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# Optimal Management of a Fishery with Bycatch 

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## Optimal Management of a Fishery with Bycatch


#### Abstract

Bycatch is often a concern in sustainable fisheries management due to its contribution to overfishing problems. This paper examines bycatch in a multi-sector fishery in which the gear of one sector is imperfectly selective while in the other it is perfectly selective. In the model, the two stocks are biologically independent so the fisheries are only linked through the nonselective harvest externality. We expand on prior work that addresses bycatch by examining this problem and its solution in a dynamic context. Using an application to a commercial-recreational fishery, we show that the two-sector harvest policy that maximizes social welfare may be cyclical rather than steady-state.


Keywords: Bioeconomics, fisheries, bycatch, multispecies fishery, nonselective harvests

## 1. Introduction

Bycatch is often a concern in sustainable fisheries management due to its contribution to overfishing problems. Bycatch occurs when fishing gear is nonselective, so that both targeted and non-targeted species are caught from the same effort. It is not uncommon for bycatch to be discarded, often with a substantial mortality rate, and it is estimated that over one-quarter of all globally harvested weight is lost in this manner (Alverson et al., 1994). Indeed, in some fisheries more biomass is discarded than kept (Harrington et al., 2005). The contribution of bycatch to overfishing is severe enough that it is considered one of the greatest threats to marine species (Kappel, 2005).

The global extent of the bycatch problem has attracted a substantial amount of economic research (Pascoe et al., 2010). Much of this work is motivated by problems in which the stock subjected to bycatch is itself valuable and targeted commercially (e.g. Anderson, 1994; Boyce; 1996; Herrera, 2005) or caught recreationally (e.g. Ward, 1994) in multi-sector fisheries, or even enjoyed through existence values (e.g. Hoagland and Jin, 1997). In general, managing fisheries with bycatch using traditional harvest quota policies will be inefficient (Androkovich and Stollery, 1994; Abbott and Wilen, 2009; Holland, 2010).

The existing literature on multi-sector fisheries experiencing bycatch offers insights into the use of various regulatory instruments, including total allowable catches (TACs), individual quotas, taxes and bycatch reduction devices. However, only a few of these papers have examined these mechanisms using dynamic bioeconomic models (Ward, 1994; Androkovich and Stollery, 1994; Herrera, 2005; Skonhoft et al., 2012). Otherwise, there is presently little research that accounts for the role that stock dynamics play in managing a fishery with bycatch. Prior work also assumes or suggests that the optimal equilibrium management regime will be a steady
state-that is, constant through time (e.g. Ward, 1994; Androkovich and Stollery, 1994; Reithe, 2006). ${ }^{1}$ Outside the context of multi-sector bycatch problems, though, it is widely recognized that optimal fisheries management policies can be cyclical rather than steady-state, particularly for multispecies systems (Fenichel et al., 2010) and when harvesting is nonselective (Tahvonen, 2009).

This paper presents a bioeconomic model of bycatch in a joint commercial-recreational fishery. That is, it analyzes a multispecies system in which the commercial and recreational sectors harvest from two biologically independent fish stocks. The commercial industry's effort is nonselective and produces bycatch while the recreational harvest is selective. Managers use a quota system to regulate effort in the commercial and recreational sectors in order to maximize the net present value of the fisheries.

Our analysis offers several contributions to the economics of multispecies harvesting generally and bycatch management in particular. First, the solution to the dynamic multi-sector harvesting problem is derived analytically. Second, we build on the existing bycatch literature by numerically examining the transition path to the long-run equilibrium. Third, we find that the optimal equilibrium harvesting policy is cyclical, transitioning between states of intensive and relaxed exploitation on the parts of the two harvesting sectors. We relate this result to prior research on the role of physical and biological capital in renewable resource extraction problems (Wirl, 1992; Liski et al., 2001; Horan et al., 2008). During the cycle we find a corner solution may be reached, which could involve temporarily shutting down one harvesting sector. This characteristic is consistent with a pattern of pulse fishing although it has not yet been identified in a model of separate harvesting sectors exploiting a multispecies system.

[^0]Our investigation is motivated by a bycatch problem observed in the Great Lakes (Johnson et al., 2004a). Although smaller in scope-boat crews tend to include between one and three people-freshwater fisheries, like marine fisheries, are known to use nonselective gear, such as a gill or trap nets (Raby et al., 2011). In the Great Lakes, these methods are used by commercial fishers to harvest lake whitefish (Coregonus clupeaformis) but result in the incidental catch of lake trout (Salvelinus namaycush), a species targeted by an active and valuable recreational fishing sector. Thus, the nonselective harvesting gear of commercial fishers imposes a potentially significant technological externality on recreational anglers.

## 2. Model

We develop a model of harvesting from a two-species system. The following assumptions are consistent with numerous multispecies fishery problems although they are adopted specifically to fit the motivating case study. Ecologically, the biomass of these species, denoted $s$ (lake trout) and $w$ (lake whitefish), do not interact. The species $s$ is susceptible to bycatch. There are two harvesting sectors: one is commercial and the other is recreational. The commercial sector uses a non-targeted technology to harvest $w$ (such as with a gill or seine net) whereas the recreational sector uses a targeted technology to harvest $s$ (a fishing line). We denote effort in the recreational fishery as $a$ and effort in the commercial fishery as $b$. Fishery dynamics take the form
(1) $\dot{s}=r_{s}\left(1-s / k_{s}\right) s-z s a-q_{s} s b$
(2) $\dot{w}=r_{w}\left(1-w / k_{w}\right) w-q_{w} w b$,
where $r_{i}$ is the intrinsic growth rate and $k_{i}$ is the carrying capacity for stock $i=s, w, q_{i}$ is the catchability coefficient of stock $i=s, w$ for commercial fishing and $z$ is the catchability coefficient of stock $s$ for recreational fishing. The first term in each of eq. (1) and eq. (2) is the
total growth in stock biomass prior to any harvesting mortality. The remaining terms in eq. (1) and eq. (2) measure recreational and commercial harvests.

