Optimal Portfolio Management of Urban Water*

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October 2013

Abstract

Cities have been investing heavily in recent years to augment their water supply, focusing on either stormwater harvesting or desalination. A more optimal strategy is to consider all sources of water jointly, thereby allowing for hedging of supply risks. A portfolio model of urban water supply is derived, consisting of reservoirs, stormwater harvesting and desalination, which takes into account the uncertainties of water flows and rainfall as well as the relative costs of associated with the three types of water sources. Calibrating the model to Melbourne's existing water supply system, a significant result is that optimal contributions are dependent on current water stocks, thereby providing insights into the appropiateness of historical water augmentation decisions. It is found that, with the exception of reservoirs, observed contributions deviate from optimal contributions for stormwater harvesting and desalination with the results suggesting a need for future investments to target stormwater harvesting ahead of desalination technology. Moreover, the optimal portfolio is found to vary throughout the year with desalination being the preferred supplementary supply source to reservoirs during summer months, while harvested stormwater is the preferred choice during winter. This result has implications for augmentation investments to mitigate seasonal water supply shortages.

KEYWORDS: urban water supply, portfolio choice, stormwater harvesting, desalination, reservoir

*Acknowledgements: We would like to thank to the Victorian Centre for Climate Change Adaptation Reserach (VCCCAR) for financial support. We also acknowledge helpful comments on earlier versions of the paper by Pedro Gomis-Porqueras, John Freebairn and participants at the annual conferences of the European Association of Environmental and Resource Economists, and the Australian Agricultural and Resource Economics Society.

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1 Introduction

The majority of cities around the world experience some form of water shortage, where urban water demand exceeds supply now, in the near, or medium term. Climate change is likely to exacerbate this problem further. This problem is recognized by water managers as highlighted by the recent surge in investments to secure future water supply (Rygaard et al., 2011). However, what type of water supply infrastructure should be invested in is far from obvious, as the optimal portfolio of assets is determined by current and projected precipitation and inflow patterns as well as a number of other economic and natural factors.

The main contribution of this paper is to consider all water sources jointly, by specifying a dynamic portfolio model to determine the optimal contributions to total water supply from three types of assets: desalination or water recycling plants, water reservoirs and stormwater harvesting systems. The flow of water from each asset is modelled as a gamma distribution, which is characterized by an average supply flow and a measure of uncertainty. To allow for differences in water supply technologies the contributions of each asset to the total water stock are adjusted by supply costs. A fundamental result of the paper is that optimal contributions to total water stock depend on current water stock, thereby providing insights into the appropriateness of historical water augmentation decisions. Calibrating the model to an existing water supply system, the results show that, with the exception of reservoirs, observed shares deviate from optimal shares for stormwater harvesting and desalination. In particular, the results suggest a need for future investments to target stormwater harvesting ahead of desalination technology. Moreover, the optimal portfolio is found to vary throughout the year with desalination being the preferred supplementary supply source to reservoirs during summer months, while harvested stormwater is relative more important during winter. This result has implications for augmentation investments to mitigate seasonal water supply shortages.

Importantly, the modelling framework takes into account a number of characteristics that distinguish the three types of water supply assets. The water supply from desalination and water recycling plants, while being subject to high unit costs, can be guaranteed in as much as it is independent of rainfall. As a result, investment in such assets may be considered as risk-free. The amount of water harvested in large reservoirs and decentralized stormwater harvesting systems, on the other hand, is dependent on technological as well as natural factors, in particular inflows and rainfall. Importantly, the decentralized nature of stormwater harvesting implies that rain may be captured when and where it falls as opposed to reservoirs, which are subject to locality constraints. Furthermore, the high degree of impermeability of urban centres means that a much larger proportion of rainwater may be harvested even after long periods of no rain. By contrast, most of the rain falling in a reservoir catchment area after a lengthy period of dry and warm weather may evaporate or be absorbed by the vegetation and in the soil. Hence, reservoirs and stormwater harvesting systems are likely to differ in the risk characteristics, whereas desalination plants bear zero supply risk. By design stormwater harvesting systems are small in scale compared with reservoirs, while desalination or recycling plants tend to produce water on demand.

The management of urban water supply infrastructure is studied extensively in the literature. Oezelkan et al. (1997) solve the problem of optimal investment and management of a water reservoir under supply uncertainty, where water flows are assumed to be normally distributed, while Feiring et al. (1998) optimize water reservoir management for the dual purpose of supplying water and energy. The economic efficiency of alternative water supply assets is investigated by Pickering et al. (2007) for rainwater tanks and Salibya et al. (2009) Fletcher et al. (2007) for desalination technology. However, none of these water supply assets are mutually exclusive. Indeed a combination of different types of water supply assets may be necessary to generate the desired amount of water cost efficiently. This is recognized by Kirsch et al. (2009) and Kasprzyk et al. (2009), who investigate the role of tradeable water products in securing urban water supply in the short and medium term. In situations where urban water markets are either nonexistent or insufficiently developed, investing in water products to secure urban water supply may not be a realistic option. With any investment to secure future supply, important questions arise with respect to the optimal timing. In this context, the use of real option theory has been advocated to determine the optimal timing of investing in new reservoirs (Michailidis and Mattas, 2007) or desalination plants (Borison et al., 2008; PC, 2011). In contrast, this study determines the optimal composition of the urban water supply portfolio, which should proceed issues associated with the timing of investment.

