AE89019 September 1989

A Control Theory Approach to Optimal Farm Debt/Equity Structure

by

James A. Larson
David L. Watt*

*Larson is Graduate Research Assistant, Department of Agricultural Economics, Oklahoma State University and Watt is Associate Professor, Department of Agricultural Economics, North Dakota State University.

Table of Contents

A Control Theory Approach to
Optimal Farm Debt/Equity Structure

Policy makers, researchers, and farmers are concerned with factors that influence the financial structure of the farm business. Their preoccupation stems from the problems farmers have had adjusting their finances to the volatile economic environment of the 1970s and 1980s. Declining asset values, stagnant returns, and rising real interest rates were evidenced in the 1980s. This economic setting severely disrupted debt/equity structures developed in the prosperous 1970s. Farm financial stress and insolvency problems have, as a consequence, increased dramatically (Johnson et al.). These problems have caused a search for effective managerial and/or policy responses to financial stress.

Barry et al. (1987) suggest that decision makers view the farm as tending towards a preferred structure of its finances and income generating activities. However, periodic economic shocks impede the achievement of a desired financial structure and, ultimately, the survival of the farm business. The success of the farm depends on an appropriate capital structure to withstand the recurrent boom-bust cycles of agriculture.

Analysis of farm capital structure in a dynamic adjustment framework could be used to formulate strategies to help farmers achieve a preferred financial position. This paper presents a control theory model to explore relationships among income, capital structure, and risk and their effects on farm profitability, growth, and survival. Optimal capital structure is analyzed using farm level data in a dynamic context that adds to the theory of the farm firm.

This paper is divided into the following sections: a brief review of farm financial structure literature, a discussion of finance and risk relationships influencing capital structure that are specified in the control model, the theoretical framework, the empirical framework, the data for the model, the intertemporal debt/equity equilibrium and sensitivity analysis results, and the summary and conclusions of this study.

Review of Literature

There has been a gradual evolution in the theory of farm capital structure. Each contribution has been cohesive to the development of the theory. Baker demonstrates the effects of credit and debt on the production equilibrium of the farm. Optimal farm organization is determined by product-price ratios, farmer expectations, and liquidity preferences. The effects of leverage and liquidity are explored by Baker and Hopkin. They analyze: 1) the influence of debt on the equilibrium position of the firm, 2) the implications of equilibrium for resource allocation, and 3) the dynamics of firm growth and achieving desired levels of income and net worth accumulation using debt. Gabriel and Baker present a framework to link farm production and investment choices with the financing decision using a risk constraint. They hypothesize that farmers seek to balance total risk between business risk (variability in returns) and financial risk (increased variation of return to equity from debt financing). Barry et al. (1981) derive a mean-variance financial structure model to show how credit risk influences debt use for a risk-averse decision maker. They demonstrate how debt use is influenced by farmer risk preferences and the relationships associated with returns and interest cost.

A conceptual debt/equity decision framework for a risk-averse farmer is presented by Collins (1985a). The farmer's objective is to maximize the expected utility of return to equity. Initial equity position, expected rate of return to assets, variance of return to assets, anticipated increases in asset values, and cost of debt are considered by the decision maker when choosing an optimal debt-equity ratio. Collins (1985b) develops a control model of the debt/equity structure for a farm business. The solution yields a steady state where return to equity equals the risk-adjusted discount rate for all time periods with a constant debt/equity time-path. Robison et al. construct a farm capital structure model to maximize expected utility of ending period wealth. They study how credit reserves influence the behavior of proprietary firms. Farm capital structure theory has, for the most part, focused on a static analysis of debt use. However, immediate adjustment to equilibrium by the farm is not possible. Requisite to the analysis of capital structure is a dynamic framework. Dynamic optimization should be expected to be consistent with static theory.

Factors Influencing Farm Capital Structure

A dynamic farm capital structure model should consider finance and risk characteristics unique to farm firms. This section is a discussion of relationships specified in the control framework.

Decision makers have limited capital resources that can be extended through debt financing. The debt-equity ratio and farm size chosen by the decision maker influences the ability of the farm to grow and survive. Increasing debt/equity may endanger farm survival during periods of low returns due to reduced liquidity reserves (Baker and Hopkin). Debt financing is influenced by internal and external capital rationing constraints (Gabriel and Baker). Prior studies of farm debt-equity ratios suggest managers do not always use outside financing to the point where external capital rationing becomes a constraint (Barry and Baker). Farmers limit debt to minimize exposure of equity capital to losses. Aversion to debt is a form of risk behavior and is a significant alternative decision maker response to uncertainty (Barry and Baker). Debt financing is a decision variable used to determine desired farm size and exposure to risk (Collins 1985a).

Variability in farm income, expected capital gains, and interest costs are important factors affecting profitability, survival, and growth (Shepard and Collins). Farmers have little influence over changes in resource or product prices that affect farm income returns (Barry and Fraser). Capital gains from farm asset appreciation are only realized when an asset is sold. Yet in farming, borrowing decisions have been frequently based on expected income and changes in asset values. Featherstone and Baker indicate decline in asset values are exacerbating farm financial liquidity risk. Interest rates are also beyond the control of decision makers. High borrowing costs reduce the amount of money available for consumption and investment and decrease the rate of equity accumulation. Lower interest rates reduce interest and debt payments, permit a satisfactory level of consumption, and allow growth in equity and farm size (Patrick and Eisgruber).

The relationships between rates of returns before interest cost and the interest rate are an important influence on willingness to use debt. Returns, either from expected farm income or capital gains, greater than interest cost are an incentive to increase leverage to take advantage of growth in farm size and increased profitability. However, changes in the returns and interest cost environment can have a significant negative impact on farm profitability and survival. This is especially true when the expected rates of returns become less than the expected interest rate for extended periods of time.

Consumption-investment preferences of the decision maker and family living expenses both impact farm profitability, growth, and survival. Maximizing equity accumulation restricts consumption of current income and requires a higher, more vulnerable debt-equity ratio to improve returns. Consuming too much current farm income may retard the growth rate of the farm to the point where it may not survive (Baker and Hopkin; Boehlje and White).

Farm size as measured by total farm assets has a demonstrable impact on profitability and survival (Sheppard and Collins). Tweeten indicates that the average cost function is relatively constant over a wide range above a minimum viable size farm. This implies constant rates of return as farm size increases above the minimum viable farm size. Also, the farm must be of a minimum size to support the needs of the farm family or it fails to be a viable economic unit. Income over expenses generated by the farm is not all available for reinvestment in the operation. A minimum level of income must be used for family consumption in any year. Patrick and Eisgruber assert that the farm business and the family are not independent due to considerations of time, limited capital, and uncertainty.

To summarize, the following relationships are specified in the control model: 1) decision maker risk preferences associated with desired debt/equity and farm size; 2) variability associated with expected farm income returns, changes in asset values, and interest rates; and 3) the consumption-savings and family living expense preferences of the decision maker.

Theoretical Framework

An optimal control model portrays the development of a system over time and derives the optimum level of one or more decision variables. The condition of a system at any point in time is described by system state variables. Control or decision variables are time-dependent variables determined, at least in part, by the decision maker. Changes in the state variable over time are defined by an equation of motion that is a function of the state and decision variables. The objective is to maximize the expected value of the sum over time of the decision maker's temporal utility function which is a function of time and the decision and state variables. The purpose of the control model is to maximize the decision maker's objective function subject to the equation of motion and the initial state of the system (Zilberman).

The assumptions and limitations of portfolio theory in financial analysis are well documented (Robison and Brake; Barry and Baker; King and Robison; Selly; Robison and Barry). Mean-variance (EV) is an attractive framework for determining the optimal structure of farm enterprises, assets, and liabilities for a risk averse decision maker. The EV framework is founded in economic and finance theory and approximates a risk averse decision maker (Tobin; Markowitz). Decision maker preferences are based on a normal distribution of returns and costs that are fully specified by their means and variances.

The objective of the control model is to maximize present value of risk-adjusted farm income using debt as the decision variable, subject to the farmer's preference to reinvest farm income. An equation of risk-adjusted farm income using mean-variance as the risk penalty defines the objective. The equation of motion is income times the marginal propensity to reinvest income into the farm operation, either to equity for growth or to retire debt.

