from farmer's* i *point of view) from the pesticide use to an alternative pest-control technology is*

$$T_i^N = -\frac{1}{nb\beta_i^{N^2}(\alpha-1)}\left[(\alpha-1)\ln\left(\frac{1}{\beta_i^N e_{\max}}\right)+\ln\left(\frac{2\left(nb\beta_i^{N^2}+\rho\right)}{\beta_i^N\rho(2-\alpha n)}\right)\right].$$

[5] *It increases with* $e_{\max}$.

Since T_i^N is different from infinity, some effectiveness is left over before switching to an alternative pest-control technology. This effectiveness reflects the need of pesticide effectiveness of the alternative pest-control technologies.

The optimal switching time (from farmer's i point of view) from the pesticide use to an alternative pest-control technology is a function of all parameters. We check that the optimal switching time (from farmer's i point of view) increases with the initial effectiveness of the pesticide, $e_{\max}$.

[5] We assume that the set of parameters is such that: $T_i^N > 0$.

4 The intertemporal production externality

To value the efficiency of the previous MPNE, we need to specify the socially optimal solution. Indeed, the MPNE should be inefficient since farmers do not take into account both the impact of their own pesticide application on the pesticide effectiveness and the impact of the other farmers' pesticide application on the effectiveness. The implication is that they may overuse pesticide in order to benefit from their effectiveness before the other farmers apply them. In order to check this, we first characterize the socially optimal solution and then compare it to the MPNE.

4.1 The socially optimal solution

We assume here that an agricultural authority seeks to maximize the aggregated profit of the n farmers. This authority thus chooses the amount of pesticide applied by each farmer i at time t, $(a_{i,t})_{i=1}^{n}$, and the switching time to an alternative pest-control technology, T, that maximizes the present value of the future aggregated profits. To make comparisons easily, we follow a similar resolution method as for the MPNE, we make the same variable transformation as previously and we again assume symmetrical solutions. The agricultural authority solves the following problem:

$$\max \int_0^{T_i} n\left[c_t\right]^{\alpha} \exp(-\rho t)dt + nE_T \exp(-\rho T)$$

$$s.t.: \dot{e}_t = -\frac{nbc_t^2}{e_t}$$

$$e_0 = e_{\max}$$

The Hamiltonian of this problem in current value is:

$$H(c_t, e_t, \lambda_t) = n\left[c_t\right]^{\alpha} - \frac{\lambda_t nbc_t^2}{e_t}$$

where λ_t reflect the social shadow price of the stock of pesticide effectiveness. We know from the concavity of this Hamiltonian and from the Mangasarian conditions that the optimal solution of the problem satisfies:

$$\alpha n c_t^{\alpha-1} = \frac{2\lambda_t nbc_t}{e_t} \tag{8}$$

$$\dot{\lambda}_t = \rho\lambda_t - \frac{\lambda_t nbc_t^2}{e_t^2} \tag{9}$$

$$\dot{e}_t = -\frac{bnc_t^2}{e_t} \tag{10}$$

$$\left(nc_t^{\alpha} - \frac{\lambda_t bnc_t^2}{e_t} + \frac{n}{\rho}\partial_t e_t - ne_t \right)_{t=T} = 0 \tag{11}$$

In order to be able to easily compare this solution with the MPNE, we choose to assume again a rule such that $c_t = \beta e_t$ and we show that it is possible to find a $\beta^* > 0$ such that conditions (8), (9) and (10) are checked. We finally show that:

Proposition 2 *There exists a unique β^* such that the decision rule $\beta^* e_t$ is a socially optimal rule of pesticide application that maximizes the profit of farmers.*

(*i*) *The socially optimal pesticide application is $a_t^* = \beta^{*2} e_{\max} \exp(-nb\beta^{*^2} t)$ where $\beta^* = \sqrt{\frac{\rho}{bn(2-\alpha)}}$. The resulting time path of the pesticide effectiveness is $e_t^* = e_{\max}\exp(-nb\beta^{*^2} t)$.*

(ii) The socially optimal switching time from the pesticide use to an alternative pest-control technology is $T^* = -\frac{1}{nb\beta^{*2}(\alpha-1)}\left[(\alpha-1)\ln\left(\frac{1}{\beta^* e_{\max}}\right) + \ln\left(\frac{2\left(nb\beta^{*^2} + \rho\right)}{\beta^* \rho (2-\alpha)}\right)\right]$.[6] *It increases with $e_{\max}$.*

We see from standart computations that the socially optimal paths of pesticide application and of pesticide effectiveness decline over time as for the MPNE paths.

The socially optimal switching time from the pesticide use to an alternative pest-control technology is a function of all parameters. We check that the variations of the socially optimal switching time is the same as in the MPNE case with respect to the initial effectiveness of the pesticide, $e_{\max}$.

4.2 The inefficiencies of the MPNE solution

The expressions of T^* and T_i^N are too complex to be compared on analytical grounds. We can only conclude that they are different. We have run a set of simulations in order to derive more conclusions.[7] The results are summarized in Table 1, 2, 3, 4 and 5 of Appendix A.4. The main conclusion is that the swtiching time from pesticide use to an alternative pest-control technology is later in the socially optimal case than in the MPNE:

$$T^* > T_i^N$$

In order to explain this result it is important to compare the pesticide application and effectiveness in both cases. We now turn to this comparison.

For clarity sake, we first reason on the same time $t \le T_i^N < T^*$ for the MPNE case and for

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[6] We assume that the set of parameters is such that: $T^* > 0$.

[7] The idea of the simulations is to put values on our parameters. The parameterization is not made on empirical basis because we do not want to restrict ourselves to a specific case. As a consequence, we run a set of simulations for different feasible values of the parameters.

the socially optimal case. Equations (4) and (8) are usual marginal conditions of long-run profit maximization. They state that, for each period of time, the marginal profit of pesticide application is equal to its marginal cost. The cost, that is a measurement of future losses implied by increasing resistance, is not the same in the MPNE case, $\frac{2\lambda_{i,t} b\phi_i(e_t)}{e_t}$, and in the socially optimal case, $\frac{2\lambda_t nbc_t}{e_t}$. Indeed, it is a private cost in the MPNE case and it is a collective cost in the socially optimal case. Furthermore, the shadow price (λ) differs in both cases. The difference between both can easily be understood by comparing the dynamic of the co-state variable in the MPNE case:

$$\dot{\lambda}_{i,t} = \rho\lambda_{i,t} - \lambda_{i,t} b\left(\beta_i^2 - (n-1)\beta_i^2\right)$$

with the one in the socially optimal case:

$$\dot{\lambda}_t = \rho\lambda_t - \lambda_t bn\beta^2$$

In the MPNE case, farmers value the impact of their own pesticide use on the pesticide effectiveness, $\lambda_t b\beta^2$. Whereas the social planner values the impact of each farmer pesticide use on the pesticide effectiveness: $\lambda_t bn\beta^2$. In addition, in the MPNE case, $-(n-1)\beta^2$ reflects the assumption according to which farmers use Markovian strategy: each one knows perfectly that the others use a rule that is a function of the pesticide effectiveness at each period of time. This term implies that in the MPNE case each farmer is going to use more pesticide than it would have been socially optimal to use. The idea is that in order to benefit from effective pesticide, it is optimal for each individual farmer to use pesticide before the other farmers.

These intuitions are analytically confirmed by Proposition 3.

Proposition 3 *The comparison between the MPNE and the socially optimal path without considering the switching time from the pesticide use to an alternative pest-control technology allows us to conclude that:*

(i) the amount of pesticide applied in the MPNE case is higher than the amount applied in the socially optimal case until the time, $t^a = \frac{(-2+\alpha)(-2+n\alpha)}{\rho 2(n-1)} \ln\left(\frac{n(-2+\alpha)}{(-2+n\alpha)}\right) > 0$, *after which the amount applied in the MPNE case can be lower than the amount applied in the socially optimal case:* $a_t^N > a_t^* \forall t < t^a \leq T_i^N < T^*$.

(ii) the effectiveness of the pesticide is always lower in the MPNE case than in the socially optimal case: $e_t^N < e_t^* \forall t \leq T_i^N < T^*$.

Conclusion (ii) confirms the previous intuitions: in the MPNE case, farmers overexploit the common that, here, is the effectiveness of the pesticide. The implication is that the effectiveness of the pesticide is lower in the MPNE case than in the socially optimal case. At first glance, one can be quite surprised by the second part of result (i) that states that the amount of pesticide applied in the MPNE case can be lower that the amount applied in the socially optimal case. This result is valid after a first regime during which the amount of pesticide applied in the MPNE case is higher than the amount applied in the socially optimal case, which fits more closely to the intuitions. This can help us explain the surprising part of the results. Indeed, in a first phase, farmers apply more pesticide in the MPNE case than in the socially optimal case. By doing so, they contribute to the decrease of the stock of pesticide effectiveness until a time at which they begin to apply less pesticide than in the socially optimal case because of pesticide reducing drastically in efficiency: this is the second phase. Another interpretation of t^a can be to consider that this is the time of overexploitation of the stock of pesticide effectiveness in the MPNE case.

The time t^a is a function of the number of farmers, n, of a proxy of the elasticity of the profit to the effective amount of pesticide applied, α, and of the discount rate, ρ. Basic comparative static allows us to conclude that:

Remark 2 *The time t^a decreases with the number of farmers, n, and with the discount rate, ρ.*

These results are direct when one keeps in mind the interpretation of t_a as the time of overexploitation of the stock of pesticide effectiveness. Indeed, when the discount rate increases, the value of the future decreases and the stock of pesticide effectiveness is exhausted early. In the same way, when the number of farmers increases, the stock of pesticide effectiveness is exhausted early.