The rest of the paper is organized as follows. Section 2 derives closed-form solutions for a three-asset model of water supply investments, where the flow returns from two assets are subject to risk as modelled by correlated but not identical gamma processes, following Wilks (1990) and Groisman et al. (1999). In contrast to financial portfolio models based on Merton (1969) where the optimal shares are constant, the optimal shares in the specified water model are dependent on water stocks. This model is applied to Melbourne's water system that recently came under stress due to a major drought, which led to the decision to build a large desalination plant. In Section 3, rainfall and reservoir inflow data for Melbourne from 1915 to 2010 are used to estimate the moments of the gamma distributions. These estimates are used in Section 4, along side parameter values that characterize Melbourne's water supply system, to calibrate the model and contrast water shares for different supply situations. Concluding remarks are provided in Section 5.

2 A Dynamic Model of Water Assets

A dynamic portfolio model of urban water is developed that optimizes the supply from three types of water sources. These consist of inflows into reservoirs with water stock $S_r = \kappa_r K_r$, where $0 \leq \kappa_r \leq 1$ is a proportion of the fixed reservoir capacity K_r which allows for situations where reservoir levels are below capacity. Two alternative sources of water are considered: harvested stormwater with stock $S_h = \kappa_h K_h$, where K_h is the annual capacity and $0 \leq \kappa_h \leq 1$ represents the proportion of the year when stormwater harvesting operates at full capacity. The final alternative water source is desalinated or recycled water with stock $S_d = \kappa_d K_d$, where K_d represents total production capacity and κ_d allows for the asset being under-utilized. An important feature of the model is the role of uncertainty from inflows and from rain for the total water stock as well as for cost heterogeneity when deriving an optimal portfolio of urban water supply.

2.1 Water Stock Dynamics and Consumption

The change in the stocks of reservoirs, harvested stormwater and desalination between t and $t + dt$, are specified respectively as

$$\begin{aligned} dS_r &= \mu_r dt + \sigma_r dz_r \\ dS_h &= \mu_h dt + \sigma_h dz_h \\ dS_d &= \mu_d dt + \sigma_d dz_d, \end{aligned} \tag{1}$$

where $\{\mu_r, \mu_h, \mu_d\}$ represent the average flows per annum of the three water sources, and $\{dz_r, dz_h, dz_d\}$ are random variables capturing the uncertainty in water flows over time with the strength of the uncertainty determined respectively by the annualized volatility parameters $\{\sigma_r, \sigma_h, \sigma_d\}$. The stochastic variables dz_i, $i = \{r, h, d\}$ are standardized to have 0 mean and variance dt. In the analysis that follows, it is assumed that desalination provides a reliable supply of water that is not subject to the uncertainty of rainfall and water inflows in general. Formally this amounts to imposing the restriction $\sigma_d = 0$, thereby effectively making water from desalination risk-free from the perspective of guaranteeing water flow. To allow for interactions amongst water flows of reservoirs and stormwater harvesting within the time interval dt, the shocks are assumed to be dependent, $dz_r dz_h = \rho dt$, where $\rho = \sigma_{r,d} / (\sigma_r \sigma_d)$, is the annualized correlation coefficient and $\sigma_{r,d}$ is the covariance between the two shocks.

Given the water stocks of reservoirs (S_r), harvested stormwater (S_h), and desalination (S_d), the total stock of water for a city with a given portfolio of water supply at time t, W, weighted by supply costs, is defined as

$$W = \sum_i \frac{c}{c_i} N_i S_i, \qquad i = \{r, h, d\}, \tag{2}$$

where N_i is the number of assets of water stock S_i held at time t, and cc_i^{-1} is the ratio of the average unit price of water to the average total cost of supply from asset i, which is assumed to be constant. Weighting the contribution to total water stock from asset i by its cost of supply recognizes the heterogeneity of water supply technologies and allows for the same amount of water supplied from a cheaper source to make a greater contribution to total weighted water stock than if it were supplied through a more expensive technology.

From (2) the change in the total water stock is given by

$$dW = \sum_i \frac{c}{c_i} dN_i S_i + \sum_i \frac{c}{c_i} dN_i dS_i + \sum_i \frac{c}{c_i} N_i dS_i, \qquad i = \{r, h, d\}. \tag{3}$$

The first two terms of this expression represent capital investments in water supply as they incorporate changes to the number of assets, dN_i, that draw from a particular water source. To capture the trade-off between such infrastructure expansions and short term consumption, capital investments needed to augment water supply in terms of consumption sacrifices in water are expressed as

$$\sum_i \frac{c}{c_i} dN_i S_i + \sum_i \frac{c}{c_i} dN_i dS_i = -\eta x dt, \qquad i = \{r, h, d\}, \tag{4}$$

where $x(t)$ is water consumption and η is a level parameter. The parameter η represents the rate of substitution between current consumption and the future consumption stream that results from the augmentation of the water supply.