The commercial sector generates profits

$$
\begin{equation*}
\Pi=\left(p_{w} q_{w} w+p_{s} q_{s} s\right) b-c_{b} b \tag{3}
\end{equation*}
$$

where $p_{i}$ is the dockside or wholesale price of harvests from $i=s, w$ and $c_{b}$ is the marginal cost of effort. The term in parentheses in eq. (3) is total revenue from a given unit of effort, although in general the revenue from stock $s$ is expected to be minimal; $p_{s}$ is frequently zero in fisheries with bycatch.

The effort from recreational fishing generates social benefits $U=u(a, s)$, where $u$ is increasing in $a$ and $s$. Following the standard approach in the recreation demand literature we assume $u(\cdot)$ is linear in arguments; note that this assumption is also convenient because it yields a net benefits function analogous to commercial profits and thus our results extend to the situation of a joint commercial-commercial, two-species fishery. Specifically, net benefits from recreational fishing are
(4) $V=u z s a-c_{a} a$,
where $c_{a}$ is the marginal cost of effort.

## 3. Optimal Fisheries Management

We now turn to the problem of optimal fisheries management. Deriving the solution for multispecies systems can be difficult (Clark, 1976), so to facilitate an answer we continue to treat each sector as a homogenous group of participants and assume that management is costless and capital is perfectly malleable (Clark et al., 1979). The fisheries management agency's problem is
to maximize the discounted net benefits from the commercial and recreational sectors. Assuming a discount rate of $\rho$, this is stated as

$$
\begin{align*}
& \max _{a, b} \int_{0}^{\infty}\left[\left(p_{w} q_{w} w+p_{s} q_{s} s-c_{b}\right) b+\left(u z s-c_{a}\right) a\right]^{-\rho t}  \tag{8}\\
& \text { subject to(1), (2), } 0 \leq a \leq a_{\max }, 0 \leq b \leq b_{\max }, s(0)=s_{0}, w(0)=w_{0}
\end{align*}
$$

We use the maximum principle to solve this problem (Clark, 1976), where the Hamiltonian expression is used to derive the necessary conditions for the optimal controls $a, b$. The Hamiltonian for (8) is

$$
\begin{equation*}
H=\left(u z s-c_{d}\right) a+\left(p_{w} q_{w} w+p_{s} q_{s} s-c_{b}\right) b+\lambda_{s} \dot{s}+\lambda_{w} \dot{w}, \tag{9}
\end{equation*}
$$

where $\lambda_{i}$ is the adjoint variable or shadow value for stock $i=s, w$. This is a linear control problem, so the optimal choice of $a$ and $b$ is determined by a pair of switching functions,
(10) $\frac{\partial H}{\partial a}=u z s-c_{a}-\lambda_{s} z s=\sigma_{a}$,

$$
\begin{equation*}
\frac{\partial H}{\partial b}=p_{w} q_{w} w+p_{s} q_{s} s-c_{b}-\lambda_{s} q_{s} s-\lambda_{w} q_{w} w=\sigma_{b} . \tag{11}
\end{equation*}
$$

The right-hand-side (RHS) of eq. (10) is the marginal net benefits from recreational fishing effort. When $\sigma_{a}>0$ recreational effort should be set to its maximum limit and when $\sigma_{a}<0$ recreational effort should be set to zero (or, alternatively, some minimum feasible amount). When $\sigma_{a}=0, a$ should be set at its singular value, $a_{S V}$, which is determined by the combinations of $s$ and $w$ that satisfy $\sigma_{a}=0$. Similarly, the RHS of eq. (11) is the marginal net benefits from commercial effort. When $\sigma_{b}>0$ commercial effort should be set to its maximum limit and when $\sigma_{b}<0$ commercial effort should be set to zero. When $\sigma_{b}$ vanishes, $b$ should be set at its singular value $b_{S V}$, which is determined by the combinations of $s$ and $w$ that satisfy $\sigma_{b}=0$.

Two more equations complete the set of conditions required for a solution. These are the adjoint equations, $\dot{\lambda}_{i}=\rho \lambda_{i}-(\partial H / \partial i)$ for $i=s, w$, which can be re-expressed as a pair of goldenrule conditions. For stock $s$, the golden rule condition is

$$
\begin{equation*}
\rho=\frac{\dot{\lambda}_{s}}{\lambda_{s}}+\frac{u z a}{\lambda_{s}}+\frac{p_{s} q_{s} b}{\lambda_{s}}+\left[r_{s}\left(1-2 s / k_{s}\right)-z a-q_{s} b\right] \tag{12}
\end{equation*}
$$