To achieve our comparisons, we now need to compare the paths of both the pesticide application and effectiness after T_i^N. Between T_i^N and T^*, both the pesticide effectiveness and application tend to zero in the MPNE case because of the switch to alternative pest-control technology. The consequence is that, between T_i^N and T^*, both the pesticide effectiveness and application must be higher in the socially optimal case than in the MNPE case.

Finally, we can conclude from simulations presented in Table 1, 2, 3, 4 and 5 of Appendix A.4 that (i) the final (at T) pesticide application is lower in the socially optimal case than in the MNPE case, $a^*_{T^*} < a^N_{T^N}$, and (ii) that the final pesticide effectiveness is lower in the MNPE case thant in the socially optimal case, $e^*_{T^*} > e^N_{T^N}$.

To sum up, we have shown that the switching time from the pesticide use to an alternative pest-control technology is earlier in the MPNE case than in the socially optimal case. This can be explained by the fact that the over-application of pesticide by farmers in the MPNE case considerably reduces the pesticide effectiveness and their profits. This means that, in the MPNE case, it is more interesting for farmers to switch to alternative pest-control technologies sooner than in the socially optimal case since the common constituted by the pesticide effectiveness is already exhausted.

5 Toward the restoration of the socially optimal solution

Once the inefficiencies that are at work have been brought to the fore, the question of their internalization remains. In this section, we propose to try some strategies in order to restore efficiency. We have two inefficiencies here:

- the intertemporal production externalities that are brought to the fore by the difference between the MPNE and socially optimal paths of pesticide application and effectiveness,

- the switching time from the pesticide use to an alternative pest-control technology that is later in the socially optimal case than in the MPNE case.

Since there is two inefficiencies, one can think of two instruments in order to correct them. We will first check if tax or subsidy designed in order to internalize externalities are able to reach the optimal switching time from the pesticide use to an alternative pest-control technology. We will show that it is not the case and we will look for a policy that is able to restore both the socially optimal paths and switching time.

5.1 Obtaining the socially optimal paths of pesticide application and effectiveness: equivalence between tax and subsidy

The externality leads to suboptimal paths both for the amount of pesticide applied and for the pesticide effectiveness. Two ways exist in order to internalize this externality and to obtain the socially optimal paths of pesticide application and effectiveness: it is possible either to tax the profits from pesticide application or to subsidize the ambient pesticide effectiveness. In this subsection, we propose to ignore the horizon of time that will be the subject of the next subsection.

In the tax case, the idea is that a tax, τ, on the profits generated by pesticide application will give farmers incentives to reduce pesticide application. The Hamiltonian in current value becomes:

$$H(c_{i,t}, e_t, \lambda_t) = (1-\tau)\left[c_{i,t}\right]^{\alpha} - \lambda_{i,t} b\left(\frac{c_{i,t}^2}{e_t} + \sum_{j\neq i}\frac{\phi_j(e_t)^2}{e_t}\right)$$

Only equation (4) and (7) are modified with respect to the MPNE case. The first order condition now states that, for each period of time, the marginal net profit of pesticide application is equal to its marginal cost that now includes the tax:

$$\alpha\phi_i(e_t)^{\alpha-1} = 2\lambda_{i,t} b\frac{\phi_i(e_t)}{e_t} + \tau\alpha\phi_i(e_t)^{\alpha-1} \tag{12}$$

Proposition 4 *The optimal tax on profits,* $\tau^* = 1-\left(\frac{\alpha n-2}{n(\alpha-2)}\right)^{\frac{\alpha}{2}-1} > 0$, *is such that there exists a MPNE for which the equilibrium strategy of farmer* i *is equal to the socially optimal one. The resulting time path of the pesticide effectiveness is also equal to the socially optimal one.*

We see that the optimal tax, τ^*, is a function of the number of farmers, n, and of a proxy of the elasticity of the farmers' profit to the effective amount of pesticide applied, α. Some basic computations allow us to conclude that:

Remark 3 *The optimal tax on profits decreases with* α *and increases with* n.

As a consequence, when the elasticity of the farmers' profit to the effective amount of pesticide applied increases, the optimal tax decreases since farmers are more reactive to the price signal. Furthermore, when the number of farmers increases, the competition between them is stronger in the MPNE case and there is a need for a stronger fiscal incentive in order to restore efficiency.

In the subsidy case, the intuition is that since farmers know perfectly how their pesticide application has an impact on the pesticide effectiveness, subsidizing the ambient level of pesticide effectiveness by σ_t will give them incentives to reduce their pesticide application. Such a subsidy scheme assumes that, at each time, the agricultural authority perfectly knows the pesticide effectiveness. The Hamiltonian in current value becomes:

$$H(c_{i,t}, e_t, \lambda_t) = \left[c_{i,t}\right]^{\alpha} + \sigma_t e_t - \lambda_{i,t} b\left(\frac{c_{i,t}^2}{e_t} + (n-1)\frac{\phi_j(e_t)^2}{e_t}\right)$$

Only equations (5) and (7) are modified with respect to the MPNE case. The dynamic of the co-state variable is now reduced by the value of the subsidy which means that farmers are going to give more value to the pesticide effectiveness since the co-state variable enters in a negative way into the Hamiltonian function:

$$\dot{\lambda}_{i,t} = \rho\lambda_{i,t} - \sigma_t - \partial_{e_t} H(\phi_i(e_t), e_t, \lambda_{i,t}) \qquad (13)$$

It is precisely the main aim of this subsidy.

Proposition 5 *The optimal ambient subsidy,* $\sigma_t^* = (n-1)\alpha e_t^{\sigma^{\alpha-1}} \beta_i^{\sigma^{\alpha}} > 0$*, is such that there exists a MPNE for which the equilibrium strategy of farmer* i *is equal to the socially optimal one. The resulting time path of the pesticide effectiveness is also equal to the socially optimal one.*

As for the optimal tax, the optimal subsidy, σ_t^*, is a function of the number of farmers, n, and of a proxy of the elasticity of the farmers' profit to the effective amount of pesticides applied, α. In addition, because the subsidy is proportional to the pesticide efficiency, it is a function of the discount rate, ρ, of the time, t, of the sensitivity of the pesticide efficiency to the amount of pesticide applied, b, and of the initial effectiveness of the pesticide, $e_{\max}$.

Remark 4 *The optimal ambient subsidy on the pesticide effectiveness increases with* t, n *and* ρ *and decreases with* b *and* $e_{\max}$.

The sensitivity of the subsidy to the number of farmers can be interpreted as the one of the tax. Furthermore, the subsidy increases with time because it is proportional to the pesticide effectiveness that decreases with time. This decrease has to be compensated by the subsidy in order to slow down the pesticide application. It is for the same reason that the subsidy decreases with the initial effectiveness of the pesticide and with the sensitivity of the pesticide efficiency to the amount of pesticide applied. The subsidy also increases with the discount rate. Equation (13) helps to understand this. We see that the subsidy intervenes in the definition of the shadow price of the stock of pesticide effectiveness. Since this shadow price increases over time with the discount rate, the subsidy must decrease with the discount rate to compensate the effect of the discount rate increase on the shadow price.

The fact that the optimal subsidy is a function of time can be a problem for implementation purposes. Indeed this means that it has to be adjusted at each period of time by the regulator. We can thus conclude that tax and subsidy are only theoretically equivalent. Both are able to make farmers behavior result in the socially optimal paths of pesticide application and effectiveness but the question remains as to whether both schemes allow for a socially optimal switch from pesticide application to an alternative pest-control technology.

5.2 Obtaining the socially optimal switching time: tax versus subsidy?

Once the equivalence between tax and subsidy has been shown for the path of pesticide application and effectiveness, what remains to check is if the socially optimal switching time can be obtained with such schemes. Intuitively, we cannot expect to reach this result. This can be seen with the modified transversality conditions:

$$\left((1-\tau)c_{i,t}^{\alpha}-\frac{\lambda_{i,t}bnc_{i,t}^{2}}{e_t}+\frac{1}{\rho}\partial_t e_t-e_t\right)_{t=T_i}=0 \qquad (14)$$

for the tax case and:

$$\left(c_{i,t}^{\alpha}+\sigma_t e_t-\frac{\lambda_{i,t}bnc_{i,t}^{2}}{e_t}+\frac{1}{\rho}\partial_t e_t-e_t\right)_{t=T_i}=0 \qquad (15)$$

for the subsidy case. The fiscal schemes seem to have an impact on the switching time from pesticide application to an alternative pest-control technology. Proposition 6 confirms this intuition.

Proposition 6 *(i) In the tax case, the switching time from the pesticide use to an alternative pest-control technology is* $T_i^{\tau}=-\frac{1}{nb\beta_i^{*2}(\alpha-1)}\left[(\alpha-1)\ln\left(\frac{1}{\beta_i^{*}e_{\max}}\right)+\ln\left(\frac{2\left(nb\beta_i^{*2}+\rho\right)}{\beta_i^{*}\rho n(2-\alpha)}\right)+\frac{\alpha}{2}\ln\left(\frac{n(\alpha-2)}{(\alpha n-2)}\right)\right]$.[8] *As before, it increases with* $e_{\max}$.

(ii) In the subsidy case, the switching time from the pesticide use to an alternative pest-control technology is $T_i^{\sigma}=-\frac{1}{nb\beta_i^{*2}(\alpha-1)}\left[(\alpha-1)\ln\left(\frac{1}{\beta_i^{*}e_{\max}}\right)+\ln\left(\frac{2\left(nb\beta_i^{*2}+\rho\right)}{\beta_i^{*}\rho\left(2+\alpha(n-2)\right)}\right)\right]$.[9] *As before, it increases with* $e_{\max}$.

(iii)The switching time in the subsidy case is always lower than the switching time in the socially optimal case: $T^{*}>T_i^{\sigma}$.

If we recall,

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[8] We assume that the set of parameters is such that: $T_i^{\tau}>0$.