The farm has a portfolio of two assets: 1) equity E(t), the state variable and 2) debt D(t), the control or decision variable. A Hamiltonian is formed for maximization (Pontryagin et al.), and solved to yield a simultaneous first order nonhomogeneous differential equation system with two unknowns: equity and debt. Solving the equation system yields the general solution which is composed of the particular integrals and complementary solutions of equity and debt. Particular integrals are a point, E(p) and D(p), at which $\dot{E}(t)=\dot{D}(t)=0$ and is defined as the intertemporal equilibrium (steady state).[1] Complementary solutions are the time paths from an initial farm financial state of equity and debt towards the equilibrium values, E(p) and D(p). The system is solved simultaneously to find the optimal level of debt to maximize farm income. Since both time paths have the same characteristic roots expressions--r1 and r2--equity and debt must converge towards or move away from equilibrium. The equilibrium of a pair of linear differential equations is stable if $\lim_{t \to \infty} E(t)=E(p)$ and $\lim_{t \to \infty} D(t)=D(p)$. For a system with constant initial farm financial state values, equity (E(0)) and debt (D(0)), the intertemporal equilibrium is stable if the characteristic roots, r1 and r2, are real and distinct and have values that are $r2<r1<0$ (Kamien and Schwartz).

This control problem is specified using an infinite time horizon and is "autonomous" because time enters only through a discount term. The transversality condition used to define a boundary condition on the solution is replaced by the assumption that the optimal solution approaches a steady state. If the environment of the system is stationary by hypothesis, then it is reasonable to assume that the optimal solution will tend to settle down over time (Kamien and Schwartz). However, some autonomous control problems do not have a stable equilibrium (Arrow).

Empirical Framework

The objective is to maximize present value of risk-adjusted farm income. The equation of motion is the farmer's preference to reinvest farm income. Debt (D(t)) is the control and equity (E(t)) is the state variable. The costate variable lambda (L) describes the marginal change in the state variable equity. The farm decision maker is assumed to have utility approximated by the negative exponential:

(1) $$U(I)=1-e^{-2\alpha\pi}.$$

Maximizing the expected value of a negative exponential integrated over a normally distributed function is equivalent to maximizing (Freund 1956)

(2) $$E(U(\pi))=E(\pi)-\alpha\cdot E(\pi^2),$$

where $E(U(\pi))$ is expected utility of income, $E(\pi)$ is expected income, and (α) is degree of risk aversion under exponential utility. A more risk averse individual places a larger value on α. Variance of income (σ_π^2) is equal to $E(\pi-E(\pi))^2=E(\pi^2)-E(\pi)^2$, where ($\pi$) is observed income and $E(\pi)$ is the expected income (Selly 1984). Solving for $E(\pi^2)$ and substituting into (2) yields expected utility that is a function of expected income and variance:

(3) $E[U(\pi)]=E(\pi)-\alpha\cdot\sigma_\pi^2.$

Expected farm income $E[\pi(t)]$ at time t is defined as

(4) $E[\pi(t)]=E[r\cdot E(t)+(r-i)\cdot D(t)-F],$

with variance of

(5) $\sigma_\pi^2=\sigma_r^2\cdot(E(t)+D(t))^2+\sigma_i^2\cdot D(t)^2-2\cdot(E(t)+D(t))\cdot D(t)\cdot\sigma_{ri},$

where (r) is normally distributed expected returns to farm assets before interest cost and taxes plus expected change in farm asset value, E(t) is equity at time t, (i) is normally distributed expected rate of interest on farm debt, D(t) is debt at time t, (F) is cash family target consumption, σ_π^2 is variance associated with the equity and debt portfolio, σ_r^2 is the variance of returns to farm assets before interest cost, σ_i^2 is the variance of interest rate paid on farm debt, and σ_{ri} is covariance of returns and interest cost. The expected utility maximizing farm income equation is:

(6) $$E[U(\pi(t)]=E[r\cdot E(t)+(r-i)\cdot D(t)-F]$$
$$-\alpha\cdot[\sigma_r^2\cdot(E(t)+D(t))^2+\sigma_i^2\cdot D(t)^2$$
$$-2\cdot(E(t)+D(t))\cdot D(t)\cdot\sigma_{ri}].$$

The first order condition for debt D(t) from (6) is

(7) $$D(t)^* = \frac{r-i-2\cdot\alpha\cdot E(t)\cdot(\sigma_r^2-\sigma_{ri}),}{2\cdot\alpha\cdot(\sigma_r^2+\sigma_i^2-2\cdot\sigma_{ri})}$$

which yields these comparative static results if $r>i$ and $r,i<1$:

$$\frac{dD(t)^*}{dr} = \frac{1}{2\cdot\alpha\cdot(\sigma_r^2+\sigma_i^2-2\cdot\sigma_{ri})} > 0. \tag{8}$$

$$\frac{dD(t)^*}{di} = \frac{-1}{2\cdot\alpha\cdot(\sigma_r^2+\sigma_i^2-2\cdot\sigma_{ri})} < 0. \tag{9}$$

$$\frac{dD(t)^*}{d\alpha} = \frac{-r+i}{2\cdot\alpha^2\cdot(\sigma_r^2+\sigma_i^2-2\cdot\sigma_{ri})} < 0. \tag{10}$$

$$\frac{dD(t)^*}{d\sigma_r^2} = \frac{-r+i-2\cdot\alpha\cdot E(t)\cdot(\sigma_r^2-\sigma_{ri})}{2\cdot\alpha\cdot(\sigma_r^2+\sigma_i^2-2\cdot\sigma_{ri})^2} < 0. \tag{11}$$

$$\frac{dD(t)^*}{d\sigma_i^2} = \frac{-r+i+2\cdot\alpha\cdot E(t)\cdot(\sigma_r^2-\sigma_{ri})}{2\cdot\alpha\cdot(\sigma_r^2+\sigma_i^2-2\cdot\sigma_{ri})^2} < 0. \tag{12}$$

$$\frac{dD(t)^*}{d\sigma_{ri}} = \frac{r-i+\alpha\cdot E(t)\cdot(\sigma_i^2-\sigma_r^2)}{\alpha\cdot(\sigma_r^2+\sigma_i^2-2\cdot\sigma_{ri})^2}. \tag{13}$$

Optimum debt is positively related to return to assets before interest cost and negatively related to interest cost. Risk aversion is inversely related to optimum debt when returns are greater than interest cost. Variance of returns and interest cost have a negative association with optimum debt.

Comparative static properties for covariance is indeterminate. These comparative static relationships are consistent with those reported by Barry et al. (1981), and are contrasted with study results where applicable.

The farm income discount equation is (Collins 1985b)

(15) $$e^{-\delta \cdot t} = e^{-(i+\beta \cdot D/E)t},$$

where (i) is the expected real interest rate on debt, (ß) is an estimated coefficient of farm solvency for a specified real rate of interest and debt/equity category, (D) is initial debt, (E) is initial equity, and t is time. The function discounts farm income to present value using the interest on debt as a cost of capital for the farm. A risk premium for farm insolvency is added to discount for the probability of foregone future earning due to farm failure. Farm solvency is a Poisson probability distribution representing time until insolvency. The risk premium is an estimated beta coefficient of farm solvency multiplied by the farm's beginning debt/equity. A probability distribution of farm solvency for a specified real interest rate and beginning debt/equity is generated using Monte Carlo simulation. The probability distribution is regressed against time and debt/equity to yield an estimate of beta.

A Hamiltonian is formed from the farm income and risk discount equations. The first part,

(16) $$J = \int_{t=0}^{\infty} e^{-(\delta \cdot t)} \cdot [r \cdot E(t) + (r-i) \cdot D(t) - F$$
$$- \alpha \cdot (\sigma_r^2 \cdot (E(t)+D(t))^2 + \sigma_i^2 \cdot D(t)^2$$
$$- 2 \cdot (E(t)+D(t)) \cdot D(t) \cdot \sigma_{ri}]dt,$$

is the expected present value of risk-adjusted farm income. The infinite time horizon recognizes risk-adjusted farm income for all future generations operating the farm.

The second part,

(17) $\dot{E}(t)=s\cdot(r\cdot E(t)+(r-i)\cdot D(t)-F),$

is the equation of motion--the state variable equity--and describes the portion of farm income retained and added to equity in time t. The letter (s) is the marginal propensity to reinvest farm income.