where the opportunity cost of investing resources elsewhere in the economy, i.e. $\rho$, is equated with the rate of return from managing $s$. The first three RHS terms, respectively, are the capital gain or loss from changes in $s$, the marginal recreational value of $s$ harvests and the marginal commercial value of $s$ bycatch, all normalized by the shadow value of $s, \lambda_{s}$. The second and third terms are positive and imply that an increase in the value of harvesting $s$ raises the return from $s$. The final three terms, in brackets, express the marginal growth in $s$. Note that whether the controls $a$ or $b$ raise or lower the rate of return from $s$ will be determined by the normalized prices $u / \lambda_{s}$ and $p_{s} / \lambda_{s}$. Greater recreational (commercial) effort will lower the return from $s$ when the recreational (commercial) value of $s$ is less than its shadow value. Eq. (10) can be rearranged to show that recreational effort should only be allowed when it raises the rate of return on $s$. However, there is no such guarantee for commercial effort and for values of $\lambda_{s}$ that lie between $u$ and $p_{s}$ harvests by the recreational industry will raise the rate of return on the stock but harvests by the commercial industry will lower it. Thus, it is unclear if the two harvest activities should be used as substitutes or compliments in optimally managing $s$.

The golden rule condition for $w$ is

$$
\begin{equation*}
\rho=\frac{\dot{\lambda}_{w}}{\lambda_{w}}+\frac{p_{w} q_{w} b}{\lambda_{w}}+\left[r_{w}\left(1-2 w / k_{w}\right)-q_{w} b\right] \tag{13}
\end{equation*}
$$

where the rate of return on investing is equated with the rate of return from managing $w$. The first two RHS terms, respectively, are the capital gain or loss from changes in $w$ and the marginal
commercial value of $w$ harvests, normalized by $\lambda_{w}$. The term in brackets is the marginal growth in $w$. Note that whether the control $b$ raises or lowers the rate of return from stock $w$ will be determined by the relative price $p_{w} / \lambda_{w}$. Eq. (13) says that an increase in commercial effort will raise the rate of return on $w$ when the price of the harvest exceeds the shadow value and vice versa. As suggested by eq. (11), commercial effort will tend to be applied when it raises the return from $w$.

The complete solution to problem (8) will generally consist of a series of corner and interior control values. To start, consider the choice of $a$. There are two corner solutions, $a=0$ and $a=a_{\max }$. To simplify the analysis, assume that $a_{\max }$ is large enough to never be binding. Thus, if recreational fishing is going to be optimal for an extended period of time it requires the switching function in eq. (10) to vanish, i.e. $\sigma_{a}=0$. This means that $\lambda_{s}$ must obey

$$
\begin{equation*}
\lambda_{s}(s)=\frac{u z s-c_{a}}{z s}, \tag{14}
\end{equation*}
$$

which implicitly defines the singular (i.e. interior) solution for $a$. Eq. (14) says the optimal amount of recreational fishing occurs when the shadow value of $s$ equals the net marginal benefits from recreational fishing normalized by the technical returns to effort. This means that if $a>0$ is optimal then $s$ must move in the same direction as its user cost.

An explicit optimal value for $a$ can be derived by using eqs (1), (12) and (14). First, take the time derivative of $\lambda_{s}(s)$ to get $\dot{\lambda}_{s}(s)=\left(c_{d} / z s\right) \dot{s} / s$. Second, substitute the expressions for $\lambda_{s}(s)$, $\dot{\lambda}_{s}(s)$ and $\dot{s}$ into eq. (12). We can then derive the following:

$$
\begin{equation*}
\Omega(s, b)=u r_{s}\left(1-s / k_{s}\right)-\left[\left(z u s-c_{a}\right) / z s\right]\left(\rho+r_{s} s / k_{s}\right)-\left(p_{s}-u\right) q_{s} b . \tag{15}
\end{equation*}
$$

The singular solution for $a$ implies $\Omega(s, b)=0$. A "partial singular" solution arises when $\Omega(s, 0)=$ 0 . In this latter case, we can use $\Omega(s, 0)=0$ to express $s$ as a function of the parameters. This
means that the partial singular solution for $a$, which we denote as $a_{P S}$, is a steady state equilibrium. To reach $a_{P S}$, the system should proceed along a most-rapid-approach-path (MRAP) using bang-bang controls-that is, using $a=0$ or $a=a_{\max }$.

A partial singular solution in $b$ can also arise, when $\sigma_{a}<0$ and $\sigma_{b}=0$. When the switching function for $b$ vanishes we can derive an expression for $\lambda_{w}$

$$
\begin{equation*}
\lambda_{w}\left(s, w, \lambda_{s}\right)=\frac{p_{w} q_{w} w+p_{s} q_{s} s-c_{b}-\lambda_{s} q_{s} s}{q_{w} w} . \tag{16}
\end{equation*}
$$

The first three terms in the numerator of eq. (16) measure the marginal profits of commercial fishing effort. The last term is the shadow value of $s$ weighted by the technical returns to commercial effort in the fishery for $s$ and is a value of the bycatch externality. We can solve for the partial singular solution for $b$ using eq. (16) along with eqs (1), (2), (12) and (13). First, take the time derivative of $\lambda_{w}\left(s, w, \lambda_{s}\right)$ to get $\dot{\lambda}_{w}\left(s, w, \lambda_{s}, a, b\right)$. Second, substitute these expressions into eq. (13) and fix $a=0$ to derive $\lambda_{s}(s, w)$. Third, take the time derivative of $\lambda_{s}(\cdot)$ to get $\dot{\lambda}_{s}(s, w, b)$. Fourth, substitute $\lambda_{s}(\cdot)$ and $\dot{\lambda}_{s}(\cdot)$ into eq. (12) and solve for $b$ to obtain $b_{P S}(s, w)$. The partial singular solution for $b$ is therefore a feedback rule that defines a number of potentially optimal paths in $s$ - $w$ space. Intuitively, it is optimal to use such a path because $b$ affects both states and achieving the right balance requires small, constant adjustments in the control.