[9] We assume that the set of parameters is such that: $T_i^{\sigma}>0$.

$T^{*} = -\frac{1}{nb\beta^{*^2}(\alpha-1)}\left[(\alpha-1)\ln\left(\frac{1}{\beta^{*}e_{\max}}\right)+\ln\left(\frac{2\left(nb\beta^{*^2}+\rho\right)}{\beta^{*}\rho(2-\alpha)}\right)\right]$, the switching time in the socially optimal case, and

$T_i^{N} = -\frac{1}{nb\beta_i^{N^2}(\alpha-1)}\left[(\alpha-1)\ln\left(\frac{1}{\beta_i^{N}e_{\max}}\right)+\ln\left(\frac{2\left(nb\beta_i^{N^2}+\rho\right)}{\beta_i^{N}\rho(2-\alpha n)}\right)\right]$, the switching time in the MPNE case, we see that they are different from the switching times obtained in the subsidy and tax cases. The equations of the switching times are too complex to conclude in a general way on the ranking of switching times. We can only conclude that a subsidy decreases the switching time with respect to the socially optimal case.

It is well known that taxes and subsidies have distributional effects. In order to delete these distributional effects, we then propose to add to the fiscal schemes a lump-sum transfer. Such a modification will have an effect only on the transversality condition. We obtain the results stated in Proposition 7.

Proposition 7 *(i) When we add a lump-sum transfer to the subsidy, we reach the socially optimal paths of pesticide application and effectiveness, and the socially optimal swtiching time.*
(ii) When we add a lump-sum transfer to the tax, we again reach the socially optimal paths of pesticide application and effectiveness but not the socially optimal switching time.

Technically, for the tax case with a lump-sum transfer, the tax also intervenes in the first order condition that is used to derive the switching time (see equation 12). Equation 12 is different from the first order condition of the socially optimal case (see equation 8). Indeed, the shadow price of the pesticide effectiveness, λ_i, is also a function of the tax to be taken into account. It explains why the switching time is formally different from the socially optimal one. More intuitively, the inefficiency of the lump-sum transfer in the tax case is due to the non-linear effect of the tax on the shadow price of the pesticide effectiveness.

The comparison between the switching times in the different cases is unclear. The general conclusions that we can derive are related to the sensitivity of the differences to the parameter values. The conclusions are summarized in the following remark.

Remark 6 *The differences between all switching times are increasing with or independant of parameters* $e_{\max}$ *and* b.

In order to obtain more interesting results, we have run a set of simulations. For the following set of parameters,

$$b=1,\ e_{\max}=1000,\ \rho=0.04,\ \alpha=0.001 \text{ and } n=50$$

we obtain the following ranking:

$$T^{N} < T^{\tau} < T^{\sigma} < T^{*} < T^{\tau LS} \tag{16}$$

In order to test the robustness of this ranking, we have run a set of simulations in which parameters n, α and ρ vary. The results are summarized in Table 6, 7, 8, 9 and 10 of Appendix A.14. Having in mind Remark 6, we have also run a set of simulations in which parameters $e_{\max}$ and b are lower than the previous set.

The simulations show that the differences between all switching times decrease with the discount rate, ρ, and are independant of the sensitivity of the pesticide efficiency to the amount of pesticide applied, b. This last result must come from the low values taken in the simulations but we know from Remark 6 that these value are the only one for which the ranking in (16) can be reversed.

Furthermore, the difference between the switching time in the socially optimal case and the switching time in the subsidy case, $T^* - T^\sigma$, is the lowest one. This means that the ambient subsidy leads to a solution very close to the socially optimal case, even when no lump-sum transfer is associated to the subsidy.

We can also observe from the simulations that the difference, on the one hand between the switching time in the tax case with a lump-sum transfer and the socially optimal case, $T^{\tau ls} - T^*$ and, on the other hand between the switching time in the tax case with a lump-sum transfer and the subsidy case, $T^{\tau ls} - T^\sigma$, are reversed for high number of agents, n, for high elasticity of the farmers' profit to the effective amount of pesticide applied, α, and for low initial effectiveness of the pesticide, $e_{\max}$. This means that the ranking in (16) is not robust to these parameter values since $T^{\tau ls}$ can become lower than T^* or lower than T^σ. Another result shows the instability of the tax case: when $e_{\max}$ is small, the switching time from the pesticide use to an alternative pest-control technology in the tax case, T^τ, become lower than in the MPNE case, T^N.

Finally, the main implication of all these simulations is that a robust ranking is as follows:

$$T^N < T^\sigma < T^*$$

This means that a subsidy on the ambient pesticide effectiveness fails to achieve the socially optimal switching time from pesticide use to an alternative pest-control technology. Proposition 7 shows that if a lump-sum transfer is added to the subsidy, the socially optimal solution is achieved. This result seems to be mainly due to the fact that a lump-sum transfer deletes distributional effects induced by the subsidy. However, a tax on farmers'profits generated from pesticide application, even when a lump-sum transfer is added to it, fails to achieve the socially optimal solution. This result is mainly due to the form of the schemes τ^* and σ^* that can be seen in Propositions 4 and 5. The tax is more basic than the subsidy since the tax does not depend on the effectiveness of pesticide over time (the tax is only a function of parameters α and n); the subsidy can be adjusted over time and with respect to the other parameters. As a consequence, a scheme that is designed with respect to the ambient effectiveness of pesticide has better chance to achieve the socially optimal solution (pesticide application and effectiveness path plus switching time to an alternative pest-control technology) than a scheme that is not designed like this.

6 Conclusion and discussion

This paper is concerned with the question of the internalization of intertemporal production externalities. We concentrate on pest resistance: it is well known that the amount of pesticide applied by farmers decreases their effectiveness over time. Developments in dynamic game theory literature show how it is possible to assume that, on the one hand, each farmer is perfectly aware of the impact of his decisions on the pesticide effectiveness. On the other hand, we assume that each farmer does not know how the other farmers act. We thus model the intertemporal production externalities in the pesticide case as a differential game. Furthermore, we add to the picture a scrap value of the switch from the pesticide use to an alternative pest-control technology.

The first step is to bring to the fore the intertemporal production externalities that are at play. In order to do this, we solve and compare two problems for the complete time horizon (and not only at the steady state as it is commonly the case): the MPNE one and the socially optimal one. We show that the effectiveness of the pesticide is always (over time) lower in the MPNE case than in the socially optimal case. We thus propose to assimilate the stock of pesticide effectiveness to a common that is overexploited by farmers because of externalities. The second step concerns the switching time from the pesticide use to an alternative pest-control technology. We show that it is later in the socially optimal case that in the MNPE case.

We then look for the fiscal schemes that can be implemented in order to internalize these intertemporal production externalities. We show that both a tax and a subsidy are able to internalize them. We then explore the full potential of a dynamic model by studying the times of switch from pesticide application to an alternative pest-control technology. We show that tax and subsidy do not lead to the same switching time. When one adds a lump-sum transfer to the subsidy, the switching time from the pesticide use to an alternative pest-control technology becomes the same one as in the socially optimal case. This result underlines the importance of designing an additional redistributive policy in order to couterbalance the unwanted effects that a fiscal scheme can have.We show that this result is not validated in the tax case: when one adds a lump-sum transfer to the tax, the socially optimal switching time from the pesticide use to an alternative pest-control technology is not reached. This is mainly due to the non-linear effect of the tax on the shadow price of the pesticide effectiveness.

The lack of effectiveness of taxes in the pesticide case was recently confirmed empirically by Skevas *et al.* (2012): they shew that both pesticide taxes and a price penalty on pesticide impact on water organisms are unable to considerably reduce pesticide use. They obtained the same result in the subsidy cases as in the tax cases. Contrary to this last result, we show that a subsidy can be efficient. However, the subsidies that they studied are not the same as the one studied in our work. They considered a subsidy on the use of low-toxicity pesticides, subsidies on research and development of low-toxicity alternatives, and subsidies on R&D of more environmental friendly pesticides. We rather consider an ambient subsidy: a subsidy that is a function of the ambient effectiveness of pesticide. We show that, in a dynamical framework and when a lump-sum transfer is added to this ambient subsidy, both the socially optimal paths of pesticide effectiveness and application, and the switching time from the pesticide use to an alternative technolgy can be reached.

The study of the switching time to an alternative pest-control technology can have interesting implications from a public policy perspective. Indeed, if the aim of the policy-maker (an agricultural authority in our work) is simply to internalize intertemporal production externalities, it is necessary to implement basic internalizing schemes. However, if the policy-maker is also concerned by the socially optimal switch from the pesticide use to an alternative pest-control technology, an ambient subsidy is better than a tax.

Furthermore, if the policy-maker has environmental goals in mind, implications are different. If we assume that alternative pest-control technologies to pesticide application are more environmentally friendly, the internalizing policy could be considered as not being necessary. Indeed, we show that a subsidy postpones the switching time to alternative pest-control technologies. This is true if a huge quick application of pesticides does not harm environment more than a low long application. One interesting extension of our work would be to add an environmental damage linked with the application of pesticide in order to control these effects. Such an extension is beyond the scope of our work that concentrate on production externalities. It is why it is left for a future work on environmental externalities of pesticide use.

Appendix

A.1 Proof of remark 1

In order to solve the modified game, we first observe from the first order condition (4) that: $\lambda_{i,t} = \alpha A_i e_t^{\alpha-1} \Leftrightarrow \dot{\lambda}_{i,t} = \alpha(\alpha-1)A_i e_t^{\alpha-2}\dot{e}_t$ where A_i is a new constant. We then have: $\dot{\lambda}_{i,t} = -\alpha(\alpha-1)A_i e_t^{\alpha-1} bn\beta_i^2$.