By combining (3) and (4), it follows that

$$dW = \sum_i \frac{c}{c_i} \mu_i N_i dt + \sum_i \frac{c}{c_i} \sigma_i N_i dz_i - \eta x dt, \qquad i = \{r, h, d\}. \tag{5}$$

Let the physical share of water supply from asset i be defined as

$$\theta_i = \frac{c N_i S_i}{c_i W}, \qquad i = \{r, h, d\}, \tag{6}$$

where the $\theta_i s$ satisfy the normalization restriction

$$\theta_r + \theta_h + \theta_d = 1. \tag{7}$$

Using (6) in (5) the change in the stock of water is now expressed in terms of the shares

$$dW = \sum_i \mu_i \theta_i \frac{W}{S_i} dt + \sum_i \sigma_i \theta_i \frac{W}{S_i} dz - \eta x dt. \tag{8}$$

As desalinated water in (1) is treated as risk-free, the change in the total weighted water stock is rewritten as

$$dW = [a_r \theta_r W + a_h \theta_h W - \eta x] dt + \frac{\sigma_r}{S_r} \theta_r W dz_r + \frac{\sigma_h}{S_h} \theta_h W dz_h, \tag{9}$$

which uses the share normalization restriction in (7), and

$$a_r = \frac{\mu_r}{S_r} - \frac{\mu_d}{S_d} \tag{10}$$

$$a_h = \frac{\mu_h}{S_h} - \frac{\mu_d}{S_d}, \tag{11}$$

representing respectively the excess flows relative to stock from reservoirs and stormwater harvesting over desalination.

To complete the specification of water consumption x, an iso-elastic utility function with constant average risk aversion and zero transaction costs is assumed

$$U(x) = \eta \frac{x^{1-\gamma}}{1-\gamma}, \tag{12}$$

where γ is the constant relative risk aversion parameter. Consumers are risk averse for $\gamma > 0$, exhibiting logarithmic preferences for $\gamma = 1$, and relatively high risk aversion for $\gamma > 1$. Values of $\gamma < 0$, correspond to risk loving preferences.

2.2 Model Solution

The objective is to choose consumption (x) and the shares allocated to reservoirs (θ_r) and stormwater harvesting (θ_h) to maximize the present value of the utility stream from water consumption for a population growing at rate ξ and having a discount rate of δ

$$\begin{gathered} \max_{x,\theta_r,\theta_h} E \int_0^\infty \left(e^{(\xi-\delta)t} \eta \frac{x^{1-\gamma}}{1-\gamma} \right) dt \\ \text{subject to (9) and } W(0) = W_0. \end{gathered} \tag{13}$$

As the utility function in (12) and the water dynamics given in (9) are independent of t, the solution of (13) is based on solving the following dynamic programming problem (see, for example p.248 in Kamien and Schwartz (1981))

$$\begin{aligned} (\delta - \xi) V \quad = \quad & \max_{x,\theta_r,\theta_h} \left(\eta \frac{x^{1-\gamma}}{1-\gamma} + \left[a_r \theta_r W + a_h \theta_h W - \eta x \right] V_W \right. \\ & \left. + \frac{1}{2} \left(\theta_r^2 \frac{\sigma_r^2}{S_r^2} + \theta_h^2 \frac{\sigma_h^2}{S_h^2} + 2 \frac{\sigma_{r,h}}{S_r S_h} \theta_r \theta_h \right) W^2 V_{WW} + O\left(dt^2\right) \right), \end{aligned} \tag{14}$$

where V is the maximum obtainable value from the maximization problem defined above, and $\sigma_{r,h}$ is the covariance of the stochastic processes defined in (1). Maximizing with respect to x, expresses the optimal level of water consumption as

$$x = [V_W]^{-\frac{1}{\gamma}}. \tag{15}$$

Maximizing the right-hand side of (14) with respect to the shares associated with reservoirs and stormwater harvesting, θ_r and θ_h, yields the linear system of equations

$$\begin{aligned} a_r W V_W + \left(\frac{\sigma_r^2}{S_r^2} \theta_r + \frac{\sigma_{r,h}}{S_r S_h} \theta_h \right) W^2 V_{WW} &= 0 \\ a_s W V_W + \left(\frac{\sigma_h^2}{S_h^2} \theta_h + \frac{\sigma_{r,h}}{S_r S_h} \theta_r \right) W^2 V_{WW} &= 0, \end{aligned}$$

with solutions

$$\theta_r = -\left[k_r a_r - k a_h \right] \frac{V_W}{W V_{WW}} \tag{16}$$

$$\theta_h = -\left[k_h a_h - k a_r \right] \frac{V_W}{W V_{WW}}, \tag{17}$$

where

$$k_r = \frac{S_r^2}{\left(1-\rho^2\right) \sigma_r^2}, \quad k_h = \frac{S_h^2}{\left(1-\rho^2\right) \sigma_h^2}, \quad k = \rho \sqrt{k_r k_h}. \tag{18}$$

Substituting the optimality expressions (15), (16) and (17) for x, θ_r and θ_h respectively, into the left hand-side of (14), yields the ordinary differential equation

$$\left(\delta - \xi\right) V = \frac{\gamma}{\left(1-\gamma\right)} \eta \left[V_W\right]^{\frac{\gamma-1}{\gamma}} - \frac{1}{2} \left(k_r a_r^2 - 2 k a_r a_h + k_h a_h^2 \right) \frac{V_W^2}{V_{WW}}. \tag{19}$$

A closed-form solution for (19) is given by

$$V\left(W\right) = A W^{1-\gamma}, \tag{20}$$

where A is a function of the parameters of the model. To derive A, equation (20) is used to rewrite (19) as

$$\left(\left(1-\gamma\right) A\right)^{\frac{-1}{\gamma}} = \frac{\left(\delta-\xi\right)}{\gamma\eta} - \frac{\left(1-\gamma\right)}{\gamma^2} \frac{1}{2\eta} \left(k_r a_r^2 - 2 k a_r a_h + k_s a_h^2 \right), \tag{21}$$

which provides a solution for A.

