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A SIMPLIFIED METHOD FOR DETERMINING THE CONDITION OF WHITE-TAILED DEER
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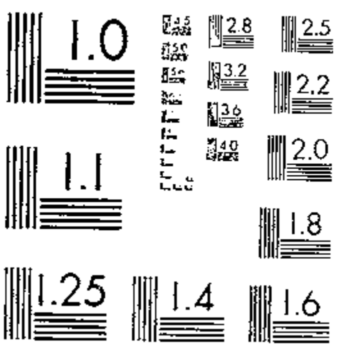

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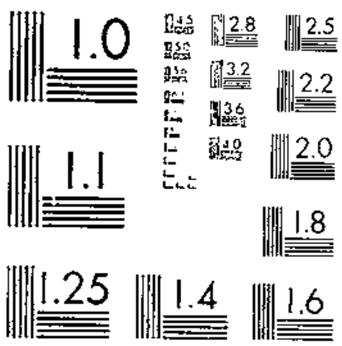

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UNITED STATES DEPARTMENT OF AGRICULTURE
WASHINGTON, D. C.

A Simplified Method For Determining the Condition of White-Tailed Deer Herds in Relation to Available Forage[1]

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CONTENTS

INTRODUCTION

The United States is working its range lands to the limit to obtain the greatest possible production of meat, leather, and wool for war. At the same time, however, the President has said:

> It is necessary in wartime to conserve our natural resources and keep in repair our national plant. We cannot afford waste or destruction, for we must continue to think of the good of future generations of Americans.

[1] Submitted for publication June 1942.

[2] The authors especially want to thank E. A. Goldman, senior biologist, Fish and Wildlife Service, for his assistance in the field work and review and helpful suggestions in the preparation of the manuscript. For their review and suggestions for the improvement of the manuscript the authors also want to express appreciation to H. L. Shantz, chief, division of wildlife management; Glen Mitchell, wildlife management, Region 6; M. A. Mattoon, assistant regional forester, Region 7 (all of the Forest Service); Stanley P. Young, senior biologist, Fish and Wildlife Service; and Richard Gerstell, chief, division of research, Pennsylvania Game Commission. To Randal McCain, now wildlife staffman on the Allegheny National Forest, goes credit for cleaning the skulls collected and for taking most of the photographs illustrating use of the dental formulae.

Thanks are due the Measurements Section of the Branch of Research, Forest Service, for generously furnishing the necessary computational aid in analyzing the data. In particular, the authors are grateful to Miss Marion Sandomire, formerly of the Office of Information, United States Department of Agriculture, not only for help in this connection but also for useful and stimulating suggestions on theoretical points.

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Full production therefore should go hand in hand with range protection and conservative management, but while much progress has been made in the proper utilization of the range by livestock, the present responsibility for managing deer herds falls on depleted staffs, and it is essential that only the minimum work required for good management be undertaken. This publication presents the results of a long series of measurements and describes a simplified method which would save an immense amount of effort on the part of management men.

Livestock operators, in addition to their observations of grazing areas, are able to check range and animal productivity by the number of young and the weights and prices obtained at the market. Similarly, big-game managers have long been concerned with the determination of a method which would readily reveal the age classes and quality of the herds. To this end, various workers have collected information on one or more characters, such as weight, antler development, and body measurements. Although these investigators have developed some guiding principles, there still remained the problem of testing all methods on a common sample. The study reported here undertook to compare the several methods and through statistical analysis to determine which was the most reliable.

The work was begun on the Allegheny National Forest in Pennsylvania because local hunters contended that the deer were getting smaller. Over a 5-year period, 1935-39, a mass of data was obtained on the white-tailed deer herd, records being taken on 1,787 deer and including four antler measurements: Spread, circumference of main beam, length of tines, and number of tines. The body measurements taken were total length, length of tail, and length of hind foot. In addition, dressed weight was recorded, and age was determined by dental development and condition.

The statistical analysis revealed that the length of hind foot was the most desirable measurement. It can be readily and accurately taken, and the age can then be determined by simple reference to a chart. From that information, the game manager can rapidly appraise the condition of the herd, since, combined with sex determination, he will have the age classes and their relative physical quality.

The simplified method herein reported appears sufficiently accurate for practicable deer-herd management in many localities, and every effort has been made therefore to make it available during the war period when it is seriously needed to prevent possible waste of animal and plant resources (fig. 1).

Forest range resources within the last generation have yielded an increasing crop of deer and elk, there being more deer now in some areas than at any previously known time. On the national forests alone, which contain about a third of the Nation's deer, more than 31 million pounds of dressed venison can be harvested annually without decreasing the breeding herds. This is equivalent to the annual production of cattle and calves on farms in the States of Maine, New Jersey, and Delaware.

Since it is clear that the war will materially reduce control by hunting—the most useful method of controlling the deer herds—it is highly desirable that expanding game populations be appraised so that State game departments may take whatever action is necessary. Control by nature—through starvation, disease, and kill by predatory animals—is wasteful of meat and leather supplies that can pro-

F 323503

FIGURE 1.—It is necessary for the deer-herd manager to have a reasonably accurate check on the increases and condition of the animals.

vide for many civilian and war needs. Obviously, management is necessary, the traditional belief that deer and elk roam over vast areas having been disproved during recent years by tagging and other studies which have revealed that individual animals and herds instinctively limit their search for food to the general localities in which they were born. Thus, even before nature's relentless processes take effect, big game may damage shrubs, young trees, and other browse plants to a point where animal-made deserts result, and finally will compete for the range needed by domestic livestock.

Moreover, most of the plants browsed by deer are difficult to re-establish and are slow in growth, so that once they have been depleted, restoration of the good hunting associated with them becomes an expensive, long-time, and tedious process. It is the adequate, yet limited, attention and care that can be given within the next few seasons which will obviate later costly and disheartening correction on critical areas.

THE DEER HERD AND RANGE CONDITIONS

The results of the work reported in this bulletin reflect to some degree the condition of both the deer herd and the area.

The Allegheny National Forest is located in northwestern Pennsylvania and has a gross area of some 740,000 acres, with a net area of some 500,000 acres of Government-owned land. The topography is rolling and hilly, the altitude varying from approximately 1,000 to 2,000 feet above sea level. The forest is made very accessible by some 700 miles of Federal, State, and forest highways, 300 miles of forest foot trails, and well over 500 miles of gas lines, oil pipe lines, and telephone and power lines. More than 40,000 people reside within the forest boundary. A few live on farms, but the majority make their homes in numerous small towns and communities where they are actively engaged in some one of the many industrial establishments, such as sawmills, furniture factories, glass plants, tanneries, and wood-distillation plants, or in some phase of the petroleum industry, as the forest contains thousands of oil and gas wells.

White pine, hemlock, and hardwoods originally covered the area. Early logging operations took the white pine, then the hemlock, and later the hardwoods. At the present time 43.5 percent of the timber on the Government-owned land falls in the 0–20-year age class, 40.5 percent in the 21–40-year age class, and 11 percent in the 41–80-year age class. A total of 84 percent is in the younger age classes (0–40 years). Prior to Government ownership, a good part of the total acreage, especially in the southern half of the forest had been burned over several times. The cover on such areas consists of poorer timber species, principally aspen (*Populus tremuloides* and *P. grandidentata*) and pin cherry (*Prunus pensylvanica*) with a ground cover of bracken fern, goldenrod, mosses, lycopodium, and grasses. Thousands of acres on the forest support scanty growth of these species (figs. 2 and 3). The cut-over areas that have not been too severely burned would normally come back to good stands of hardwoods. Excessive deer browsing interrupts the natural succession of forest cover and may have a detrimental effect on the resulting stand.

The forest type is commonly known as the "northern hardwood" and varies in composition, but the characteristic species are: Beech (*Fagus grandifolia*), yellow birch (*Betula lutea*), black birch (*Betula lenta*), and sugar maple (*Acer saccharum*). Mixed with these principal

F 31905

FIGURE 2.—Winter. Typical growth (aspen and pin cherry) on the Allegheny Forest after repeated burns. It is low in winter carrying capacity for deer.

F 321228

FIGURE 3.—Early spring. This picture shows typical older growth (mostly pin cherry but some aspen) after repeated burns. It is low in carrying capacity for deer.

species are: Black cherry (*Prunus serotina*), white ash (*Fraxinus americana*), oaks (*Quercus* sp.), yellowpoplar (*Liriodendron tulipifera*), cucumber (*Magnolia acuminata*), American basswood (*Tilia americana*), hickory (*Carya* sp.), soft maple (*Acer rubrum*), black tupelo (*Nyssa sylvatica*), and elm (*Ulmus* sp.). The following species are also found: Water beech (*Carpinus caroliniana*), alder (*Alnus* sp.), serviceberry (*Amelanchier canadensis*), sassafras (*Sassafras officinale*), striped maple (*Acer pensylvanicum*), viburnum (*Viburnum* sp.), dogwood (*Cornus* sp.), ironwood (hophornbeam) (*Ostrya virginiana*), and witch-hazel (*Hamamelis virginiana*).

Various mixtures of the above species occur, depending upon site. On the higher ridges there exists a subtype containing higher percentages of oak, hickory, and black tupelo. Chestnut, prior to the blight, was common in this subtype. Sprouts from old stumps still persist but are annually blighted back.

The estimated deer population on the forest during the period of the study is shown in table 1.

In December 1938, Pennsylvania had a State-wide antlerless deer hunting season of 6 days, and some 24,000 animals were killed on the Allegheny Forest. This heavy kill, however, did not equal the annual

TABLE 1.—*The estimated size of the deer herd on the Allegheny National Forest at the close of the hunting season by years for the 5-year period 1935–39*

Year	Estimated population	Deer per 1,000 acres	Year	Estimated population	Deer per 1,000 acres
1935	23,000	31.25	1938	40,000	55.07
1936	39,000	52.50	1939	37,000	50.90
1937	43,000	58.06			

increase for the 2 previous years, which demonstrates the difficulty of controlling extremely heavy deer populations by hunting. In 1937 the average for the forest was 1 deer to every 17 acres, and on areas of heavy concentration, particularly in the southern part of the forest, it was estimated that there was 1 deer to every 5 or 6 acres. The best estimate for proper stocking of the forest range is probably 1 deer for every 40 to 50 acres, or a total of some 15,000 deer. With an average population of 1 deer to every 18 acres for the last 5 years and excessive populations prior to that time, it is probable that the herd should be reduced to at least 10,000 animals in order to permit normal recovery of food and cover plants. The average area on the forest shows a definite deer line and such species as hemlock, soft maple, rhododendron, laurel, devil's-walkingstick, and sumac have in many places been killed out by overbrowsing (fig. 4).

Since 1938, on a good part of the forest the deer have consumed their supply of summer food early and have begun to eat their winter food before the normal time. This has resulted in rather heavy winter loss even during mild winters, such as 1936–37 and 1937–38. The two successive antlerless deer-hunting seasons (1938 and 1939) greatly reduced the herd in the southern part of the forest, but at the close of the 1939 season, there remained more deer than the depleted environment could carry and recover. Since 1938, a rapid increase in the northern herd has been very evident, and in 1939 it was estimated that this herd was rapidly approaching the 1937 peak of the southern herd (an average of 1 deer to every 17 acres and 1 to every 5–6 acres on concentration areas).

Reforestation on thousands of acres of burned-over land on the Allegheny Forest has been materially affected by deer browsing, which occurs principally during the winter months. Deer damage on plantations runs as high as 94 percent.[3]

F 408759

FIGURE 4.—Hemlock heavily browsed by deer. The "deer line" is very evident.

[3] VAN NORT, A. C. DEER AND RABBIT DAMAGE STUDY—ALLEGHENY NATIONAL FOREST. Third Annual Progress Report. U. S. Forest Serv., 8 pp. 1937. [Unpublished.]

The planting of wildlife food species to alleviate the food shortage was unsuccessful because the deer ate the plants almost as fast as they were planted. A reduction in the number of deer must precede any plan to rehabilitate the environment on the Allegheny National Forest.

Winters in the area are cold with an average annual snowfall of about 50 inches, but ordinarily the snow does not lay deep in the woods. Deer do not "yard up" as they do farther north, but during severe cold weather and deep snow, such as occurred in 1935–36, they concentrate on the areas providing the best shelter, principally in hemlock reproduction, rhododendron, and laurel thickets. At such times they try to subsist on these species and hundreds of them die from malnutrition (fig. 5). Losses occur principally among the younger age classes, especially fawns, perhaps because they cannot reach as high as the older animals.

During 1938 nose flies made their appearance for the first time; many animals were infested and many died. In 1939, when the nose fly (*Cephanomia sp.*) appeared in epizootic proportions, a high percentage of the herd was badly infested and thousands died. In the middle of the following summer, those that survived were in pitifully poor condition, for, as predicted, nature finally had taken a hand in an effort to balance the deer herd and save the environment.

Excluding both male and female fawns, as of December 31, 1937, the sex ratio (after the hunting season) was approximately 1 male to 11 females. Distribution was uneven, and by actual count on some of the census areas, the ratio was found to be as high as 1 male to 30 females.

During the 1937 hunting season, hunters commonly reported seeing from 50 to 75 does in 1 day and not a single buck. The buck law has

F 320526

FIGURE 5.—Deer dead from malnutrition, the result of a continued diet on hemlock browse, March 1936, Allegheny National Forest.

been in effect on the area for some 32 years, during which time only 4 antlerless deer-hunting seasons (2 to 6 days each) have been held. These antlerless deer-hunting seasons have always been strongly protested by sportsmen throughout the State (*5*).[4] As Pennsylvania's open season on legal male deer (male deer with two or more points to one antler) comes at the end of the rutting season, it seems that even the annual killing of an extremely high percentage of legal male deer does not materially decrease the breeding potentiality.

TECHNIQUE AND FIELD WORK

Personnel and Seasons

The data which follow were collected during five hunting seasons (1935 to 1939) by the following personnel of the Forest Service and the Fish and Wildlife Service: 1935 and 1936—Barry C. Park and John Pearce, Forest Service; 1937—Barry C. Park and Theodore C. Fearnow, Forest Service, and E. A. Goldman, Fish and Wildlife Service; 1938 and 1939—Barry C. Park and Randal McCain, Forest Service.

Each year the measurements were taken by two two-man field parties. Each party consisted of a qualified wildlife technician who took the measurements, and one assistant to help handle the carcasses. Although more costly, this procedure assured a higher degree of uniformity and accuracy than would have been possible by other means. The objective of the study required accurate data on a good cross section of the herd. Several thousand deer could have been measured annually if it had been advisable to use untrained personnel.

Each of the eight management units (see frontispiece map) was covered by car. All deer found on cars and at camps along the roads were weighed and measured. No selection whatsoever entered into the sample; the deer were taken in the order found. The work, however, was planned each year so as to obtain insofar as possible an equal number of weights and measurements in each management unit in proportion to the size of the area.

Measurements were taken on a total of 1,787 white-tailed deer. The number and sex of those taken during each of the five hunting seasons are given in table 2.

Table 2.—*Number and sex of legal deer examined during the hunting seasons of 1935–39*

Sex of deer	Hunting season of—					
	1935 [1]	1936 [2]	1937 [3]	1938 [4]	1939 [5]	Total
	Number	*Number*	*Number*	*Number*	*Number*	*Number*
Male	244	263	300	...	256	1,063
Female	130			191	124	445
Fawns (male and female)	64			143	72	279
Total	438	263	300	334	452	1,787

[1] Dec. 2 to 14, inclusive.
[2] Dec. 1 to 12, inclusive.
[3] Nov. 20 to Dec. 11, inclusive.
[4] Nov. 28 to Dec. 3, inclusive (antlerless deer only).
[5] Dec. 1 to 15, inclusive (antlerless in 2 counties, Dec. 14 and 15).

Antler Measurements

Antler measurements were taken in accordance with the New York Zoological Society Standards (*6*) as follows:

[4] Italic numbers in parentheses refer to literature cited, p. 50.

Greatest spread, measured between perpendiculars at extreme width of antlers at right angles to the center line of the skull.

Circumference of main beam, taken midway between the basal snag and the first fork.

Total length of antler, taken in the inside curve of the antler from basal snag to tip of longest prong.

Antler tines were not counted unless they protruded at least 1 inch (2.54 cm.).

Body Measurements

Body measurements were taken under the personal supervision of E. A. Goldman as follows:

Total length, the distance from the tip of the nose to the end of the tail vertebrae (not including terminal hairs of tail). The unskinned animal was laid on its side with the head extended forward so that the nose was brought into as nearly a straight line with the back as possible, the tapeline being passed from the end of the nose over the top of the head directly to the top of the shoulders, and along the backbone to the end of the bones in the tail (not along curvature of neck or side of body).

Hind foot, measured from the hock (heel or ankle joint) to tip of the hind hoof or longest claw (fig. 6).

F 407956

Figure 6.—Typical Allegheny National Forest male deer bagged during the 1939 hunting season, showing (center) hind foot measurement.

Tail, measurement was made of the length of the tail vertebrae (not ends of hairs), when the tail was held straight or bent upward at right angles to the back.

All measurements were taken in centimeters.

Weight

The weights taken were of dressed carcasses (viscera removed). All figures on live weight given in annual progress reports were computed by using W. T. Hornaday's formula (7):

> The dressed weight being given, in pounds, add to it five ciphers, divide by 78612, and the result will be the live weight, in pounds.

In the field, live weights were computed by dividing the dressed weight by four and adding this figure to the dressed weight, since observations proved the weight of entrails to be approximately one-fifth of the live weight (fig. 7).

F 407957

FIGURE 7.—Above-average mature male deer (5½-year age class) with dressed weight of 155 pounds and a very large set of antlers. The antler spread was 53 cm., circumference of main beam 10 cm., and length of antler 60 cm.