We also know from equation (5) and from the assumption on symmetry of solutions that: $\dot{\lambda}_{i,t} = \rho\lambda_{i,t} + \lambda_{i,t} b(n-2)\beta_i^2$. We replace $\lambda_{i,t}$ by $\alpha A_i e_t^{\alpha-1}$ in this equation and we obtain: $\dot{\lambda}_{i,t} = \alpha A_i e_t^{\alpha-1}\left(\rho + b(n-2)\beta_i^2\right)$.

We now look for A_i that solves:

$$-\alpha(\alpha-1)A_i e_t^{\alpha-1} bn\beta_i^2 = \alpha A_i e_t^{\alpha-1}\left(\rho + b(n-2)\beta_i^2\right)$$

$$\Leftrightarrow \rho\alpha A_i e_t^{\alpha-1} = -\alpha A_i e_t^{\alpha-1} b\beta_i^2(n\alpha-2)$$

$$\Leftrightarrow \rho = -b\beta_i^2(n\alpha-2) \qquad (17)$$

Before going further, we express β_i with respect to A_i. To do so, we depart from the first order condition (4) in which we put $\lambda_{i,t} = \alpha A_i e_t^{\alpha-1}$:

$$\alpha c_{i,t}^{\alpha-1} - 2\alpha A_i e_t^{\alpha-2} b c_{i,t} = 0$$

$$\Leftrightarrow c_{i,t}^{\alpha-2} = 2A_i e_t^{\alpha-2} b$$

$$\Leftrightarrow c_{i,t} = e_t (2bA_i)^{\frac{1}{\alpha-2}}$$

We assume that $c_{i,t} = \beta_i e_t$. The implication is that $\beta_i = (2bA_i)^{\frac{1}{\alpha-2}}$. We now put this expression in equation (17) and we obtain:

$$\left(\frac{\rho}{b(2-n\alpha)}\right)^{\frac{1}{2}} = (2bA_i)^{\frac{1}{\alpha-2}}$$

$$\Leftrightarrow \left(\frac{\rho}{b(2-n\alpha)}\right)^{\frac{\alpha-2}{2}} = 2bA_i$$

$$\Leftrightarrow A_i = \frac{\left(\frac{\rho}{b(2-n\alpha)}\right)^{\frac{\alpha-2}{2}}}{2b} := A_i^N$$

It directly results from this that:

$$\beta_i^N = \sqrt{\frac{\rho}{b(2-n\alpha)}}$$

We can now look for the equilibrium paths of the modified game. We begin with equation (6) that can now be written as: $\dot{e}_t = \frac{ne_t\rho}{n\alpha-2}$ with $e_0 = e_{\max}$

The solution of this differential equation is:

$$e_t^N = e_{\max}\exp\left(\frac{n\rho t}{n\alpha-2}\right) = e_{\max}\exp\left(-nb\beta_i^{N^2}t\right)$$

that gives:

$$c_{i,t}^N = \sqrt{\frac{\rho}{b(2-n\alpha)}}e_{\max}\exp\left(\frac{n\rho t}{n\alpha-2}\right) = \beta_i^N e_{\max}\exp\left(-nb\beta_i^{N^2}t\right)$$

and

$$a_{i,t}^N = -\frac{\rho}{b(-2+n\alpha)}e_{\max}\exp\left(\frac{n\rho t}{-2+n\alpha}\right) = \beta_i^{N^2}e_{\max}\exp\left(-nb\beta_i^{N^2}t\right)$$

A.2 Proof of proposition 1

We depart from first order condition (4): $\alpha c_{i,t}^{\alpha-1} - 2\lambda_{i,t}b\frac{c_{i,t}}{e_t} = 0 \Leftrightarrow \lambda_{i,t} = \frac{c_{i,t}^{\alpha-2}\alpha e_t}{2b}$ that we put into transversality condition (7):

$$c_{i,T_i}^{\alpha} - \frac{c_{i,T_i}^{\alpha}\alpha n}{2} + \frac{1}{\rho}\partial_t e_t\,|_{t=T_i} - e_{T_i} = 0$$

$$\Leftrightarrow c_{i,T_i}^{\alpha} = \frac{2}{(2-\alpha n)}\left(-\frac{1}{\rho}\partial_t e_t\,|_{t=T_i} + e_{T_i}\right)$$

$$\Leftrightarrow \left(\beta_i^N e_{\max}\exp\left(-nb\beta_i^{N^2}T_i\right)\right)^{\alpha} = \frac{2e_{\max}}{(2-\alpha n)}\left(\frac{nb\beta_i^{N^2}}{\rho}+1\right)\exp\left(-nb\beta_i^{N^2}T_i\right)$$

$$\Leftrightarrow \left(\exp\left(-nb\beta_i^{N^2}T_i\right)\right)^{\alpha-1} = \frac{2e_{\max}^{1-\alpha}\left(nb\beta_i^{N^2}+\rho\right)}{\beta_i^{N^{\alpha}}\rho(2-\alpha n)}$$

$$\Leftrightarrow \exp\left(-nb\beta_i^{N^2} T_i\right) = \left[\frac{2e_{\max}^{1-\alpha}\left(nb\beta_i^{N^2}+\rho\right)}{\beta_i^{N^\alpha}\rho\left(2-\alpha n\right)}\right]^{\frac{1}{\alpha-1}}$$

$$\Leftrightarrow -nb\beta_i^{N^2} T_i = \frac{1}{\alpha-1}\ln\left[\frac{2e_{\max}^{1-\alpha}\left(nb\beta_i^{N^2}+\rho\right)}{\beta_i^{N^\alpha}\rho\left(2-\alpha n\right)}\right]$$

$$\Leftrightarrow T_i = -\frac{1}{nb\beta_i^{N^2}\left(\alpha-1\right)}\left[\left(\alpha-1\right)\ln\left(\frac{1}{\beta_i^N e_{\max}}\right)+\ln\left(\frac{2\left(nb\beta_i^{N^2}+\rho\right)}{\beta_i^N\rho\left(2-\alpha n\right)}\right)\right] := T_i^N$$

Our assumption on symmetrical farmers insures that there is no possible deviation from this switching time.

Some basic computations then lead to:

$$\partial_{e_{\max}} T_i^N = -\frac{-2+n\alpha}{e_{\max}n\rho} > 0$$

A.3 Proof of proposition 2

We use a similar method as the one used for the proof of Remark 1 and Proposition 2.

(i) We first observe from the first order condition (8) that: $\lambda_t = \alpha A e_t^{\alpha-1} \Leftrightarrow \dot{\lambda}_t = \alpha(\alpha-1)Ae_t^{\alpha-2}\dot{e}_t$ where A is a new constant. We then have: $\dot{\lambda}_t = -\alpha(\alpha-1)Ae_t^{\alpha-1}bn\beta^2$.

We also know from equation (9) that: $\dot{\lambda}_t = \rho\lambda_t - \lambda_t bn\beta^2$. We replace λ_t by $\alpha A e_t^{\alpha-1}$ in this equation and we obtain: $\dot{\lambda}_t = \alpha A e_t^{\alpha-1}\left(\rho - bn\beta^2\right)$.

We now look for A that solves:

$$-\alpha(\alpha-1)Ae_t^{\alpha-1}bn\beta^2 = \alpha A e_t^{\alpha-1}\left(\rho - bn\beta^2\right)$$

$$\Leftrightarrow \rho\alpha A e_t^{\alpha-1} = -\alpha A e_t^{\alpha-1} b\beta^2 n(\alpha-2)$$

$$\Leftrightarrow \rho = -nb\beta^2(\alpha-2) \qquad (18)$$

Before going further, we express β with respect to A. To do so, we depart from first order condition (8) in which we put $\lambda_t = \alpha A e_t^{\alpha-1}$:

$$n\left(\alpha c_t^{\alpha-1} - 2\alpha A e_t^{\alpha-2} bc_t\right) = 0$$

$$\Leftrightarrow c_t^{\alpha-2} = 2Ae_t^{\alpha-2}b$$

$$\Leftrightarrow c_t = e_t(2bA)^{\frac{1}{\alpha-2}}$$

We assume that $c_t = \beta e_t$. The implication is that $\beta = (2bA)^{\frac{1}{\alpha-2}}$. We now put this expression in equation (18) and we obtain:

$$\left(\frac{\rho}{bn(2-\alpha)}\right)^{\frac{1}{2}} = (2bA)^{\frac{1}{\alpha-2}}$$

$$\Leftrightarrow \left(\frac{\rho}{bn(2-\alpha)}\right)^{\frac{\alpha-2}{2}} = 2bA$$

$$\Leftrightarrow A = \frac{\left(\frac{\rho}{bn(2-\alpha)}\right)^{\frac{\alpha-2}{2}}}{2b} := A^*$$

It directly results from this that:

$$\beta^* = \sqrt{\frac{\rho}{bn(2-\alpha)}}$$

We can now look for the equilibrium paths of the modified game. We begin with equation (10) that can now be written as: $\dot{e}_t = \frac{e_t\rho}{\alpha-2}$ with $e_0 = e_{\max}$

The solution of this differential equation is:

$$e_t^* = e_{\max}\exp\left(\frac{\rho t}{\alpha-2}\right) = e_{\max}\exp\left(-nb\beta_i^{*^2}t\right)$$

that gives:

$$c_t^* = \sqrt{\frac{\rho}{bn(2-n\alpha)}}e_{\max}\exp\left(\frac{\rho t}{\alpha-2}\right) = \beta_i^* e_{\max}\exp\left(-nb\beta_i^{*^2}t\right)$$

and

$$a_t^* = -\frac{\rho}{nb(-2+\alpha)}e_{\max}\exp\left(\frac{\rho t}{-2+\alpha}\right) = \beta_i^{*^2}e_{\max}\exp\left(-nb\beta_i^{N^2}t\right)$$