The optimal control function for water consumption is obtained by using (20) and (21) in (15), to give

$$x = \frac{W}{\eta} \left[\frac{\left(\delta-\xi\right)}{\gamma} - \frac{\left(1-\gamma\right)}{\gamma^2} \frac{1}{2} \left(k_r a_r^2 - 2 k a_r a_h + k_h a_h^2 \right) \right]. \tag{22}$$

Similarly, using (20) and (21) in (16) and (17) yields

$$\theta_r = [k_r a_r - k a_h] \frac{1}{\gamma} \tag{23}$$

$$\theta_h = [k_h a_h - k a_r] \frac{1}{\gamma}, \tag{24}$$

as the optimal contributions of reservoirs, θ_r, and stormwater harvesting systems, θ_h, to total water supply. The optimal solution for the share allocated to desalination is $\theta_d = 1 - \theta_r - \theta_h$, by using the normalization condition (7).

2.3 Economic Interpretation

The optimal level of water consumption given in (22), and the optimal portfolio shares for reservoirs in (23) and harvested stormwater in (24), are all functions of the underlying parameters of the model. The properties of the model are discussed for the special case where the stochastic processes governing reservoir inflows and rainfall are independent, that is $\sigma_{r,h} = 0$. From (18) this means that $k = 0$, resulting in the optimal shares in (23) and (24) simplifying to

$$\theta_r = \frac{a_r}{\gamma} \frac{S_r^2}{\sigma_r^2} \tag{25}$$

$$\theta_h = \frac{a_h}{\gamma} \frac{S_h^2}{\sigma_h^2}. \tag{26}$$

These expressions show that the larger is the average excess flow of water for reservoirs and stormwater harvesting to desalination, the greater is the optimal contribution to total stock from the riskier sources of water

$$\frac{\partial \theta_r}{\partial a_r}, \frac{\partial \theta_h}{\partial a_h} > 0.$$

The magnitude of the optimal contributions in (25) and (26) are controlled by the size of the risk associated with each water source. A water source with the same excess water flow, but with higher risk in terms of reliability of water flow, would optimally have a smaller role in the portfolio

$$\frac{\partial \theta_r}{\partial \sigma_r}, \frac{\partial \theta_h}{\partial \sigma_h} < 0.$$

Moreover, the more risk averse society is with respect to water supply, the smaller are the contributions of the riskier sources of water relative to desalination

$$\frac{\partial \theta_r}{\partial \gamma}, \frac{\partial \theta_h}{\partial \gamma} < 0, \frac{\partial \theta_d}{\partial \gamma} > 0.$$

In the extreme case of infinite risk aversion the water portfolio is entirely based on desalinated water, $\theta_d = 1$, as

$$\lim_{\gamma \to \infty} \theta_r, \theta_h = 0.$$

An important feature of the optimal allocation of water is that the shares in general vary over time as they are a function of their respective water stocks,

$$\frac{\partial \theta_r}{\partial S_r}, \frac{\partial \theta_h}{\partial S_h} > 0.$$

This is in contrast to the optimal solution that arises from the Merton (1969) portfolio model, where the shares are constant. A sufficient condition for the allocations to be time invariant is that $\frac{\mu_r}{S_r}, \frac{\mu_h}{S_h}, \frac{\mu_d}{S_d}, \frac{\sigma_r}{S_r}, \frac{\sigma_h}{S_h}$ are constant. However, by allowing these ratios to vary, the current model allows for dynamic contributions over time to be a function of the initial conditions of water stocks.

The optimal consumption of water in (22) for the special case where inflows and rain are independent, $\sigma_{r,h} = 0$, is

$$x = \frac{W}{\eta} \left[\frac{(\delta - \xi)}{\gamma} - \frac{(1 - \gamma)}{2} \left(\frac{S_r^2}{\gamma^2 \sigma_r^2} a_r^2 + \frac{S_h^2}{\gamma^2 \sigma_h^2} a_h^2 \right) \right], \tag{27}$$

where the last term in round brackets represents the importance of risky water supply assets in the water supply portfolio. Closer examination of this expression reveals that for optimal consumption to be positive, γ must exceed a threshold, $\gamma_{\min}$, where $\gamma_{\min} = 1$ for $\delta = \xi$ and $\gamma_{\min} > 1$ when $\delta < \xi$. Over the range of positive consumption, $\gamma > \gamma_{\min}$, optimal consumption decreases in the level of risk aversion

$$\frac{\partial x}{\partial \gamma} < 0.$$

Equation (27) also shows that consumption increases linearly with the total stock of water in (2)

$$\frac{\partial x}{\partial W} > 0.$$

Finally, increases in the contribution to the total water stock from risky supply sources reduces optimal water consumption as

$$\frac{\partial x}{\partial \left(\frac{S_r^2}{\gamma^2 \sigma_r^2} a_r^2 + \frac{S_h^2}{\gamma^2 \sigma_h^2} a_h^2 \right)} < 0.$$