Weights were taken with tested ice scales with a weighing capacity of 300 pounds. A tally of estimated weights made in 1935 indicated that the average hunter overestimated the weight of fawns 25 pounds and of legal males 50 pounds (fig. 8). In 1936 another tally of hunters' weights for their kills disclosed estimates ranging from 10 pounds underweight to 75 pounds overweight, which clearly indicates the inaccuracy of hunter-estimated weights. The weighing crews found that experience improved their estimates, but they were never very consistent, a fact that attests the unreliability of all estimated weights.

The majority of the animals were "dressed," and the dressed weight (viscera removed) was obtained, but often the liver, lungs, and heart remained in the carcass.

It was determined from an average of a number of actual weights that the weight of the liver, lungs, and heart equals approximately one-fifth of the total weight of the entrails, and this amount was

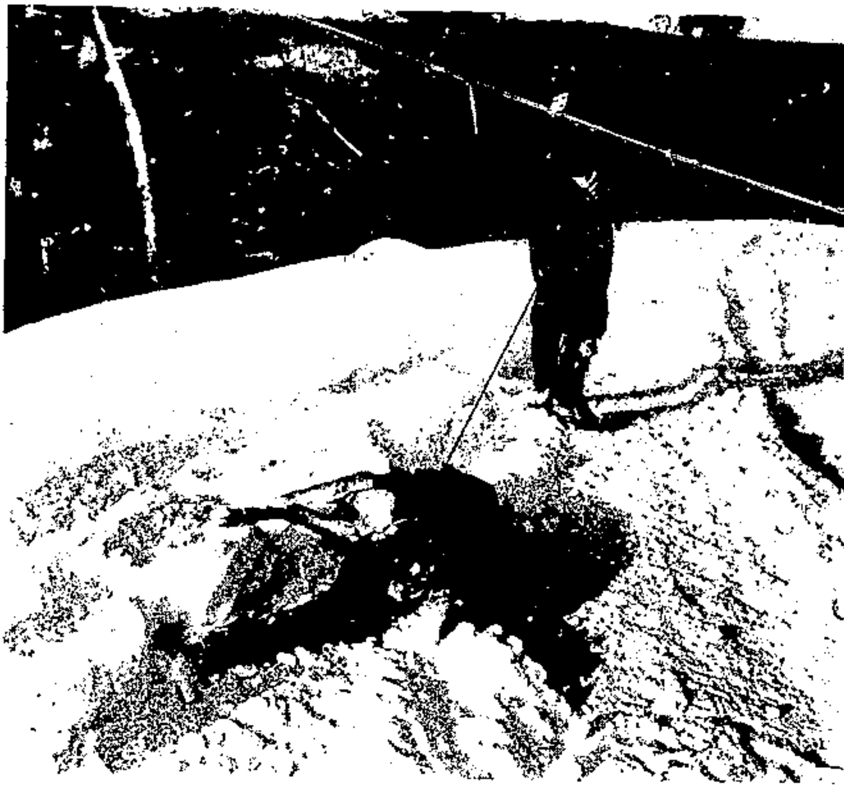

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FIGURE 8.—The average hunter generally overestimated the weight of the deer he brought in by from 25 to 50 pounds or more.

allowed when it was necessary to compute dressed weight. (See table 3.)

TABLE 3.—*Allowances used in computing dressed weight of deer carcasses*

Dressed weight in pounds	Allowance for liver, lungs, and heart	Dressed weight in pounds	Allowance for liver, lungs, and heart
	Pounds		*Pounds*
186 to 200	11	110 to 139	7
166 to 185	10	80 to 109	6
150 to 165	9	Less than 80	5
140 to 149	8		

Trees, trophy racks, etc., were not always convenient, and therefore a tripod similar to that used on an automobile wrecker was built on the bed of a pick-up truck. The tripod extended beyond and high enough above the truck body to allow a deer carcass to swing free of the ground, provided it was properly slung. By using a quarter-inch rope about 8 feet long (ends tied together forming a loop), deer carcasses were slung as shown in figure 9.

F 406302

FIGURE 9.—Tripod on pick-up truck showing method of slinging deer carcass on scales. The loop was placed ahead of one front foot and well back on the rump, thereby preventing the sling from coming together in the center as one rope.

SIZE OF SAMPLE

The number of legal deer weighed compared to the kill for the 5-year period is given in table 4. It indicates that the sample is large enough to be representative of conditions for the entire area. The Allegheny Forest was divided into eight management units based upon distribution of the deer herd (see frontispiece map). The census figures used for determining distribution were obtained by a series of game drives representing a 1-percent sample of the gross area of the forest [5] (*8*). In order to obtain the best possible representative sample, weighing crews covered each management unit thoroughly, and obtained, insofar as possible, an equal proportionate number of weights and measurements in each one.

Field observations on hunter concentrations during the 5 years (1935-39) revealed that "hunter distribution" on relatively small areas lags 1 year behind deer-herd distribution when the deer herd has been substantially reduced by hunting. Hunters naturally return to

[5] PARK, BARRY C. GAME DRIVES, ALLEGHENY NATIONAL FOREST, 1935-36. 15 pp., illus. 1936. [Unpublished.]

TABLE 4.—*Number of legal deer weighed compared to kill for 5-year period, 1935–39*

Year	Legal deer killed	Deer weighed and measured	Sample weighed and measured
	Number	*Number*	*Percent*
1935	14,800	438	2.96
1936	4,300	263	6.14
1937	5,700	300	5.30
1938	24,000	334	1.39
1939	9,500	452	4.76
5-year total or average	58,300	1,787	3.07

areas where they previously have been successful, and their success entices others to the same areas the following season.

During the period covered by this study, the hunting effort was more intense on the southern half of the forest (management units 5, 6, 7, and 8), but the hunting effort within management units was fairly evenly distributed. Table 5 shows the number of hunters and their kill during the period of the study. The figures were obtained from an actual tally made by numerous contact stations located at the main entrances to the Allegheny National Forest during the open deer seasons.

TABLE 5.—*Number of hunters and their kill for the 5 years 1935–39*

Year	Kill	Hunters	Successful hunters
	Number	*Number*	*Percent*
1935	14,800	[1] 133,000	11.1
1936	4,300	[2] 40,000	10.8
1937	5,700	[3] 42,000	13.6
1938	24,000	[4] 71,000	33.8
1939	9,500	[5] 70,000	13.5
5-year total or average	58,300	356,000	16.3

[1] Season for legal male and antlerless deer.
[2] Season for legal male deer only.
[3] Season for legal male deer only.
[4] Season for legal antlerless deer only.
[5] Season for legal male and antlerless deer in 2 counties.

AGE

Age was taken by dental formula as follows (*1*)[6] (fig. 10):

Age 9 months—

$$\text{Dental formula } \frac{0 \quad 0 \quad 3 \quad 1}{4 \quad 0 \quad 3 \quad 1}$$

The middle pair of incisors have been replaced by permanent incisors, and remainder of incisors and premolars are milk teeth. First molar is well developed and the second partially erupted.

Age 18 to 21 months (considered as 1½-year age class)—

$$\text{Dental formula } \frac{0 \quad 0 \quad 3 \quad 2}{4 \quad 0 \quad 3 \quad 2}$$

Incisors all have been replaced by permanent teeth. Premolars are all milk teeth, but much worn. First and second molars are fully developed, but the third molar may be only partially erupted.

[6] RUSH, W. M. DENTITION OF MULE DEER (ODOCOILEUS HEMIONUS). Yellowstone National Park. 2 pp. illus. 1932. (Unpublished.)

Age 31 to 55 months (considered as 2½ years to 4½ years inclusive)—

$$\text{Dental formula } \frac{0}{4}-\frac{0}{0}-\frac{3}{3}-\frac{3}{3}$$

Permanent dentition is complete. Ridges of the enamel are sharp and well above dentine.

Age 67 to 103 months (considered as 5½ years to 9½ years)—

$$\text{Dental formula } \frac{0}{4}-\frac{0}{0}-\frac{3}{3}-\frac{3}{3}$$

Ridges of the enamel are no longer sharp, and rise only slightly and gradually above the dentine.

Age 115 months plus (considered as 9½ years plus)—

$$\text{Dental formula } \frac{0}{4}-\frac{0}{0}-\frac{3}{3}-\frac{3}{3}$$

The crowns of the premolars and molars rise only about one-eighth of an inch above the gums. The grinding surfaces are worn practically smooth with no appreciable ridges of enamel remaining.

Ages of deer from 9 to 30 months may be determined with a reasonable degree of accuracy by the dental formula. At about 30 months there is complete permanent dentition after which time the age must be determined by the degree of wear on the grinding surfaces of the dentition. This may be a variable factor depending upon the type of food eaten, etc. To obtain accurate information on ages above 30 months, it would be necessary to make a collection of skulls of

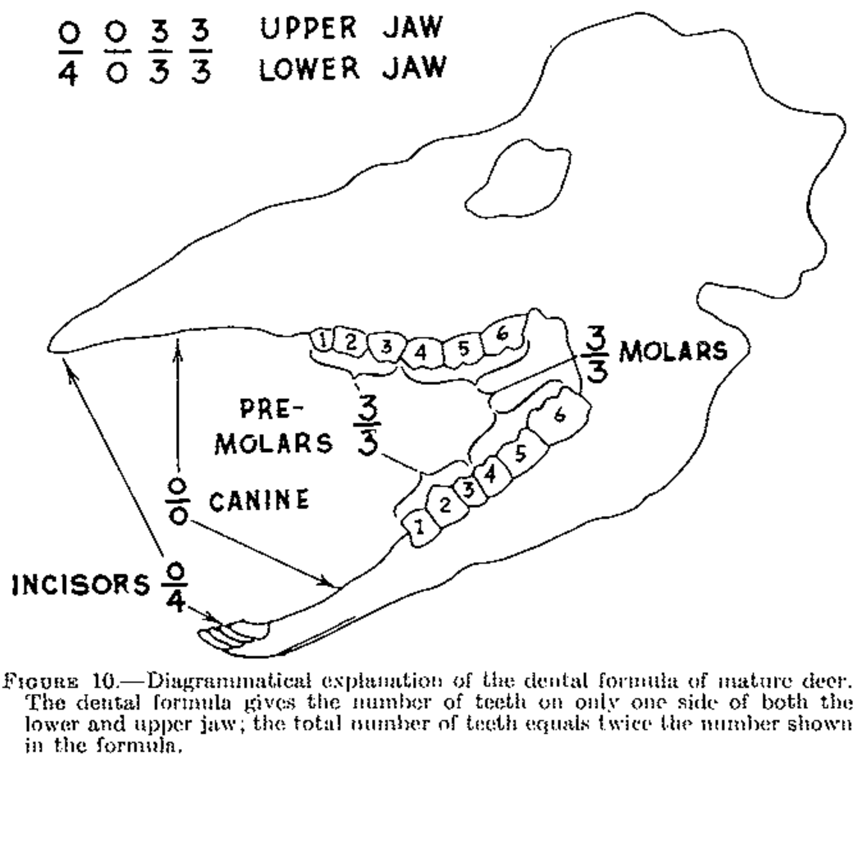

Figure 10.—Diagrammatical explanation of the dental formula of mature deer. The dental formula gives the number of teeth on only one side of both the lower and upper jaw; the total number of teeth equals twice the number shown in the formula.

known age classes. Such a collection would necessitate fawn tagging and would require many years to complete. Pending the collection of such data the grouping of deer into broad age classes, such as given herein, was considered to be sufficiently accurate for the purposes of this work.

However, it was realized that if the data were to be of any scientific value, the field work must be done by reliable, technically trained individuals. A high degree of uniformity was obtained in spite of adverse field conditions. Inspection of the teeth of frozen carcasses was made possible by the use of an ordinary screw driver for prying open the lips, and a small pocket flashlight. Very few deer were found to be in the 9½-year-and-over class; therefore, only four age classes were used in compiling the data, 9 months, 1½ years, 2½ to 4½ years, and 5½ years and over (figs. 11, 12, 13, 14, 15, 16).

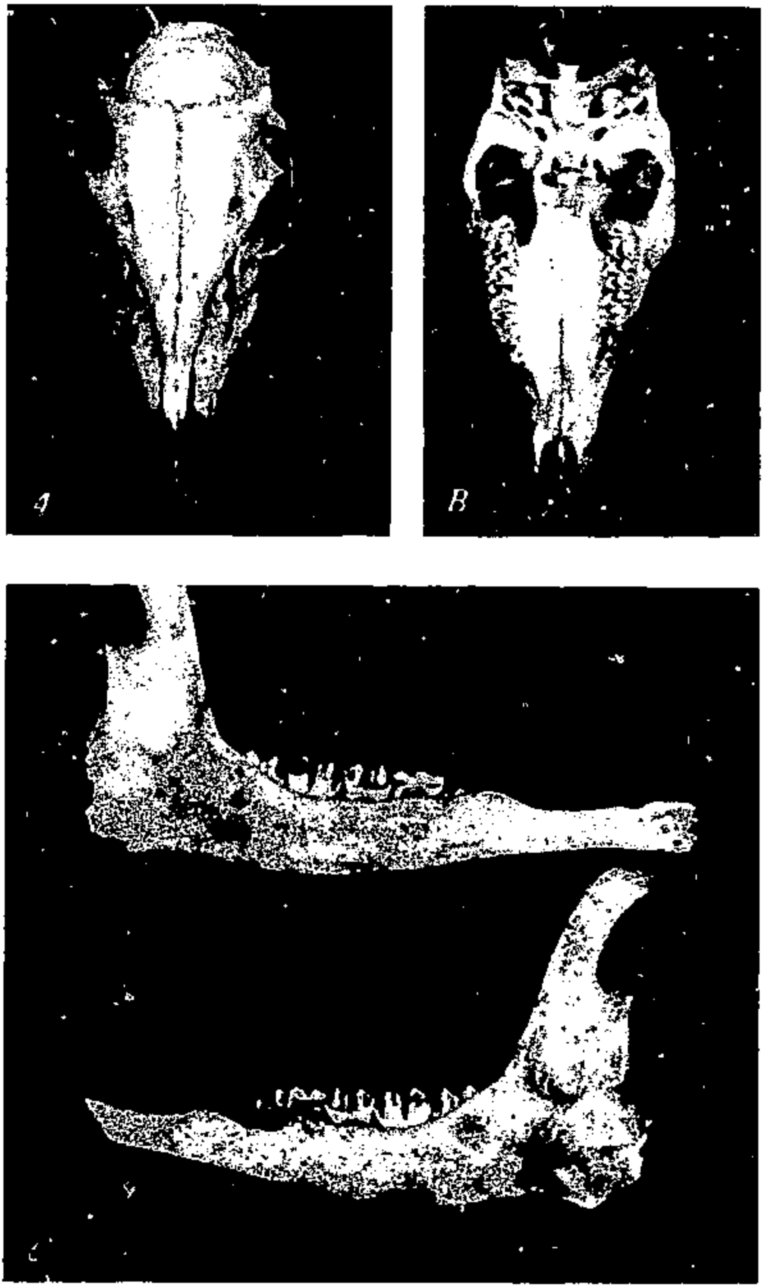

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FIGURE 11. *A*, Top of skull, boney protrusions being the first years' antlers "buttons," of a "button buck" male deer, of the 9-month age class (includes all fawns). *B*, Bottom of skull. *C*, Inside and outside of the lower jaw showing three premolar milk teeth, the first molar well-developed, and the second molar partially erupted. These premolars and molars are all milk teeth and are much smaller than the permanent premolars and molars shown in figures 13 and 14.

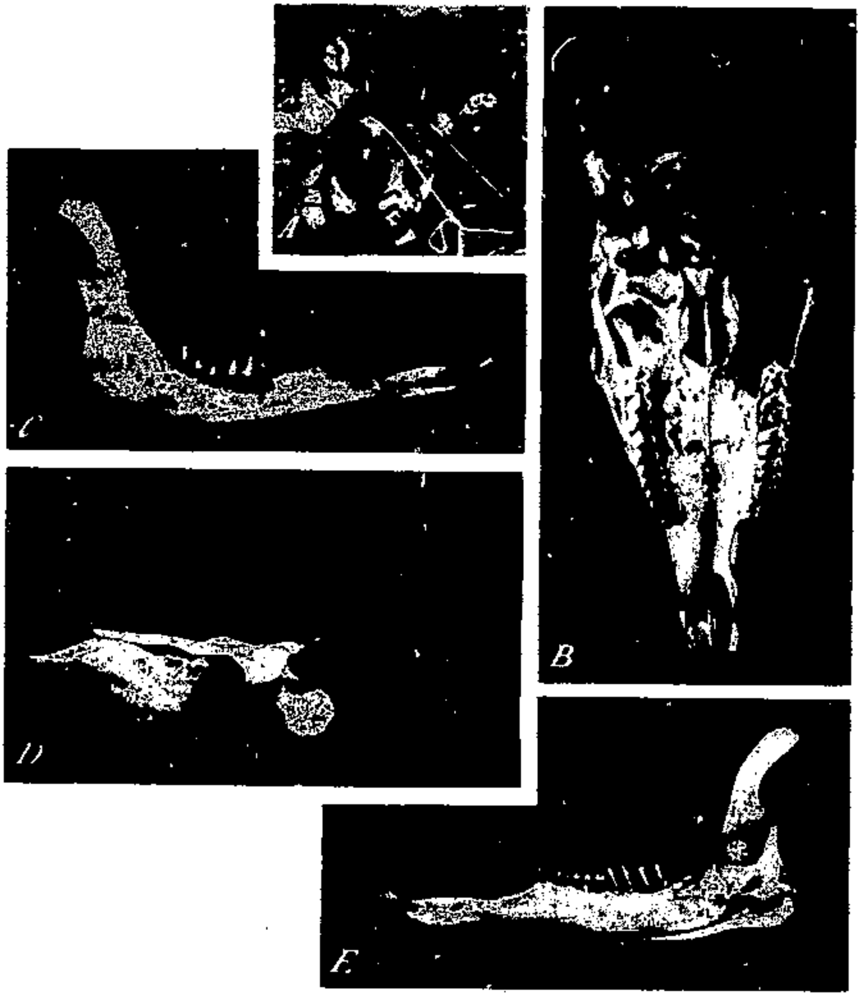

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FIGURE 12.—*A*, Head of a typical spike buck, of the 1½-year age class; *B*, upper jaw of skull; *C*, lower jaw. In the lower jaw the incisors are all permanent teeth while the premolars and molars are all milk teeth, much worn. The third molar is not entirely through, but the jawbone is much more elongated than in figure 11; *D* and *E* show the upper and lower jaws of the skull of an unusually large 1½-year-old, four-point buck. In the upper jaw the permanent premolars are pushing out behind much-worn milk teeth; the second premolar was so slightly attached that it was lost in cleaning the skull, leaving the permanent second premolar plainly visible. In the lower jaw, the permanent premolars are pushing out the milk teeth.