(ii) We depart from first order condition (8): $\alpha c_t^{\alpha-1} - 2\lambda_t b\frac{c_t}{e_t} = 0 \Leftrightarrow \lambda_t = \frac{c_t^{\alpha-2}\alpha e_t}{2b}$ that we put into transversality condition (11):

$$nc_T^\alpha - \frac{c_T^\alpha \alpha n}{2} + \frac{n}{\rho}\partial_t e_t\,|_{t=T} - ne_T = 0$$

$$\Leftrightarrow c_T^\alpha = \frac{2}{(2-\alpha)}\left(-\frac{1}{\rho}\partial_t e_t\,|_{t=T} + e_T\right)$$

$$\Leftrightarrow \left(\beta_i^* e_{\max}\exp\left(-nb\beta_i^{*^2}t\right)\right)^\alpha = \frac{2e_{\max}}{(2-\alpha)}\left(\frac{nb\beta^{*^2}}{\rho}+1\right)\exp\left(-nb\beta^{*^2}T\right)$$

$$\Leftrightarrow \left(\exp\left(-nb\beta^{*^2}T\right)\right)^{\alpha-1} = \frac{2e_{\max}^{1-\alpha}\left(nb\beta^{*^2}+\rho\right)}{\beta^{*^\alpha}\rho(2-\alpha)}$$

$$\Leftrightarrow \exp\left(-nb\beta^{*2}T\right) = \left[\frac{2e_{\max}^{1-\alpha}\left(nb\beta^{*2}+\rho\right)}{\beta^{*\alpha}\rho\left(2-\alpha\right)}\right]^{\frac{1}{\alpha-1}}$$

$$\Leftrightarrow -nb\beta^{*2}T = \frac{1}{\alpha-1}\ln\left[\frac{2e_{\max}^{1-\alpha}\left(nb\beta^{*2}+\rho\right)}{\beta^{*\alpha}\rho\left(2-\alpha\right)}\right]$$

$$\Leftrightarrow T = -\frac{1}{nb\beta^{*2}\left(\alpha-1\right)}\left[\left(\alpha-1\right)\ln\left(\frac{1}{\beta^{*}e_{\max}}\right)+\ln\left(\frac{2\left(nb\beta^{*2}+\rho\right)}{\beta^{*}\rho\left(2-\alpha\right)}\right)\right] := T^{*}$$

Some basic computations then lead to:

$$\partial_{e_{\max}}T^{*} = -\frac{-2+\alpha}{e_{\max}\rho} > 0$$

A.4 Results of the simulations comparing the solutions in the socially optimal case and in the MPNE case

We have run some simulations in order to see how the differences between the socially optimal solutions and the MPNE solutions vary according to parameter values. We obtain the results in Table 1, 2, 3, 4 and 5.

	$n=5$	$n=50$	$n=500$	$n=1999$
$T^{*}-T^{N}$	+	+	+	+
$a_{T^{*}}^{*}-a_{T^{N}}^{N}$	-	-	-	-
$e_{T^{*}}^{*}-e_{T^{N}}^{N}$	+	+	+	+
$a_{T^{N}}^{*}-a_{T^{N}}^{N}$	+	+	+	+

Table 1: Differences for the set of parameters: $b=1$, $e_{\max}=1000$, $\alpha=0.001$ and $\rho=0,04$.

	$\rho=0.004$	$\rho=0.04$	$\rho=0.4$	$\rho=0.9$
$T^{*}-T^{N}$	+	+	+	+
$a_{T^{*}}^{*}-a_{T^{N}}^{N}$	-	-	-	-
$e_{T^{*}}^{*}-e_{T^{N}}^{N}$	+	+	+	+
$a_{T^{N}}^{*}-a_{T^{N}}^{N}$	+	+	+	+

Table 2: Differences for the set of parameters: $b=1$, $e_{\max}=1000$, $\alpha=0.001$ and $n=50$.

	$\alpha = 10^{-6}$	$\alpha = 0.0001$	$\alpha = 0.001$	$\alpha = 0.0039$
$T^* - T^N$	+	+	+	+
$a^*_{T^*} - a^N_{T^N}$	-	-	-	-
$e^*_{T^*} - e^N_{T^N}$	+	+	+	+
$a^*_{T^N} - a^N_{T^N}$	+	+	+	+

Table 3: Differences for the set of parameters: $b = 1$, $e_{\max} = 1000$, $n = 50$ and $\rho = 0.04$.

	$e_{\max} = 10000$	$e_{\max} = 1000$	$e_{\max} = 100$	$e_{\max} = 50$
$T^* - T^N$	+	+	+	+
$a^*_{T^*} - a^N_{T^N}$	-	-	-	-
$e^*_{T^*} - e^N_{T^N}$	+	+	+	+
$a^*_{T^N} - a^N_{T^N}$	+	+	+	+

Table 4: Differences for the set of parameters: $b = 1$, $\alpha = 0.001$, $n = 50$ and $\rho = 0.04$.

	$b = 1000$	$b = 100$	$b = 1$	$b = 0.001$
$T^* - T^N$	+	+	+	+
$a^*_{T^*} - a^N_{T^N}$	-	-	-	-
$e^*_{T^*} - e^N_{T^N}$	+	+	+	+
$a^*_{T^N} - a^N_{T^N}$	+	+	+	+

Table 5: Differences for the set of parameters: $\alpha = 0.001$, $e_{\max} = 1000$, $n = 50$ and $\rho = 0.04$.

A.5 Proof of proposition 3

Firstly, we know from the assumptions that:

$n\alpha > \alpha$

$\Leftrightarrow b(2 - n\alpha) < b(2 - \alpha)$

$$\Leftrightarrow \frac{\rho}{b(2 - n\alpha)} > \frac{\rho}{b(2 - \alpha)}$$

$\Rightarrow {\beta^N}^2 > {\beta^*}^2$

(i) We look for a crossing point:

$a^N_t = a^*_t$

$$\Leftrightarrow -\frac{\rho}{b(-2 + n\alpha)} e_{\max} \exp\left(\frac{n\rho t}{-2 + n\alpha}\right) = -\frac{\rho}{nb(-2 + \alpha)} e_{\max} \exp\left(\frac{\rho t}{-2 + \alpha}\right)$$

$$\Leftrightarrow \frac{n(-2+\alpha)}{(-2+n\alpha)} = \exp\left(\frac{\rho t}{-2+\alpha} - \frac{n\rho t}{-2+n\alpha}\right)$$

$$\Leftrightarrow \frac{n(-2+\alpha)}{(-2+n\alpha)} = \exp\left(\rho t\left(\frac{-2+n\alpha-n(-2+\alpha)}{(-2+\alpha)(-2+n\alpha)}\right)\right)$$

$$\Leftrightarrow \frac{n(-2+\alpha)}{(-2+n\alpha)} = \exp\left(\rho t\left(\frac{2(n-1)}{(-2+\alpha)(-2+n\alpha)}\right)\right)$$

$$\Leftrightarrow t = \frac{(-2+\alpha)(-2+n\alpha)}{\rho 2(n-1)} \ln\left(\frac{n(-2+\alpha)}{(-2+n\alpha)}\right) := t^a$$

We then look for the sign of this crossing point. Firstly, we know from the assumptions that: $\frac{(-2+\alpha)(-2+n\alpha)}{\rho 2(n-1)} > 0$. Furthermore, we know that:

$$-2n < -2$$
$$\Leftrightarrow -2n + \alpha n < -2 + \alpha n$$
$$\Leftrightarrow n(-2+\alpha) < (-2+n\alpha)$$
$$\Leftrightarrow \frac{n(-2+\alpha)}{(-2+n\alpha)} > 1$$
$$\Leftrightarrow \ln\left(\frac{n(-2+\alpha)}{(-2+n\alpha)}\right) > 0$$

We can then conclude that: $t^a > 0$.

We now have to compare $a_t^N = a_t^*$ before and after this crossing point. $t = 0$ occurs before t^a. Firstly, we know that $a_0^N = -\frac{\rho}{b(-2+n\alpha)} e_{\max}$ and $a_0^* = -\frac{\rho}{nb(-2+\alpha)} e_{\max}$. We also know that :

$$n(-2+\alpha) < (-2+n\alpha)$$
$$\Leftrightarrow \frac{\rho}{n(-2+\alpha)} > \frac{\rho}{(-2+n\alpha)}$$
$$\Leftrightarrow -\frac{\rho}{nb(-2+\alpha)} e_{\max} < -\frac{\rho}{b(-2+n\alpha)} e_{\max}$$
$$\Leftrightarrow a_0^* < a_0^N$$

We can thus conclude that: $a_t^N > a_t^* \forall t < t^a < T_i^N$.

It is more complicated to conclude analytically for $t > t^a$. That is why we use simulations presented in Tables 1, 2, 3, 4 and 5 of Appendix A.4. We observe from these simulations that $a_{T^N}^* > a_{T^N}^N$. We can thus conclude that: $a_t^N < a_t^* \forall T_i^N > t > t^a$.