An interior or "double singular" solution arises when eqs (14) and (16) are satisfied. In this case, we can use $\Omega(s, b)=0$ to derive a feedback rule for $b$

$$
\begin{equation*}
b_{D S}(s)=\frac{\left[\left(z u s-c_{a}\right) / z s\right]\left(\rho+r_{s} s / k_{s}\right)-u r_{s}\left(1-s / k_{s}\right)}{\left(p_{s}-u\right) q_{s}} . \tag{17}
\end{equation*}
$$

where the ${ }_{D S}$ signifies that the expression is only optimal in the double singular case. In eq. (17), the bracketed term in the RHS numerator is the shadow value of $s$, so the optimal choice of $b_{D S}(s)$ explicitly depends on the marginal value of the bycatch stock. In fact, $b_{D S}(s)$ depends only
on the value of $s$, rather than $w$ and $s$, implying that the choice of $b$ is determined by the conservation status of the bycatch stock, which is consistent with some historical management policies. ${ }^{2}$ Implicitly, however, $b_{D S}(s)$ depends on $w$ through changes in $s$ and therefore the double singular solution for $a$.

We derive the double singular solution for $a$ by rewriting eq. (16) as

$$
\begin{equation*}
\lambda_{w}(s, w)=\frac{p_{w} q_{w} w+p_{s} q_{s} s-c_{b}-\left[\left(z u s-c_{a}\right) / z s\right] q_{s} s}{q_{w} w} \tag{18}
\end{equation*}
$$

substituting $\lambda_{w}(s, w)$ and $\dot{\lambda}_{w}(s, w)$ into eq. (12) and rearranging:

$$
\begin{align*}
a_{D S}(s, w) & =\frac{r_{s}\left(1-s / k_{s}\right)}{z}-\frac{q_{s}}{z} \cdot b_{D S}(s)- \\
& \frac{q_{w} w}{q_{s} s} \cdot \frac{\left[\left(p_{w} q_{w} w+p_{s} q_{s} s-c_{a}-q_{s} s\left(u-c_{a} / z s\right)\right) / q_{w} w\right]\left(\rho+r_{w} w / k_{w}\right)+p_{w} r_{w}\left(1-w / k_{w}\right)}{\left(p_{s}-u\right) z} \tag{17}
\end{align*}
$$

The first term is a measure of the biotechnical productivity (BTP) of stock $s$ in the recreational fishing industry (Clark, 1976). This term suggests that recreational effort should increase with higher BTP. The second term accounts for the commercial sector and says that as commercial effort increases recreational effort should decrease, although the size of this response depends on the relative catchability of $s$ in the two industries. The final term accounts for changes in $w$ and says, among other things, that recreational effort should fall as the returns to commercial fishing rise.

The double singular solution therefore consists of a pair of feedback rules for the controls. This candidate optimal management strategy yields the system (1) and (2) with substitutions $b=b_{D S}(s)$ and $a=a_{D S}(s, w)$. The dynamics of the double singular solution are illustrated for several different systems in phase plane diagrams in Figure 1. The model is

[^1]parameterized using hypothetical values similar to those in our numerical example. The $\dot{s}=0$ isocline is the threshold for changes in $s: \dot{s}>0$ for combinations of $s$ and $w$ to the right of the isocline and $\dot{s}<0$ for combinations of $s$ and $w$ to the left of the isocline. A similar interpretation holds for the $\dot{w}=0$ isocline and $w: \dot{w}>0$ for combinations of $s$ and $w$ to the right of the isocline and $\dot{w}<0$ for combinations of $s$ and $w$ to the left of the isocline. A steady state forms at the intersection of these isoclines.

The phase planes in Figure 1 suggest that the optimal harvest policy will be sensitive to the severity of the bycatch problem and that only in certain cases could the double singular solution be long run optimal. Figure 1(a) assumes the catchability of $s$ for commercial fishing is one-eighth the catchability of $w$. The various paths defined by the double singular solution, marked by phase arrows in the figure, cycle around an unstable node. Figure 1(b) assumes the catchability of $s$ for commercial fishing is one-half the catchability of $w$, and in this case there is no equilibrium node. All paths in these two figures spiral toward extreme values, suggesting that the system will eventually crash if commercial and recreational fishing are pursued simultaneously in the long run. Figure 1(c) assumes the catchability of $s$ for commercial fishing is equivalent to the catchability of $w$. This system is characterized by a saddle point equilibrium that suggests one commercial-recreational fishing strategy may be long-run optimal: move the system to the saddle path and then proceed along this path to equilibrium (in fact, we find this is actually a suboptimal strategy in the sensitivity analysis to our numerical example).

The solution is determined by considering several different harvest strategies. In general, there are four such strategies: no fishing, recreational fishing-only, commercial fishing-only and simultaneous commercial-recreational fishing. We can rule out the first, but the other three are candidate long-run strategies that must be compared numerically. As Figure 1 suggests, the joint
commercial-recreational strategy alone is unlikely to be long-run feasible. However, it could be used in a solution that combines different strategies in a piece-wise fashion.

