Pollution-Income Dynamics

by

Ramón López and Sang W. Yoon

WP 14-02

Department of Agricultural and Resource Economics

The University of Maryland, College Park

Last revised: February, 2014

Pollution-Income Dynamics

Ramón E. López*
University of Maryland and University of Chile

Sang W. Yoon**
University of Maryland

JEL classification number: O1, Q2

Keywords:

Environmental Kuznets curve, Elasticity of marginal utility, Elasticity of substitution

Abstract

This paper shows that the dynamic properties of the pollution-income relationship under an optimal pollution tax depends on three key factors, namely the degree of temporal and inter-temporal flexibility in consumption and the elasticity of substitution among production inputs. This paper derives general conditions for eluding the limits to growth showing that they require rather stringent assumptions which the existing literature has failed to identify.

* 3125 Symons Hall, University of Maryland, College Park, MD 20740 and Faculty of Economics and Business, University of Chile, Santiago, Chile (e-mail: rlopez@umd.edu)

** 2200 Symons Hall, University of Maryland at College Park, MD 20740 (e-mail: syoon123@umd.edu).

1. Introduction

We examine the relationship between pollution and income in a dynamic general equilibrium framework with endogenous growth in a multi-output context. Previous theoretical literature has assumed a single final good thus ignoring the output composition effect and has often modeled production using a Cobb-Douglas specification (e.g. López, 1994; Stokey, 1998; Andreoni and Levinson, 2001; Johansson and Kriström, 2007). However, empirical evidence shows that the structure of consumption, not merely its level, is important in affecting the pollution-income relationship (Grossman and Krueger, 1995), and the Cobb-Douglas specification is often rejected (Chirinko, 2008). Figueroa and Pasten (2013) is one of the few analyses that allow for more general functional forms for consumer preferences and production functions. While their analysis constitutes an important generalization of earlier models in several respects, it is static in the sense that output expansion is exogenous and it still considers only one final consumer good, thus neglecting the output composition effect.

The most important conclusion of the existent theoretical literature is that the so-called environmental Kuznets process (EKC), where pollution first increases with income but beyond a certain income level it secularly declines, constitutes a plausible description of the pollution-income relationship. That is, the limits to growth would in this case be overcome. Below we show that this optimistic conclusion requires rather stringent assumptions often ignored by the literature. Consumer preferences both temporal and inter-temporal, and/or production technologies must have a high degree of flexibility for an EKC to be a relevant for an economy that taxes pollution optimally.

2. The Model

The economy produces two goods: a clean and a dirty one. The dirty-good production generates pollution as a byproduct while production of the clean good involves no pollution. Let k denote the total man-made composite clean input available at time t. The composite input includes human and physical capital. Henceforth, we refer to k as capital, which is momentarily distributed between the clean industry and dirty industry. Let k_d denote the amount of capital employed in the dirty sector. The flow of pollution from the dirty sector is represented by *x*. Following López (1994), and Copeland and Taylor (2005), we regard pollution as a factor of production, its price being determined by a pollution tax. Let $F(k_d, x)$ represents the production technology of the dirty-good sector, which is characterized by the constant elasticity of substitution (CES) function,

$$(1) \qquad y_d = F\left(k_d, x\right) = \left[\alpha k_d^{-\frac{1-\omega}{\omega}} + (1-\alpha)x^{-\frac{1-\omega}{\omega}}\right]^{-\frac{\omega}{1-\omega}},$$

where ω represents the elasticity of substitution between capital and pollution. The dirty sector produces only final goods. The output of the clean-good sector is assumed to depend only on the capital input and is governed by the linear technology

$$(2) \qquad y_c = A(k - k_d).$$

This sector produces the final good and new capital. If we normalize the price of the clean good to unity ($p_c = 1$), the economy's budget constraint is,

$$(3) \qquad \dot{k} = A(k - k_d) + pF\left(k_d, x\right) - c - \delta k,$$

where $p \equiv p_d / p_c$ is the relative price of the dirty good, $c \equiv c_c + pc_d$ is the total-consumption expenditure expressed in units of the clean good, δ is the rate of capital depreciation, and $\dot{k} \equiv dk/dt$ is the net capital accumulation. The sum of the first two terms on the right-hand side on (3) represents the income of the economy expressed in units of the clean good. The gross capital accumulation, $\dot{k} + \delta k$, is equal to net savings (income less consumption), also in units of the clean good.[1]

The consumer's indirect utility function is

---

[1] We assume that investment in capital is irreversible. Once the economy builds capital, it cannot be transformed back into consumption goods.

$$u = \frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a},$$

where c denotes the total-consumption expenditure, $e(1,p)$ is the unit (dual) expenditure function or cost-of-living index, and a is a parameter that is equal to the elasticity of marginal utility (*EMU*).[2] The indirect utility function is assumed to be increasing and strictly concave in c.