Combining (16) and (17) yields:

(18) $$H(D,E,L)=e^{-\delta\cdot t}\cdot[r\cdot E(t)+(r-i)\cdot D(t)-F$$
$$-\alpha\cdot(\sigma_r^2\cdot(E(t)+D(t))^2+\sigma_i^2\cdot D(t)^2-2\cdot(E(t)+D(t))\cdot D(t)\cdot\sigma_{ri})]$$
$$+L\cdot s\cdot(r\cdot E(t)+(r-i)\cdot D(t)-F)$$

The first order necessary conditions are:

(19) $$\frac{dH}{dD}=e^{-\delta\cdot t}\cdot[r-i-2\cdot\alpha\cdot\sigma_r^2\cdot E(t)-2\cdot\alpha\cdot\sigma_r^2\cdot D(t)$$
$$-2\cdot\alpha\cdot\sigma_i^2\cdot D(t)+\alpha\cdot 2\cdot\sigma_{ri}\cdot E(t)+2\cdot\alpha\cdot 2\cdot\sigma_{ri}\cdot D(t)]+L\cdot s\cdot(r-i)=0;$$

(20) $$\frac{dH}{dE}=e^{-\delta\cdot t}\cdot[r-2\cdot\alpha\cdot\sigma_r^2\cdot E(t)-2\cdot\alpha\cdot\sigma_r^2\cdot D(t)+\alpha\cdot 2\cdot\sigma_{ri}\cdot D(t)+L\cdot s\cdot r=-L;$$

(21) $$\frac{dH}{dL}=s\cdot[r\cdot E(t)+(r-i)\cdot D(t)-F]=\dot{E}(t).$$

Solving from the first order conditions yields two first order differential equations for debt and equity. The differential equation for change in farm debt with respect to time is

(22) $$\dot{D}(t) = E(t)\cdot\left[\frac{\sigma_r^2\cdot[s\cdot(2\cdot r-(r-i))-\delta]+\sigma_{ri}\cdot(\delta-2\cdot s\cdot r)}{(2\cdot\sigma_{ri}-\sigma_i^2=-\sigma_r^2)}\right]$$

$$+D(t)\cdot\left[\bar{\delta}+s\cdot r\cdot\left[\frac{\sigma_r^2+\sigma_i^2-2\cdot\sigma_{ri}}{2\cdot\sigma_r^{i}-\sigma_i^2-\sigma_r^2}\right]\right]+\left[\frac{\bar{\delta}\cdot(r-i)+2\cdot a\cdot s\cdot F\cdot(\sigma_{ri}-\sigma_r^2)}{2\cdot a\cdot(2\cdot\sigma_{ri}-\sigma_i^2-\sigma_r^2)}\right].$$

The differential equation for change in equity over time is

(23) $\dot{E}(t)=E(t)\cdot s\cdot r+D(t)\cdot s\cdot(r-i)+s\cdot F.$

These two equations form a simultaneous first order nonhomogeneous differential equation system with two unknowns: E(t) and D(t). Solving the equation system yields the general solution which is a linear combination of the particular integrals and the complementary solution. The particular integrals, D(p) and E(p), are the intertemporal equilibrium values of debt and equity. The complementary solutions, D(c(t)) and E(c(t)), are the time paths towards equilibrium from an initial farm financial state.

(24) $D(t)=D(c(t))+D(p)$

(25) $E(t)=E(c(t))+E(p)$

The solution to the system is found by introducing an initial condition for equity and debt.

Data

A farm situation was defined to represent the finance and risk environment encountered by a typical North Dakota grain operation. North Dakota Farm Business Management Program records, Federal Land Bank, and Production Credit Association data were used to develop the financial structure of the representative farm. Firm level financial data more accurately reflects the finance and risk environment experienced by farm decision makers in North Dakota.

Of the farms enrolled in the program with six years of consecutive financial data for the period 1980 to 1985 (Rourke 1988), eighteen had similar financial characteristics and obtained most of their income from crop operations. Data from these farms were used as a basis to calculate present and future financial conditions for a representative farm.

The initial balance sheet is typical of the asset size and debt/equity structure for a North Dakota grain farm (Table 1). The farm has a moderate level of debt against total assets (26 percent) of which 70 percent are in long-term liabilities and 30 percent are in intermediate- and short-term obligations. These proportions are held constant for the control model analysis.

Projected finance and risk variables for the farm are presented in Table 2. The decision maker is assumed to exhibit risk averse behavior in forming expectations. Income returns to farm assets are defined as total farm receipts minus production expenses except interest cost. Returns are divided by farm assets to yield a rate of return to farm assets before interest cost. Average nominal return for the six-year period was 18.47 percent. Between 1980 and 1985, North Dakota farm assets values declined an average of 4.5 percent per year and inflation as measured by the GNP price deflator averaged 6.05 percent. Expected returns are adjusted for the change in asset value and inflation to yield the expected real return of 7.92 percent. Variance of rate of return is measured using whole-farm income rate of return variation. The average rate of return variation for the six year period represents income risk in the model.

Unfortunately, farm recordkeeping data do not correctly reflect the interest cost environment of an individual firm. The data measure end of the year financial performance. Consequently, interest rates and interest rate variation are biased upward, especially for farms with large operating loans relative to longer- term debt. Instead, Federal Land Bank (FLB) and Production Credit Association (PCA) interest rate data (1980-1985) are used to approximate expected interest rates (Table 2). FLB and PCA data reflects the typical interest rate environment encountered by the firm, i.e., the lender practices of negotiating the operating loan annually and adjusting the long-term financing interest rate rarely more than once a year. Variance is calculated using six annual observations from each of the two sources. A weighted average of the two is used for interest rate variance (70 percent long term and 30 percent for intermediate and short term).

The average coefficient of variation over the eighteen farms for rate of return (86.34) is about seven times larger than the weighted average interest rate coefficient of variation (12.56) over the six years. Greater risk is experienced from changing farm income than from changes in the cost of debt. Covariance of returns and interest cost is measured using correlation and standard deviations. The data derived correlation between rates of return and interest (0.10) was not statistically significant. An assumption of no correlation would result in a loss of information so, 0.10 is used.

The risk-adjusted discount rate is the real interest rate on debt (5.94 percent) plus the risk premium for solvency (3.43 percent). The risk premium is an estimated beta coefficient of farm solvency (9.53 percent) multiplied by the farm's beginning debt/equity (0.36). A probability distribution of farm solvency for the expected real interest rate and beginning debt/equity was generated from the Monte Carlo simulation. The probability distribution was regressed against time and debt/equity to yield beta.

The Cash family target consumption level could be thought of as a required return to management, family labor, and capital. In the presented scenerio, the farmer reinvests 95 percent of farm income defined by equation 4.

Results and Analysis

An intertemporal equilibrium solution was found using the initial balance sheet, finance, and risk data from Tables 1 and 2 (Table 3). Negative characteristic roots indicate that the solution is stable and the time paths of debt and equity move toward equilibrium. At equilibrium, total farm assets increased by $66,079, or by about 16 percent. The growth in assets was equally divided between increases in debt by $32,288 and equity by $33,791. Debt/equity rose from 0.36 to 0.41 and debt/assets increased from 0.26 to 0.29.

The increase in farm asset size at equilibrium is consistent with farm size trends in the state. However, the control model results indicate that near term growth is dependant on a reduction in debt to provide the financial strength for this increase in assets (Figure 1). Even though returns are above interest cost, we expect the risk aversion, business risk and financial risk measures in the model to have a negative impact on debt until equity position improves. The next section of this paper presents a sensitivity analysis of the equilibrium. This analysis is used to examine the validity of the model results and show the impact of changes in financial variables on financial structure.

Sensitivity Analysis

The behavior of equilibrium equity and debt when one variable at a time is changed from the initial financial state (Tables 1 and 2) is examined in this section. Equilibrium levels are shown for efficiency, since there is a unique path to each equilibrium for each value of the variable being evaluated. The objectives of this sensitivity analysis are to: 1) verify the structure of the model by determining if results are consistent with the comparative static analysis of farm financial structure; 2) examine relationships between income, capital structure, and risk and their effects on farm profitability, growth, and survival, and 3) discuss effective managerial and/or policy responses to financial stress suggested by the results.