3 Empirical Inflows and Rainfall Distributions

The dynamic water flow equations in (1) that arise from either rainfall or inflows, are specified as functions of the mean parameters μ_i and the volatility parameters σ_i. To estimate these parameters a flexible empirical distribution is needed to capture the changing seasonal patterns of reservoir inflows and rainfall to capture flows of harvested stormwater over time. Following Wilks (1990) and Groisman et al. (1999) a gamma distribution is specified

$$g\left(r;\alpha,\beta\right) = \left(\frac{r}{\beta}\right)^{\alpha-1} \exp\left[-\frac{r}{\beta}\right] \frac{1}{\beta\Gamma\left[\alpha\right]}, \qquad r \geq 0, \quad \alpha,\beta > 0, \tag{28}$$

where r represents either rainfall or inflows, and α and β represent the shape and shift parameters respectively. Given the properties of the gamma distribution the means and standard deviations of the flow equations in (1), are related to the gamma parameters respectively as $\mu_i = \alpha\beta$ and $\sigma_i^2 = \alpha\beta^2$. An advantage of this choice of distribution is that is provides robust estimates of its moments as it takes into account extreme observations. The parameters of the gamma distribution are estimated by maximum likelihood methods by maximizing the following log-likelihood for a sample of T observations

$$\begin{aligned} \ln L &= \ln g\left(r_t;\alpha,\beta\right) \\ &= \left(\alpha - 1\right) \ln\left(\frac{r_t}{\beta}\right) - \frac{r_t}{\beta} - \ln\beta - \ln\Gamma\left[\alpha\right]. \end{aligned} \tag{29}$$

As the log-likelihood function in (29) is nonlinear in the parameters α and β, an iterative gradient algorithm is used to compute the maximum likelihood estimates where the derivatives of the gradients are computed numerically. All computations are performed using GAUSS Version 10, with the optimizer based on the software MAXLIK.

To estimate the parameters of the gamma distribution in (29) data on inflows to reservoirs, obtained from Melbourne Water, and on rainfall, obtained from the Australian Bureau of Meteorology, are used. The data on inflows consist of monthly reservoir inflows beginning January 1915 and ending December 2010, for the four major water reservoirs servicing Melbourne: Maroondah, O'Shannassy, Upper Yarra and Thomson. Aggregating across the four reservoirs for each month yields an aggregate monthly data set of total inflows into Melbourne's reservoirs. A plot of total inflows into all reservoirs per month is given in Figure 1 from 1915 to 2010.

The data on rainfall consist of daily precipitation from 1 January 1915 to 31 December 2010 for six weather stations across Melbourne: Lovely Banks, Meredith, Portarlington, Toorourrong, Yan Yean and Wallaby Creek. These stations are chosen for being High Quality Climate Sites, which are used for climate projections by the Bureau of Meteorology.[1] The daily data are converted into monthly rainfall by aggregating the daily rainfalls within each month. The monthly rainfall data are then averaged across the six stations for each month. Figure 2 provides a plot of the average rainfall data per month across the six sites from 1915 to 2010.

Table 1 provides some descriptive statistics on monthly reservoir inflows and rainfall. Inflows and harvested stormwater exhibit large variations over the year. August has the highest inflows into reservoirs with a monthly average of

[1] A map of HQCS Network in Victoria is available from the Australian Government Bureau of Meteorology at http://www.bom.gov.au/climate/change/hqsites/; accessed 12/12/11).

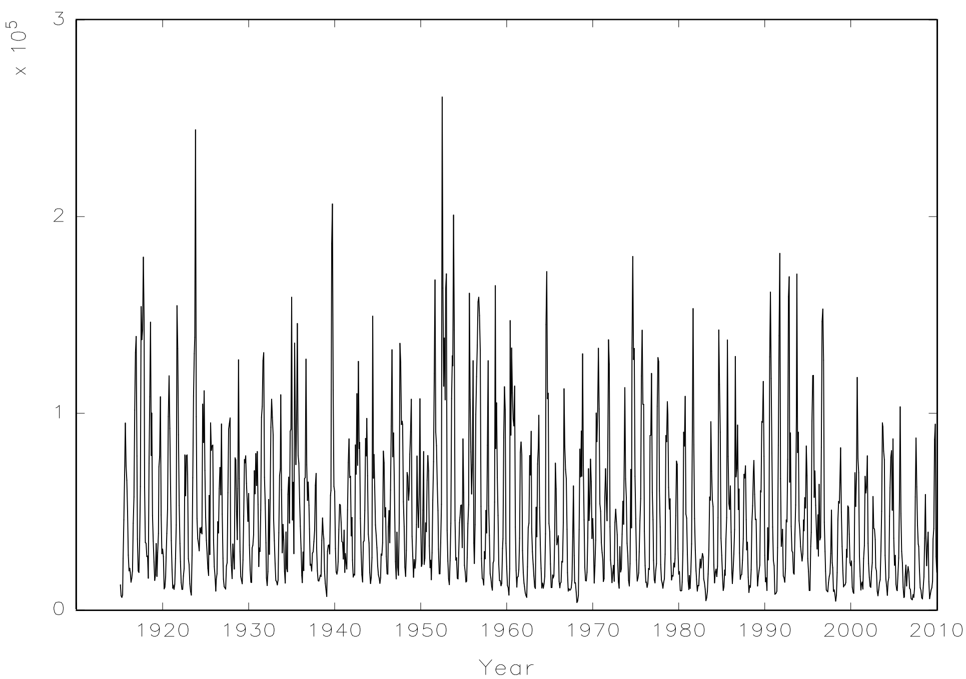


Figure 1: Reservoir water per month, in ML, total of 4 sites, January 1915 to December 2010.