The consumer's underlying preferences are described by a CES utility function so that the unit expenditure function is given as

$$e(1,p) = \left[\gamma_c + \gamma_d p^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

where σ is the consumption elasticity of substitution between a dirty good and clean good, and $\gamma_c > 0$ and $\gamma_d > 0$ are fixed parameters. Consumer demand for the clean good c_c and dirty good c_d can be retrieved from the indirect utility function using Roy's identity. The optimal level of c is determined by the inter-temporal optimization, as detailed below. We assume for analytic convenience that the environmental damage is separable with consumption in consumer welfare, and can be represented as $v(x) = \frac{x^{1+\eta}}{1+\eta}$, where η>0 is a fixed parameter. Then the consumer's instantaneous welfare is

$$U \equiv \frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a} - \frac{x^{1+\eta}}{1+\eta},$$

where a is the elasticity of marginal utility of income (*EMU*).

We assume that the discount rate ρ is fixed. When the government regulates pollution emissions in an optimal way, the competitive economy behaves "as if" it maximizes the present discounted value of the utility function,

[2] If $a < 1$ we adopt a positive utility scale such that $0 < u < \infty$, while we scale the utility index to $-\infty < u < 0$ when $a > 1$.

$$\int_0^\infty \left\{ \frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a} - \frac{x^{1+\eta}}{1+\eta} \right\} \exp(-\rho t) dt ,$$

subject to the budget constraint (3), and the initial condition $k = k_0$. The consumer chooses the levels of c and x at each point in time. The government imposes a pollution tax in a socially optimal way and reimburses the tax revenue in a lump-sum way to the consumer. The above optimization implies the following current-value Hamiltonian function,

$$H = \frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a} - \frac{x^{1+\eta}}{1+\eta} + \lambda\left[A(k-k_d) + pF(k_d, x) - c - \delta k\right],$$

where λ is the shadow price of capital (also equal to the marginal utility of consumption).

The following first-order conditions to the optimization program are necessary:

(4) $$e(1,p)^{a-1} c^{-a} = \lambda ,$$

(5) $$pF_1(k_d, x) - A = 0 ,$$

(6) $$-v_1(x) + \lambda p F_2(k_d, x) = 0 ,$$

(7) $$\frac{\dot{\lambda}}{\lambda} = -\left[A - \rho - \delta\right] = -M ,$$

(8) $$\dot{k} = A(k-k_d) + pF(k_d, x) - c - \delta k ,$$

(9) $$\lim_{t\to\infty} \lambda k(t) e^{-\rho t} = 0,$$

where a subscript number reflects the first derivative with respect to the corresponding argument in functions of more than one variable. The optimal pollution tax is equal to the marginal rate of substitution between the pollution and consumption expenditure, and is thus $\tau \equiv v_1(x)/\lambda$.

Using Roy's identity, we can derive the consumer demand for the dirty good from the indirect-utility function, which is $c_d = \dfrac{\gamma_d p^{-\sigma} c}{\gamma_c + \gamma_d p^{1-\sigma}}$. We then have the following market clearing condition for the dirty good:

(10) $$F\left(k_d, x\right) = \frac{\gamma_d p^{-\sigma} c}{\gamma_c + \gamma_d p^{1-\sigma}},$$

while the rate of growth of production of the dirty good is

(11) $$\hat{F}\left(k_d, x\right) = S_k \widehat{\left(\frac{k_d}{x}\right)} + \hat{x}.$$

Noting that the market for the dirty good must clear at all points in time, it follows that the growth-rate of production and demand for the dirty good must be equal (e.g., $\hat{c}_d = \hat{F}(k_d, x)$) . Hence, using Lemma 2, (1) and (11), we arrive at

(12) $$z\hat{p} + S_k \widehat{\left(\frac{k_d}{x}\right)} + \hat{x} = \frac{M}{a},$$

where $z \equiv \frac{s(p)}{a} + (1 - s(p))\sigma > 0$.