Equilibrium farm capital structure is highly sensitive to changes in the expected real rate of return to assets (r). Varying (r) from 7.5 to 9.9 percent indicates returns are positively correlated with farm size. At the low end, the farm is financed totally by equity. At the high end, returns exceed risk adjusted interest rates and 100 percent debt financing is optimal (Figure 2). A positive relationship between equilibrium debt and (r) is consistent with the comparative static outcome. The intertemporal equilibrium values of equity and debt become nonsensical when (r) is sufficiently close to the interest rate (i) to force the farm to be financed with negative debt. This is consistent with investment theory. A rational decision maker would not invest in the farm operation when this relationship is expected. An increase in the difference between (r) and (i) results in a larger equilibrium farm asset size and an increase in the steady state debt-equity ratio. These imply increased income and increased equilibrium risk level. Leverage is greater than one when returns are greater than 8.3 percent.

Altering the interest rate (i) from 3.9 to 6.4 percent yields an inverse relationship with steady state debt and a positive relationship with steady state equity (Figure 3). Declining equilibrium debt with higher interest rates parallels the comparative static result. Interest rates above 6.4 percent produces negative equilibrium debt values and presents an impossible investment option to the decision maker. Debt/asset reaches 1.00 when (i) approaches 4.0 percent. The ratio of debt-to-equity is greater than one when (i) less than 5.5 percent.

These outcomes suggest that debt financing as an investment option becomes too risky when expected (i) approaches expected (r). The ability to accumulate net worth through equity and increase farm asset size is clearly impaired as the gap between (r) and (i) becomes smaller or negative. This relationship is consistent with empirical studies suggesting that an increase in (i) reduces income for consumption-savings purposes and lowers the rate at which equity can be accumulated.

If returns exceed interest cost and there is an absence of decision maker risk aversion and other constraints, the optimum levels of debt and equity are infinite. However, non-zero risk aversion makes optimal farm size finite. As risk aversion rises, the concern for insolvency requires a lower leverage position for the farm. The use of debt as an investment option decreases dramatically. Changing the risk aversion coefficient (α) from the initial value yields an inverse relationship with equilibrium debt (Figure 4). The behavior of steady state debt with risk aversion is consistent with the comparative static result. Optimum farm asset size is also inversely related to risk aversion. A risk aversion coefficient greater than 0.000016 results in a steady state debt-equity ratio less than one.

Policymakers have traditionally focused on farm income and input cost subsidization as ways to improve farm survival. The two primary policy tools used are target/loan price and interest rate subsidies. However, these programs have unintended effects on decision maker risk attitudes. The analysis shows that subsidies designed to increase returns and/or reduce interest costs provide incentives to use more debt and increase farm size. The effect is to increase exposure to risk and reduce the probability of survival. Clearly, the events of the 1970s and 1980s show how farmers react to such incentives. Perhaps diverting some federal expenditures to education programs to improve farmer understanding of the trade-offs between debt and survival would be more effective.

Another measure of risk is variance of returns to farm assets before interest cost (σ_r^2). Varying σ_r^2 from 1.4 to 5.4 percent demonstrates a negative relationship with steady state debt and equity (Figure 5). These negative relationships are consistent with the behavior of the risk aversion coefficient. Further the association of debt with variance is consistent with the comparative static result. The optimal debt level decreases more rapidly than equity with increases in business risk. Equilibrium debt/equity becomes less than one when variance is greater than 3.18 percent. Although equity increases with variances of returns, farm asset size declines. This implies that return stabilizing farm policies encourage greater leverage and larger farms.

The sensitivity of the variance of interest rate (σ^2_i) from the representative level of 0.01 percent to more than 2 orders of magnitude larger at 2.81 percent is very small (Figure 6). The resulting negative relationship with debt parallels the comparative static outcome. The equilibrium debt-equity ratio declines from 0.42 to 0.36 over the range examined. Optimal farm asset size is also inversely related to interest variance.

Rising variance is generally considered to increase the farm decision maker's exposure to risk. Model results clearly show the significant impact that business risk has on farm capital structure. Results demonstrate that volatile farm income conditions that put a farmer's equity at greater risk may warrant a smaller farm asset size and debt-equity ratio. This relationship conflicts with the establishment or growth stage farmer goal of increasing farm size through debt financing. However, subsidized interest rates to beginning farmers would promote growth that would be rational and affordable. The impact of interest rate variation on financial structure is less in the model. FLB and PCA interest rate data may not completely reflect the risk from debt financing.

Correlation between returns and interest cost was varied between -1.0 and 1.0 (corresponding to covariance of -.2075 percent and .2075 percent) to analyze the impact on capital structure. Although equity declines, covariance has a slight positive impact on equilibrium debt and farm size (Figure 7). Barry et al. (1981) state that larger correlation between (r) and (i) causes lower portfolio variance. Smaller portfolio variance should allow a larger equilibrium farm size and increase the ability to carry more debt. Decision makers interested in farm size growth would want to choose a portfolio of assets that minimizes variance. The covariance between (r) and (i) has the expected but very small impact on size of operation $484,741 to $486,189 and leverage (debt/equity of .411 to .418). This response is probably small because of the very small variance in interest rates in the initial model.

Summary and Conclusions

The purpose of this paper is to present a dynamic model of the debt/equity structure of the farm firm. An optimal control model is developed from determinants of farm financial structure. A representative financial situation is constructed for a typical North Dakota grain farm. Equilibrium farm financial structure results indicate a larger asset size farm with a higher debt-equity ratio than at the initial state. The calculated time path to equilibrium indicates an initial reduction in debt level to enable equity growth. Where measurable, dynamic results were consistent with comparative static outcomes. The rate of return, interest rate, risk attitude, and variance of rate of return have the most significant impact on farm capital structure, profit and survival. Variance of interest rate and covariance between return in assets and interest rate have little impact.

The management implications consistent with static theory is that growth-oriented managers should seek low interest rates and high, consistent returns on assets. Variation in interest rate is of minor consequence compared to average level of interest. In other words, the good manager should be willing to accept significant variation in interest rates through variable rates in exchange for a lower expected rate.

One policy implication is that increased efforts to make farmers more sensitive to risk would have a higher payoff in terms of farm survival and keeping the equilibrium number of farms close to current numbers.

Using empirical data in a very basic optimal control model provides reasonable results with applications and meaningful implications. The dynamic framework provides a realistic adjustment prescription for the farm manager while maintaining a manageable level of required information.

Footnote

[1] $\dot{E}(t)$ and $\dot{D}(t)$ are the partial derivatives of equity and debt with respect to time.

TABLE 1. INITIAL FINANCIAL BALANCE SHEET FOR A REPRESENTATIVE NORTH DAKOTA GRAIN FARM, (1986)

Assets	
Long-Term Assets (Land, Buildings, etc.)	$293,619
Intermediate- and Short-Term Assets (Machinery, Equipment, Inventory, Financial Reserves, etc.)	$125,836
Total Assets	$419,455
Liabilities	
Long-Term Debt	$ 77,028
Short- and Intermediate-Term Debt	$ 33,012
Total Liabilities	$110,040
Net Worth	
Operator's Equity	$309,415
Financial Ratios	
Debt-to-Asset Ratio	0.26
Debt-to-Equity Ratio	0.36

Source: Rourke 1988.

TABLE 2. EXPECTED FINANCE AND RISK VARIABLES FOR THE REPRESENTATIVE NORTH DAKOTA GRAIN FARM, (1986)

Item	Value
Rate of Return to Farm Assets[a]	0.1847
Variance	0.0428
Standard Deviation	0.2069
Coefficient of Variation[b]	86.34
Change in Farm Asset Values[c]	-0.045
Expected Inflation Rate[d]	0.0605
Interest Rate[e]	
Long-term Debt (Federal Land Bank)	0.076
Variance	0.000049
Standard Deviation	0.007686
Coefficient of Variation	10.11
Intermediate and Short-Term Debt (Production Credit Association)	0.089
Variance	0.000221
Standard Deviation	0.016288
Coefficient of Variation	18.30
Risk-Adjusted Discount Rate	0.0937
Family target consumption	$30,000
Risk Aversion Coefficient	0.00002
Marginal Propensity to Reinvest Farm Income	0.95

Source: [a]Before interest payments, Rourke, 1988.
[b]Calculated by summing the coefficient of variation for each farm and dividing by 18, the number of farms.
[c]North Dakota Agricultural Statistics, various issues.
[d]Council of Economic Advisors to the President, 1986.
[e]Agricultural Statistics, various issues.