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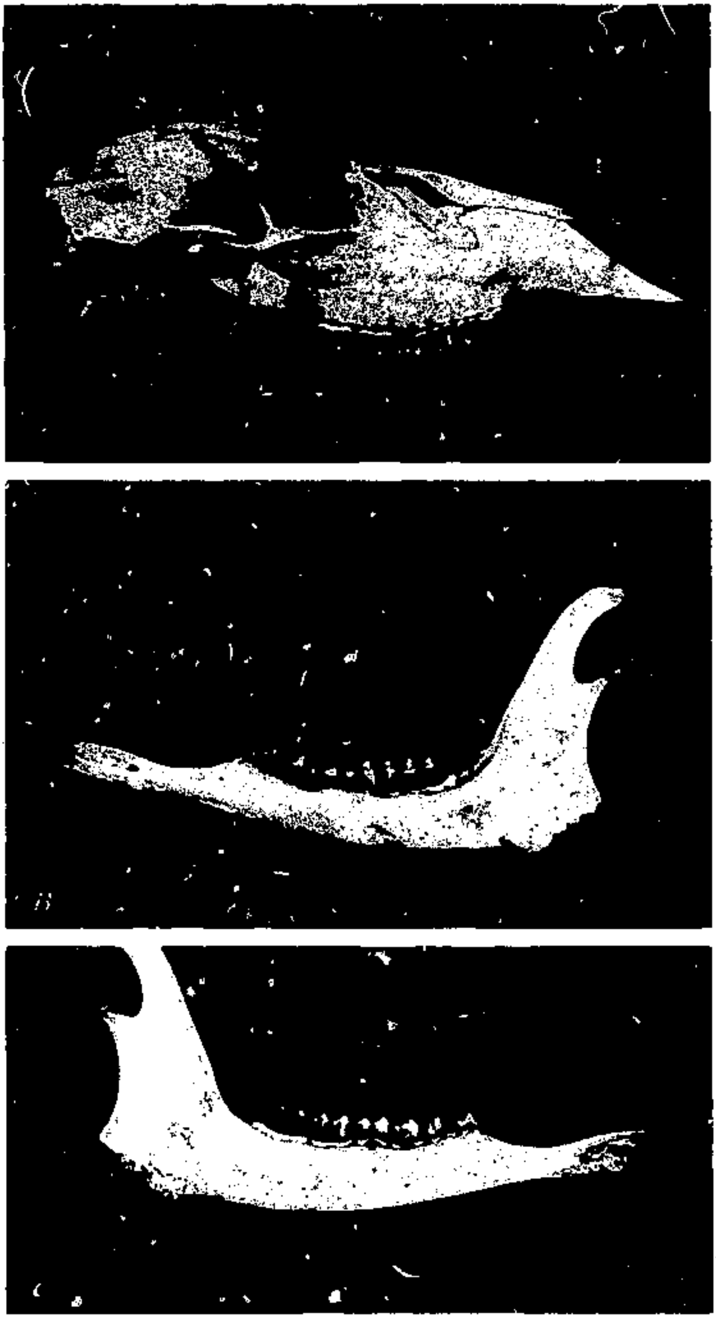
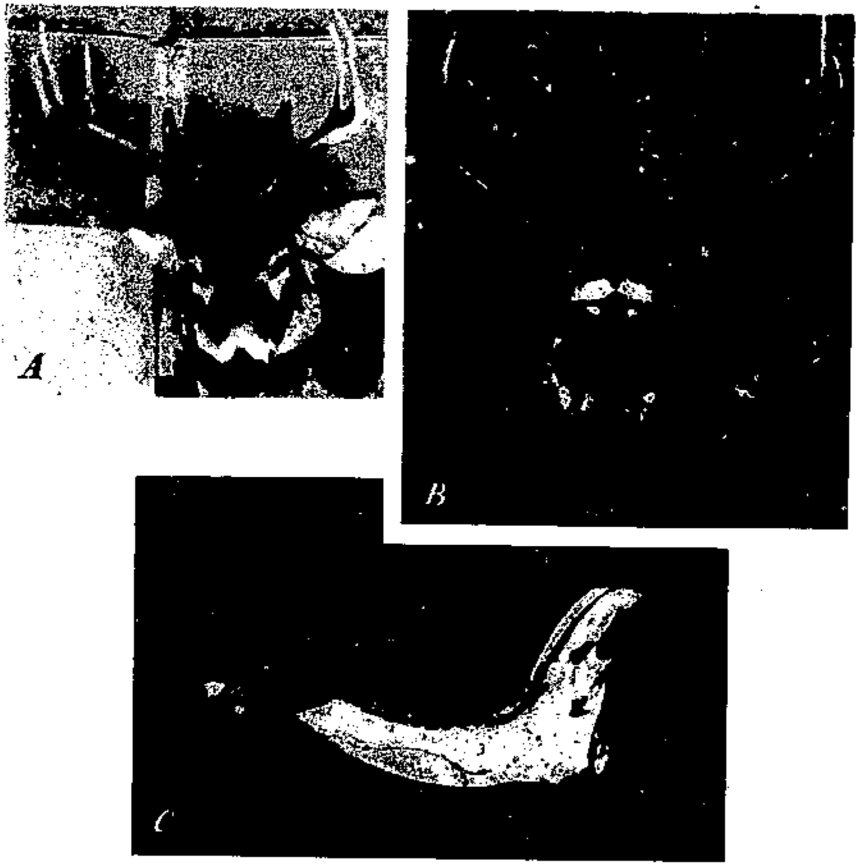


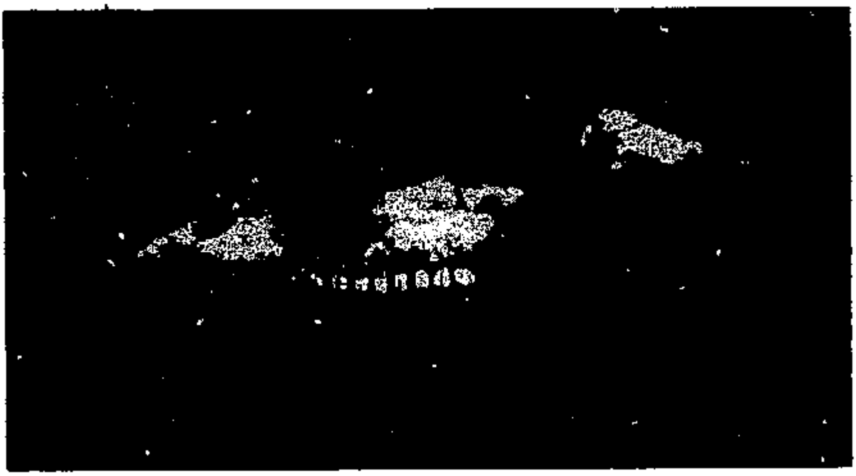
F 407970, 407972, 407971

FIGURE 13.—*A*, The complete set of permanent premolars and molars of a typical doe deer of the 2¼- to 4¼-year age class. *B*, and *C*, The outside and the inside of the lower jaw, respectively, show sharp ridges of enamel well above the dentine, the teeth being long and only slightly worn.



F 407955, 407960, 407964

FIGURE 14.—Dental formula used in age determination. *A*, Head of a typical mature male deer of the 5½-year-plus age class showing evenly developed antlers, 5 points on each side. *B*, Upper jaw of the skull. *C*, Lower jaw. The ridges of enamel are no longer sharp and rise only slightly and gradually above the dentine. The teeth are much worn and are only about half as high above the gum line as those shown in figure 13.



F 407973

FIGURE 15.—Upper jaw of the skull of an old, mature doe of the 9½-year-plus age class. The crowns of the premolars rise only slightly above the gums, and the grinding surfaces are worn practically smooth with no appreciable ridges of enamel remaining.

F 410963

FIGURE 16.—Comparative size of the skulls shown in figures 12 and 14. *A*, Head shown in figure 12, *A*, *B*, *C*. *B*, Head shown in figure 12, *D* and *E*. *C*, Head shown in figure 14.

SUBSPECIES OF WHITE-TAILED DEER

The Game Commission of the Commonwealth of Pennsylvania kept no record of the location of deer plants during the early period of restocking. It appears certain that at least part of the deer stocked on the Allegheny National Forest came from Michigan and a few may have come from New York. There is little reliable information to support the belief that any of the animals came from States farther south than northern Virginia and West Virginia.[7] The lack of definite information as to the origin of the deer herd made it necessary to determine the subspecies of white-tailed deer existing on the forest before comparisons of weight and measurements could be made. The Fish and Wildlife Service assigned E. A. Goldman, senior biologist, to the project, and he worked with the field crews during the 1937 season. Mr. Goldman's report [8] states:

> The deer vary in the depth of color tone in all of the Allegheny National Forest units, some being distinctly tawny, while others are grayer in the same winter pelage, but these differences are within the usual range of individual variation in the subspecies. Comparison of measurements from the eight forest units by age classes reveals only differences attributable to individual variation. The conclusion is therefore reached that all of the deer on the forest are referable to a single subspecies. No specimens were obtainable for actual comparison, but the data taken point to indentification as the northern race, or subspecies, of the Virginia white-tailed deer (*Odocoileus virginianus borealis*). Typical *borealis* is from Maine, but animals from as far west as Michigan do not appear to be appreciably different.

STATISTICAL TECHNIQUE

Definitions of the statistical terms as well as detailed descriptions of techniques used in the analysis will be expressed in algebraic form. From these it should be possible to either check the analyses made or to apply the techniques to data of a similar nature.

The description of the field technique indicates that no personal bias entered into the selection of the deer on which measurements were taken. Hence, the procedure used fulfilled the fundamental requirements for randomness, and the samples may be considered random. On this assumption, the statistical theory and deductions therefrom adapted to random sampling are considered to be applicable to these data.

Frequency Distribution and Curves

Measurements of any single character made on individuals taken at random from a population are known to differ among themselves, and hence in their original form will appear as a mass of unorganized figures. For example, in 1936, 1937, and 1939, dressed weight was recorded on 452 mature male deer, each year's record being in a random order insofar as magnitude of weight was concerned. These data must be condensed and presented in an orderly manner before their significance is apparent.

Rearranging them, reading in the order of size and thereby giving the entire range of weights, would help somewhat but would not be sufficient. Since it would not necessarily be important to know each individual deer weight correct to the pound, the whole range of weights may be divided into a number of subranges of equal size. The

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[7] Ruff, F. J. The white-tailed deer of the Pisgah National Game Preserve, North Carolina. U. S. Forest Serv., South. Region, 249 pp., illus. 1938. [Processed.]

[8] Goldman, E. A. Deer investigations on Allegheny National Forest. 7 pp. 1937. [Unpublished.]

frequency of individuals—in this case, deer—in each subrange may then be recorded. Such a record or table is called a frequency distribution; the subrange is the class interval, and the midpoint of class interval is one-half the sum of the two limits of that interval.

Diagrams may be of considerable value as an aid in grasping the meaning of the frequency distributions. It was found most convenient in this study to use frequency polygons. Along the horizontal base line, a scale representing the equal class intervals was laid off. The vertical scale, expressed in percent, represented the proportion of the total number of items (deer) whose measurements (dressed weight) fell within a class interval. The percent of the total number of items occurring in each class interval was computed. The percents were then measured vertically at the midpoint or center of the class intervals on the base lines. These successive points were connected with straight lines, going one interval below the lowest class for which there was a count and one point above the highest class.

The first step in the organization and analysis of quantitative data, such as that collected on this project, would be the formation of appropriate frequency distributions. The next step would be the computation of numerical constants which would most nearly describe the character being measured. In the section which follows, methods for computing the two most useful, mean average and the standard deviation, will be given. The first measures the central tendency of the distribution around which the value of the items cluster, and the second is a measure of the "scatter" about this central value.

Mean and Standard Deviation

If $x_1, x_2, \ldots . . x_N$ be the individual measurements of a particular character on a sample of N deer, then the arithmetic mean (or average) will be the sum of the individual items divided by the number of items. That is, $\bar{x}=\frac{x_1+x_2+\ldots+x_N}{N}$ or written more simply

$$\bar{x}=\frac{\Sigma x}{N} \quad (1)$$

The best measure of the dispersion or spread of the individual items in the sample is the standard deviation, which is the square root of the variance. In turn, the variance is the sum of the squares of the deviations of the individual items from the arithmetic mean divided by the number of items less one. Expressed algebraically, variance becomes $s=\frac{\Sigma(x-\bar{x})^2}{N-1}$ and the standard deviation, $s=\sqrt{\frac{\Sigma(x-\bar{x})^2}{N-1}}$.

It can be shown that $\Sigma(x-\bar{x})^2=\Sigma x^2-\frac{(\Sigma x)^2}{N}$ where $\Sigma x^2=x^2_1+x^2_2+\ldots x^2_n$ and Σx as in (1) above.

Hence, for ease of computation, the formula for s may be written

$$s=\sqrt{\frac{\Sigma x^2-\frac{(\Sigma x)^2}{N}}{N-1}} \quad (2)$$

Consider the dressed weights on the sample in 1938 of 10 female deer, aged 1½ years, from the northern district. The mean is

$$\bar{x}=\frac{79+81+88+83+72+70+81+93+74+91}{10}=81.2$$

$$s=\sqrt{\frac{(79^2+81^2+\ldots+91^2)-\frac{(812)^2}{10}}{9}}\sqrt{\frac{66{,}486-65{,}934.4}{9}}=\sqrt{\frac{551.6}{9}}$$

$$s=7.8$$

Comparison of Two Means

The Student t test has been extended to determine whether two samples may be regarded as belonging to the same population (3).

If $x_1, x_2, \ldots x_{N_1}$ and $x'_1\ x'_2 \ldots x'_{N_2}$ be two samples, the means will be found as in formula 1. The variance is estimated by pooling the sums of squares from the two samples,

$$s^2=\frac{1}{N_1+N_2-2}\{\Sigma(x-\bar{x})^2+\Sigma(x'-\bar{x}')^2\} \quad (3)$$

$$\text{and}\ \ t=\frac{\bar{x}-\bar{x}'}{s}\sqrt{\frac{N_1\,.\,N_2}{N_1+N_2}} \quad (4)$$

the degrees of freedom for entering the t table to estimate the significance of the difference in the two means will be $n=N_1+N_2-2$.

Example: Compare sample above dressed weight of 1½-year-old females, northern district, 1938, with southern deer of same sex, age, and year.

$$\bar{x}=81.2,\ \bar{x}'=77.4,\ N_1=10,\ N_2=11$$

$$s^2=\frac{1}{10+11-2}[551.6+1112.54]$$

$$s^2=87.586316,\ t=\frac{81.20-77.36}{9.359}\sqrt{\frac{10\ .\ 11}{10+11}}$$

Hence, $t=0.94$. With $n=19$ the chance is somewhat better than 4 in 10 that the difference could be explained by random sampling.

Analysis of Variance

The basic treatment of the technique known as the analysis of variance was developed by R. A. Fisher (*3*). Stated briefly, if two or more groups of factors are causing variation in a set of measurements, then it is possible to separate the variance ascribable to one group of causes from the variance ascribable to other groups. A rigid test of the relative importance of such causes may then be made to which probabilities are assignable. For these data—considering age as a possible cause of variation—the number of items (deer) varies from age to age. The algebraic identity expressing the fact that the sum of squares of the deviations of all the values of x from their general means $\bar{x}$, may be broken up into two parts, the first representing the sum of squares of the deviations of the means of the age groups from the general mean, each multiplied by the number in that age group; while the second is the sum of squares of the deviations of the observation on each deer from the mean of its age group.

With p designating a particular age group, n_p the number of deer in that group, and $\bar{x}_p$ its mean of a particular character, then

$$\Sigma(x-\bar{x})^2=\Sigma n_p(\bar{x}_p-\bar{x})^2+\Sigma\Sigma(x-\bar{x}_p)^2 \qquad (5)$$

Considering dressed weight on the male deer for which there were four age classes, the computations for the analysis of variance in dressed weight will be shown in two parts, variance due to age and variance within age. The ratio of these two variances will be a test of the relative importance of age in the dressed weight of deer (table 6).

Let the number of age groups be k, represented by I, II, III, IV, $k=4$, $n_{I}=105$, $n_{II}=140$, $n_{III}=440$, $n_{IV}=225$, the total $N=910$.

The degree of freedom for the between-ages sums of squares will be one less than the number of age groups, $k-1=3$.

The degrees of freedom within age groups will be the pooled degrees of freedom within each age; that is, $104+139+439+224=906$.

A check, here, is that the between and within degrees of freedom should add to one less than the total number of items.

Using the identity given under (2) above for $\Sigma(x-\bar{x})^2=\Sigma x^2-\frac{(\Sigma x)^2}{N}$ the computations for the sums of squares become simple arithmetic, tabulated in table 7.