A switch from commercial-recreational fishing to an alternative strategy may naturally arise due to the nature of the feedback rules. In general, the double singular strategy cannot be pursued when a feedback rule yields a value outside the bounds of a control (e.g. $a<0$ ). Clark (1976), following Arrow (1964), refers to this as a blocked interval problem. We can formulate the set of $(w, s)$ that produce blocked intervals by examining the boundaries $a_{D S}(s, w)=0$ and $b_{D S}(s)=0 .{ }^{3}$ A blocked interval for commercial effort occurs at

$$
\begin{equation*}
\frac{\left[\left(z u s-c_{a}\right) / z s\right]\left(\rho+r_{s} s / k_{s}\right)-u r_{s}\left(1-s / k_{s}\right)}{\left(p_{s}-u\right) q_{s}}=0, \tag{18}
\end{equation*}
$$

which implicitly defines a boundary along $s, s_{T}$. Assuming $p_{s}<u$, if $s$ exceeds $s_{T}$ then operating both commercial and recreational fishing cannot be an optimal strategy. Similarly, we can identify a boundary for recreational effort, $w_{T}(s)$. When $w$ lies beyond $w_{T}(s)$ the commercial-recreational fishing strategy cannot be optimal.

## 4. Numerical example

We consider a numerical example to illustrate the characteristics of a solution. The parameters for the numerical example are listed in Table 1. These values are based on a bycatch problem in the Laurentian Great Lakes, although the example is better viewed as a guide to formulating a bycatch management strategy. The fisheries management agency can be thought of as a state, provincial or tribal resource authority. ${ }^{4}$ The agency is responsible for a region of a lake that

[^2]contains the stocks and spawning grounds for two species, lake trout and lake whitefish, which are important to Great Lakes anglers and several Great Lakes coastal communities (Taylor and Ferreri, 2002). Historically, both lake trout and lake whitefish were harvested commercially but due to a combination of intensive harvesting and invasive species the two fisheries collapsed in the mid- $20^{\text {th }}$ century. Commercial fishing for these species was strictly regulated thereafter, allowing lake whitefish to make an extensive recovery in the upper Great Lakes (Huron, Michigan and Superior). Lake whitefish is now the top commercial fish in the Great Lakes, both by weight and by value. Despite careful management, naturally-reproducing lake trout stocks remain small in Lakes Michigan and Huron, although the species is considered rehabilitated in Lake Superior (Taylor and Ferreri, 2002).

Management of lake trout and lake whitefish harvests by state, provincial and tribal resource agencies is significantly influenced by concerns over lake trout rehabilitation. The lakes are divided into management zones in which total harvest is regulated through TACs that are divided into individual quotas for commercial harvesters (Ebener et al., 2008). At present, commercial harvesters primarily target lake whitefish but harvesting gear is not perfectly selective and lake trout are the principal bycatch species from this harvest. ${ }^{5}$ Due to restrictive fisheries regulations nearly all commercial harvest of lake trout is bycatch. Nevertheless, targeted

[^3]recreational harvest of lake trout is permitted, with yields regulated through angler-day harvest quotas.

We begin by investigating the solution when the system is defined by the benchmark parameters in Table 1. The model was implemented in Mathematica 9.0. The initial stock levels are the open access equilibrium values (i.e. relatively low compared with the optimal equilibrium values) to gain insights into the dynamics of recovering depleted stocks. Following the benchmark problem, we consider several alternative parameter arrangements as a sensitivity analysis.

### 4.1. Benchmark scenario

The solution to the bioeconomic problem is found by comparing the three candidate harvest strategies. Based on our analytical analysis we can expect that: (1) the recreational fishing-only strategy will use a MRAP to reach equilibrium, (2) the commercial fishing-only strategy will proceed along a trajectory defined by its partial singular feedback rule and (3) the commercialrecreational fishing strategy is not likely to be long-run feasible by itself but, due to blocked intervals, it could form part of a piece-wise-defined solution.

First, consider commercial fishing-only ( $a=0, b=b_{P S}(s, w)$ ) as the long run solution. We find that the dynamics defined by this strategy yield a system that is characterized by a saddle point equilibrium. ${ }^{6}$ All paths not leading to the saddle point yield outcomes that imply the eventual eradication of lake whitefish or abandoning commercial fishing, so the only feasible strategy is to follow the saddle path to equilibrium. Movement to the saddle path as performed using bang-bang controls. Initially, the two stocks are low ( $s_{0}=166663, w_{0}=389009$ ) so it is optimal to cease all commercial harvests. Once the stocks have recovered to the point that the

[^4]system lies on a part of the saddle path commercial fishing is initiated in accordance with the feedback rule $b_{P S}(s, w)$. Thereafter, the system is guided to the equilibrium at $s^{*}=436106, w^{*}=$ 1139617. We find that this solution yields $\$ 5$ million in present value net benefits.

Next, consider the solution using the recreational fishing-only ( $a=a_{P S}, b=0$ ) strategy. The MRAP to equilibrium requires an initial moratorium on lake trout fishing until the stock reaches $s^{*}=324642$ and thereafter further stock recruitment is offset by recreational harvesting. Lake whitefish is unaffected by this harvest strategy and recovers to its carrying capacity. This solution yields $\$ 17$ million in present value net benefits.

Finally, we examine the solution that involves both commercial and recreational fishing ( $\left.a=a_{D S}(s, w), b=b_{D S}(s)\right)$. A diagram of the solution is presented in Figure 2. The boundaries defining the regions in which the feedback rules for the controls take on feasible values are marked by dashed lines. Within the shaded regions either $a_{D S}(s, w)<0$ or $b_{D S}(s)<0$; only when the system occupies a point in the unshaded region should commercial-recreational harvesting be pursued. An equilibrium is formed at the intersection of the zero isoclines in this unconstrained region. However, as demonstrated by the phase arrows, there is no trajectory that can move the system to this equilibrium.