TABLE 3. INTERTEMPORAL EQUILIBRIUM FINANCIAL BALANCE SHEET FOR THE REPRESENTATIVE NORTH DAKOTA GRAIN FARM (1986)

Assets	
Long-Term Assets	$339,874
Short- and Intermediate-Term Assets	$145,660
Total Assets	$485,534
Liabilities	
Long-Term Debt	$ 99,630
Short- and Intermediate-Term Debt	$ 42,698
Total Liabilities	$142,328
Net Worth	
Operator's Equity	$343,206
Financial Ratios	
Debt-to-Asset Ratio	0.29
Debt-to-Equity Ratio	0.41
Characteristic Root Values	
r1	-0.0242
r2	-0.0565

References

Arrow, Kenneth J. Applications of Control Theory to Economic Growth. Technical Report No. 2, Institute of Mathematical Studies in Social Sciences, Stanford University, 1967.

Baker, C. B. "Credit in the Production Organization of the Firm."American Journal of Agricultural Economics. 50(1968):507-520.

Baker, C. B. and John A. Hopkin. "Concepts of Finance Capital for a Capital-Using Agriculture." American Journal of Agricultural Economics. 51(1969): 1055-1064.

Barry, Peter J. and C. B. Baker. "Reservation Prices on Credit Use: A Measure of Response to Uncertainty." American Journal of Agricultural Economics. 53(1971): 222-227.

Barry, Peter J. and C. B. Baker. "Financial Responses to Risk in Agriculture." Risk Management in Agriculture, Peter J. Barry Editor, Iowa State University Press, Ames, Iowa, 1984, pp. 183-199.

Barry, Peter J., C. B. Baker, and Luis R. Sanint, "Farmers' Credit Risks and Liquidity Management." American Journal of Agricultural Economics, 63(1982):216-227.

Barry, Peter J., Paul N. Ellinger, and Vernon R. Eidman. "Firm Level Adjustments to Financial Stress." Agricultural Finance Review. 47(1987):72-99.

Barry, Peter J. and Donald R. Fraser. "Risk Management in Primary Agricultural Production: Methods, Distribution, Rewards, and Structural Implications." American Journal of Agricultural Economics. 58(1976): 286-295.

Boehlje, Michael D. and T. Kelly White. "A Production-Investment Decision Model of Farm Firm Growth." American Journal of Agricultural Economics. 51(1969): 546-564.

Collins, Robert A. "Expected Utility, Debt-Equity Structure and Risk Balancing." American Journal of Agricultural Economics. 67(1985a): 627-629.

Collins, Robert A. "A Dynamic Model of the Debt-Equity Structure of a Proprietary Firm." Risk Analysis for Agricultural Production Firms: Concepts, Information Requirements and Policy Issues, Department of Agricultural Economics, Michigan State University, East Lansing, Michigan, Staff Paper 85-85, November 1985b, pp. 151-159.

Council of Economic Advisors to the President. Economic Report of the President. U.S. Government Printing Office, July 1986.

Featherstone, Allen M., and Timothy G. Baker. "An Examination of Farm Sector Real Asset Dynamics: 1910-85." American Agricultural Economic Journal, 69(1987):532-546.

Freund, Rudolf J., "The Introduction of Risk into a Programming Model," Econometrica, 24(1956):253-263.

Gabriel, Stephen C. and C.B. Baker. "Concepts of Business and Financial Risk." American Journal of Agricultural Economics. 62(1980):560-564.

Kamien, Morton I. and Nancy L. Schwartz. Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management. North Holland, New York, 1981, pp. 331.

Johnson, James D, Mitchell J. Morehart, and Kenneth Erickson. "Financial Conditions of the Farm Sector and Farm Operators." Agricultural Finance Review. 47(1987):1-18.

King, Robert P. and Lindon J. Robison. "Risk Efficiency Models." Risk Management in Agriculture, Peter J. Barry Editor, Iowa State University Press, Ames, Iowa, 1984, pp. 68-81.

Markowitz, H. M. Portfolio Selection. John Wiley and Sons, New York, 1959.

Pontryagin, L.S., R.V. Boltyanshii, R.V. Gambreldzi, and E.F. Misenken. The Mathematical Theory of Optimal Processes. Oxford: Pergamon Press, 1964.

Patrick, George F. and Ludwig M. Eisgruber. "The Impact of Managerial Ability and Capital Structure on Growth of the Farm Firm." American Journal of Agricultural Economics. 50(1968):491-506.

Robison, Lindon J. and Peter J. Barry. The Competitive Firm's Response to Risk. Macmillon, 1986, pp. 336.

Robison, Lindon J. and J. R. Brake. "Application of Portfolio Theory to Farmer and Lender Behavior." American Journal Of Agricultural Economics. 61(1979):158-64.

Robison, Lindon J., Peter J. Barry, and William G. Burghardt. "Borrowing Behavior under Financial Stress by the Proprietary Firm: A Theoretical Analysis." Western Journal of Agricultural Economics. 12(1987):144-151.

Rourke, Gary. "Capital Structure and Financial Management Practices of North Dakota Farmers." Unpublished Masters thesis, North Dakota State University, Fargo, 1988.

Selly, Roger. "Decision Rules in Risk Analysis." Risk Management in Agriculture, Peter J. Barry Editor, Iowa State University Press, Ames, Iowa, 1984, pp. 53-67.

Shepard, Lawrence E. and Robert Collins. "Why do Farmers Fail? Farm Bankruptcies 1910-1978." American Journal of Agricultural Economics. 64(1982):609-616.

Tobin J. L. "Liquidity Preference as a Behavior toward Risk." Review of Economic Studies. 25(1958):65-86.

Tweeten, Luther G. "Theories Explaining the Persistence of Low Resource Returns in A Growing Farm Economy." American Journal of Agricultural Economics. 51(1969): 798-817.

U.S. Department of Agriculture, Economic Research Service. Agricultural Statistics, Various Issues. U.S. Government Printing Office

U.S. Department of Agriculture, Statistical Reporting Service. North Dakota Agricultural Statistics, Various Issues.

Zilberman, David. "The Use and Potential of Optimal Control Models in Agricultural Economics." Western Journal of Agricultural Economics. 7(1982):395-406.