The analysis of variance table (table 8) is readily made up from these tabulations. The "total" is taken directly from the last column last line, 673,318.46; the "within" is the sum of the four; the "between" will be the difference of these two, or it may be found directly by subtracting the total correction factor from the sum of the four correction factors. The expression $\frac{(\Sigma x)^2}{n}$ is called the correction factor.

TABLE 6.—*The general form of the analysis of variance; the ratio of the between- and within-age variances provides a test of the importance of age in the dressed weight of deer*

Source of variance	Degrees of freedom	Sum of squares
Between ages	$k-1$	$\Sigma n_p(\bar{x}_p-\bar{x})^2$
Within ages	$\Sigma(n_p-1)=N-k$	$\Sigma\Sigma(x-\bar{x}_p)^2$
	$N-1$	$\Sigma(x-\bar{x}_p)^2$

TABLE 7.—*Tabulation showing computations for the sums of squares; for dressed weight, Σx and Σx^2 are compiled from the individual items as in equations (1) and (2)*

Item	Age class				Total
	I (0 months)	II ($1\frac{1}{2}$ years)	III ($2\frac{1}{2}$ to $4\frac{1}{2}$ years)	IV ($5\frac{1}{2}$ years and over)	
Number	105	140	440	225	910
Σx	5,706.00	13,383.00	51,455.00	30,258.00	100,802.00
Σx^2	318,448.00	1,295,723.00	6,102,635.00	4,122,494.00	11,839,300.00
$\frac{(\Sigma x)^2}{n_p}$	310,080.34	1,279,319.21	6,017,311.42	4,069,095.84	11,165,981.54
$\Sigma(x-\bar{x})^2$	8,367.66	16,403.79	85,323.58	53,398.16	673,318.46

TABLE 8.—*Analysis of variance made up from data in table 7*

Source of variance	Degrees of freedom	Sum of squares	Mean square	F
Between ages	3	509,825.27	169,942.	941.74
Within ages	906	163,493.19	180.456	
Total	909	673,318.46		

The sum of squares divided by the corresponding degrees of freedom gives the mean square. F is the ratio $\frac{169{,}942}{180.456}$ which equals 941.74. Tables of F values for pertinent probability levels are available (*4, Table V*). Entering this table, with n_1, always the larger mean square, equal to 3 and n_2, the less, 906, we find that the largest F, which is still many times smaller than our F, is at the 0.1 percent level; that is, there is 1 chance in 1,000 that so large a ratio could be attributed to random sampling. Thus, for all practical purposes, it may with certainty be said that age does influence the dressed weight of a deer.

TEST OF THE EFFICIENCY OF A SINGLE CHARACTER FOR CLASSIFYING INDIVIDUAL DEER BY AGE

Let $\bar{x}_{pI}$, $\bar{x}_{pII}$, $\bar{x}_{pIII}$, $\bar{x}_{pIV}$ be the means for the character p for the four age classes, and s_p be the standard deviation computed from the pooled-within sums of squares. The standard deviations for each character are given in tables 14 and 15. For dressed weight, using all male deer,

$N=910$

$$\bar{x}_I=54.34 \qquad \bar{x}_{III}=116.94$$
$$\bar{x}_{II}=95.59 \qquad \bar{x}_{IV}=134.48 \qquad s_a=13.433$$

Then

$$\frac{\bar{x}_{II}-\bar{x}_I}{2s_a}=1.535,\ \frac{\bar{x}_{III}-\bar{x}_{II}}{2s_a}=0.795,\ \frac{\bar{x}_{IV}-\bar{x}_{III}}{2s_a}=0.653$$

The probabilities corresponding to these are 0.12, 0.43, and 0.51 (*4, I*). These are the probabilities of an observation falling outside the range $-x$ to $+x$, that is, both tails of the normal curve; and since we are interested only in one tail of the curve at a time, each is divided by 2. The resulting values are 0.06, 0.21, and 0.26. The chance is 6 to 100 that a 9-month-old deer will be classified as a 1½-year-old animal, or that a 1½-year-old deer will be put in the 9-month class; 21 out of 100 that a 1½-year-old deer will be classified as a 2½- to 4½-year-old, or the older animals will be called 1½-year-olds; and 26 out of 100 of the 2½- to 4½-year-old deer will be classed as overmature, or that proportion of the oldest deer will be classified as the next younger. Hence, the final probabilities for misclassifying by age groups on the basis of dressed weight alone are 0.06, 0.27, 0.47, and 0.26.

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DISCRIMINANT FUNCTION, A COMPOUND OF TWO OR MORE CHARACTERS AS A CRITERION FOR CLASSIFYING DEER BY AGE (3)

(a) *Algebraic expression of procedure used.*

If a linear function of the several deer measurements showing the most distinct change with age be represented by the expression

$$Y=\lambda_1x_1+\lambda_2x_2+\ .\ .\ .\ +\lambda_px_p, \tag{6}$$

then the coefficients $\lambda_1, \lambda_2\ .\ .\ .\ \lambda_p$
are the solution of the equations

$$\lambda_1\ [x_1^2]+\lambda_2[x_1x_2]+\ .\ .\ .\ +\lambda_p[x_1x_p]=B_1$$
$$\lambda_1[x_1x_2]+\lambda_2[x_2^2]\ +\ .\ .\ .\ +\lambda_p[x_2x_p]=B_2$$
$$\lambda_p[x_1x_p]+\lambda_2[x_2x_p]+\ .\ .\ .\ +\lambda_p[x_p^2]\ =B_p$$

where $[x_qx_p]$ is a simplified scheme for writing the pooled sums of squares and of products for the age classes. The sums of squares and cross-products set forth in Fisher's article 29 (3) for the standard regression equation might have been used instead, for the resulting coefficients would have been proportional to the λ's above, but since a precise study of variances within groups was wanted, these values already compiled were utilized.

The B's are the regressions of the means of the particular measurement on age, that is,

$$B_p=\frac{\sum_{A=1}^{IV}(\bar{x}_{pA}-\bar{x}_p)(a_A-a)}{\sum_{A=1}^{IV}(a_A-\bar{a})^2} \tag{7}$$

It was necessary to assign values to the age elements, ^aA occurring in the expressions for the B's. The logarithms of the numbers 1, 2, 3, 4 seemed appropriate for this, since the means of each of the characters plotted against these gave curves which were reasonably linear in nature. Therefore,

$$a_I=0.000,\ a_{II}=0.30103,\ a_{III}=0.477121,\ \text{and}\ a_{IV}=0.602060$$

The standard deviation, σ_p, of the discriminant function will be

$$\sigma_p=\sqrt{\frac{\lambda_1B_1+\lambda_2B_2+\ .\ .\ .\ +\lambda_pB_p}{N-p-k+1}} \tag{8}$$

where p is, of course, the number of characters making up the discriminant function and k the number of age groups. The standard error for the λ's is:

$$\sigma_{\lambda_p}=\sigma_p\sqrt{C_{pp}} \tag{9}$$

and to test its precision:

$$t=\frac{\lambda_p}{\sigma\lambda_p} \tag{10}$$

(b) *Procedure applied to data for female deer.*

The data on the female deer will be used to illustrate this method when all three characters measured were included. Hence,

$$p=3,\ k=4,\ \text{and}\ N=425$$

If from table 19 we copy the pooled sums of squares and products for the left-hand side of the equations; and, for the right-hand, unity is substituted for each of the B's and 0 for the others, the solutions obtained constitute the reciprocal matrix of multipliers,

$$35{,}814.95329\lambda_{11}+16{,}960.17128\lambda_{12}+3{,}288.332836\lambda_{13}=1,0,0$$
$$33{,}459.76905\lambda_{12}+2{,}382.619283\lambda_{13}=0,1,0$$
$$1{,}494.238007\lambda_{13}=0,0,1$$

where $\lambda_{11}=c_{11},\ c_{12},\ c_{13}$
$\lambda_{12}=c_{12},\ c_{22},\ c_{23}$
$\lambda_{13}=c_{13},\ c_{23},\ c_{33}$

This is the standard procedure outlined by Fisher in his article 29 (*3*).

Solving the three sets of equations, we have the matrix of multipliers reciprocal to the sum of squares and products within ages (table 9).

While these are the c values from which the λ's and their standard deviations are derived, the three B's must be computed first. As previously stated, the assumption is made:

	I	*II*	*III*	*IV*	
$a=$	0.00000	0.301	0.477	0.602	then $\bar{a}=0.345$
and $(a-\bar{a})$	-0.345	-0.044	0.132	0.257	

TABLE 9.—*c values from which the λ's and their standard deviations are derived*

Item	Dressed weight	Length of body	Length of hind foot
Dressed weight	$0.0^{4}4175378902$		
Length of body	$-.0^{4}1649390540$	$0.0^{4}4023036044$	
Length of hind foot	$-.0^{4}0558628210$	$-.0^{4}2785098400$	$0.0^{3}8579809980$

The denominators of the expression for the B's as in formula (7) are constant, $\Sigma(a_A-\bar{a})^2$ and equal to

$$(-0.345)^2+(0.044)^2+(0.132)^2+(0.257)^2=0.204434$$

The numerators are the summation of the products of $(a-\bar{a})$ and the corresponding mean of the character. These means are given in table 20. Hence

B_1 (dressed weight) $=[(-0.345\times 51.225225)+(-0.044\times 79.851852)+(0.132\times 90.600000)+(0.257\times 98.815385)]/0.204434$

B_2 (length of body) $=[(-0.345\times 138.347748)+(-0.044\times 157.0833333)+(0.132\times 161.328462)+(0.257\times 168.600769)]/0.204434$

Likewise B_3 (length of hind foot) is computed,

Simplifying

$$B_1=79.089436$$
$$B_2=48.837969$$
$$B_3=\ 9.638495$$

Following the standard method, the λ coefficients are calculated by substituting in these formulas:

$$\lambda_1 = B_1c_{11} + B_2c_{12} + B_3c_{13}$$
$$\lambda_2 = B_1c_{12} + B_2c_{22} + B_3c_{23}$$
$$\lambda_3 = B_1c_{13} + B_2c_{23} + B_3c_{33}$$

substituting

$$\lambda_1 = (79.089436)\ (0.0^4 41753789) + (48.837969)\ (-0.0^4 16493965) + (9.638495)\ (-0.0^4 65586282)$$

Likewise λ_2 and λ_3 are computed

Thus $\lambda_1 = 0.0018645988$

$\lambda_2 = 0.0003918294$

$\lambda_3 = 0.0017222777$

Then $Y = 0.00186460x_1 + 0.00039183x_2 + 0.00172228x_3$ (11)

The standard deviation of this function is now computed by formula (8).

$$\sigma_y = \sqrt{\frac{(0.0018645988)(79.089436) + (0003918294)(48.837969) + (0.001722777)(9.638495)}{425-3-4+1}}$$

$\sigma_y = 0.0209104$

Using σ_Y and c_{11}, c_{22}, c_{33} we find in the usual way, formula (9), the standard error of each λ coefficient, and test its significance in the derived discriminant function by formula (10).

$$\sigma_{\lambda 1} = 0.0209104\sqrt{0.0^4 41753789} = 0.00013512$$
$$\sigma_{\lambda 2} = 0.0209104\sqrt{0.0^4 40230366} = 0.00013263$$
$$\sigma_{\lambda 3} = 0.0209104\sqrt{0.0^3 857980998} = 0.00061249$$

Then $t_1 = \frac{0.0018646}{0.00013512} = 13.80$

$t_2 = \frac{0.00039183}{0.00013263} = 2.95$

$t_3 = \frac{0.0017223}{0.00061249} = 2.81$

Dressed weight, indicated by t_1 has by far the major influence in indicating age of deer. The influence of the other two characters is quite negligible in the presence of dressed weight.

To determine the efficiency of this function in estimating deer age, it is necessary to find its mean value for each age. This is done by substituting the mean x's in (11). For the 9-months' group:

$$Y_I = (0.0018646)(51.2252) + (0.00039183)(138.3477) + (0017223)(41.4631)$$

$Y_I = 0.22114$

Similarly, $Y_{II} = 0.28908$, $Y_{III} = 0.31159$, $Y_{IV} = 0.33204$

To simplify the work of computing the discriminant functions for a series of deer measures, one coefficient may be taken as unity and the others adjusted accordingly by dividing by that coefficient. For example, divide each of the coefficients in (11) by the length of body figure, 0.00039183, and the new expression will be

$$Y' = 4.7587x_1 + x_2 + 4.3955x_3$$

The same procedure is now used as for the single variable case as in the test of the efficiency of a single character for classifying individual deer by age p. 25.

$$\frac{Y_{II}-Y_{I}}{2\sigma_y}=1.625$$

$$\frac{Y_{III}-Y_{II}}{2\sigma_y}=0.538$$

$$\frac{Y_{IV}-Y_{III}}{2\sigma_y}=0.489$$

One-half of the probabilities corresponding to these are 0.06, 0.30 and 0.31, and the probabilities of misclassifying a deer using this discriminant function as the criterion are 0.06, 0.36, 0.61, and 0.31.

(c) *Omission of a character from a derived discriminant function.*

It is desirable to make comparable tests of discriminant functions which contain fewer than the maximum number of characters, particularly to omit characters whose λ's are shown to be of little importance according to Fisher's article 29.1 (*3*). A new *c*-matrix is calculated using the formula

$$c'_{pq}=c_{pq}-\frac{c_{pr}c_{qr}}{c_{rr}}$$

where *r* is the character to be omitted. Computations are most easily carried out if this is expressed in fractional form

$$c'_{pq}=\frac{c_{pq}c_{rr}-c_{pr}c_{qr}}{c_{rr}} \qquad (12)$$

Consider omitting x_3, the length of body, in our example. Then

$$c'_{11}=\frac{c_{11}c_{22}-c_{12}c_{12}}{c_{22}} \text{ etc.}$$

Substituting values from our *c*-matrix:

$$c'_{11}=\frac{(0.0^{4}4175378902)(0.0^{4}4023036644)-(-0.0^{4}1649396540)^{2}}{0.0^{4}4023036644}$$

$$c'_{11}=0.0^{4}3371476867$$

The new *c'*-matrix is:

$$\begin{matrix} 0.0^{4}3371476867 & \\ -0.0^{4}5375947980 & 0.0^{3}7549589610 \end{matrix}$$

From this point the method is the same, as in the three-character case. The results appear along with those of other combinations in table 21.

(d) *Factor U for ranking discriminant functions.*

$$\text{By definition } U=\frac{\text{slope of the function}}{\sigma_y} \qquad (13)$$

where the slope of the function is $\Sigma\lambda_p B_p$

For the female deer with three measurements forming the compound function for discriminating age

$$U=\frac{(0.0018646)(79.089)+(0.00039183)(48.838)+(0.0017223)(9.6385)}{0.02091}$$

This computation is simplified if it is noted that the numerator is the same as that for the variance of the function

$$U=8.76$$

(c) *Procedure in application of discriminant function to a series of deer measurements.*

(1) Compute the function Y for each deer by substituting its measurements in the derived discriminant function.

(2) Substitute the proper mean values from table 20 to compute

$$\overline{Y}_{I},\ \overline{Y}_{II},\ \overline{Y}_{III},\ \overline{Y}_{IV}$$

(3) Find $\frac{\overline{Y}_{II}-\overline{Y}_{I}}{2}$, $\frac{\overline{Y}_{III}-\overline{Y}_{II}}{2}$ and $\frac{\overline{Y}_{IV}-\overline{Y}_{III}}{2}$

(4) Then the limits for the age classes are:

$$\text{Age class I: } \overline{Y}_{I}+\frac{\overline{Y}_{II}-\overline{Y}_{I}}{2};$$

$$\text{Upper limit—}\overline{Y}_{II}+\frac{\overline{Y}_{III}-\overline{Y}_{II}}{2}$$

Age class II:

$$\text{Lower limit—}\overline{Y}_{I}-\frac{\overline{Y}_{II}-\overline{Y}_{I}}{2}$$

etc.

Applying the discriminant function derived for the three measurements on the female deer to the same data from which it was derived,

$$Y=4.7587x_1+x_2+4.3955x_3,$$

gives age group ranges with the corresponding actual counts as shown in table 10.

Here 4 percent were wrongly classified in age I, 33 percent in II, 57 percent in III, and 32 percent in IV. Our corresponding theoretical probabilities were 0.06, 0.36, 0.61, and 0.31 percent, a slight overestimate of error in the first three ages. It is, of course, true that this particular test would not be conclusive since the data were the same for the derivation of the formula and the test of it.

TABLE 10.—*Age group ranges and actual counts*

Theoretical age groups based on Y	Age class— I		II		III		IV	
	Number	*Percent*	*Number*	*Percent*	*Number*	*Percent*	*Number*	*Percent*
821.4 and above	0	0	1	1.9	35	26.9	89	68.5
766.5 to 821.3	0	0	15	27.8	56	43.1	30	23.1
651.1 to 766.4	4	3.6	36	66.7	38	29.2	11	8.5
651.0 and below	107	96.4	2	3.7	1	.7	0	0
Actual collected	111	-------	54	-------	130	-------	130	-------

STATISTICAL ANALYSIS OF DATA

As previously stated the reason for analyzing the deer measurements—age, body, and antler—was to find, if possible, an efficient criterion for age determination which would be independent of the dental formula. There can now be discussed the results of applying the methods of analyses outlined in the section on statistical technique, pp. 21 to 30, to these data with this purpose in mind.

Extent of Data

The number of animals from which the particular data were obtained is shown in table 11. Age was not obtained for the 438 deer weighed and measured in 1935.