The constrained regions and stability properties in Figure 2 suggest that a solution is formed by a cyclical combination of harvest strategies. Beginning at the open access equilibrium point, which lies in the constrained region for $a$, we find it is optimal to forgo any harvesting to allow the system to recover (trajectory A). ${ }^{7}$ Once the system reaches threshold $b$, the recreational

[^5]fishing-only strategy is used to balance the growth in the lake trout stock. ${ }^{8}$ The system then moves up the $b=0$ boundary (trajectory B) as the whitefish stock continues to increase. This completes the part of the solution that defines the recovery of the two stocks. The next phase involves a piece-wise continuous cycle combining the commercial-recreational fishing strategy (trajectory C) and the recreational fishing-only strategy. The double singular feedback rule should be applied once the recovered system crosses the zero isocline for $s$. Initially, the stock of whitefish improves as lake trout decline, leading to a rapid increase in commercial harvests but a modest change in recreational harvests (Figure 3). As the two stocks start to contract, the reduction in recreational harvests is tracked by commercial harvests, which peaked later owing to the longer whitefish recovery. Now with underharvesting, whitefish and lake trout begin to recover. Prior to the system reaching the $b=0$ boundary it is optimal to cease commercial fishing and increase recreational fishing due to the premature switching principle (Horan and Wolf, 2005). Eventually, the system reaches the $b=0$ boundary and recreational effort is maintained so that the lake trout stock is stabilized, until it is optimal to allow commercial fishing again and renew the piece-wise cycle just described. This solution yields $\$ 19$ million in present value net benefits, which makes the management program utilizing commercial-recreational fishing firstbest.

The use of a piece-wise cycle in the joint commercial-recreational harvesting equilibrium is somewhat surprising, due to the availability of a selective harvest technology. ${ }^{9}$ However, this result is explained by a combination of the harvest technologies and the nature of the two stocks as biological capital. Commercial fishing is imperfectly targeted and imposes significant costs on

[^6]recreational fishing. A manager ignorant of the externality in question could do better by cutting back on commercial effort. However, the solution is not quite as simple as correcting an externality because reductions in commercial harvesting act as an investment into the stock of whitefish. Temporarily, the manager should allow the commercial sector to overharvest (and to protect lake trout the recreational sector should underharvest). After heavily exploiting whitefish the manager should wind down and briefly pause commercial harvests to re-invest. Eventually, the growing opportunity costs of a small commercial sector exceed the benefits and whitefish harvests can continue. Ultimately, because the manager can never directly target whitefish and stock adjustment is slow, it is best to dynamically balance the returns of the two sectors.

The solution we identify is akin to that of several papers. Wirl (1992) also looks at a twostate problem that involves a nontargeted control and demonstrates the potential optimality of cyclical control policies. Liski et al. (2001) analyze increasing returns in the management of a single-stock fishery to show that cyclical harvest strategies can become optimal in the presence of harvest adjustment costs. Similar to Liski et al., we find cyclical harvesting is linked to adjustment costs, although in our case these costs are due to a technological externality and ecological feedbacks. Horan and Wolf (2005) and Horan et al. (2008) examine wildlife management problems that involve interacting populations and feeding, a non-targeted control. Both Horan and Wolf and Horan et al. find that the nature of feeding makes it difficult to control the system, making cyclical feeding and harvesting generally optimal. Similarly, we find that the optimal equilibrium strategy must be cyclical if lake whitefish harvests, which are non-targeted, and lake trout harvests are to be used simultaneously.

The results of this section are intended as a guide but the model can be adjusted to suit some practical matters we have thus far ignored. For example, the solution in the numerical
example suggests that management of bycatch in multi-sector fisheries may include periods of intense effort followed by moderate effort and moratoria. In practice it may be impractical to shut down a fishery, but the model can be amended to allow some minimum amount of commercial or recreational activity above zero. This would not affect the general form of the solution, although it would slow the time it takes for a fishery to recover to the desired level.

### 4.2. Sensitivity analysis

We examine the solutions to several alterative parameter arrangements to test the robustness of the cyclical equilibrium strategy. In the first scenario, the intrinsic growth rate of lake trout is halved. This adjustment is of interest because lake trout stocks have been slow to recover historically. In this case, because lake trout have no commercial value the present value net benefits of the optimal commercial fishing-only strategy remain unchanged from its benchmark counterpart at $\$ 5$ million. The recreational fishing-only strategy is now worth $\$ 8$ million, about half as much compared with the benchmark. Although it remains first-best, the solution utilizing both commercial and recreational fishing is also worth less, about $\$ 9$ million. This solution is illustrated in Figure 4, which shows that the initial recovery trajectory is much longer (trajectory A) compared with the benchmark. Furthermore, although a cyclical equilibrium strategy remains optimal, the piece-wise cycle now involves a commercial fishing-only portion (trajectory D). This adjustment from the benchmark case is intuitive: with a lower lake trout growth rate it becomes optimal to divert effort away from recreational fishing and into commercial fishing.