Watt/4.2/dynamic.doc
mmu/1-89

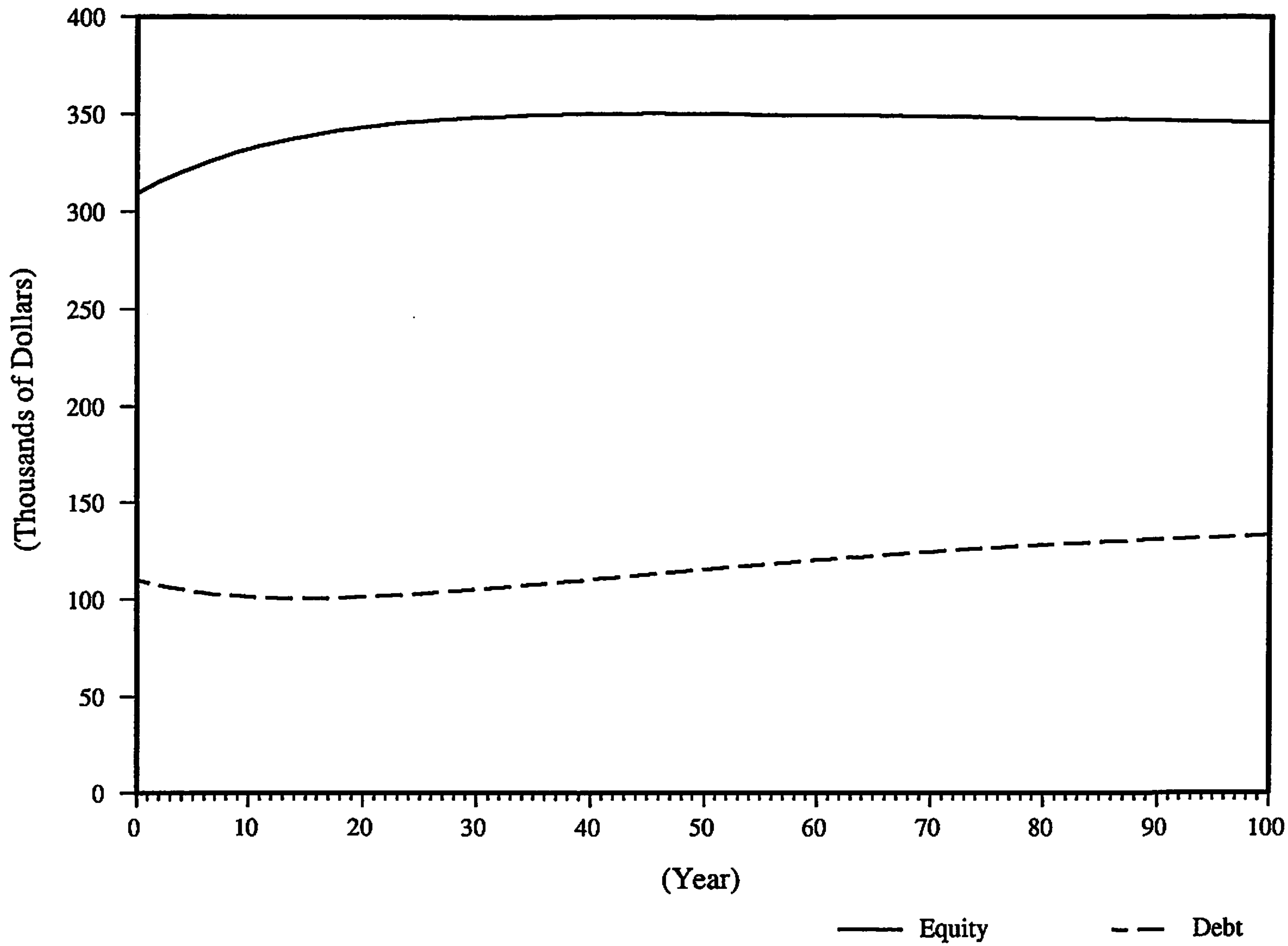

Figure 1. Time Paths of Debt and Equity to Equilibrium

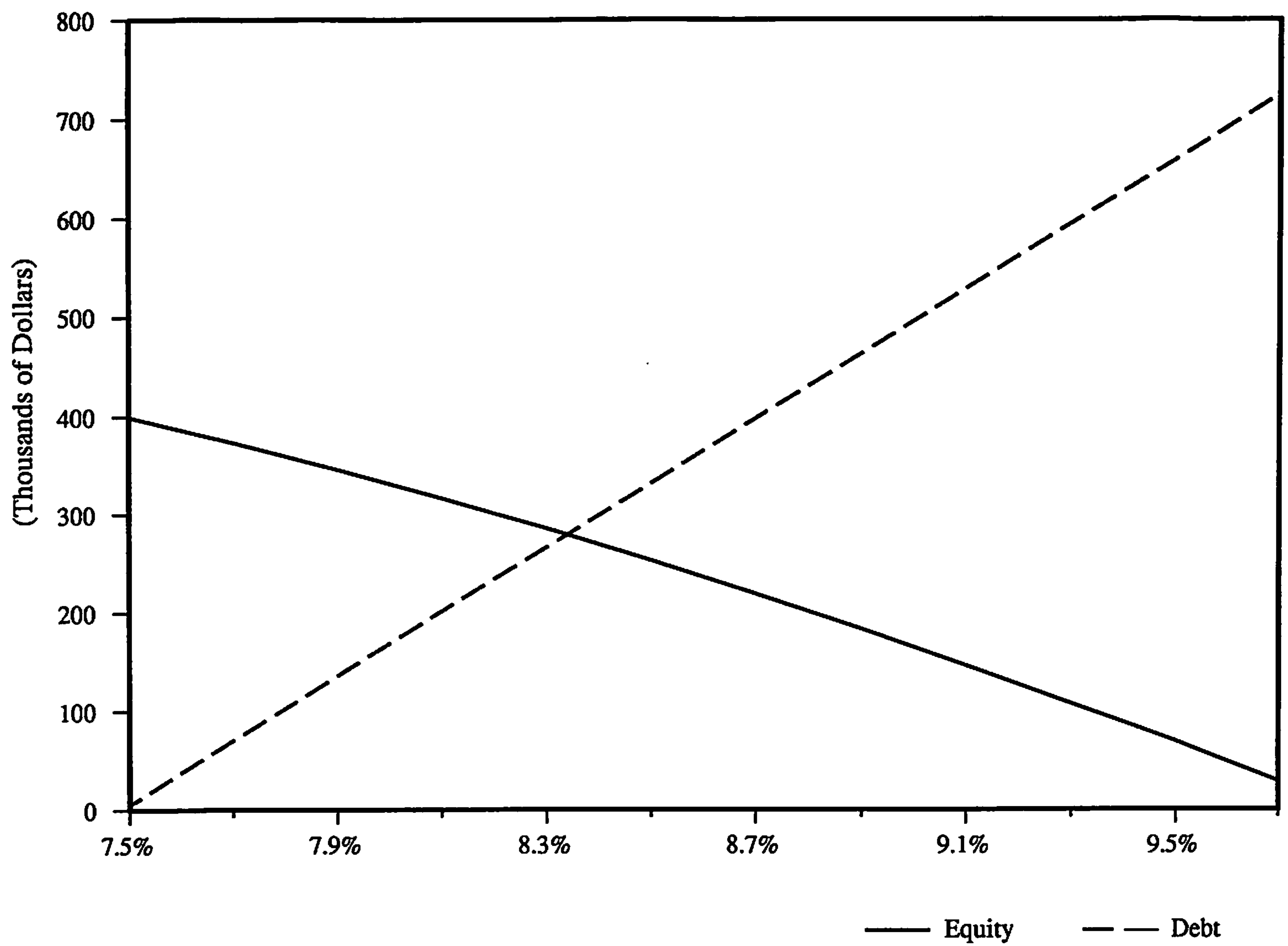

Figure 2. Rate of Return to Farm Assets Before Interest Cost