(ii) We know from (i) that:

$$\frac{\rho}{b(2-n\alpha)} > \frac{\rho}{b(2-\alpha)}$$

$$\Leftrightarrow \sqrt{\frac{\rho}{b(2-n\alpha)}} > \sqrt{\frac{\rho}{b(2-\alpha)}}$$
$$\Leftrightarrow \beta^N > \beta^*$$
$$\Leftrightarrow -nb\beta^N t < -nb\beta^* t$$
$$\Leftrightarrow e_{\max}\exp(-nb\beta^N t) < e_{\max}\exp(-nb\beta^* t)$$
$$\Leftrightarrow e^N < e^*$$

A.6 Proof of remark 2

$$\partial_n t^a = \frac{(\alpha-2)\left(2(1-n)+n(\alpha-2)\ln\left(\frac{-2+n\alpha}{(\alpha-2)n}\right)\right)}{2n\rho(-1+n)^2} < 0$$
$$\partial_\rho t^a = \frac{(-2+n\alpha)(\alpha-2)\ln\left(\frac{-2+n\alpha}{(\alpha-2)n}\right)}{2\rho^2(-1+n)} < 0$$

A.7 Proof of proposition 4

The beginning of this proof is similar to the one of Remark 1 except that the first order condition is now given by equation (12). This implies modifications of the proof when this equation is used to express β_i with respect to A_i:

$$(1-\tau)\alpha c_{i,t}^{\alpha-1} - 2\alpha A_i e_t^{\alpha-2} b c_{i,t} = 0 \text{ with } \lambda_{i,t} = \alpha A_i e_t^{\alpha-1}$$
$$\Leftrightarrow (1-\tau)c_{i,t}^{\alpha-2} = 2A_i e_t^{\alpha-2} b$$
$$\Leftrightarrow c_{i,t} = e_t\left(\frac{2bA_i}{1-\tau}\right)^{\frac{1}{\alpha-2}}$$

We assumed that $c_{i,t} = \beta_i e_t$. The implication is that $\beta_i = \left(\frac{2bA_i}{1-\tau}\right)^{\frac{1}{\alpha-2}}$. We now put this expression in equation (17) and we obtain:

$$\left(\frac{\rho}{b(2-n\alpha)}\right)^{\frac{1}{2}} = \left(\frac{2bA_i}{1-\tau}\right)^{\frac{1}{\alpha-2}}$$
$$\Leftrightarrow \left(\frac{\rho}{b(2-n\alpha)}\right)^{\frac{\alpha-2}{2}} = \frac{2bA_i}{1-\tau}$$
$$\Leftrightarrow A_i = \frac{(1-\tau)\left(\frac{\rho}{b(2-n\alpha)}\right)^{\frac{\alpha-2}{2}}}{2b}$$

Remark 7 *Let us observe that* $A_i = A^* \Leftrightarrow \tau^* = 1 - \left(\frac{(\alpha n-2)}{n(\alpha-2)}\right)^{\frac{\alpha}{2}-1}$. *We know that* $\frac{n(-2+\alpha)}{(-2+n\alpha)} > 1$ *and that* $\frac{\alpha}{2} - 1 < 0$. *We can then conclude that* $1 - \left(\frac{(\alpha n-2)}{n(\alpha-2)}\right)^{\frac{\alpha}{2}-1} > 0$.

When $\tau = \tau^*$, $A_i^\tau = \frac{\left(\frac{\rho}{bn(2-\alpha)}\right)^{\frac{\alpha-2}{2}}}{2b}$ and $\beta^\tau = \sqrt{\frac{\rho}{bn(2-\alpha)}}$.

We can now look for the equilibrium paths of the modified game. We begin with equation (10) that can now be written as: $\dot{e}_t = \frac{e_t \rho}{\alpha - 2}$ with $e_0 = e_{\max}$

The solution of this differential equation is:

$$e_t^\tau = e_{\max} \exp\left(\frac{\rho t}{\alpha - 2}\right) = e_t^*$$

that gives:

$$c_t^\tau = \sqrt{\frac{\rho}{bn(2-\alpha)}} e_{\max} \exp\left(\frac{\rho t}{\alpha - 2}\right) = c_t^*$$

and

$$a_t^\tau = -\frac{\rho}{nb(-2+\alpha)} e_{\max} \exp\left(\frac{\rho t}{-2+\alpha}\right) = a_t^*$$

A.8 Proof of remark 3

$$\partial_\alpha \tau^* = -\frac{\left(\frac{\alpha n-2}{n(\alpha-2)}\right)^{\frac{\alpha}{2}} (\alpha-2)^2}{(\alpha n-2)^2} < 0$$

$$\partial_n \tau^* = -\frac{\left(\frac{\alpha n-2}{n(\alpha-2)}\right)^{\frac{\alpha}{2}} n(\alpha-2)\left((\alpha n-2)\ln\left(\frac{\alpha n-2}{n(\alpha-2)}\right) + 2(1-n)\right)}{(\alpha n-2)^2} > 0$$

A.9 Proof of proposition 5

The beginning of the proof is the same as in the proof of Remark 1. We then know from equation (13) and from the assumption on the symmetry of solutions that:

$\dot{\lambda}_{i,t} = \rho\lambda_{i,t} - \sigma_t + \lambda_{i,t} b(n-2)\beta_i^2$. We replace $\lambda_{i,t}$ by $\alpha A_i e_t^{\alpha-1}$ in this equation and we obtain: $\dot{\lambda}_{i,t} = \alpha A_i e_t^{\alpha-1}\left(\rho + b(n-2)\beta_i^2\right) - \sigma_t$.

The next step is to look for A_i that solves:

$$-\alpha(\alpha-1)A_i e_t^{\alpha-1} bn\beta_i^2 = \alpha A_i e_t^{\alpha-1}\left(\rho + b(n-2)\beta_i^2\right) - \sigma_t \tag{19}$$

Remark 8 *Let us observe that:* $\left((13) \Leftrightarrow (9)\right) \Leftrightarrow \ \sigma_t^* = (n-1)2\alpha A_i e_t^{\alpha-1} b\beta_i^2$

Let us set $\sigma_t = \sigma_t^*$ and go further:

$$\begin{aligned}
&(19) \Leftrightarrow -\alpha(\alpha-1)A_i e_t^{\alpha-1} bn\beta_i^2 = -\alpha A_i e_t^{\alpha-1}(-\rho + b\beta_i^2 n) \\
&\Leftrightarrow \alpha A_i e_t^{\alpha}\rho = -\alpha(\alpha-2)A_i e_t^{\alpha} bn\beta_i^2 \\
&\Leftrightarrow \rho = -b\beta_i^2 n(\alpha-2)
\end{aligned} \tag{20}$$

We know from the proof of Remark 1 that $\beta_i = (2bA_i)^{\frac{1}{\alpha-2}}$. We now put this expression in equation (20) and we obtain:

$$\left(\frac{\rho}{bn(2-\alpha)}\right)^{\frac{1}{2}} = (2bA_i)^{\frac{1}{\alpha-2}}$$

$$\Leftrightarrow \left(\frac{\rho}{bn(2-\alpha)}\right)^{\frac{\alpha-2}{2}} = 2bA_i$$

$$\Leftrightarrow A_i = \frac{\left(\frac{\rho}{bn(2-\alpha)}\right)^{\frac{\alpha-2}{2}}}{2b} := A_i^*$$

It directly results from this that:

$$\beta_i^{\sigma} = \sqrt{\frac{\rho}{bn(2-\alpha)}}$$

We can now look for the equilibrium paths of the modified game. We begin with equation (10) that can now be written as: $\dot{e}_t = \dfrac{e_t\rho}{\alpha-2}$ with $e_0 = e_{\max}$

The solution of this differential equation is:

$$e_t^{\sigma} = e_{\max}\exp\left(\frac{\rho t}{\alpha-2}\right) = e_t^*$$

Remark 9 *Since* $\sigma_t = (n-1)\alpha e_t^{\sigma^{\alpha-1}} \beta_i^{\sigma^{\alpha}}$ *, we then directly have*

$$\sigma_t^* = (n-1)\alpha\left(e_{\max}\exp\left(\frac{\rho t}{\alpha-2}\right)\right)^{\alpha-1}\left(\frac{\rho}{bn(2-\alpha)}\right)^{\frac{\alpha}{2}} > 0$$

We finally have:

$$c_t^\sigma = \sqrt{\frac{\rho}{bn(2-\alpha)}}e_{\max}\exp\left(\frac{\rho t}{\alpha-2}\right) = c_t^*$$

and

$$a_t^\sigma = -\frac{\rho}{nb(-2+\alpha)}e_{\max}\exp\left(\frac{\rho t}{-2+\alpha}\right) = a_t^*$$

A.10 Proof of remark 4

$$\partial_t\sigma_t^* = \frac{\rho(\alpha-1)(n-1)\alpha\left(e_{\max}\exp\left(\frac{\rho t}{\alpha-2}\right)\right)^{\alpha-1}\left(\frac{\rho}{bn(2-\alpha)}\right)^{\frac{\alpha}{2}}}{\alpha-2} > 0$$

$$\partial_\rho\sigma_t^* = \frac{(n-1)\alpha\left(e_{\max}\exp\left(\frac{\rho t}{\alpha-2}\right)\right)^{\alpha}\left(\frac{\rho}{bn(2-\alpha)}\right)^{\frac{\alpha}{2}}\exp\left(-\frac{\rho t}{\alpha-2}\right)(\alpha^2-2\alpha+2\rho t\alpha-2\rho t)}{e_{\max}\rho(\alpha-2)} > 0$$

since $\alpha^2-2\alpha+2\rho t\alpha-2\rho t<0$

$$\partial_n\sigma_t^* = -\frac{\alpha\left(e_{\max}\exp\left(\frac{\rho t}{\alpha-2}\right)\right)^{\alpha}\left(\frac{\rho}{bn(2-\alpha)}\right)^{\frac{\alpha}{2}}\exp\left(-\frac{\rho t}{\alpha-2}\right)(\alpha n-\alpha-2n)}{2e_{\max}n} > 0$$

$$\partial_b\sigma_t^* = -\frac{(n-1)\alpha^2\left(e_{\max}\exp\left(\frac{\rho t}{\alpha-2}\right)\right)^{\alpha}\left(\frac{\rho}{bn(2-\alpha)}\right)^{\frac{\alpha}{2}}\exp\left(-\frac{\rho t}{\alpha-2}\right)}{2be_{\max}} < 0$$

$$\partial_{e_{\max}}\sigma_t^* = \frac{(\alpha-1)(n-1)\alpha\left(e_{\max}\exp\left(\frac{\rho t}{\alpha-2}\right)\right)^{\alpha-1}\left(\frac{\rho}{bn(2-\alpha)}\right)^{\frac{\alpha}{2}}\exp\left(-\frac{\rho t}{\alpha-2}\right)}{e_{\max}^2} < 0$$