In the second scenario, the price of lake whitefish and the bycatch rate are both doubled relative to the benchmark. This would represent a situation in which a commercial sector with considerable economic value imposes a severe technological externality on the other harvesting
sector. Pursuing the commercial fishing-only strategy leads to smaller equilibrium stock levels $\left(s^{*}=355882, w^{*}=1029661\right)$ but a considerably higher return, with present value net benefits of $\$ 14$ million, relative to the benchmark. The new parameters do not affect the optimal recreational fishing-only strategy, which yields $\$ 17$ million. The optimal joint commercial-recreational fishing plan, which is illustrated in Figure 5, is worth $\$ 22$ million. Qualitatively, the solution to the joint problem here is similar to the benchmark, although the piece-wise cycle in equilibrium is substantially larger and takes about twice as long to complete.

Further increases in the bycatch parameter will qualitatively alter the solution. We find that if this parameter were quadrupled relative to the benchmark case that a saddle point equilibrium would characterize the solution to the joint commercial-recreational fishing problem. However, this equilibrium lies in the region where $a=0$, which suggests that moving to the steady state (i.e. along a saddle path) is not optimal. Any feasibly optimal piece-wise cycle must involve the commercial fishing-only strategy, but pursuing this strategy in the $a=0$ region leads to eradication of both fish stocks, which cannot be optimal. This suggests that the joint commercial-recreational fishing strategy is suboptimal, in favor of either the commercial fishingonly or recreational-fishing only strategy. In this case, the latter yields higher present value net benefits.

### 4.3. Gear modification cost-benefit analysis

The present value calculations from the model can be used to judge the efficiency of policies that would use gear modification to increase harvest selectivity and reduce bycatch in local fisheries. Substantial reductions in the amount of global bycatch could be achieved through gear modification, and in some fisheries gear modification can result in nearly the complete
elimination of bycatch (Hall and Mainprize, 2005; Johnson et al., 2004a). Gear modification remains an attractive regulatory option for fisheries management agencies, in part because the use of taxes and transferable quotas can be controversial in practice (McCay, 2004; Abayomi and Yandle, 2012) or impractical given regulatory costs, particularly for small fisheries (Squires et al., 1998).

As a simple example, consider a gear modification that would completely eliminate the bycatch in our benchmark scenario. This would effectively separate the two fishing sectors and result in $\$ 22$ million in present value net benefits from both commercial and recreational fishing, respectively, or $\$ 3$ million more than the joint commercial-recreational fishing strategy with bycatch. The fisheries could therefore be better off on the whole if this gear modification cost less than a few million dollars.

## 5. Conclusion

This paper shows that management of multi-stock fisheries exploited by imperfectly selective gear may involve cyclical harvesting policies. We develop a model of a two-species system in which bycatch is produced by one of two harvesting sectors. In the long run, optimal management balances the return from the fisheries by fluctuating the harvest levels in the two sectors: as one sector reduces exploitation of the resource, the other sector winds up its harvests. Furthermore, corner solutions can arise because a sector is limited to a minimum harvest of zero. Our results support several previous findings made outside of the bycatch literature that cyclical rather than steady-state management is optimal in the presence of capital adjustment costs. In this case, a technological externality in harvesting the resource creates ecological adjustment
costs. A cyclical harvest policy is optimal because it is difficult to separate the ecological spillover from the targeted-stock effect with changes in the harvest levels.

Incorporating the economic impacts of bycatch into decision making can increase the value of fisheries linked through harvest technologies. Furthermore, using economic models to determine optimal harvest policies can be useful in designing efficient regulations, particularly for small (e.g. freshwater or inland) fisheries where implementing alternative policies may be prohibitively expensive. As we showed, jointly managing different harvesting sectors is preferable to operating only one fishery unless the bycatch problem is severe. Gear modification may be practical if the alternative harvest technology is relatively inexpensive and will substantially reduce bycatch levels. An effective, flexible bycatch reduction policy could combine harvest quotas, gear subsidies and other incentives to encourage the adoption of more selective harvest technologies.

Our model presents a bycatch problem in a dynamic, albeit stylized, bioeconomic framework. This captures the essential features of many coastal and freshwater bycatch problems and may be generally adequate for fisheries, including marine fisheries, in which two harvesting sectors with fairly malleable capital investments find themselves competing for aquatic biomass due to the use of nonselective gear.

Extensions of the model could prove useful in understanding the economic nature of bycatch. Stocking is a common management device that would increase the controllability of a system like the one studied here. We expect that a steady state harvest policy would be more likely when combined with stocking. Consideration of nonuse values may be important, as many bycatch problems involve endangered, charismatic species with significant existence value. Finally, the failure of many collapsed fisheries to recover despite protective moratoria, such as
with lake trout, may suggest that the assumption of compensatory growth is inappropriate.
Incorporating depensation into the model could add realism to our application and would at least add significant time to the recovery of the system. Further insight into these matters is left to future research.