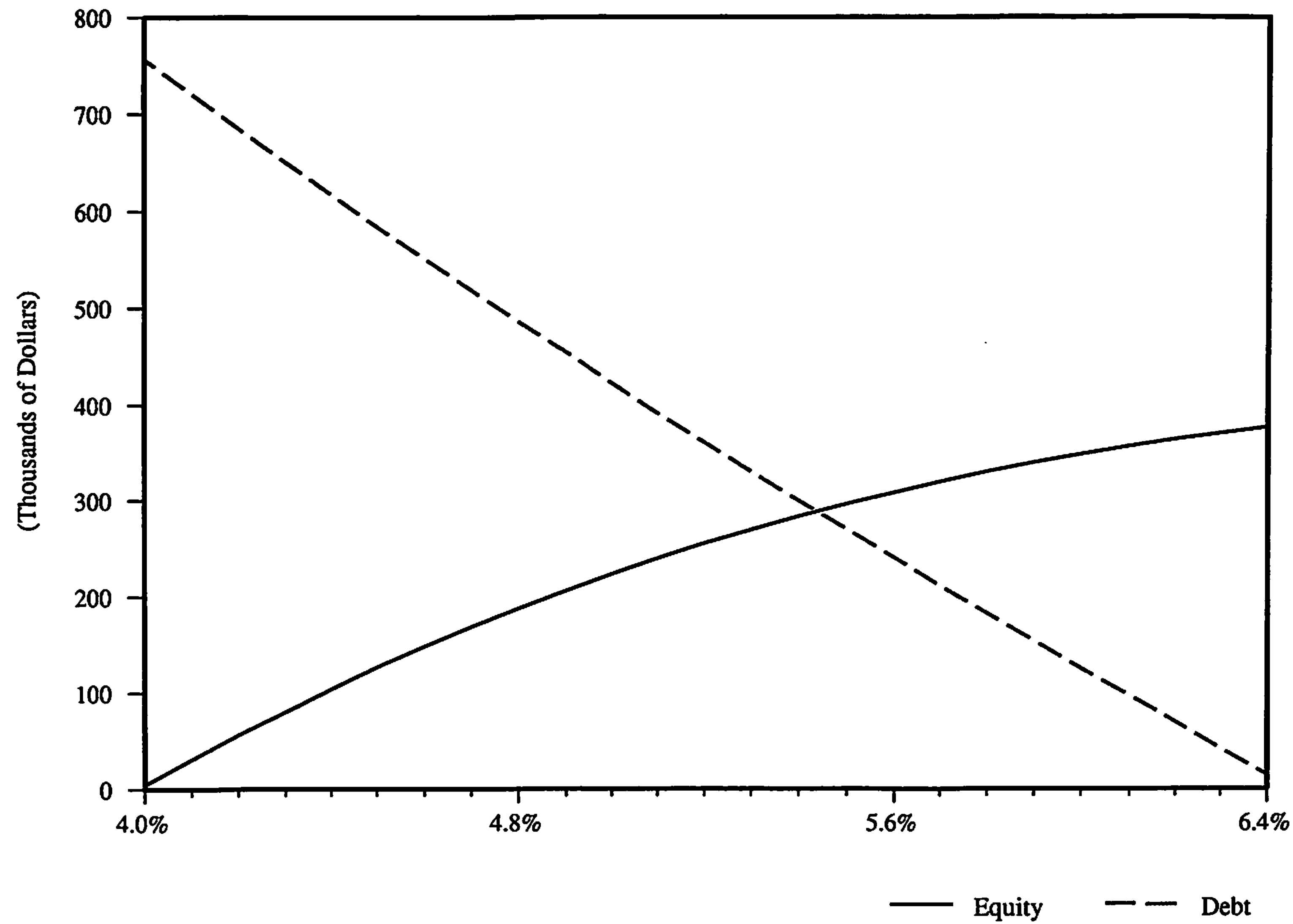

Figure 3. Interest Rate on Farm Debt

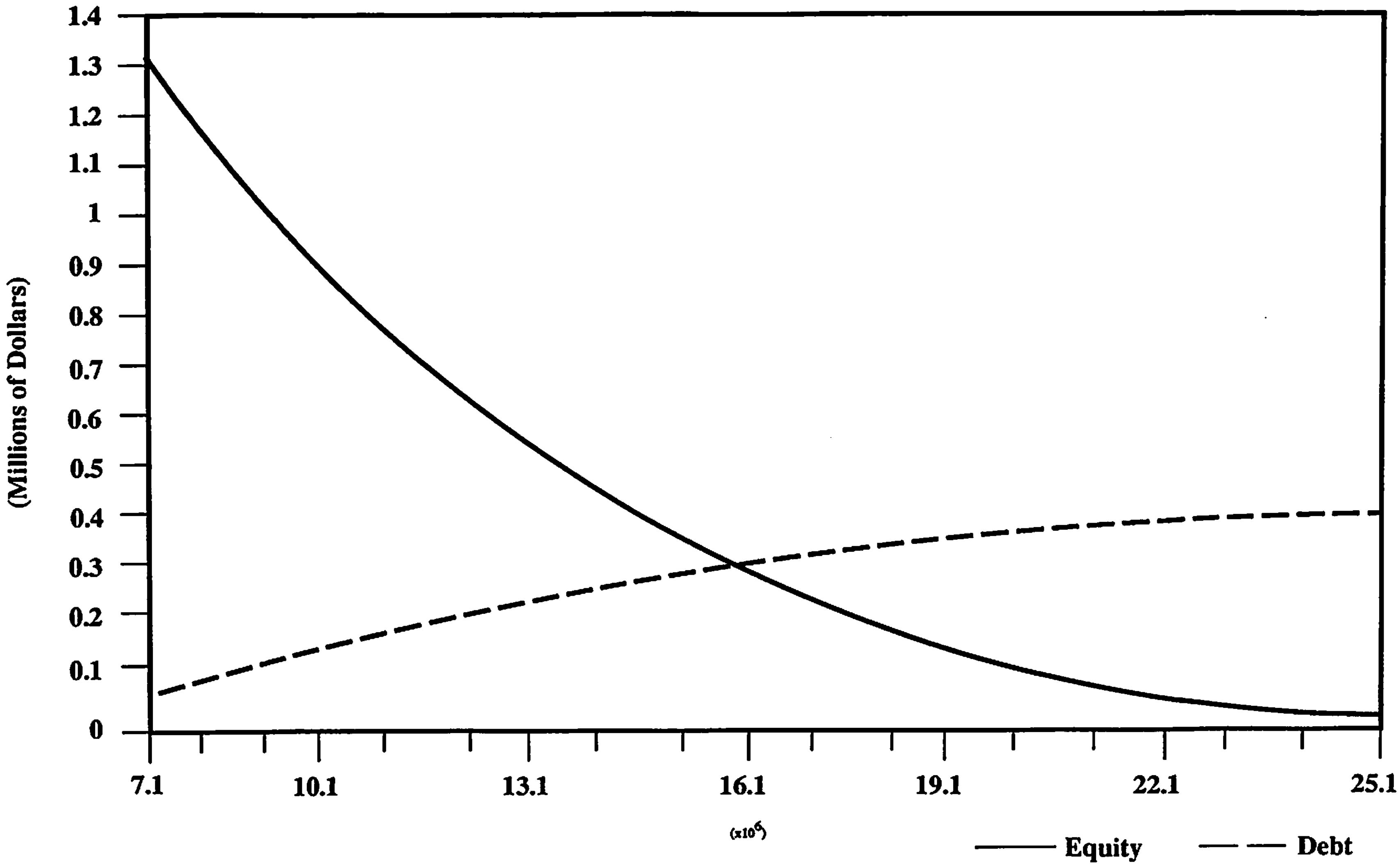

Figure 4. Risk Aversion Coefficient

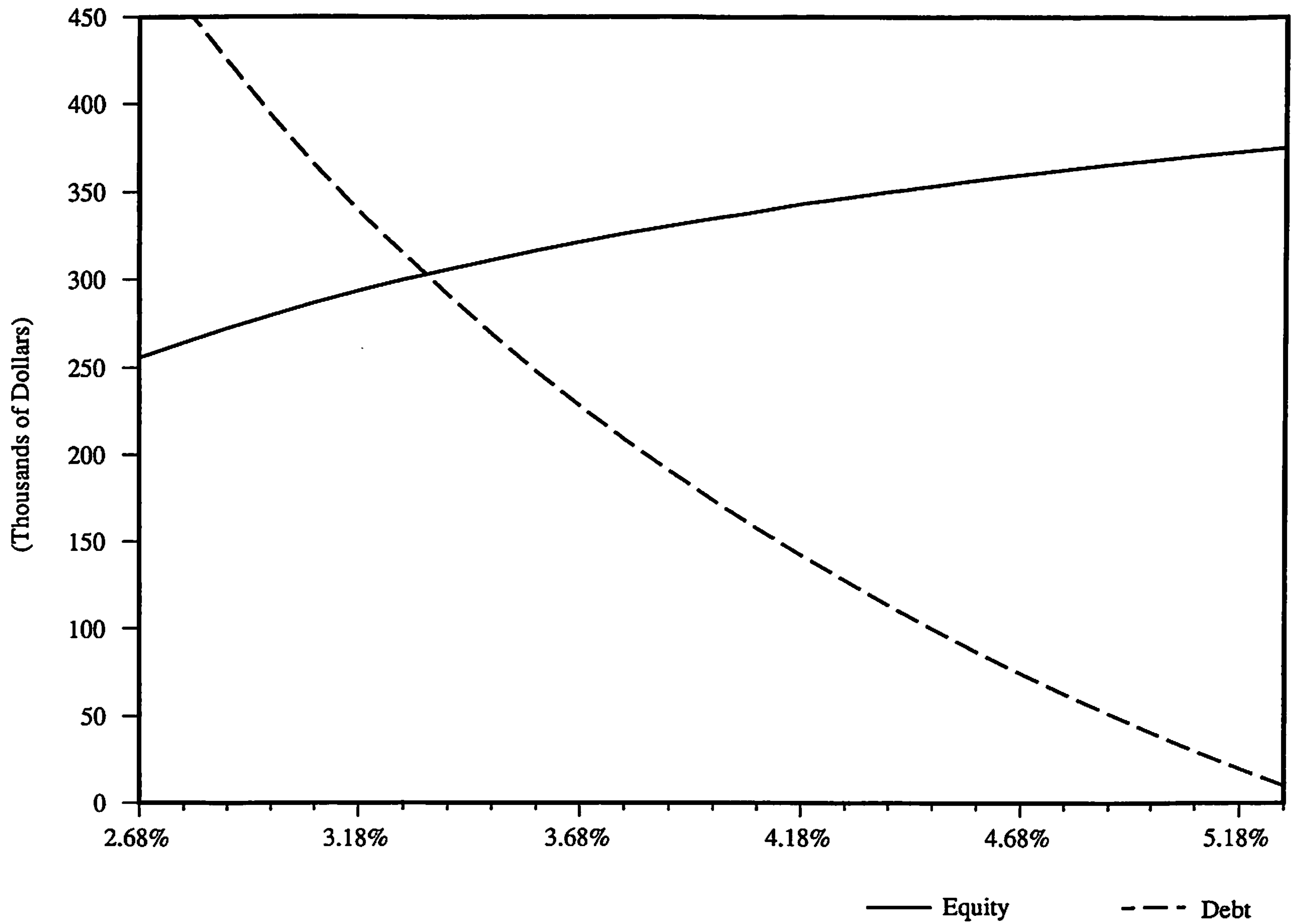