Table 11.—*Number of animals included in determining the relation of age to weight, body, and antler measurements*

Sex and year	Age class				Total
	I (9 months)	II (1½ years)	III (2½ to 4½ years)	IV (5½ years and over)	
Female:	*Number*	*Number*	*Number*	*Number*	*Number*
1938	75	21	84	85	265
1939	36	33	46	45	160
Total females	111	54	130	130	425
Male:					
1936	0	27	191	45	263
1937	0	76	154	69	299
1938	69	0	0	0	69
1939	36	37	107	112	292
Total males	105	140	452	226	923

On the female deer, the three characters measured in both 1938 and 1939 were dressed weight, x_1; length of body, x_2; and length of hind foot, x_3. Length of tail was measured in 1938, but not repeated on the 1939 deer, since an analysis showed no difference in this measurement by age classes.

Four antler measurements were taken on the male deer; spread of antler, x_4; circumference of main beam, x_5; length of antler, x_6; and number of points, x_7. Of the three body measurements, dressed weight was taken on all the male deer, while length of body and length of hind foot were not measured on the 1936 deer and on some of the 1937 deer. Of the 502 deer (males), for which measurements of all seven characters were available, 105, 64, 179, and 154 were in classes I, II, III, and IV, respectively.

Throughout the analysis, the order of these characters has, for convenience, remained unchanged and with the same subscripts as given in the preceding paragraphs.

TABLE 12.—*Frequency distributions of female deer by character classes within age groups (1938 and 1939)*

Dressed weight					Length of body					Length of hind foot				
Midpoint of class interval	Age class: 9 months	Age class: 1½ years	Age class: 2½–4½ years	Age class: 5½ years and over	Midpoint of class interval	Age class: 9 months	Age class: 1½ years	Age class: 2½–4½ years	Age class: 5½ years and over	Midpoint of class interval	Age class: 9 months	Age class: 1½ years	Age class: 2½–4½ years	Age class: 5½ years and over
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Pounds	*Number*	*Number*	*Number*	*Number*	*Centimeters*	*Number*	*Number*	*Number*	*Number*	*Centimeters*	*Number*	*Number*	*Number*	*Number*
122.45				2	187.45				2	52.45				
117.45			3	8	182.45				11	51.45			1	5
112.45				14	177.45		1	5	18	50.45			3	1
107.45			7	18	172.45		3	21	35	49.45		1	3	21
102.45			10	21	167.45		9	27	23	48.45		4	20	26
97.45		1	20	23	162.45		8	25	19	47.45		7	29	35
92.45		6	31	19	157.45		10	25	13	46.45		17	23	21
87.45		8	29	14	152.45	5	13	15	4	45.45	2	13	22	10
82.45		13	21	9	147.45	23	7	7	4	44.45	8	9	15	6
77.45		12	4	1	142.45	17	2	2	1	43.45	20	2	6	
72.45	1	10	4	1	137.45	23		1		42.45	18	2	5	
67.45	4	2			132.45	27				41.45	31		1	
62.45	11	2	1		127.45	12	1			40.45	19		2	1
57.45	30				122.45	3		1		39.45	5		1	
52.45	25				117.45					38.45	2			
47.45	17				112.45	1				37.45	2			
42.45	12									36.45	2			
37.45	10									35.45	1			
32.45	1									34.45	1			
Total	111	54	130	130		111	54	130	130		111	51	130	130

TABLE 13.—*Frequency distribution of male deer by body measurements within age groups (1936, 1937, 1938 (9 months only), 1939)*

Dressed weight

Midpoint of class interval (1)	Age class: 9 months (2)	Age class: 1½ years (3)	Age class: 2½–4½ years (4)	Age class: 5½ years and over (5)
Pounds	*Number*	*Number*	*Number*	*Number*
187.45				1
182.45				
177.45				2
172.45				2
167.45				5
162.45			1	6
157.45			1	7
152.45			5	10
147.45			12	23
142.45			9	26
137.45			19	28
132.45			38	27
127.45			50	34
122.45		2	55	21
117.45		6	56	16
112.45		4	61	9
107.45		19	58	3
102.45		20	41	4
97.45		25	33	1
92.45		22	8	1
87.45		17	5	
82.45		14	1	
77.45	1	6		
72.45	10	2		
67.45	6			
62.45	12			
57.45	22			
52.45	20			
47.45	20			
42.45	11			
37.45	3			
Total	105	110	452	226

Length of body

Midpoint of class interval (6)	Age class: 9 months (7)	Age class: 1½ years (8)	Age class: 2½–4½ years (9)	Age class: 5½ years and over (10)
Centimeters	*Number*	*Number*	*Number*	*Number*
242.45				1
227.45				1
222.45				3
207.45				13
197.45			5	16
192.45			11	31
187.45		2	19	25
182.45		4	33	29
177.45		9	36	16
172.45		11	39	7
167.45		16	22	4
162.45	2	11	11	5
157.45	4	6	1	
152.45	15	4	1	
147.45	13			
142.45	24		1	
137.45	20	1		
132.45	17			
127.45	7			
122.45	3			
Total	105	64	179	154

Length of hind foot

Midpoint of class interval (11)	Age class: 9 months (12)	Age class: 1½ years (13)	Age class: 2½–4½ years (14)	Age class: 5½ years and over (15)
Centimeters	*Number*	*Number*	*Number*	*Number*
55.45				1
54.45				4
53.45			5	7
52.45			11	22
51.45		2	24	41
50.45		3	46	26
49.45		11	38	22
48.45		14	33	15
47.45		18	18	10
46.45	3	9	3	5
45.45	6	4	1	1
44.45	16	1		
43.45	18	1		
42.45	21			
41.45	23			
40.45	9			
39.45	5			
38.45	2			
37.45	2	1		
Total	105	64	179	154

TABLE 13.—*Frequency distribution of male deer by body measurements within age groups (1936, 1937, 1938 (9 months only), 1939)*—Continued

Spread of antlers				Circumference of main beam				Length of antlers				Number of points			
Midpoint of class interval	1½ years	2½–4½ years	5½ years+	Midpoint of class interval	1½ years	2½–4½ years	5½ years+	Midpoint of class interval	1½ years	2½–4½ years	5½ years+	Midpoint of class interval	1½ years	2½–4½ years	5½ years+
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Centimeters	*Number*	*Number*	*Number*	*Centimeters*	*Number*	*Number*	*Number*	*Centimeters*	*Number*	*Number*	*Number*	*Centimeters*	*Number*	*Number*	*Number*
				12.45		1	3	62.45				11.45		3	18
63.45			3	11.45		2	14	57.45			4	9.45		12	30
60.45			5	10.45		12	44	52.45		3	6	8.45	2	103	67
57.45			6	9.45		29	59	47.45		11	26	7.45	2	65	59
54.45	1	2	15	8.45	2	105	56	42.45		26	49	6.45	11	89	18
51.45		3	19	7.45	5	162	34	37.45		67	53	5.45	10	67	22
48.45		18	37	6.45	38	99	15	32.45	4	147	46	4.45	25	66	5
45.45		30	38	5.45	56	40	1	27.45	11	120	32	3.45	27	31	6
42.45		47	43	4.45	31	2		22.45	48	56	10	2.45	42	15	1
39.45		40	29	3.45	3			17.45	40	18		1.45	16	1	
36.45	3	88	17	2.45				12.45	21	4			5		
33.45	4	66	7	1.45				7.45	11						
30.45	6	59	4		5				5						
27.45	19	34	1												
24.45	24	26													
21.45	28	19	2												
18.45	28	7													
15.45	16	12													
12.45	5	1													
9.45	1														
6.45															
3.45	5														
Total	140	452	226		140	452	226		140	452	226		140	452	226

COMPARISON OF MEASUREMENTS BY DISTRICT AND YEAR

On the basis of year and district, the deer measured fall into 12 categories within sex and age. Eight of these were for the males and four for the females.

The initial step in the organization and analysis of the data was to form frequency distributions of each character within several categories. Tables 12 and 13 give those for combined years and districts. Figures 17, 18, and 19 are the corresponding frequency polygons or curves.

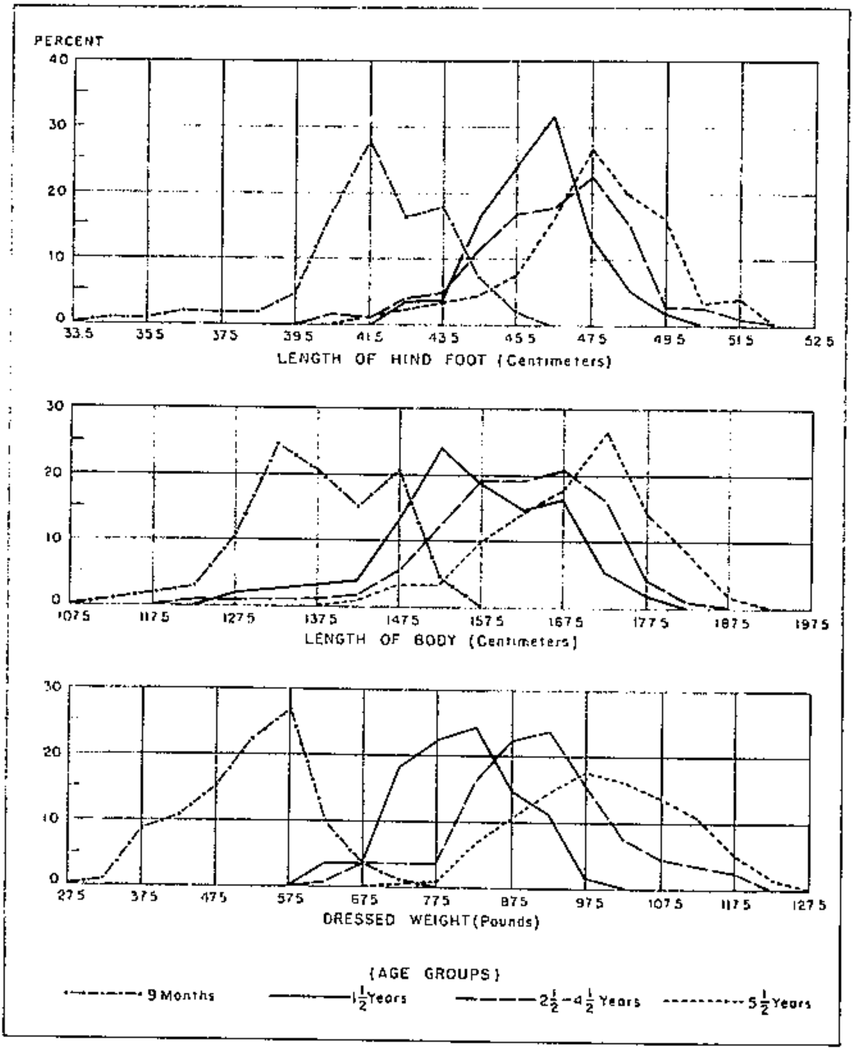

FIGURE 17.—Frequency curves of body measurements of the sample female deer within age groups for 1938 and 1939.

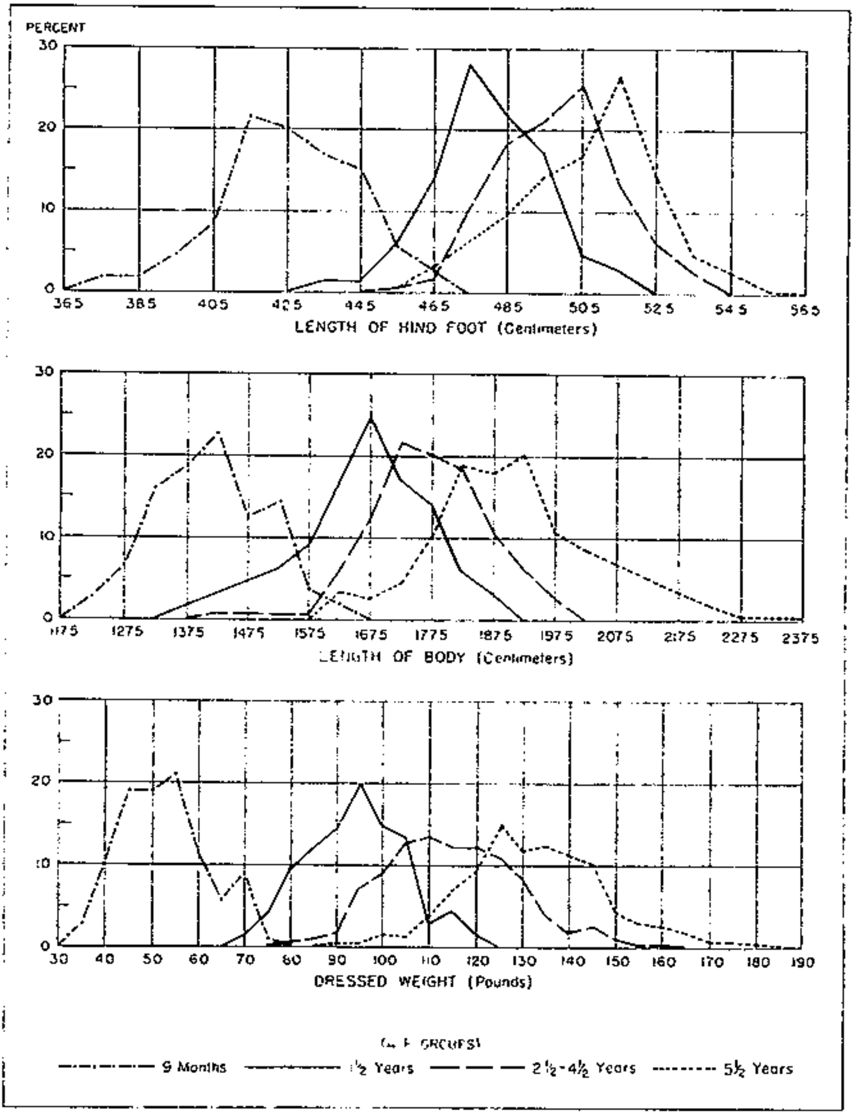

FIGURE 18.—Frequency curves of body measurements of the sample male deer within age groups for 1936, 1937, 1938, and 1939 (1938 data for 9 months-old only).

Tables 14 and 15 give the number of deer, the arithmetic mean (average), and standard deviation of each measurement by age groups within these subclasses. The number of deer range from 2 to as many as 105.

TABLE 14.—*Mean (average) and standard deviation of measurements of female deer by year, district and age classes*

Year, district, and age class (1)	Deer in sample	Dressed weight		Length of body		Length of hind foot		Length of tail	
		Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1938									
	Number	*Pounds*	*Pounds*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*
Northern district:									
I—9 months	41	48.8	6.4	135.9	5.4	41.6	1.7	21.6	2.6
II—1½ years	10	81.2	7.8	156.9	12.1	46.2	1.8	22.2	1.9
III—2½–4½ years	48	88.5	9.7	159.5	10.7	46.0	1.7	23.3	2.6
IV—5½ years and over	50	94.7	10.2	167.0	9.5	47.5	1.9	23.9	2.6
Southern district:									
I—9 months	34	50.9	9.3	138.9	8.3	40.6	2.5	21.7	2.9
II—1½ years	11	77.4	10.6	154.2	9.4	44.0	1.7	24.6	3.1
III—2½–4½ years	36	89.9	8.0	160.3	8.0	45.0	1.8	23.8	2.9
IV—5½ years and over	35	101.3	9.8	168.6	8.9	47.4	1.8	24.2	4.3
1939									
Northern district:									
I—9 months	15	52.9	5.7	140.5	6.4	41.5	1.4	...	...
II—1½ years	19	77.9	7.2	157.1	8.9	45.8	1.2	...	...
III—2½–4½ years	29	93.1	9.9	164.0	9.1	47.1	2.0	...	...
IV—5½ years and over	27	101.1	10.9	170.5	8.9	47.3	2.2	...	...
Southern district:									
I—9 months	21	55.4	8.8	140.0	7.0	42.5	1.6	...	...
II—1½ years	14	83.5	6.5	159.4	8.4	45.7	1.0	...	...
III—2½–4½ years	17	93.4	7.9	162.5	5.2	47.0	1.6	...	...
IV—5½ years and over	18	102.2	9.5	170.2	6.6	47.8	1.1	...	...