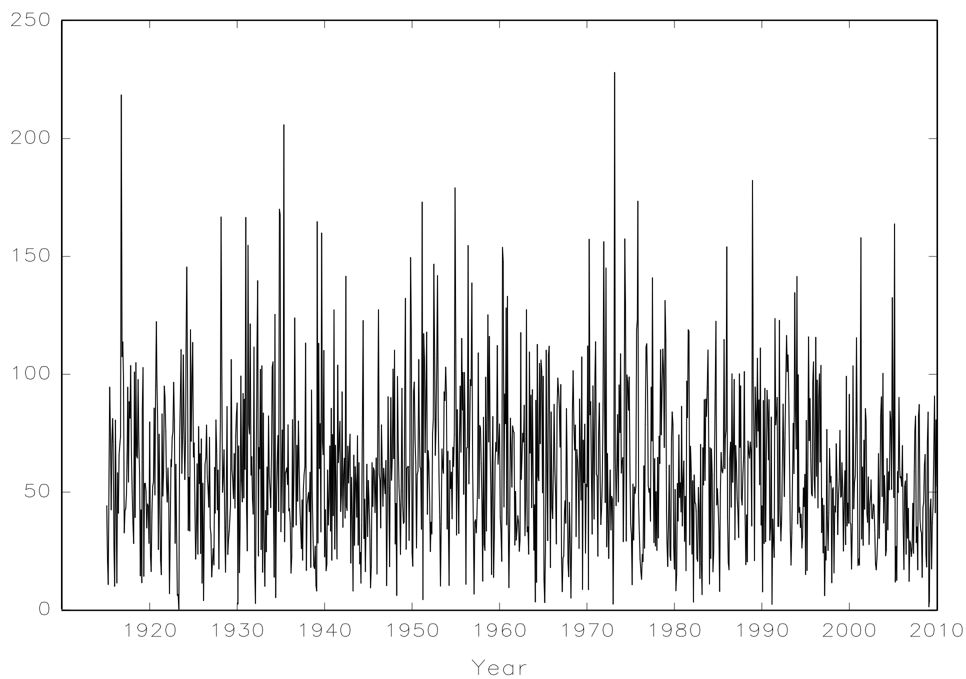


Figure 2: Rainfall per month, in *mm*, average of 6 sites, January 1915 to December 2010.

Table 1:

Descriptive statistics of reservoir inflows (in ML per month) and rainfall (in mm per month), January 1915 to December 2010. Reservoir inflows are based on total inflows per month across 4 sites. Rainfall based on average precipitation per month across 6 sites.

Month	Inflows (ML)				Rainfall (mm)			
	Mean	SD	Min	Max	Mean	SD	Min	Max
Jan.	24291	14335	5443	126692	44	26	1	127
Feb.	16912	9194	3684	64859	49	44	2	228
March	16356	8313	4472	50754	47	34	4	157
April	21133	17286	5548	135676	60	38	0	206
May	33270	26546	9484	149401	63	32	5	155
June	48583	33924	11203	260871	62	25	9	147
July	68868	34311	14902	163658	65	24	21	124
Aug.	90244	40075	21994	186336	72	28	16	160
Sept.	89291	35940	19267	206494	70	31	19	219
Oct.	78766	38781	10956	244167	74	35	12	182
Nov.	56952	30324	10411	162965	67	37	13	182
Dec.	39392	24408	5806	170884	58	35	2	167
Total p.a.$^{(a)}$	584058	195424	163337	1272555	729	147	425	1060

(a) Based on annual data.

$90244ML$ over the period 1915 to 2010, while March has the lowest inflows on average of $16356ML$. In the case of rainfall it is October that has the highest precipitation and January the lowest. The total inflow across all months of the year is $584058ML$ with a standard deviation of $195424ML$, and for rainfall the total per annum is $729mm$ with a standard deviation of $147mm$.

The parameter estimates of α and β for the gamma distributions for reservoir inflows and rainfall for each month are given Table 2. The estimates of the shape parameter for both distributions for all months are $\widehat{\alpha} > 1$ implying a hump-shaped distribution. The scale parameter estimates show some variation over the months to reflect the change in the spread of the distributions in inflows

Table 2:

Parameter estimates for α and β of reservoir inflows (in ML per month) and rainfall (in mm per month), January 1915 to December 2010. Reservoir inflows are based on total inflows per month across 4 sites. Rainfall based on average precipitation per month across 6 sites.

Month	Inflows		Rainfall	
	α	β	α	β
Jan.	4.416	5500.828	2.185	20.335
Feb.	4.277	3954.134	1.373	35.469
March	4.648	3518.987	1.975	23.560
April	2.795	7561.184	2.359	25.357
May	2.541	13093.56	3.544	17.772
June	3.092	15712.92	5.715	10.766
July	4.136	16650.89	6.918	9.354
Aug.	4.961	18189.19	5.640	12.722
Sept.	6.196	14412.25	5.759	12.112
Oct.	4.205	18731.04	3.843	19.281
Nov.	3.778	15074.82	3.309	20.163
Dec.	3.691	10671.64	2.536	22.736
Total p.a.$^{(a)}$	8.878	65790.561	23.802	30.615

(a) Based on annual data.

and precipitation over the year. These properties of the water distributions are further highlighted in Figures 3 and 4 for inflows and rain respectively, which give the empirical distributions for selected months over the period 1915 to 2010, together with the estimated gamma distributions using the estimates in Table 2.

Figure 3: Distribution of reservoir inflows (in ML) for selected months.