Figure 5. Variance of Rate of Return to Farm Assets Before Interest Cost

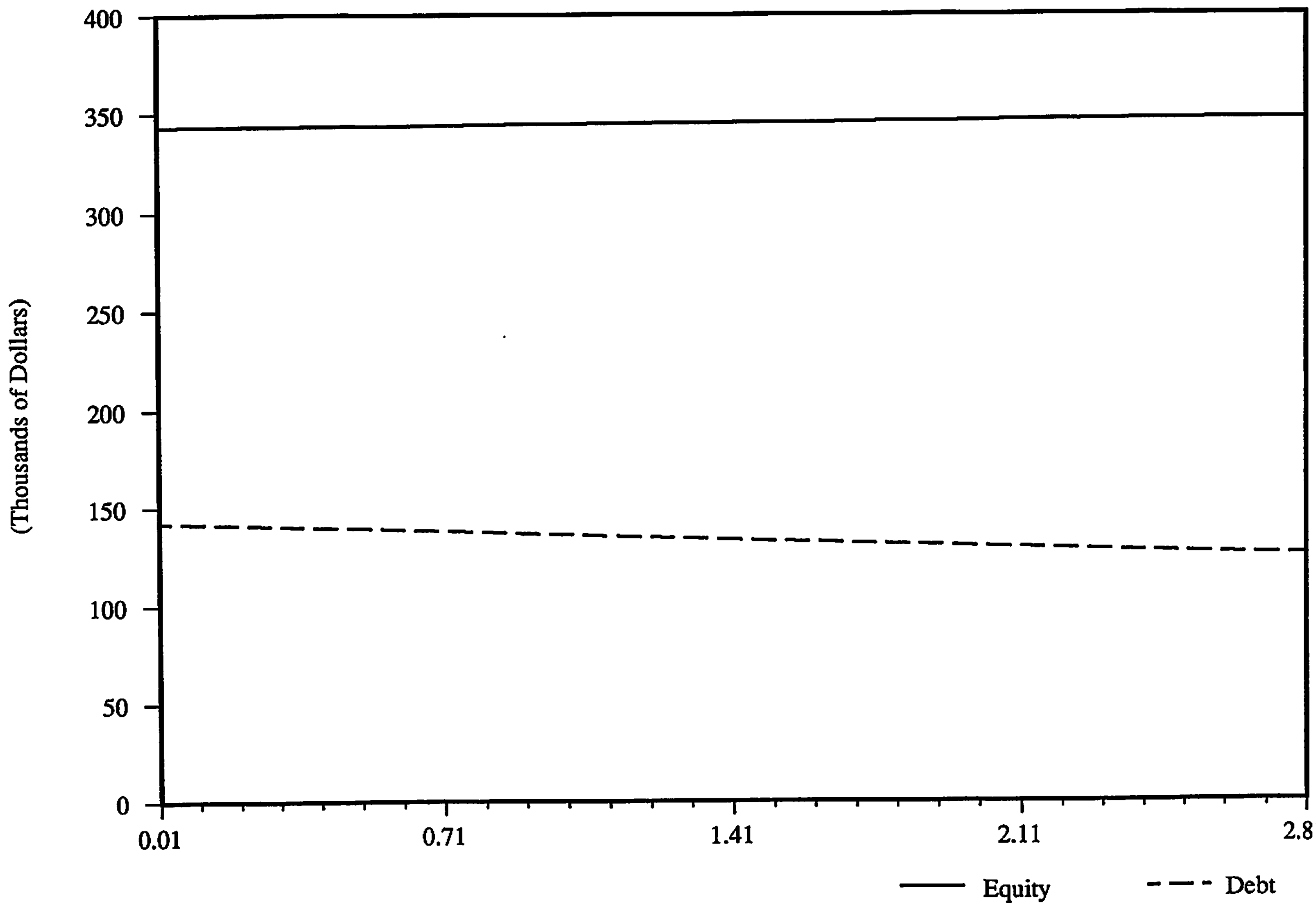

Figure 6. Variance of Interest Rate on Farm Debt

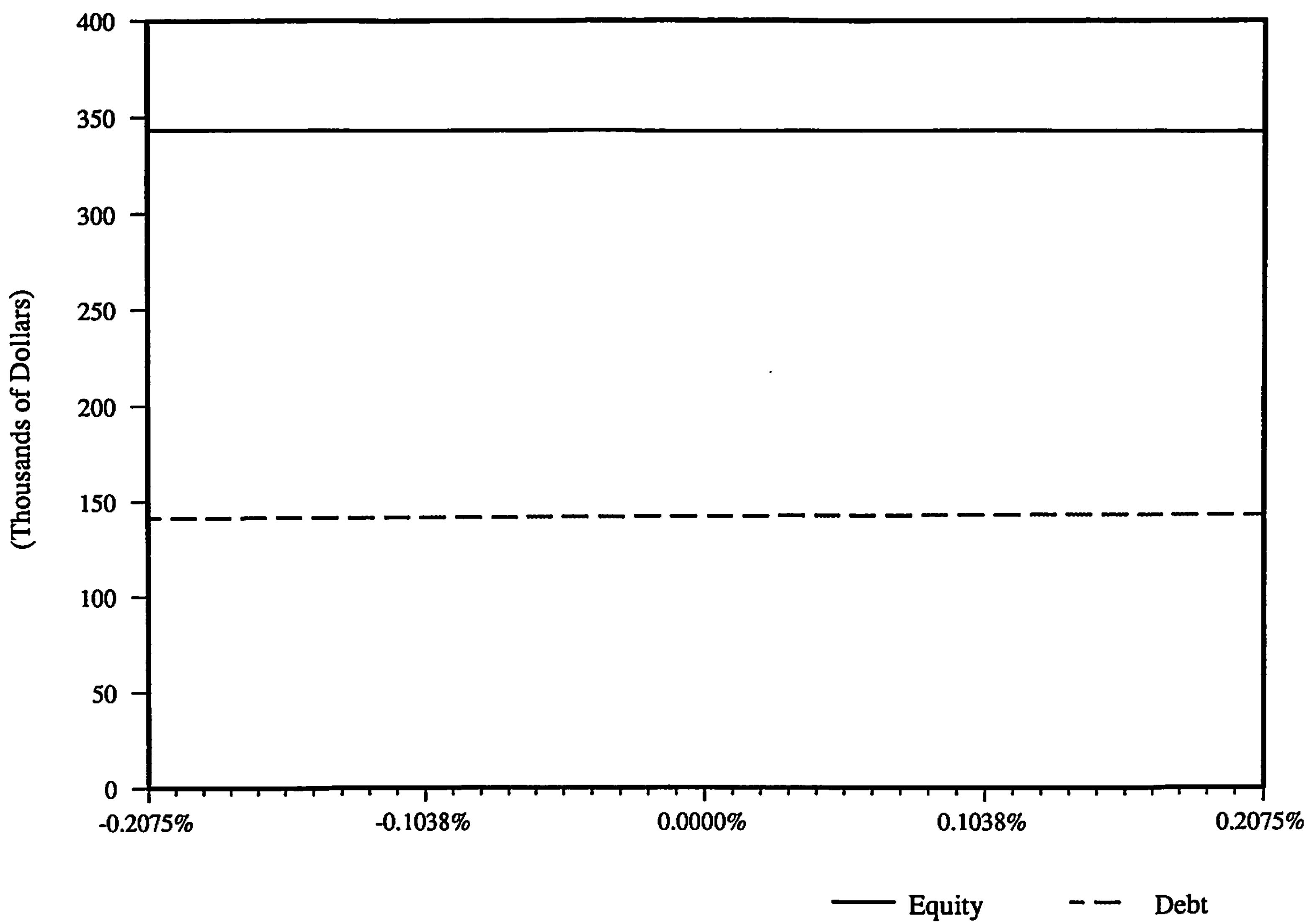

Figure 7. Covariance of Rates of Return and Interest