TABLE 15.—*Mean (average) and standard deviation of measurements of male deer by year, district, and age classes*

Year, district, and age class	Deer in sample	Dressed weight		Body measurements						Antler measurements							
				Length of body		Length of hind foot		Length of tail		Spread of antler		Circumference of main beam		Length of antler		Number of points	
		Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
1936	*Number*	*Pounds*	*Pounds*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*
Northern district:																	
I—1½ years	25	99.5	10.8							23.3	4.6	5.9	0.9	21.6	5.0	4.1	1.7
II—2½–4½ years	86	118.0	16.8							33.7	8.1	7.7	1.4	32.0	8.2	6.1	2.0
III—5½+ years	33	133.7	15.8							42.2	9.5	9.3	1.5	41.2	9.5	8.1	2.0
Southern district:																	
I—1½ years	2	103.5	6.4							17.8	2.5	4.8	.4	16.3	1.8	2.0	0
II—2½–4½ years	105	113.1	13.8							29.8	7.8	7.2	1.2	28.9	6.4	5.7	2.0
III—5½+ years	12	141.6	17.1							45.1	6.9	9.5	1.7	41.6	7.9	9.6	1.9
1937																	
Northern district:																	
I—1½ years	32	97.0	9.4	169.4	8.5	48.0	1.8	24.4	2.5	23.4	5.4	5.7	.8	21.5	4.6	3.9	1.6
II—2½–4½ years	83	120.9	13.4	179.2	10.3	49.6	1.5	25.4	3.6	34.9	7.3	7.5	1.3	32.6	6.9	6.4	1.8
III—5½+ years	47	136.7	16.0	185.5	12.5	50.0	1.5	26.8	3.7	42.7	6.5	8.9	1.5	39.0	6.1	7.7	1.4
Southern district:																	
I—1½ years	44	91.8	9.6	165.7	12.3	46.5	3.0	25.7	2.1	19.3	4.7	5.6	.8	17.5	4.8	3.0	1.3
II—2½–4½ years	71	115.1	13.4	184.9	7.7	49.4	1.5	25.7	3.2	31.4	6.1	7.2	1.1	28.7	6.5	5.9	1.8
III—5½+ years	22	136.9	17.6	200.4	15.4	51.4	.9	27.9	3.7	41.2	3.4	8.8	1.5	37.2	4.6	7.7	1.4
1938																	
Northern district:																	
9 months	32	49.1	5.6	136.4	7.1	41.6	1.6	21.1	1.8								
Southern district:																	
9 months	37	54.4	9.2	140.2	8.7	41.7	1.7	22.4	3.1								
1939																	
Northern district:																	
I—9 months	21	57.0	9.1	143.0	8.2	43.3	1.7										
II—1½ years	16	94.5	11.6	169.4	9.0	47.8	1.1			17.6	13.3	4.1	2.5	15.4	10.0	2.6	
III—2½–4½ years	58	118.2	11.9	177.3	8.9	49.5	1.8			34.6	5.4	7.5	1.0	32.0	5.0	6.4	
IV—5½+ years	82	132.1	15.1	182.8	9.1	50.1	2.1			40.8	6.9	8.7	1.2	39.0	6.8	7.6	
Southern district:																	
I—9 months	15	61.6	7.9	149.3	6.2	43.9	1.0										
II—1½ years	21	96.8	13.3	166.7	8.0	47.7	1.9			14.9	5.6	4.5	1.2	12.6	5.0	2.6	
III—2½–4½ years	49	117.1	10.5	180.2	7.3	49.9	1.5			33.9	5.3	7.5	1.0	31.6	5.7	6.9	
IV—5½+ years	30	132.8	12.5	189.8	7.7	50.6	2.0			41.3	5.7	9.1	1.3	40.4	7.1	8.0	

The standard deviation indicates the spread or variation of the individual items which make up the mean or average. In a normal distribution, two-thirds of the items will fall within the range of the mean plus and minus the standard deviation. Formulas (1) and (2), p. 22 indicate the procedure for calculating mean and standard deviation. From these figures it is at once evident that, except for the length of tail, there is a consistent trend upward within the subgroups in the means of the characters measured with an increase in age. Comparable subgroups for males and females are present only in the 9-month animals in 1938 and all ages in 1939. Except for the 9-month ages, the mean values for males are, in every case, larger than the females.

E. A. Goldman, in his report, states that white-tailed deer reach full maturity at about 5 years. Between 70 and 80 percent of the deer measured were immature (table 23) and the majority of those in the mature class were probably between 5½ and 6½ years old. If a higher percentage of the animals had been older, making data on the 9½-years-plus age class available, the upward trend in these calculations might have leveled off at the 5½-year age class.

There does not seem to be any definite trend in the amount of variation, either by years or district, except possibly slightly more variation in the northern antlers than the southern ones within the same year and age group. This substantiates the tentative conclusion made after 3 years of work (*9*), that the deer herd would be reduced by some natural cause before any definite genetic herd deterioration occurred. Further discussion of this point will be found in the section on the application of results and use of data, pp. 47 to 57.

Table 16 should be studied in connection with these group means, as it records a precise test of the difference in group means average as computed by use of formula 4.

If there is considerable variation in the individual items within a population, the means from two random samples from this population might differ considerably. Hence, in order to judge whether or not two samples may be considered as coming from the same population, it is necessary to take into consideration the individual variations within those two samples. If the means differ significantly, the samples cannot be said to be from the same population. In the example given in the discussion of statistical technique, pp. 21 to 30, $t=0.94$ for a comparison of dressed weight between the 1938 northern and southern female deer 1½ years old. P lies between 0.4 and 0.5. This means approximately in 4 out of 10 cases a t of this size might be due to random sampling only. Roughly, we might say that t should be at least 2.5 if a real difference exists between the two groups.

Since the population was much denser in the southern district than in the northern district, the two were analyzed separately and compared to see if differences might be significant. Of the 63 comparisons between northern and southern males (table 16, columns 2, 3, 4), 17 had t values large enough to be considered as significant. In 7 of these, the southern animals had the larger means. Only 2 of the 24 comparisons of female northern and southern deer (columns 11 and 12) were high enough to be significant; in 1 the southern average was larger, in the other the northern was larger. At least one t value above the limit would be expected, even if all were from the same population, when as many as 24 samples were taken. It is concluded,

TABLE 16.—*Values of t in test*[1] *of differences in characters measured between districts and years within age groups for male and female deer*

Character and age group	Males									Females			
	North vs. South			1936–37		1936–39		1937–39		North vs. South		1938–39	
(1)	1936 (2)	1937 (3)	1939 (4)	North (5)	South (6)	North (7)	South (8)	North (9)	South (10)	1938 (11)	1939 (12)	North (13)	South (14)
Dressed weight:													
9 months	...	[2] 2.84	1.56	...	...	...	...	[2] 4.84	[2] 4.83	1.15	0.97	3.99	2.75
1½ years	0.51	2.34	.55	0.92	1.69	0.70	0.97	.80	1.72	.94	2.31	1.40	2.85
2½–4½ years	2.22	2.92	.47	1.03	.98	.05	1.81	1.05	.07	.69	.11	3.00	2.42
5½ years and over	1.40	.05	.18	.83	.73	.48	1.84	1.61	.98	2.98	.33	3.89	.52
Length of body:													
9 months	...	[2] 1.91	2.51	...	...	...	...	[2] 3.08	[2] 3.68	1.51	.18	1.93	.93
1½ years	...	.91	.08	...	...	...	...	.01	.03	.58	.76	.05	1.47
2½–4½ years	...	2.50	1.81	...	...	...	...	1.05	2.67	.40	.96	2.27	.94
5½ years and over	...	3.33	3.38	...	...	...	...	1.22	3.01	.76	.11	1.56	.69
Length of hind foot:													
9 months	...	[2] .17	1.20	...	...	...	...	[2] 3.65	[2] 4.60	2.01	2.06	.29	3.18
1½ years	...	1.84	.22	...	...	...	...	.33	1.40	1.74	.41	.59	1.53
2½–4½ years	...	.85	1.20	...	...	...	...	.54	1.44	2.69	.29	2.58	3.77
5½ years and over	...	3.00	1.20	...	...	...	...	.30	1.27	.30	.76	.32	.79
Spread of antler:													
1½ years	1.66	3.45	.71	.04	.47	2.25	.71	2.13	4.30	...	...	...	...
2½–4½	3.66	3.20	.67	.04	1.48	.71	3.33	.23	2.28	...	...	...	...
5½ years and over	.92	.95	.34	.23	2.20	.88	1.81	1.50	.00	...	...	...	...
Circumference of main beam:													
1½ years	1.75	.49	.72	.97	1.41	3.37	.03	3.34	4.14	...	...	...	...
2½–4½ years	2.76	1.50	.13	.88	.29	.83	1.42	.02	1.59	...	...	...	...
5½ years and over	.53	.74	1.29	.95	1.33	1.02	1.21	.82	1.44	...	...	...	...
Length of antler:													
1½ years	1.48	3.66	1.11	.04	.37	2.64	1.88	2.93	3.77	...	...	...	...
2½–4½ years	2.88	4.29	.36	.52	.29	.01	2.47	.68	2.59	...	...	...	...
5½ years and over	1.05	1.24	.98	1.61	.82	1.39	.45	.07	.69	...	...	...	...
Number of points:													
1½ years	1.72	2.70	.02	.39	1.11	2.66	1.31	2.55	1.39	...	...	...	...
2½–4½ years	1.52	1.80	2.89	1.00	.64	.75	3.90	.21	3.32	...	...	...	...
5½ years and over	3.12	.13	1.30	1.03	3.33	1.36	2.95	.75	.69	...	...	...	...

[1] t =Student test to determine whether two samples belong to the same population.
[2] 1938 data instead of 1937.

therefore, that no untoward bias would be introduced if districts as well as years were combined within sex and age groups in making the analysis to determine the relationship between age and characters measured.

The t values, when comparing 1936 and 1937 shown in table 16, columns 5 and 6, are consistently lower than 2.5, the limit for a real difference. There is more variation when the 1939 antler figures are compared with 1936 and 1937, table 16, columns 7–10. The significant values of t were for the 1½-year animals. The 1939 antler measurements were consistently smaller for this age than those in the earlier years. For the other two ages, as shown in columns 2 and 3, the differences were quite erratic, not always occurring in the same direction.

LENGTH OF TAIL

The length of the tail is of little or no value for discriminating the age of white-tailed deer. In 1937, length of tail was measured on 142 deer; in 1938 this measurement was taken on all deer in the study, but was discontinued in 1939 because it was not significant. The means and standard deviations of these appear in tables 14 and 15 (columns 9 and 10).

For the 9-month animals, the mean of the females was slightly larger than the males in the northern district. The reverse was true in the southern district. As indicated in table 17, the only *t* large enough to give any evidence of a real difference was 2.53, and there the younger deer had the larger mean. In 7 of the 12 comparisons, the mean of the younger was the larger of the 2.

TABLE 17.—*Results of the* t *test when used in comparing successive age groups for the data available on length of tail*

Districts	*t* values for the comparison indicated			
	9 months male vs. female	9 months vs. 1½-year	1½-year vs. 2½–4½-year	2½–4½-year vs. 5½-year and over
1937				
Northern			0.97	1.62
Southern			0.01	1.97
1938				
Northern	1.01	1.11	1.21	1.21
Southern	0.98	2.53	0.72	0.41

Another test was made on the 1937 data which substantiated the results of the *t* test. The sums of squares of the deviations in tail lengths was broken down into two portions, that due to age and that within ages, and the *F*, or ratio test of the mean squares applied (table 18). A discussion of the method used is given in the section on analytical procedure, p. 24.

For age to be considered as a significant factor in tail length, *F* should be at least as large as the *F* at the 5 percent level. This would mean a chance of 1 in 20 that so large a difference would be due to random sampling. In neither the northern nor southern district was the *F* value large enough for significance. These *F*'s are surprisingly insignificant in size when compared to the *F*'s for all the other characters in the analysis to follow combining the data for district and year.

TABLE 18.—*Determination of F, or ratio test of deviations in tail lengths*

Source of variance	Degrees of freedom	Mean square	*F*	*F* at 5 percent level
Northern district:				
Between ages	2	31.69	2.57	3.10
Within ages	86	12.33		
Total	88			
Southern district:				
Between ages	2	24.40	2.54	3.19
Within ages	50	9.62		
Total	52			

DETERMINATION OF THE EFFECTIVENESS OF EACH CHARACTER FOR DISTINGUISHING THE AGE OF A DEER

The fact that for these data, all years and districts within each sex and age group might be combined without introducing untoward bias, made samples for each age class sufficiently large to assure confidence in the results. The grouped frequency distributions for each character appear in tables 12 and 13. Figures 17 and 18 show the corresponding actual probability curves (percent of total in each class interval). The method for making frequency distributions and curves was outlined in the first part of the section on analytical procedures, pp. 21–22.

These curves should be examined along with table 19 which gives the analysis of variance for each character measured, the mean square due to difference between age groups, and that within age groups. In each case, the first named mean square is considerably larger than the second. Their ratio, called F, ranges in value in table 19 from 218.99 to 941.73. To be significant at the 0.1 percent level—that is, a 1 to 1,000 chance that so large a difference might be due to random sampling and not to differences in age levels—the largest F (which depends somewhat on the degrees of freedom involved) is only 7.20. Since the smallest F was more than 200, there is no doubt that each of these seven characters does vary with age.

TABLE 19.—*Analysis of variance by age groups for male and female deer, 1936 to 1939, inclusive*

MALE

Character and source of variation	Degrees of freedom	Sum of squares	Mean square	Standard deviation	F value: Observed	F value: At 0.1 percent level
Dressed weight:						
Between ages	3	500825.27	169,942		941.73	5.50
Within ages	906	162493.19	180.456059			
Total	909	673318.46		13.433		
Length of body:						
Between ages	3	150086.41	50,021		548.93	5.50
Within ages	498	45387.22	91.138090			
Total	501	195473.63		9.5467		
Length of hind foot:						
Between ages	3	4747.50	1,582.52		495.67	5.50
Within ages	498	1589.9556	3.1926810			
Total	501	6337.5256		1.787		
Spread of antler:						
Between ages	2	40439.52	20,219.8		404.27	7.20
Within ages	802	40112.879	50.016058			
Total	804	80552.40		7.072		
Circumference of main beam:						
Between ages	2	1158.0988	579.049		223.06	7.20
Within ages	802	1279.9212	1.5959117			
Total	804	2438.02		1.2633		
Length of antler:						
Between ages	2	40104.15	20,052.1		437.45	7.20
Within ages	802	36762.24	45.838201			
Total	804	76866.38		6.7704		
Number of points:						
Between ages	2	1805.0013	902.50065		307.66	7.20
Within ages	802	2352.5888	2.9334025			
Total	804	4157.5901		1.7127		

TABLE 19.—*Analysis of variance by age groups for male and female deer, 1936 to 1939, inclusive*—*Continued*

FEMALE

Dressed weight:						
Between ages	3	151626.73	50,542		594.11	5.50
Within ages	421	35814.95	85.071148			
Total	424	187441.68		9.224		
Length of body:						
Between ages	3	58548.68	19,516		245.56	5.50
Within ages	421	33459.77	70.476886			
Total	424	92008.45		8.915		
Length of hind foot:						
Between ages	3	2231.71	777.24		218.99	5.50
Within ages	421	1494.24	3.5492596			
Total	424	3825.95		1.884		

The within sums of squares were pooled by age groups. For this to be a valid procedure, the age-group variance must be considered equal. Reference is made to the standard deviations in tables 14 and 15 and to figures 17 and 18. While there was some variation, particularly for dressed weight, it was not believed sufficient to cause any disturbance in results. The mean square resulting from the pooled within sum of squares yields the best estimate of the variance. The square root of this is the standard deviation, indicated in table 19.

SINGLE CHARACTER

Although table 19 shows that the several characters vary materially with age, it does not give the answer to the question of how much of an error might be made if the age of the individual deer was based on any one or more of these characters. If a deer were always an average animal, it would be possible to give its age group accurately. These mean values of each character for each age group appear in table 20. The frequency distributions (tables 12 and 13, figures 17 and 18) and the standard deviations (table 19) indicate considerable variation in measurements for the deer in any one age group. The standard deviation for each is found in table 19; i. e., for dressed weight of males it is 13.433, etc. The accuracy of age predictions when using a single measurement was based on these standard deviations (see p. 46).

TABLE 20.—*Means (average) of the several characters measured in the 4 age groups*

[For the sample of male and female deer used in the analysis]

Sex and body character	Age class			
	I (9 months)	II (1½ years)	III (2½–4½ years)	IV (5½ years and over)
Males ... number	105	61	179	154
x_1 Dressed weight ... pounds	51.312690	95.390625	118.502793	135.006494
x_2 Length of body ... centimeters	140.888678	167.754688	179.906890	187.340909
x_3 Length of hind foot ... do	42.273395	47.548438	49.603928	50.322078
x_4 Spread of antler ... do		17.973438	34.225346	41.510260
x_5 Circumference of main beam ... do		4.839063	7.530168	8.975325
x_6 Length of antler ... do		15.878125	31.780155	39.639610
x_7 Points ... number		2.937500	6.513966	7.720779
Females ... do	111	54	130	130
x_1 Dressed weight ... pounds	51.225225	79.851852	90.600000	98.815385
x_2 Length of body ... centimeters	138.347748	157.083333	161.328462	168.600769
x_3 Length of hind foot ... do	41.468063	45.659259	46.128462	47.452308

The procedure is straightforward. One-half the difference in the means (table 20) for two consecutive age classes is divided by the standard deviation. From table I, Fisher and Yates Statistical Tables (*4*,) the value of P corresponding to the preceding quotient x is found. Since this is the probability of an observation falling outside the range $-x$ to $+x$, and the interest is in one tail of the curve, one-half of P may be taken for the chance of a deer falling above or below the midpoint between age groups—above the smaller age class or below the larger one when judged by one measurement. For the second and third age group, the total probability of making a mistake will be the sum of two probabilities. For example, the chance of making a mistake in classifying a 1½-year deer will be the probability of its being classed in the 9-month group, plus the probability it will be put in the 2½–4½-year age group.

In table 21, appear the resulting probabilities for each character measured. Using all 4 age groups for males, if dressed weight alone were used, there would be 6 chances out of 100 of putting a 9-month deer in the 1½-year class, 27 of misclassifying a 1½-year deer, 47 for the 2½–4½-year deer, and 26 for the overage animals—not a very precise estimate, but the best of the 3 body measurements.

Since 9-month old deer do not have antlers, including them when using antler measurements would seem to distort the picture. In the first section of table 21 they have been omitted. Number of points continue poor, but now length and spread of antler are both better measures than circumference of main beam, with length slightly better. Columns 10 and 18, which will be interpreted later, give an over-all evaluation of the several probabilities. The larger this figure, the better is the character it represents.

With female deer, dressed weight, although the probabilities are too high to be satisfactory, is much better than either of the other two characters.

COMPOUNDS OF TWO OR MORE CHARACTERS FOR DETERMINATION OF AGE

Preliminary tests were made in order to select those characters which would seem to yield the most desirable function, and it seemed desirable to use as few as possible. Since all the characters are influenced by age, table 19, and the association between characters is considerable, then the selection should be on the basis of the greatest independent variation. This would be a combination of those which have the greatest influence on all others and the ones which have the least association with the others. In the case of the male animals, length of antler seemed to have a closer relationship to the other measurements, while length of hind foot was the most independent. However, it was thought worth while—for the purpose of comparison and possible practicable use of the analysis—to present the results from a series of different combinations of characters, some of which were not the most promising, as shown in table 21. Since the 9-month animals were antlerless, two separate analyses were made of the male deer—one comparable to the female deer including all age groups, but considering only the three body measurements; while in the other, the 9-month old animals and the influence of all characters except number of points were examined; the latter character was omitted because preliminary tests had shown it to be of little value.