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7. Tables and Figures

Table 1. Parameters used in the numerical example

| Parameter | Value | Notes |
| :---: | :---: | :---: |
| $r_{s}$ | 0.70 | Jensen (1991) uses this value for lake trout. |
| $r_{w}$ | 0.60 | Jensen (1991) uses this value for whitefish. |
| $k_{s}$ | 500,000 | Lake trout carrying capacity in hypothetical management zone. |
| $k_{w}$ | 2,000,000 | Whitefish carrying capacity in hypothetical management zone. |
| $q_{s}$ | 0.0000166 | Commercial effort is measured in units of fishing net lifts. Johnson et al. (2004b) report a bycatch of 28.9 lbs per lift (1000 ft ) in a part of Lake Huron. With a lake trout biomass of about $1,732,000 \mathrm{lbs}$ in the area (approximately management unit MH02 ; see Woldt et al. (2006)), this implies $28.9=q_{s}{ }^{*} 1 * 1,732,000$. |
| $q_{w}$ | 0.0000479 | Johnson et al. (2004b) report a catch of 292.3 lbs per lift (1000 ft ) in a part of Lake Huron. With a whitefish biomass of about $6,102,000 \mathrm{lbs}$ in the area (about half of management units WFH-04 and WFH-05; see Woldt et al. (2006)) this implies $292.3=q_{w}{ }^{*} 1^{*} 6,102,000$. |
| $z$ | 0.0000126 | Commercial effort is measured in units of angler days. Woldt et al. (2006) indicates that $20,000 \mathrm{lbs}$ of lake trout were harvested in management unit MI-07 from 18,500 hours of effort and a stock of about $507,000 \mathrm{lbs}$. Assuming a day of fishing averages about four hours of effort, this implies $20,000=z^{*}(18,500 / 6$ hours per day)*507,000. |
| $c_{b}$ | 30 | On the typical commercial boat a pair of workers can fish about six lifts per day. We assume the opportunity cost of fishing is $\$ 90 /$ day/worker, and this implies that $2 * 90=c_{b} * 6$. |
| $c_{a}$ | 50 | Suggests a trip cost of \$50. |
| $p_{s}$ | 0 | We assume bycatch has no value. In Michigan there was no commercial sale of lake trout in 2012 (MIDNR, 2012). |
| $p_{w}$ | 1.61 | Average per-pound value of whitefish in 2012 (MIDNR, 2012). |
| $u$ | 23.81 | Assumes net benefits of $\$ 25$ per day, which is similar to the day-trip values in Melstrom and Lupi (2013). This implies $u^{*} z^{*} s-c_{d}=30$, with $z=0.0000126, s=250,000$ and $c_{a}=50$. |



Figure 1. Phase planes of the double singular solution for various parameter combinations: (a) small bycatch parameter with unstable steady-state equilibrium, (b) moderate bycatch parameter with no steady-state equilibrium and (c) large bycatch parameter with saddle point steady-state equilibrium. Point OA is the open-access equilibrium.


Figure 2. State-space of the strategies that make up the solution to the multi-sector fishing problem. There is no fishing along path A, recreational fishing-only along path B, and both commercial and recreational fishing along path C .


Figure 3. Cyclical equilibrium harvesting. LWC is commercial lake whitefish harvest, LTR is recreational lake trout harvest and LTC is commercial lake trout harvest (bycatch).


Figure 4. State-space of the solution to the multi-sector fishing problem when the lake trout growth rate is lowered compared with the benchmark.


Figure 5. State-space of the solution to the multi-sector fishing problem when lake whitefish are more valuable and the bycatch externality is greater compared with the benchmark.


[^0]:    ${ }^{1}$ Skonhoft et al. (2012) is an exception here in that they do analytically verify that the equilibrium is a steady state, although in their paper bycatch is associated with the exploitation of an age-structured population rather than two different species.

[^1]:    ${ }^{2}$ Ebener et al. (2008) discuss the management of lake whitefish in the Great Lakes and the role of the whitefish fisheries' bycatch of lake trout. They report that in much of the Great Lakes "the focus of management is directed at lake trout so lake whitefish populations are basically managed as a by-product of lake trout rehabilitation."

[^2]:    ${ }^{3}$ Blocked intervals would also form at the upper limits $a_{D S}(s, w)=a_{\max }$ and $b_{D S}(s)=b_{\max }$. For simplicity, our analysis supposes that the solution is never constrained in this manner, i.e. the upper limits are infinite. This restriction does not significantly affect our results.
    ${ }^{4}$ For the sake of brevity this paper omits a discussion of Great Lakes fisheries policy, which involves over a dozen state, provincial, tribal and federal natural resource agencies in the United States and Canada. The responsibility for

[^3]:    managing areas of the Great Lakes varies among these authorities, although over time binational agreements (particularly through the Convention on Great Lakes Fisheries) have helped to develop and coordinate a common fisheries management strategy. See Taylor and Ferreri (2002), pp. 305-338, for a detailed history. It should also be noted that these agencies contend with problems other than bycatch (e.g. invasive species, disease, native and nonnative stocking) that can play an important role in optimal fisheries management. However, incorporating these matters into the model would cloud the analysis and diminish the relevance of the solution to fisheries where the bycatch problem predominates.
    ${ }^{5}$ Two types of gear are primarily used to harvest whitefish: gill nets and trap nets. In the numerical example we assume commercial harvesting is done by gill nets, which are less selective than trap nets. Several state natural resource agencies now mandate traps nets but gill nets were used historically and continue to be used extensively by tribal fishing operations.

[^4]:    ${ }^{6}$ In addition to the saddle point the system dynamics are shaped by a critical but unstable point.

[^5]:    ${ }^{7}$ That the system lies in this constrained region suggests pursuing the commercial fishing-only strategy might be optimal. However, the partial singular path defined by this strategy moves away from equilibrium and leads to eradication of lake whitefish, so it cannot be optimal. The only other candidate optimal strategy is to cease all fishing and allow both lake whitefish and lake trout to recover, which is consistent with the concept of a most-rapid-approach-path to equilibrium.

[^6]:    ${ }^{8}$ This is actually the equilibrium level as defined in recreational fishing-only strategy, where the partial singular control $a_{P S}$ is applied.
    ${ }^{9}$ Past research on the joint use of selective and nonselective controls has found steady state harvesting to be longrun optimal (e.g. Melstrom and Horan, 2014).