A.11 Proof of proposition 6

(i) We depart from first order condition (12):

$(1-\tau)\alpha c_{i,t}^{\alpha-1} - 2\lambda_{i,t} b \frac{c_{i,t}}{e_t} = 0 \Leftrightarrow \lambda_{i,t} = \frac{(1-\tau)c_{i,t}^{\alpha-2}\alpha e_t}{2b}$ that we put into the transversality condition (14):

$$(1-\tau)c_{i,T_i}^{\alpha} - \frac{(1-\tau)c_{i,T_i}^{\alpha}\alpha n}{2} + \frac{1}{\rho}\partial_t e_t \,|_{t=T_i} - e_{T_i} = 0$$

$$\Leftrightarrow \left(\frac{(\alpha n-2)}{n(\alpha-2)}\right)^{\frac{\alpha}{2}-1}(2-\alpha n)c_{i,T_i}^{\alpha} = -\frac{2}{\rho}\partial_t e_t \,|_{t=T_i} + 2e_{T_i}$$

$$\Leftrightarrow \left(\frac{(\alpha n-2)}{n(\alpha-2)}\right)^{\frac{\alpha}{2}} n(2-\alpha)c_{i,T_i}^{\alpha} = -\frac{2}{\rho}\partial_t e_t \,|_{t=T_i} + 2e_{T_i}$$

$$\Leftrightarrow \left(\frac{(\alpha n-2)}{n(\alpha-2)}\right)^{\frac{\alpha}{2}} n(2-\alpha)\left(\beta_i^* e_{\max}\exp\left(-nb\beta_i^{*2}T_i\right)\right)^{\alpha} = \left(\frac{nb\beta_i^{*2}}{\rho}+1\right)2e_{\max}\exp\left(-nb\beta_i^{*2}T_i\right)$$

$$\Leftrightarrow \left(\frac{(\alpha n-2)}{n(\alpha-2)}\right)^{\frac{\alpha}{2}} \left(\exp\left(-nb\beta_i^{*2}T_i\right)\right)^{\alpha-1} = \frac{2e_{\max}^{1-\alpha}\left(nb\beta_i^{*2}+\rho\right)}{\beta_i^{*\alpha}\rho n(2-\alpha)}$$

$$\Leftrightarrow \left(\exp\left(-nb\beta_i^{*2}T_i\right)\right)^{\alpha-1} = \frac{2e_{\max}^{1-\alpha}\left(nb\beta_i^{*2}+\rho\right)}{\beta_i^{*\alpha}\rho n(2-\alpha)}\left(\frac{n(\alpha-2)}{(\alpha n-2)}\right)^{\frac{\alpha}{2}}$$

$$\Leftrightarrow \exp\left(-nb\beta_i^{*2}T_i\right) = \left[\frac{2e_{\max}^{1-\alpha}\left(nb\beta_i^{*2}+\rho\right)}{\beta_i^{*\alpha}\rho n(2-\alpha)}\left(\frac{n(\alpha-2)}{(\alpha n-2)}\right)^{\frac{\alpha}{2}}\right]^{\frac{1}{\alpha-1}}$$

$$\Leftrightarrow -nb\beta_i^{*2}T_i = \frac{1}{\alpha-1}\ln\left[\frac{2e_{\max}^{1-\alpha}\left(nb\beta_i^{*2}+\rho\right)}{\beta_i^{*\alpha}\rho n(2-\alpha)}\left(\frac{n(\alpha-2)}{(\alpha n-2)}\right)^{\frac{\alpha}{2}}\right]$$

$$\Leftrightarrow T_i = -\frac{1}{nb\beta_i^{*2}(\alpha-1)}\left[(\alpha-1)\ln\left(\frac{1}{\beta_i^* e_{\max}}\right) + \ln\left(\frac{2\left(nb\beta_i^{*2}+\rho\right)}{\beta_i^*\rho n(2-\alpha)}\right) + \frac{\alpha}{2}\ln\left(\frac{n(\alpha-2)}{(\alpha n-2)}\right)\right] := T_i^{\tau}$$

Some basic computations then lead to:

$$\partial_{e_{\max}} T_i^{\tau} = -\frac{-2+\alpha}{e_{\max}\rho} > 0$$

(ii) We depart from first order condition (4): $\alpha c_{i,t}^{\alpha-1} - 2\lambda_{i,t} b \frac{c_{i,t}}{e_t} = 0 \Leftrightarrow \lambda_{i,t} = \frac{c_{i,t}^{\alpha-2}\alpha e_t}{2b}$ that we put transversality into condition (15): $\left(c_{i,t}^{\alpha} + \sigma_t e_t - \frac{\lambda_{i,t} bnc_{i,t}^2}{e_t} + \frac{1}{\rho}\partial_t e_t - e_t\right)_{t=T_i} = 0$

$$c_{i,T_i}^{\alpha}-\frac{c_{i,T_i}^{\alpha}\alpha n}{2}+\sigma_{T_i}e_{T_i}+\frac{1}{\rho}\partial_t e_t\,|_{t=T_i}-e_{T_i}=0$$

$$\Leftrightarrow\left(1-\frac{\alpha n}{2}\right)c_{i,T_i}^{\alpha}+(n-1)\alpha e_{T_i}^{\alpha}\beta_i^{*\alpha}=-\frac{1}{\rho}\partial_t e_t\,|_{t=T_i}+e_{T_i}$$

$$\Leftrightarrow\left(1-\frac{\alpha n}{2}\right)\left(\beta_i^{*}e_{\max}\exp\left(-nb\beta_i^{*2}T_i\right)\right)^{\alpha}+(n-1)\alpha\left(e_{\max}\exp\left(-nb\beta_i^{*2}T_i\right)\right)^{\alpha}\beta_i^{*\alpha}$$

$$=\left(\frac{nb\beta_i^{*2}}{\rho}+1\right)e_{\max}\exp\left(-nb\beta_i^{*2}T_i\right)$$

$$\Leftrightarrow\left(1+\frac{\alpha n}{2}-\alpha\right)\left(\beta_i^{*}e_{\max}\exp\left(-nb\beta_i^{*2}T_i\right)\right)^{\alpha}=\left(\frac{nb\beta_i^{*2}}{\rho}+1\right)e_{\max}\exp\left(-nb\beta_i^{*2}T_i\right)$$

$$\Leftrightarrow\exp\left(-nb\beta_i^{*2}T_i\right)=\left[\frac{2e_{\max}^{1-\alpha}\left(nb\beta_i^{*2}+\rho\right)}{\rho\beta_i^{*\alpha}\left(2+\alpha(n-2)\right)}\right]^{\frac{1}{\alpha-1}}$$

$$\Leftrightarrow T_i=-\frac{1}{nb\beta_i^{*2}\left(\alpha-1\right)}\ln\left(\frac{2e_{\max}^{1-\alpha}\left(nb\beta_i^{*2}+\rho\right)}{\rho\beta_i^{*\alpha}\left(2+\alpha(n-2)\right)}\right)$$

$$\Leftrightarrow T_i=-\frac{1}{nb\beta_i^{*2}\left(\alpha-1\right)}\left[\left(\alpha-1\right)\ln\left(\frac{1}{\beta_i^{*}e_{\max}}\right)+\ln\left(\frac{2\left(nb\beta_i^{*2}+\rho\right)}{\beta_i^{*}\rho\left(2+\alpha(n-2)\right)}\right)\right]:=T_i^{\sigma}$$

Some basic computations then lead to:

$$\partial_{e_{\max}}T_i^{\sigma}=-\frac{-2+\alpha}{e_{\max}\rho}>0$$

(iii) $$T^{*}-T_i^{\sigma}=-\frac{1}{nb\beta^{*2}\left(\alpha-1\right)}\left[\ln\left(\frac{2\left(nb\beta^{*2}+\rho\right)}{\beta^{*}\rho\left(2-\alpha\right)}\right)-\ln\left(\frac{2\left(nb\beta_i^{*2}+\rho\right)}{\beta_i^{*}\rho\left(2+\alpha(n-2)\right)}\right)\right]$$

$$=-\frac{1}{nb\beta^{*2}\left(\alpha-1\right)}\ln\left(\frac{\dfrac{2\left(nb\beta^{*2}+\rho\right)}{\beta^{*}\rho\left(2-\alpha\right)}}{\dfrac{2\left(nb\beta_i^{*2}+\rho\right)}{\beta_i^{*}\rho\left(2+\alpha(n-2)\right)}}\right)=-\frac{1}{nb\beta^{*2}\left(\alpha-1\right)}\ln\left(\frac{2+\alpha(n-2)}{2-\alpha}\right)$$

We know that $\left(n-1\right)\alpha>-\alpha$

$$\Leftrightarrow 2+\left(n-1\right)\alpha>2-\alpha$$

$$\Leftrightarrow\frac{2+\alpha(n-2)}{2-\alpha}>1$$

$$\Leftrightarrow \ln\left(\frac{2+\alpha(n-2)}{2-\alpha}\right) > 0$$

$$\Leftrightarrow T^* - T_i^\sigma > 0 \text{ since } \alpha - 1 < 0$$

A.12 Proof of proposition 7

(i) We directly see that when we add $LS_t = -\sigma_t^* e_t^\sigma$ in the transversality condition used in the (ii) of the proof of proposition 6, we obtain the same expression as in the socially optimal case (see the (ii) of the proof of proposition 2).