The statistical details of the procedure followed in computing the discriminant function, finding its standard deviation, test of the characters represented, and an over-all evaluation of its effectiveness are described in the section on analytical procedure p. 26. The probabilities for misclassifying deer of the different age classes, based on each discriminant function considered will be found in table 21.

Interpretation of the Summary Figures

Table 21, in which are assembled the summary figures for an appraisal of the efficiency of the single characters and their several combinations for determining age of deer, is made up of two major parts. The first half of the table gives the results from data for the three upper age classes of males, those deer with antlers, based on 397 animals; while the second half is from data in which all four age classes were represented, 502 male and 425 female deer. Animals in the 9-month class, being antlerless, were not included because they would have distorted the results reflecting the influence of antlers.

Columns 7 to 9, and 14 to 17 are the probabilities for misclassifying deer by age groups when particular character combinations are used. For example, when the three body measurements are used, there are 7 chances out of 100 of misclassifying female deer 9 months old, 38 out of 100 for 1½-year animals, 63 out of 100 for 2½-year animals, and 32 out of 100 for mature deer. The smaller the probability, the more precise are the age determinations. Columns 10 and 18, with heading U is the ratio of the slope to the standard deviation of the function. It is an over-all measure of the effectiveness of the function. The larger the U, the better it discriminates age.

A careful study of table 21 leads to the following conclusions:

From a strictly statistical standpoint, no single measurement or combination of measurements discriminates age with any high degree of precision for the middle ages. Of the body measurements, dressed weight is better than either of the other two; adding one or both of the length measurements does not improve the precision of estimate perceptibly. There is no difficulty in determining the age of the 9-month old group.

Any one of the antler measurements, except number of points, is better than dressed weight. In the three-age analysis, the length of antler is the best single measure. Combinations of two antler measurements give better estimates than one alone, the spread and length of antler being the best; but increasing the number of measurements over two does not improve the estimate. A slightly better compound of two is one body and one antler character, preferably length or spread of antler. The U for the combination, length of hind foot and length of antler, would seem to be as effective as a compound in which dressed weight was the body measurement selected.

The best combination of three is length of hind foot, spread of antlers, and length of antler. No advantage is gained by including the circumference. In dealing with the mature deer, the chance of misclassifying is only slightly less when a compound is used instead of any one single measurement. The greatest improvement of the compound discriminant function over the single character is in the two middle age groups of male deer.

TABLE 21.—*Probabilities of misclassifying male and female deer for various combinations of characters and ratio test of function*

Three age groups only (9-month deer omitted)[1]

Dressed weight	Length of body	Length of hind foot	Spread of antlers	Circumference of main beam	Length of antler	Probability of misclassification II[3]	III[4]	IV[5]	U[6]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Single character									
*						0.22	0.48	0.26	9.46
	*					.27	.62	.35	6.70
		*				.28	.70	.42	5.27
			*			.18	.45	.27	11.20
				*		.20	.47	.27	10.06
					*	.17	.43	.26	11.77
Two characters									
*	*					.19	.46	.27	9.89
*		*				.18	.46	.28	9.96
*			*			.10	.35	.25	13.01
*				*		.13	.38	.25	12.28
*					*	.11	.35	.24	13.00
	*	*				.23	.56	.33	7.60
	*			*		.13	.40	.27	11.63
		*	*			.09	.37	.28	12.87
		*		*		.12	.39	.27	11.99
		*			*	.09	.36	.27	12.97
			*	*		.10	.37	.27	12.59
			*		*	.10	.37	.27	12.80
				*	*	.11	.38	.27	12.40
Three characters									
*	*	*				.18	.46	.28	10.08
*		*		*		.11	.36	.25	12.65
*			*		*	.09	.33	.24	13.42
		*	*		*	.08	.34	.28	13.53
Four characters									
*	*	*		*		.11	.36	.25	12.67
*	*		*		*	.09	.33	.24	13.46
*		*	*		*	.09	.34	.25	13.80
*			*	*	*	.10	.34	.24	13.18
Five characters									
*		*	*	*	*	.08	.33	.25	13.86

All age groups[2]

Dressed weight	Length of body	Length of hind foot	Probability of misclassification I[7]	II[3]	III[4]	IV[5]	U[6]
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
Males							
Single character							
*			0.06	0.27	0.47	0.26	9.97
	*		.08	.34	.61	.35	8.15
		*	.07	.35	.70	.42	7.08
Two characters							
*	*		.04	.22	.44	.26	10.97
*		*	.03	.20	.45	.28	11.34
	*	*	.04	.27	.58	.35	9.81
Three characters							
*	*	*	.03	.20	.45	.28	11.61
Females							
Single character							
*			.06	.31	.61	.33	8.59
	*		.15	.56	.75	.34	5.48
		*	.13	.58	.81	.36	5.12
Two characters							
*	*		.06	.35	.61	.32	8.69
*		*	.05	.34	.61	.32	8.68
	*	*	.11	.53	.75	.33	6.48
Three characters							
*	*	*	.06	.36	.61	.31	8.76

[1] N=307
[2] N males=502; N females=425.
[3] II=1½ years.
[4] III=2½ to 4½ years.
[5] IV=5½ years and over.
[6] U=ratio of slope to standard deviation of the function.
[7] I=0 months.
* =Indicates character grouping.

The statistical analysis shows that there is a great deal of individual variation in each age group. These data are believed to be applicable to the Allegheny National Forest deer herd, but they will not apply to all white-tailed deer of the subspecies (*Odocoileus virginianus borealis*). However, the method outlined in the section on the application of results, pp. 47 to 57, for obtaining the necessary data for practicable deer management is believed applicable to any herd of the same subspecies and probably to all subspecies of Virginia white-tailed deer.

In selecting the best character for determining age, the probable accuracy which can be obtained under field conditions and the attendant zoological factors, as well as the mathematical precision must be considered. These points are covered in some detail in the next section.

APPLICATION OF RESULTS AND USE OF DATA IN PRACTICABLE DEER-HERD MANAGEMENT

Good deer-herd management requires a knowledge of the distribution of the herd by age groups. For such practical purposes, a single measurement is recommended on the basis of the results of the detailed analysis of the data.

It would be preferable to select a measurement that is easy to obtain and one that is not likely to be subject to excessive annual variation. This choice of a single measurement is important because the basis for calculation (age and one measurement) will be collected for only one relatively small sample for a single year. Data collected over a period of as long as 5 years eliminates the significance of annual fluctuation as shown from the results of this study.

It is considered necessary to determine the range of measurement for each age group for the deer herd or locality, because an analysis of data from other localities on the same subspecies of white-tailed deer indicates that all measurements may be uniformly larger or smaller. This fact does not seem to effect the relative accuracy of any one measurement as an indicator of age. For example, Cahalane's beam-diameter method (*1*) for determining the age of white-tailed deer in Michigan was applied to Allegheny deer, and it was found that the average white-tailed deer on the Allegheny National Forest of any given age class had antlers of a smaller diameter than the average white-tailed deer of the same age in Michigan. A comparison of measurements for each age class for both herds, according to the same formula used in the beam-diameter method, shows considerable uniformity. This further substantiates the conclusion that a method for determining age can be worked out for any group of deer based on a single measurement that would be statistically adequate and would be applicable, for all practicable purposes of management, for deer from that locality, but would probably not be applicable to deer from another locality. For example, Cahalane's method could be used on the Allegheny herd, but first a series of actual measurements of Allegheny deer would be necessary in order to establish the beam diameter sizes for each age group.

An appraisal of the measurements taken from a management standpoint during the course of this study will be found helpful in the selection of the measurements most applicable to a given condition (table 22).

TABLE 22.—*Mean (average) weights and measurements of sample deer by age classes (summary)* [1]

Age class, sex, and year	Size of sample	Average weight or measure: Dressed weight	Length of body	Length of hind foot	Spread of antlers	Circumference of main beam	Length of antler
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Number	*Pounds*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*	*Centimeters*
5½-year group:							
Males—1936	45	135.81			42.97	9.35	41.31
1937	70	136.76	190.25	50.45	42.22	8.97	38.43
1939	112	132.33	184.70	50.26	40.55	8.76	38.99
Total males	227	134.39	186.83	50.33	41.54	8.94	39.28
Females—1938	85	97.38	167.66	47.43			
1939	45	101.53	170.38	47.49			
Total females	130	98.84	168.61	47.45			
2½- to 4½-year group:							
Males—1936	191	115.30			31.65	7.40	29.44
1937	154	118.00	181.35	49.55	33.28	7.37	30.78
1939	107	117.67	178.61	49.66	34.27	7.50	31.83
Total males	452	116.78	179.72	49.61	32.85	7.41	30.42
Females—1938	84	89.13	159.87	45.61			
1939	46	93.22	164.00	47.08			
Total females	130	90.60	161.33	46.13			
1½-year group:							
Males—1936	27	99.78			22.89	5.81	17.58
1937	76	94.00	167.61	47.48	21.03	5.61	19.70
1939	37	95.81	167.86	47.78	16.08	4.32	13.51
Total males	140	95.59	167.75	47.63	20.08	5.31	17.52
Females—1938	21	79.19	155.50	45.48			
1939	33	80.27	158.08	45.77			
Total females	54	79.85	157.08	45.66			
9-month group:							
Males—1938	69	51.94	138.43	41.65			
1939	36	58.94	145.63	43.55			
Total males	105	51.34	140.89	42.27			
Females—1938	75	49.73	137.22	41.16			
1939	36	51.33	140.71	42.10			
Total females	111	51.23	138.35	41.46			
Total	[2] 1,349						

[1] Data for 1935 is not included because age was not taken. The 438 deer weighed and measured in 1935 would make a grand total of 1,787 animals.
[2] Includes 924 males, 425 females.

APPRAISAL OF INDIVIDUAL MEASUREMENTS

AGE

Deer weights and measurements data without age have very limited value, because all measurements increase with age up to at least maturity (see tables 20 and 22). If, for example, average weights are known, no definite assumption can be made as to the reason for an increase or decrease in average weight. If the investigator could ascertain that the mean (average) age for each sample was the same, then age might be disregarded, but in this event it would be exceedingly valuable to know the distribution of age classes in the herd.

Table 23 shows the percentage of mature and immature animals for the Allegheny National Forest by years based upon the data collected. Approximately 75 percent of the animals were immature and the other 25 percent were probably not over 6½ years old. During the whole 5-year period, 20 animals or less were found that could be classed in the 9½-year-plus age class. There is, as would be expected, a substantial increase in the number of mature deer following an antlerless deer hunting season. When a high percentage of the deer herd is immature animals, sportsmen cannot expect to bag many good heads. In analyzing the data by management units, it was learned that, generally, more mature deer were found in the areas having the smallest number of deer. This is attributed to the fact that the greater part of the hunting is done in the sections where the deer are most plentiful, thereby upsetting the normal progression of age groups.

TABLE 23. *Percentage of mature and immature animals by years*

Year	Mature	Immature		
	5½ years	2½ to 4½ years	1½ years	Total
	Percent	*Percent*	*Percent*	*Percent*
1936	17.4	72.3	10.3	82.6
1937	23.8	51.0	25.2	76.2
1939[1]	43.8	41.8	14.4	56.2
3 year average	28.0	55.0	17.0	72.0
1938 (without fawns)	47.8	41.8	10.4	52.2
1938 (fawns included)	27.8	24.3	6.2	[2] 72.2

[1] Female deer weighed and measured during the 1939 season were not included because the season on antlerless deer was not forest-wide.
[2] Includes 21.7 percent female and 20 percent male fawns.

DRESSED WEIGHT

The variation in weight of legal male and female deer, excepting fawns (table 24), is definitely influenced by the percentage of animals in the older age class. For example, the weight of legal male deer jumped from 116 pounds in 1937 to 121 pounds in 1939 following an anterless deer hunting season. The reason for this increase is readily explained by table 23, which shows that in 1939, nearly 44 percent of the kill was mature deer. This is practically double the percentage of mature deer for 1937. The same thing is true for the female deer for the years 1938 and 1939, the weights being 92 and 93 pounds; and the percentage of mature animals, 47.8 and 43.8, respectively, compared to the 1936 average weight of 87 pounds and 17.4 percent mature animals.

TABLE 24.—*Average (dressed) weight for legal deer, 1935-39*

Deer	Year				
	1935	1936	1937	1938	1939
	Pounds	*Pounds*	*Pounds*	*Pounds*	*Pounds*
Male	112	117	116		121
Female	87			92	93
Male fawns[1]	58			52	59
Female fawns[1]	56			50	54

[1] Up to 9 months.

A comparison of weight by years, as shown in tables 16 and 22, indicates that annual variation in weight of female deer in the same age class reflects food conditions. The *t* test (table 16) shows that the male weights are not significantly different by years. This is probably because all weights were taken during the hunting season, which follows rut. However, the female weights are very significant, the weight of females being therefore a better index of environmental conditions than the weight of males when weights are taken following the rutting season. In table 22, the weights for female deer in all four age classes are consistently higher in 1939 than in 1938. The 1939 season (spring,

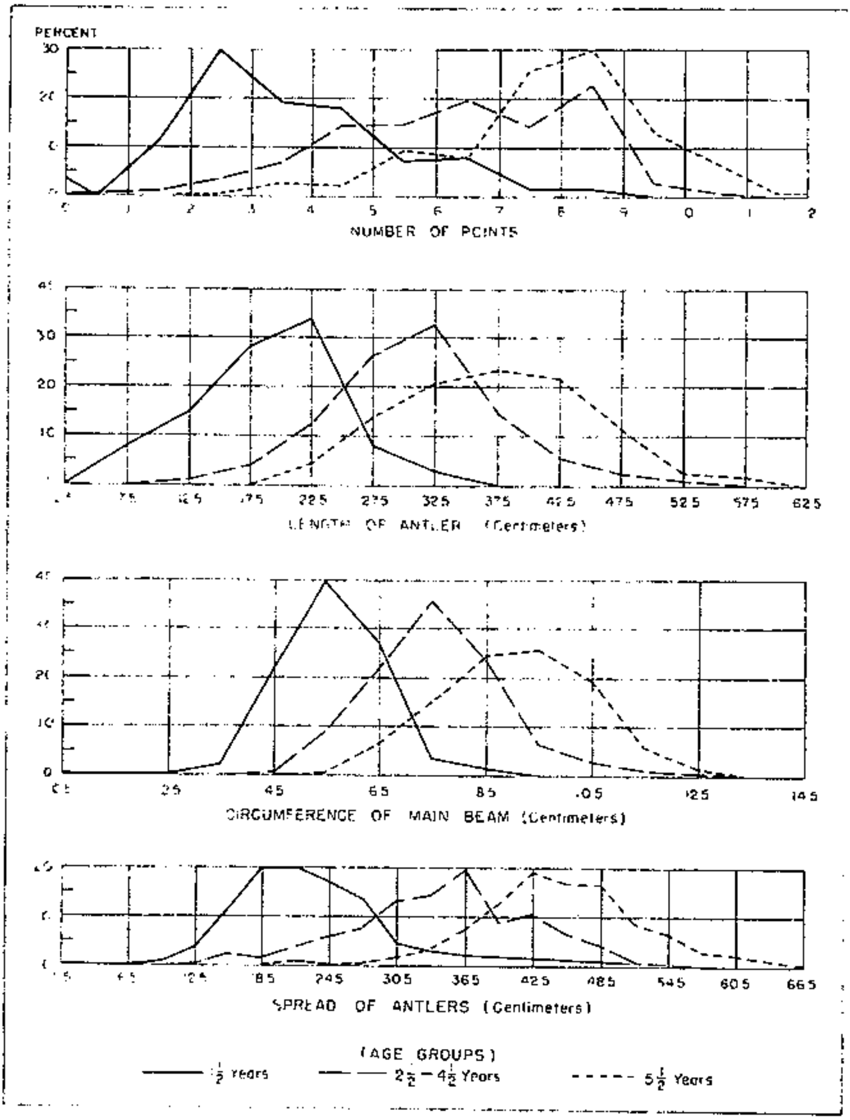

FIGURE 19. Frequency curves of antler measurements of the sample male deer within age groups for 1936, 1937, and 1939.

summer, and fall) was an excellent food-crop year, and 1938 was a poor food-crop year.

There is some correlation between weight and acres per deer as was indicated in the report based on 3 years' work (*9*); but the analysis of 5 years' data shows that this is influenced more by age than by food conditions. (See *F* test in table 19 and mean value in table 20.)

It is very evident from the findings presented herein that average weight alone is not a true indicator of the physical condition of a deer herd. However, with the addition of age and sex, it provides a very good indicator of physical condition of the herd.

On the basis of the statistical analysis in the section on statistical analysis of data, pp. 31 to 47, dressed weight is one of the best single body measurements for discriminating age, although the difference in precision between any two of the measurements, except length of tail and number of points, is too small to be consequential, insofar as its practical application to deer-herd management is concerned. Weighing a large number of deer carcasses is a rather costly undertaking, and uniform or consistent results are usually difficult to obtain because of the variation in the way carcasses are dressed out. For example, on the Allegheny Forest some hunters left in the heart, lungs, and liver, while others took them out, and still others took the liver and left the heart and lungs. Unless each carcass is examined and the proper allowance is made, the result will not be very accurate.