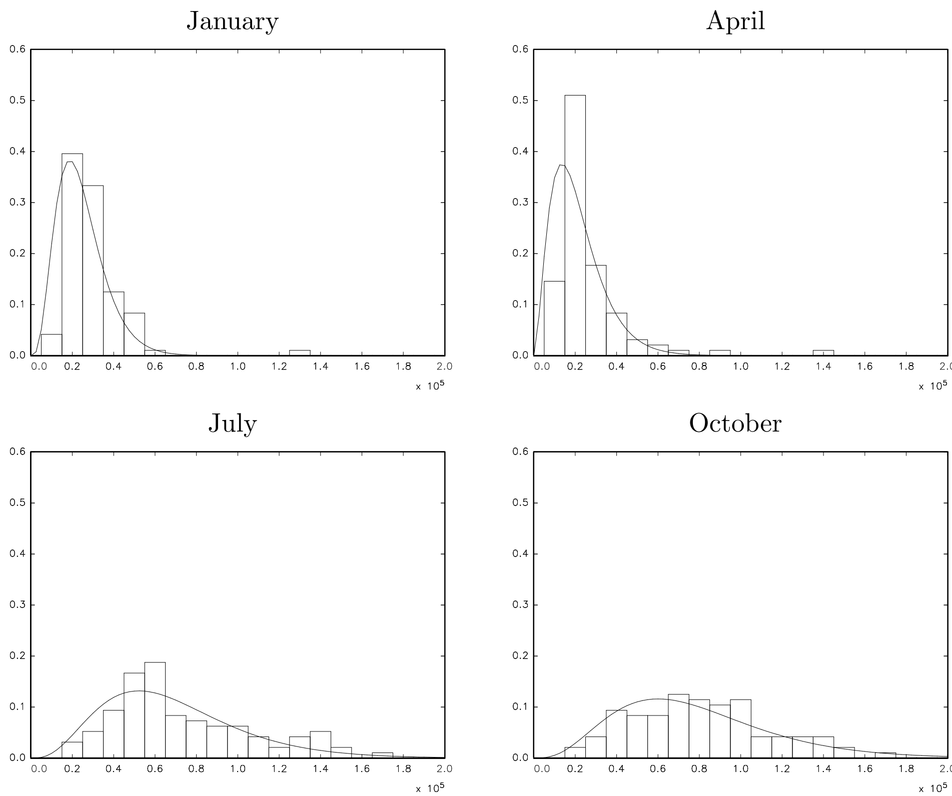
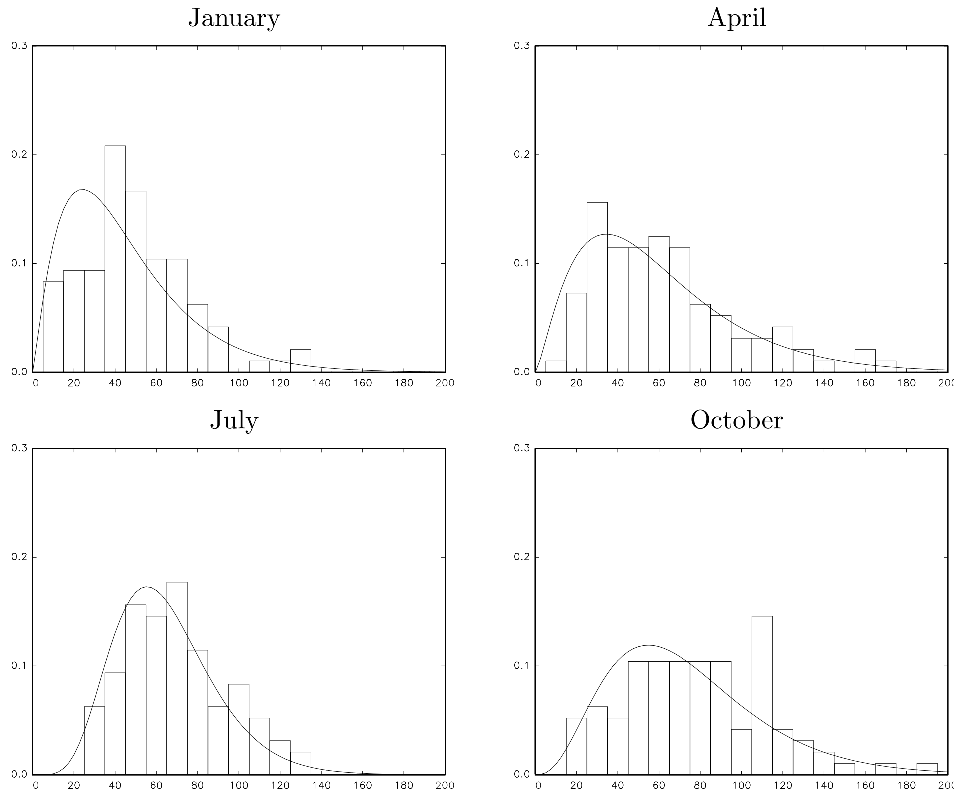


Figure 4: Distribution of rainfall (in mm) for selected months.



4 Model Calibration

The model is calibrated to Melbourne's water system. This system came under stress due to a major drought between 1997 and 2009, which led to the decision to build a large desalination plant in 2007. As consumption and reservoir levels leading up to this decision as well as prior inflows and rainfall are known, this presents an ideal setting to calibrate the model and identify the level of risk aversion of the water authorities during the drought in particular.

4.1 Choice of Model Parameter Values

The majority of Melbourne's urban water comes from the four major reservoirs used in the empirical analysis of Section 3, with a total capacity of $K_r = 1290000ML$ as shown in Table 3. In contrast to these large reservoirs, harvested stormwater is stored in smaller basins that hold only a fraction of the annual rainfall harvested. The parameter values for stormwater harvesting are based on the characteristics of four pilot projects across Melbourne. The relative holding capacities as well as the efficiency with which stormwater is harvested vary widely between the four pilot projects. The total stormwater harvesting capacity across the four sites is $K_h = 1650ML$, which assumes that storages undergo on average three full-empty cycles a year (CWSC, 2011). The third asset in the model is calibrated to Melbourne's desalination plant, which has an annual production capacity of $K_d = 150000ML$ (MW, 2013).