(ii) As in the proof of (i) of proposition 6, we depart from first order condition (12) that is unchanged. We put the expression of λ_i into the new transversality condition: $\left((1-\tau)c_{i,t}^\alpha - \frac{\lambda_{i,t} bnc_{i,t}^2}{e_t} + \frac{1}{\rho}\partial_t e_t - e_t + LS_{i,t}\right)_{t=T_i} = 0$ where $LS_{i,T_i} = \tau c_{i,T_i}^{\tau^\alpha}$ and we obtain:

$$c_{i,T_i}^\alpha - \frac{(1-\tau)c_{i,T_i}^\alpha \alpha n}{2} + \frac{1}{\rho}\partial_t e_t \,|_{t=T_i} - e_{T_i} = 0$$

$$\Leftrightarrow \left(2 - \alpha n\left(\frac{(\alpha n-2)}{n(\alpha-2)}\right)^{\frac{\alpha}{2}-1}\right) c_{i,T_i}^\alpha = -\frac{2}{\rho}\partial_t e_t \,|_{t=T_i} + 2e_{T_i}$$

$$\Leftrightarrow \left(2 - \alpha n\left(\frac{(\alpha n-2)}{n(\alpha-2)}\right)^{\frac{\alpha}{2}-1}\right)\left(\beta_i^* e_{\max}\exp\left(-nb\beta_i^{*2}T_i\right)\right)^\alpha = \left(\frac{nb\beta_i^{*2}}{\rho}+1\right)2e_{\max}\exp\left(-nb\beta_i^{*2}T_i\right)$$

$$\Leftrightarrow \left(\exp\left(-nb\beta_i^{*2}T_i\right)\right)^{\alpha-1} = \frac{2e_{\max}^{1-\alpha}\left(nb\beta_i^{*2}+\rho\right)}{\beta_i^{*\alpha}\rho\left(2-\alpha n\left(\frac{(\alpha n-2)}{n(\alpha-2)}\right)^{\frac{\alpha}{2}-1}\right)}$$

$$\Leftrightarrow \exp\left(-nb\beta_i^{*2}T_i\right) = \left[\frac{2e_{\max}^{1-\alpha}\left(nb\beta_i^{*2}+\rho\right)}{\beta_i^{*\alpha}\rho\left(2-\alpha n\left(\frac{(\alpha n-2)}{n(\alpha-2)}\right)^{\frac{\alpha}{2}-1}\right)}\right]^{\frac{1}{\alpha-1}}$$

$$\Leftrightarrow -nb\beta_i^{*2}T_i = \frac{1}{\alpha-1}\ln\left[\frac{2e_{\max}^{1-\alpha}\left(nb\beta_i^{*2}+\rho\right)}{\beta_i^{*\alpha}\rho\left(2-\alpha n\left(\frac{(\alpha n-2)}{n(\alpha-2)}\right)^{\frac{\alpha}{2}-1}\right)}\right]$$

$$\Leftrightarrow T_i = -\frac{1}{nb\beta_i^{*2}(\alpha-1)}\left[(\alpha-1)\ln\left(\frac{1}{\beta_i^* e_{\max}}\right)+\ln\left(\frac{2\left(nb\beta_i^{*2}+\rho\right)}{\beta_i^*\rho}\right)-\ln\left(2-\alpha n\left(\frac{(\alpha n-2)}{n(\alpha-2)}\right)^{\frac{\alpha}{2}-1}\right)\right] := T_i^{\tau LS}$$

Which is different from T^*.[10]

A.13 Proof of remark 6

Some basic computations lead to:

$$\partial_{e_{\max}}\left(T^\tau - T^N\right) = \partial_{e_{\max}}\left(T^\sigma - T^N\right) = \partial_{e_{\max}}\left(T^{\tau ls} - T^N\right) = \partial_{e_{\max}}\left(T^* - T^N\right) = \frac{2(n-1)}{n\rho e_{\max}} > 0$$

$$\partial_b\left(T^\tau - T^N\right) = \partial_b\left(T^\sigma - T^N\right) = \partial_b\left(T^{\tau ls} - T^N\right) = \partial_b\left(T^* - T^N\right) = -\frac{\alpha(n-1)}{bn\rho(\alpha-1)} > 0$$

$$\partial_{e_{\max}}\left(T^\tau - T^*\right) = \partial_{e_{\max}}\left(T^\tau - T^\sigma\right) = \partial_{e_{\max}}\left(T^\sigma - T^*\right) = \partial_{e_{\max}}\left(T^{\tau LS} - T^\sigma\right)$$

$$= \partial_{e_{\max}}\left(T^{\tau LS} - T^\tau\right) = 0$$

$$\partial_b\left(T^\tau - T^*\right) = \partial_b\left(T^\tau - T^\sigma\right) = \partial_b\left(T^\sigma - T^*\right) = \partial_b\left(T^{\tau LS} - T^\sigma\right)$$

$$= \partial_b\left(T^{\tau LS} - T^\tau\right) = 0$$

$$\partial_{e_{\max}}\left(T^* - T^{\tau ls}\right) = -\frac{\alpha-2}{\rho e_{\max}} > 0$$

$$\partial_b\left(T^* - T^{\tau ls}\right) = \frac{\alpha(\alpha-2)}{2b\rho(\alpha-1)} > 0$$

A.14 Results of the simulations comparing the switching times in the different cases

We have run some simulations in order to see how the differences between the switching times vary according to parameter values. We obtain the results in Table 6, 7, 8, 9 and 10.

[10] We assume that the set of parameters is such that $T_i^{\tau LS} > 0$.

	$n = 5$	$n = 50$	$n = 500$	$n = 1999$
$T^* - T^N$	284	356	365	365.5
$T^* - T^\tau$	80.5	195.5	310	345
$T^{\tau ls} - T^*$	0.5	64	-255.5	-344
$T^* - T^\sigma$	0.1	1	11	20
$T^\sigma - T^\tau$	80.5	194.5	299.5	325
$T^\tau - T^N$	203.5	160	54	20.5
$T^{\tau ls} - T^N$	285	419.5	109.5	21.5
$T^\sigma - T^N$	284	354.5	353.5	345.5
$T^{\tau ls} - T^\sigma$	0.5	65	-244.5	-324
$T^{\tau ls} - T^\tau$	81	259.5	55.5	1

Table 6: Differences for the set of parameters: $b = 1$, $e_{\max} = 1000$, $\alpha = 0.001$ and $\rho = 0,04$.

	$\rho = 0.004$	$\rho = 0.04$	$\rho = 0.4$	$\rho = 0.9$
$T^* - T^N$	3558	356	35.5	16
$T^* - T^\tau$	1956	195.5	19.5	8.5
$T^{\tau ls} - T^*$	638.5	64	6.5	3
$T^* - T^\sigma$	12	1	0.1	0.05
$T^\sigma - T^\tau$	1944	194.5	19.5	8.5
$T^\tau - T^N$	1602	160	16	7
$T^{\tau ls} - T^N$	4197	419.5	42	18.5
$T^\sigma - T^N$	3546	354.5	35.5	16
$T^{\tau ls} - T^\sigma$	650.5	65	6.5	3
$T^{\tau ls} - T^\tau$	2594.5	259.5	25.9	11.5

Table 7: Differences for the set of parameters: $b = 1$, $e_{\max} = 1000$, $\alpha = 0.001$ and $n = 50$.

	$\alpha = 10^{-6}$	$\alpha = 0.0001$	$\alpha = 0.001$	$\alpha = 0.0039$
$T^* - T^N$	355.5	355.5	356	358
$T^* - T^\tau$	195.5	195.5	195.5	195.5
$T^{\tau ls} - T^*$	0.06	6.5	64	-137
$T^* - T^\sigma$	0.001	0.1	1	11
$T^\sigma - T^\tau$	195.5	195.5	194.5	184.5
$T^\tau - T^N$	160	160	160	163
$T^{\tau ls} - T^N$	355.5	362	419.5	221
$T^\sigma - T^N$	355.5	355.5	354.5	347
$T^{\tau ls} - T^\sigma$	0.06	7	65	-126
$T^{\tau ls} - T^\tau$	195.5	203	259.5	58.5

Table 8: Differences for the set of parameters: $b = 1$, $e_{\max} = 1000$, $n = 50$

and $\rho = 0.04$.

	$e_{max} = 500$	$e_{max} = 100$	$e_{max} = 75$	$e_{max} = 50$
$T^* - T^N$	322	243	229	211
$T^* - T^\tau$	195.5	195.5	195.5	216
$T^{\tau ls} - T^*$	64	64	64	-32.5
$T^* - T^\sigma$	1	1	1	2
$T^\sigma - T^\tau$	194.5	194.5	194.5	214
$T^\tau - T^N$	126	47.5	33	-5
$T^{\tau ls} - T^N$	385.5	307	292.5	178.5
$T^\sigma - T^N$	320.5	242	227.5	209.5
$T^{\tau ls} - T^\sigma$	65	65	65	-30.5
$T^{\tau ls} - T^\tau$	259.5	259.5	259.5	183.5

Table 9: Differences for the set of parameters: $b = 1$, $\alpha = 0.001$, $n = 50$ and $\rho = 0.04$.

	$b = 0.9$	$b = 0.5$	$b = 0.1$	$b = 0.001$
$T^* - T^N$	356	356	356	356
$T^* - T^\tau$	195.5	195.5	195.5	195.5
$T^{\tau ls} - T^*$	64	64	64	64
$T^* - T^\sigma$	1	1	1	1
$T^\sigma - T^\tau$	194.5	194.5	194.5	194.5
$T^\tau - T^N$	160	160	160	160
$T^{\tau ls} - T^N$	419.5	419.5	419.5	419.5
$T^\sigma - T^N$	354.5	354.5	354.5	354.5
$T^{\tau ls} - T^\sigma$	65	65	65	65
$T^{\tau ls} - T^\tau$	259.5	259.5	259.5	259.5

Table 10: Differences for the set of parameters: $\alpha = 0.001$, $e_{max} = 1000$, $n = 50$ and $\rho = 0.04$.

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