From (5), we also have that $\hat{p} + \hat{F}_1(k_d, x) = 0$,

(13) $$\hat{p} - \frac{1}{\omega}\left(1 - S_K\right)\widehat{\left(\frac{k_d}{x}\right)} = 0.$$

Finally, differentiating the (6) with respect to time, we obtain

(14) $$-\eta\hat{x} + \hat{p} + \frac{1}{\omega} S_K \widehat{\left(\frac{k_d}{x}\right)} = M.$$

The equation system (12), (13), and (14) simultaneously solves for the three endogenous variables $\hat{p}$, $\widehat{\left(\frac{k_d}{x}\right)}$, and $\hat{x}$,

(15) $$\hat{p} = \frac{\frac{M}{\omega}\left(1 - S_k\right)\left[\frac{\eta}{a} + 1\right]}{|W|} \geq 0$$

(16) $$\widehat{\left(\frac{k_d}{x}\right)} = \frac{M\left[\frac{\eta}{a} + 1\right]}{|W|} > 0$$

(17) $$\hat{x} = \frac{\frac{M}{\omega} T(p)}{|W|} \,,$$

where $T(p) \equiv \frac{1}{a}[1 - s(p)(1 - S_k)] - S_k\omega - (1 - s(p))(1 - S_k)\sigma$, $|W| \equiv \frac{1}{\omega}\left[(1 - S_k)(1 + z\eta) + S_k\right] + \eta S_k$,

$s(p) = \frac{pc_d}{pc_d + c_c}$ and $S_k = \alpha\left[(1-\alpha)\left(\frac{k_d}{x}\right)^{\frac{1-\omega}{\omega}} + \alpha\right]^{-1}$.[3]

Eq. (15) implies that the price of dirty goods continuously increases over time if the economy has sufficiently strong growth potential ($A > \rho + \delta$ or, equivalently, if $M > 0$). This is partly due to the fact that, under optimal regulation, the price of the dirty good depends on the marginal social cost of pollution $(v_1(x)/\lambda)$, which, at the given level of pollution, is increasing over time as λ falls. The increasing price of the dirty good induces consumers to increase the clean-good–dirty-good consumption ratio. This triggers a structural change in production and leads to the output-composition effect, where production of the dirty good declines relative to that of the clean good.

Eq. (16) shows that the so-called technique effect takes place along the optimal-growth path. Thus, (15) and (16) imply that closed economy must rely on both the output composition and technique effects as a way to counter the scale effect caused by positive economic growth. The net result which is described by (17) is, in general, ambiguous and critically dependent on the dynamics of $s(p)$ and $S_k(p)$, in addition to the consumption elasticity of substitution, σ, the production elasticity of substitution, ω, and *EMU*.

An important issue is whether the dynamic path described by (15) to (17) allows for a positive rate of consumption growth. Proposition 1 below shows that this is indeed the case.

[3] The share of the dirty good in the consumer budget, $s(p)$, is an increasing (decreasing) function of p if $\sigma < 1$ $(\sigma > 1)$. The factor share of capital in the production of dirty goods, $S_k(p)$, is increasing (decreasing) in p if $\omega > 1$ $(\omega < 1)$.

Proposition 1: (i) *The growth rate of real consumption expenditure is:* $\widehat{\left(\frac{c}{e}\right)} = \frac{1}{a}\left[M - s(p)\hat{p}\right]$, *where* $\hat{p}$ *is given by (14).* (ii) *The rate of growth of real consumption remains positive throughout the equilibrium path for any positive* ω *and* σ .

Proof: *See Appendix.*

3. Conditions for an EKC

Sufficient conditions for the emergence of EKC can be summarized as follows.

Proposition 2: *Assume* $a > 0$, *then pollution emissions increase over a certain interval of time before eventually declining if any of the following three conditions are satisfied.*

(i) If $\sigma < Min\left[1, \frac{1}{a}\right]$ *and* $\omega > Max\left[1, \frac{1}{a}\right]$,

(ii) If $\sigma > Max\left[1, \frac{1}{a}\right]$ *and* $\omega < Min\left[1, \frac{1}{a}\right]$,

(iii) If $\sigma > \frac{(1/a) - \alpha}{1 - \alpha}$ *and* $\omega = 1$.

Proof: *See Appendix.*

Proposition 2 states that if the economy lacks flexibility in both consumer preferences and input substitution, an EKC is not feasible. If *EMU* is less than one the likelihood for the emergence of an EKC is low. In this case at least one of the elasticities of substitution must be much greater than one. The popular specification where both consumer preferences and production technologies are Cobb-Douglas may be consistent with an EKC process only if the *EMU* is greater than one, which is precisely the assumption made by most of the EKC literature (e.g., Stokey, 1998).

4. Conclusion

This paper examines the scale, composition, and technique effects of economic growth on pollution emission growth. This paper shows that the limits to growth can be eluded through a Kuznets-type process only if there is a sufficient degree of substitution flexibility in either production technology or consumer preferences. The flexibility requirements are more demanding the lower is the *EMU*. If an economy is endowed with such flexibility, then economic growth can be sustained at positive levels while pollution falls over the long run. If the elasticity of substitution between the dirty inputs and the clean inputs is much less than unity, as often reported in the empirical literature, the feasibility of sustainable growth under optimal pollution tax hinges greatly on the size of the output composition effect, an effect that has been consistently neglected in the theoretical literature.