It has been thought by some that younger animals fatten more readily than older animals, but the average gain in weight of the female deer during a good food year over a poor food year (1938-39, table 22) was 5 pounds for 9-month old deer; 3 pounds for 1½-year old deer; 4 pounds for 2½- to 4½-year old deer; and 4 pounds for the 5½-year and older deer. There is, therefore, apparently little difference in the fattening ability of mature and immature animals. The weight of fawns born following hard winters was considerably less than those born following normal winters.

The difference in the behavior of weight magnitudes in males and females is interesting. The females seem to attain maturity in weight at an earlier age than do the males. The distinction between the two upper-age groups is not so marked as in the males.

LENGTH OF BODY

This character was not the best statistical indicator of age, even though zoologically it is a very stable measurement because it is not influenced by temporary changes in the physical condition of the herd. In the field work, however, it was found that the position of the head and neck affected the total-length measurement materially. For this reason, it is not the best measurement to select for age determinations. The extreme variability in length of body may be partially attributed to the difficulty in taking the measurement.

LENGTH OF HIND FOOT

The analysis showed this character to be the most independent measurement of all; and since all characters are influenced by age and the association between characters is considerable, some consideration should be given to the character having the greatest independent variation, especially if a single measurement is used as a basis for age determination. Zoologically, the hind-foot measurement is as stable as the length of body and it is an easy measurement to take. It can be taken readily by any one, with a high degree of consistent accuracy, the chance for error being extremely slight. It is not affected by the condition of the carcass and could be taken accurately even on decayed carcasses, and after the body had disintegrated considerably.

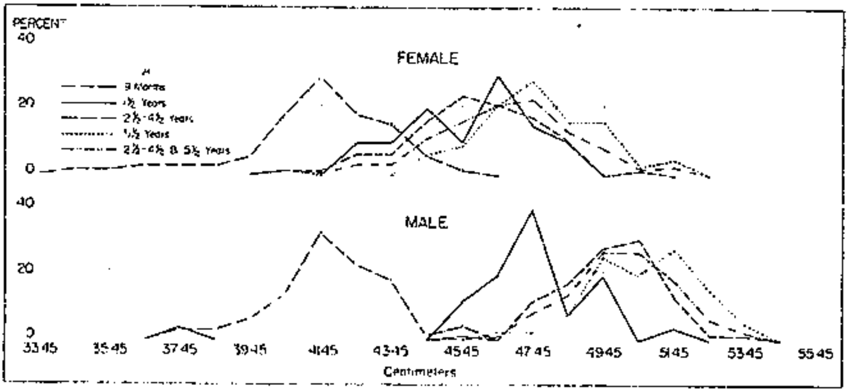

FIGURE 20.—Frequency curve of the length of the hind-foot measurements of the sample male deer for 1937 and 1938, and female deer for 1938, by age groups.

LENGTH OF TAIL

This character is similar to the number of points. It was such a poor indicator of age that it was not included in the final computations.

ANTLER MEASUREMENTS

The computations for this group of deer indicated that all antler characters, except the number of points, were good indicators of age. Statistically, the three measurements, circumference of main beam, length of antler, and spread of antler, were of almost equal value, length of antler being the best single measurement. Zoologically, all antler characters are very sensitive to the physical condition of the animal (*2*). A study of the statistical analysis by years shows a great deal of variation in antler measurements. For example, the 1939 antler measurements were consistently smaller than those of previous years for the 1½-year old class. In the other two age groups, the variation was very erratic, not always occurring in the same direction (table 22). The measurements for the entire 5-year period averaged up very well. This may or may not have been a coincidence, but it is very evident from the results of this work that all antler characters are sensitive to changes in the physical vigor of the individual animal and the herd.

PROPOSED METHOD FOR AGE DETERMINATION

To sportsmen, the trophy (antler) is all important, but the game manager is interested in finding the most practicable and reliable guide to the condition of the herd so that he can manage it to produce annually a relatively high percentage of good trophies. He is interested in obtaining the essential information about a deer herd with a minimum expenditure of time and money. By using the procedure outlined herein, this becomes possible as the following can be ascertained:

1. Percentage of mature and immature animals (deer in each age class).
2. Physical vigor of animals.
3. Sex ratio of mature and immature animals.
4. Distribution of hunters and kill.
5. Annual increase.
6. Percentage of adult does producing fawns. This can be determined by obtaining number of lactating females as the mammary glands of animals that have suckled young during the current year contain milk as late as December. The size of the glands is also a good indication.
7. Percentage of decrease for each sex and age class through hunting effort.

Since the collection of data is usually limited to deer bagged during open season, the type of season limits the extent and value of the data collected. For example, on the Allegheny National Forest the sex ratio of mature animals could not be obtained very accurately from the sample weighed and measured during the 5-year period covered by this study, because the seasons for hunting antlerless deer were shorter than seasons for legal male deer. It is possible to obtain all of the information (items 1 to 7), provided the open hunting season is for both sexes and all ages during the same period, or the equivalent of equal hunting pressure for both sexes and all ages.

The proposed procedure is made up of two distinct phases, both of which involve sampling of the herd under management. The first one is the determination of the range of the measurement for each age group and the second is the use of these standards in classifying deer. Both must be based on adequate sampling. In the first phase, in addition to the selected measurement and sex, the age of each deer must be precisely determined, although afterward this is omitted.

A. *The several steps for determining the age group ranges the first year are as follows:*

Step 1.—Obtain sex, age, and one measurement on an adequate sample of the deer herd.

Step 2.—Test the adequacy of the data for this method by preparing frequency distributions and drawing the resulting probability curves. The procedure for this has already been described, pp. 21–22.

Step 3.—Compute the mean (average) of the selected measurement for each of the age classes within each sex. This will be the sum of the measurements divided by the number in the group.

Step 4.—Set up the range for each age group. This will be based on one-half the difference between the means of two successive age groups. However, it may be best to shift the theoretical ranges somewhat to put them in line with the values read from the graphs.

Step 5.—Following the technique outlined, pp. 43 to 47, the probability of misclassifying a deer on the basis of these ranges may be exactly determined. These should be computed for each group.

B. *Use of age-group limits or ranges to determine age distribution of deer herd in future years.*

Step 1.—Obtain sex and the one measurement on sample from the deer herd under management.

Step 2.—After the data have been sorted by sex, arrange the measurements in order of magnitude and divide at those points determined under A for limits of the age groups.

Step 3.—Make a count of deer within each segment for the age distribution of deer in the sample. This will be an estimate of the proportionate distribution of the deer population by ages.

Theoretically such an estimate is at best very rough, as will be seen in a study of the example which follows.

For the first phase of the method it is essential that the sample for each age group is representative of the deer of that age. For subsequent samples it is also important that the sampling method, in addition to this, insures an equally likely opportunity for all deer of whatever age to be selected, depending on their frequency of representation. The sample must be a cross section of the herd by age distribution as well as within the age group.

One outstanding limitation affecting the accuracy of this method is that the greater the diversity in number within the age group, the less precise are the results. If all age groups are equally represented then the estimates will be fairly good; but where two successive age groups differ widely, the number in the larger group will be underestimated unless the age group in the other direction is equally larger. This fact should be taken into account in evaluating the results.

C. *Illustration of procedure.*

The successive steps in the procedure just outlined, using the length of hind foot measurement for both male and female deer, may be illustrated. The data for the earlier years, 1937 and 1938 for males, and 1938 for females, will be the basis for section A, the setting up of age-group ranges; and the 1939 data will be used for section B, the application of these ranges to estimate age distribution. Fortunately, the age distribution of the 1939 sample is known so the precision of this method can be tested. Referring to A, the first part of the proposed method, and considering the data for male deer:

Step 1.—The 1937 and 1938 (9 months only) samples were combined—as the basis for determining age-group limits for male deer—with the number in each group in the order of increasing age, or 69, 27, 72, and 42, these being the only male deer of which all measurements were made in those years.

Step 2.—Arranging the deer within each age group in the order of magnitude of the length of hind foot, actual counts are made for each class interval shown in table 13. The percent each count is of the total is then computed. These frequency distributions are shown graphically in figure 18. The midpoints of the class intervals were plotted along the base line, the percents corresponding to these midpoints on the vertical axis. Since age groups III and IV show little difference, their combined graph is also given

Step 3.—The mean for each age group was next computed. They were:

Group	Mean (centimeters)
I	41.7
II	47.1
III	49.5
IV	50.4

Step 4.—The computed upper limit for group I will be $41.7+\frac{47.1-41.7}{2}=44.4$; for group II, $47.1+\frac{49.5-47.1}{2}=48.3$; for group III, $49.5+\frac{50.4-49.5}{2}=49.9$.

Adjusting these figures to be consistent with the graph and frequency distribution table, will yield the estimate of the ranges:

Group	*Range (centimeters)*
I	below 44.9
II	45.0–47.9
III	48.0–50.9
IV	51.0 and up

It is very evident that the two upper groups cannot be segregated satisfactorily on the basis of the hind-foot measurement. The standard deviations, had they been computed as suggested earlier, would have likewise indicated this.

Combining the two age groups in question and computing new means and ranges, proceeding as before, yields:

Group	*Mean (centimeters)*	*Range (centimeters)*
I	41.7	below 44.9
II	47.1	45.0–47.9
III and IV	49.9	48.0 and up

Step 5.—How well the established ranges divide the original data into age groups may be tested by a standard procedure which will be outlined under D. Also since the age information is available on the 1939 data, which will be ranked on the basis of these limits, the test will be made later on how well the predicted coincides with the actual.

D. *Use of the age ranges determined under A to estimate age distribution of 1939 deer.*

Step 1.—Length of hind-foot measurements on the 292 male deer in the 1939 sample.

Step 2.—Arrange data in order of magnitude and make a count within the ranges set up under A for the four groups.

	I	II	III	IV	Total
Predicted	32	46	128	86	292
Observed	36	37	107	112	292

How closely the predicted is in agreement with the observed is judged by the chi-square (χ^2) test, where $\chi^2=\frac{(\text{Predicted}-\text{Observed})^2}{\text{Observed}}$

This is computed for each group and summed. It is clear that the more closely the predicted agrees with the observed, the smaller will χ^2 be. Entering a χ^2-table with n one less than the number of groups, $n=3$ for four groups and $n=2$ for three groups, the probability P corresponding to our computed χ^2 is read (*4, Table IV*). This will be the chance that so large a χ^2 may be due to random sampling. A small χ^2 is associated with a large probability, indicating a good fit. Any probability less than 0.10 (1 chance in10) will indicate a poor fit.

Now testing the agreement of the predicted distribution for the four age groups with the observed distribution,

$$\chi^2=\frac{(36-32)^2}{36}+\frac{(46-37)^2}{37}+\frac{(128-107)^2}{107}+\frac{(86-112)^2}{112}$$

$$\chi^2=0.444+2.189+4.121+6.036$$

$$\chi^2=12.789$$

For $n=3$, a χ^2 this large is associated with a probability less than 1 in 100, this is $P<0.01$. This very definitely indicates a lack of agreement. In other words, the proposed plan did not properly classify the deer into four age groups.

It will be noted that the greater part of the discrepancy was in the two upper age groups, these two χ^2's adding to more than 10. Combining these two groups a new χ^2 can be computed. The first two groups remain as before, although this is not necessarily true for age-class II. Predicted are 32, 46, and 214. Observed are 36, 37, and 219. $\chi^2=0.444+2.189+0.114=2.747$. Now $n=2$ and P lies between 0.30 and 0.20, which indicates a satisfactory agreement of predicted with the observed distribution of deer by ages.

E. *For the female deer under A:*

Step 1.—The 1938 sample of 265 deer for which age and hind-foot measurements were available consisted of 75 animals in age-class I, 21 in II, 84 in III, and 85 in IV.

Step 2.—In table 16 and figure 18 is shown the distribution of the animals on the basis of length of hind foot.

Step 3.—The means for the age groups are: I, 41.2 cm.; II, 45.4 cm.; III, 45.6 cm.; IV, 47.6 cm. Since groups II and III have practically the same means and the curves are very close together, the two are combined. The ranges will then be: I, below 43.5 cm.; II and III, 43.5 to 46.5 cm.; IV, 46.6 cm. and up. The predicted age distribution of the 1939 deer on this basis are 34, 51, and 75 as against observed values 36, 79, and 45. The χ^2 value is 30.104 and P less than 0.01. This indicates an extremely poor consistency of results.

Combining instead, the two upper-age groups (which scarcely seems reasonable on the basis of the curves alone, but which other information does justify) gives:

Group	*Mean (centimeters)*	*Range (centimeters)*
I	41.2	Below 44.0
II	45.5	44.0–45.9
III and IV	46.5	46.0 and up

The distribution of the 160 deer in 1939 will now be 34, 30, and 95 against 36, 33, and 91. Now $\chi^2=0.560$ and P lies between 0.80 and 0.70, sufficiently large for consistency.

This procedure was likewise applied to the dressed weight, length of body, and length of antler (males). An appraisal of the results for males follows:

1. Length of hind foot is reasonably satisfactory with two upper-age classes combined; P lies between 0.30 and 0.20.
2. Dressed weight was reasonably satisfactory with four age groups; P lies between 0.20 and 0.30. Combining the two upper-age groups gave a bimodal curve, which is not satisfactory.
3. Length of body is unsatisfactory.
4. Length of antler is reasonably satisfactory, as P lies between 0.30 and 0.20.

An appraisal of the results with females shows:

1. Length of hind foot is satisfactory when 2½–4½-year and 5½-year-plus are combined, for P lies between 0.80 and 0.70.

2. Dressed weight is satisfactory when 2½–4½-year and 5½-year-plus are combined, for P lies between 0.70 and 0.50.

3. Length of body is unsatisfactory.

Hence, for males there seems little choice among dressed weight, length of hind foot, and length of antler except that length of hind foot does not differ in the two upper-age groups; for females, either dressed weight or length of hind foot may be used, but only for discriminating three age groups—9 months, 1½ years, and 2½ years and over.

Because of the speed and accuracy with which length of hind foot may be measured, it is recommended as the most satisfactory to use. It is highly desirable to have as large a sample as possible, 250 to 300 deer seeming to be acceptable as a minimum. It is not as essential that the initial sample be large as it is for the subsequent ones, provided, of course, it is truly representative within each age group. Neither does the size of the age groups therein need to be in exact proportion to their representation in the population from which the sample is drawn. The reverse must be true for the later samples. There are certain pertinent facts and limitations in the use of this method which should be pointed out and emphasized. Statistically, at best, it would be classified as a rough approximation, but from the standpoint of practicable deer-herd management, it exceeds, or equals, in accuracy any known practicable method of obtaining necessary information on deer herds. The accuracy of the method is, however, materially limited by the adequacy of the sampling both as to size and representativeness. This is true both for the initial sample from which the age ranges are determined and subsequent samples to which the ranges are applied. It is very essential that such subsequent sampling provide distributions of age groups comparable to those in the population. The more nearly the age groups become the equal in size, the more precise the estimate. Disparity in age sizes will, in general, be underestimated.

A great deal of care needs to be taken to insure accuracy in measurements in the last figure recorded.

The importance of a thorough analysis of the initial sample used in setting up the age-group limits cannot be too strongly emphasized. Frequency distributions of measurements within each age class should be made, care being taken to choose the class intervals so that the most of the numbers will tend to cluster around the midpoints of these intervals. These distributions should be plotted as in fig. 18. Standard deviations should be computed and the significance of the differences in means of successive age groups tested. Likewise, the χ^2 test, as illustrated herein, should be made to determine how well the estimated limits will segregate the original sample into age groups. The results from such an analysis will serve as a basis for judging whether or not it will be worth while to take the subsequent years' samples where only sex and single measurement are recorded. It would be a waste of funds to continue further after the initial sample has been made, unless it has shown iteslf an effective tool for segregating deer into age groups. Hence, the necessity for its early careful evaluation.

CONCLUSIONS

During five consecutive hunting seasons (1935 to 1939 inclusive) on the Allegheny National Forest in Pennsylvania, 1,787 deer were measured. The measurements taken included dressed weight, length of body, length of tail, length of hind foot, circumferences of main beam, length of antler, spread of antlers, and number of points.

Provided the sample is representative, deer weights and measurements data can be a reasonably reliable basis for the determination of annual increase, percentage of mature and immature animals (age classes), sex ratio of mature and immature animals, percentage of adult does producing fawns, distribution of hunters and kill, and percentage of decrease for each sex and age class through hunting effort.

The accuracy of such information is dependent upon the adequacy of the sample obtained. An adequate sample should include a good cross section of the herd, including both sexes and all age classes, taken under equal hunting pressures, preferably during seasons open to the taking of both sexes and all ages.

Deer weights and measurements data without age have limited value as a basis for determining facts essential to good herd management. Age for data previously collected can be determined by following the method outlined herein.

There is a great deal of individual variation in white-tailed deer but the measurements taken—except length of tail and the number of points—increased with age for the four age classes considered.

For all practicable purposes of good deer management, a method has been proposed for determining age which would be a sufficiently accurate indicator of age for animals from a particular locality or herd for an indefinite period—based on any one of the body measurements, except length of tail and dressed weight, or until such time as some change occurred which affected the physical condition of the herd if based on dressed weight, or on any one of the antler measurements except number of points. Dressed weight and antler measurements are sensitive characters, which reflect to a considerable degree, the physical condition of the herd and therefore, are not as stable as body measurements for determining age. Weight and antler measurements are good indicators of the physical condition of a deer herd.

The weight of female deer is a better indicator of physical condition than the weight of male deer when weights are taken after the rutting season.

Average dressed weight without age and sex is not a true indicator of physical condition.

Actual genetic herd deterioration (body measurements), so far as could be ascertained from this study, does not occur before natural balances reduce the herd to normalcy.

Of the measurements affording reliable indices of age, length of hind foot is preferred, because it is the one that can be taken with the greatest accuracy by anyone.

By application of the procedure outlined herein, necessary information for good deer-herd management can be obtained without carrying on exhaustive and expensive deer weighing and measuring projects.

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