Reference

Andreoni, J., Levinson, A., 2001. The simple analytics of the environmental Kuznets curve. Journal of public economics 80, 269-286.

Copeland, B., Taylor, M.S., Trade and the Environment: Theory and Evidence, (NJ: Princeton University Press, 2005).

Chirinko, R., 2008. σ: The long and short of it, Journal of Macroeconomics 30, 671-686.

Figueroa B, E., Pasten, R., 2013. A tale of two elasticities: A general theoretical framework for the environmental Kuznets curve analysis. Economics Letters 119, 85-88.

Johansson, P. O., & Kriström, B.,2007. On a clear day you might see an environmental Kuznets curve. Environmental and Resource Economics 37, 77-90.

López, R., 1994.The Environment as a Factor of Production: The Effects of Trade Liberalization. Journal of Environmental Economics and Management 27, 163-184.

Stokey, N. L., 1998. Are There Limits to Growth? . International Economic Review 39, 1-31.

Appendix

Proof of Proposition 1

(i) By Roy's identity, the demand for the dirty good $c_d = \frac{c}{e(1,p)} e_2(1,p)$. Using Shephard's lemma,

$\hat{e}(1,p) = \frac{pe_2}{e}\hat{p} = s(p)\hat{p}$. Therefore, $\widehat{\left(\frac{c}{e}\right)} = \hat{c} - \hat{e} = \frac{1}{a}\left[M - s(p)\hat{p}\right]$.

(ii) The real consumption grows over time if $\hat{p} < M / s(p)$. Using (15) this inequality holds if

$$\hat{p} = \frac{(1/\omega)M(1-S_k)\left[(\eta/a)+1\right]}{(1/\omega)\left[(1-S_k)(1+z\eta)+S_k\right]+\eta S_k} < M/s(p).$$

Rearranging this inequality we have the following;

(A1) $(1-S_k)\left(\frac{\eta}{a}+1\right)s(p) < \left[(1-S_k)(1+z\eta)\right] + S_k + \eta S_k \omega$.

Since, $(S_k + \eta S_k \omega) > 0$ and $z \equiv \frac{s(p)}{a} + (1-s(p))\sigma$, (A1) is satisfied if

(A2) $\frac{\eta s(p)}{a} + s(p) < 1 + \frac{\eta s(p)}{a} + (1-s(p))\sigma\eta$. (A2) holds if $0 < (1-s(p))(1+\sigma\eta)$, which is always true for $0 < s(p) < 1$. Thus, we have $\hat{p} < (M/s(p))$ and hence consumption growth is positive for all finite $\sigma > 0$ and $\omega > 0$. Q.E.D.

Proof of Proposition 2

We note that T changes continuously in time.

(i) If $\omega > Max\left[1, \frac{1}{a}\right]$, S_k increases to 1 over time. Since $T\big|_{S_k=1} = \frac{1}{\omega a} - 1 < 0$ and $T\big|_{S_k=0} = \left(\frac{1}{a} - \sigma\right)(1-s(p))$,

EKC can emerge as long as $\sigma < Min\left[1, \frac{1}{a}\right]$.

(ii) If $\omega < Min\left[1, \frac{1}{a}\right]$, S_k decreases to 0 over time. Since $T\big|_{S_k=1} = \frac{1}{a} - \omega > 0$ and $T\big|_{S_k=0} = (1-s(p))\left(\frac{1}{a} - \sigma\right)$,

we have $T\big|_{S_k=0} = \left(\frac{1}{a} - \sigma\right) < 0$ if either $1 < \frac{1}{a} < \sigma$ or $\frac{1}{a} < 1 < \sigma$.

(iii) If $\omega = 1$, S_k remains equal to some value $0 < \alpha < 1$.

When $0 < a < 1$ then $\sigma > \frac{(1/a)-\alpha}{1-\alpha} > 1$ and $s(p)$ decreases to 0 overtime. EKC can emerge since $T\big|_{S_k=\alpha, s(p)=0} = \frac{1}{a} - \alpha - (1-\alpha)\sigma < 0$ and $T\big|_{S_k=\alpha, s(p)=1} = \alpha\left(\frac{1}{a}-1\right)$>0.

When $a > 1$ then $1 > \sigma > \frac{(1/a)-\alpha}{1-\alpha}$ and $s(p)$ decreases to 0 overtime. Since $T\big|_{S_k=\alpha, s(p)=1} = \alpha\left(\frac{1}{a}-1\right) < 0$ and $T\big|_{S_k=\alpha, s(p)=0} = \frac{1}{a} - \alpha - (1-\alpha)\sigma < 0$. The pollution level can either monotonically decrease over time, or EKC can emerge. Q.E.D.