The means (μ_i) and standard deviations (σ_i) of the flow equations in (1) are computed using the parameter estimates from the gamma distribution in Table 2. In the case of reservoirs, the parameter estimates of the total annual inflows given in the last row, are used. From the properties of the gamma distribution in (28), the estimates for reservoirs are computed as $\mu_r = 8.878 \times 65790.561 = 584089ML$, and $\sigma_r = \left(8.878 \times 65790.561^2\right)^{0.5} = 196029ML$. In contrast, the decentralized and small-scale nature of stormwater harvesting systems, which tend to have a faster stock renewal rate than reservoirs, is acknowledged by using the monthly parameter estimates in Table 2 to calculate the mean and standard deviation of harvested stormwater across each month of the year. This is done in two steps. First, the monthly estimate of the scale parameter, β, is annualized by scaling the estimate by 12. Second, the means and standard deviations are then converted into a volume measure by multiplying the rainfall estimate by catchment area and harvesting rate. Over the total catchment area for the four stormwater harvesting projects of $628ha$, rainfall is harvested at a rate of 15%, which is representative of the range of values reported in CWSC (2011). In the case of January for example, the mean is computed as $\mu_h = 2.185 \times 20.335 \times 12 \times 6.28 \times 0.05 = 167ML$, while the standard deviation is $\sigma_h = \left(2.185 \times (20.335 \times 12)^2\right)^{0.5} \times 6.28 \times 0.05 = 113ML$. The estimate of the correlation parameter is $\rho = 0.475$, which is calculated as the correlation between monthly reservoir inflows and rainfall given in Figures 1 and 2 for the total sample period 1915 to 2010. For desalination, the mean is taken as $\mu_d = 0$ as the desalination plant has not produced water since being completed. As this source of water is assumed to be riskless, the standard deviation is $\sigma_d = 0$.

The wholesale average price of water is $c = 2.47$ $\$/kL$, including headworks and transfers (MW, 2013). The total average costs of the reservoirs and stormwater harvesting are respectively $c_r = 1.39$ $\$/kL$ (MW, 2013) and $c_h = 2.69$ $\$/kL$ (CWSC, 2011). In the case of desalination the average total cost used in this calibration is $c_d = 3.00$ $\$/kL$.[2] The population growth rate for Melbourne is $\xi = 0.02$ (ABS, 2011) and a low discount rate of $\delta = 0.01$ is chosen to reflect the long term planning horizon of urban water infrastructure projects.

The relative risk aversion parameter γ, is calibrated for the year 2007, when the State Government announced the construction of a desalination plant with a capacity of 33% of remaining water stock to secure urban water supply (DSE, 2007). This announcement was made when the effects of a Millennium drought from 1997-2009 were at their most severe with reservoirs at merely 30 per cent capacity, $\kappa_r = 0.3$. This decision provides the opportunity to infer the level of risk aversion of Melbourne's water policy makers.

[2] This cost estimate is lower than the total average prices that Melbourne's water authority is currently charged for desalinated water, which ranges between $\$5.00/kL$ and $\$13.29/kL$ depending on the volume ordered (PWC, 2011). However, it is unlikely that such high prices were considered when the decsion was made to build the plant, especially given the cost of desalinated water from the existing plant in Sydney, which is $\$2.29/kL$ (NWC, 2008).

In terms of the model in Section 2, there are two types of shares. The first set of shares are based on the optimal formulae given in (23) and (24) together with the normalization (7). As these shares are cost invariant and hence only based on physical flows, they are referred to as physical shares. The second set of shares, denoted as θ_i^c, are referred to as economic shares, and allow for costs according to

$$\theta_i^c = \frac{c}{c_i}\left(\frac{\theta_i W^p}{W}\right), \tag{30}$$

where θ_i is the physical share for a particular water source, W is defined in equation (2) and W^p represents the physical stock of water which is computed as (**??**) with neutral cost weights, $c = c_r = c_h = c_d = 1$. In which case the 33% announced desalination capacity corresponds to a share of $\theta_d^c = 0.33$. The relative risk aversion parameter is determined by substituting (23) and (24) in (7), together with (10) and (11), and using (30) to convert physical shares to economic shares, resulting in

$$\gamma = \frac{(k_r - k)}{(1-\theta_d^c)}\left(\frac{\mu_r}{S_r} - \frac{\mu_d}{S_d}\right) + \frac{(k_h - k)}{(1-\theta_d^c)}\left(\frac{\mu_h}{S_h} - \frac{\mu_d}{S_d}\right), \tag{31}$$

where k_r, k_h, k are defined in (18). Given that reservoirs operated at 30% capacity, $S_r = \kappa_r K_r = 0.3 \times 1290000ML = 387000ML$. Assuming a comparable capacity utilization for stormwater harvesting systems, which in effect corresponds to harvested stormwater being available 30% of the year, then the annual stock of stormwater is $S_h = \kappa_h K_h = 495ML$. Based on these water stocks and the parameter values in Table 3 the implied risk aversion parameter is calculated as $\gamma = 15$.[3]

Finally, the choice of the level parameter on consumption is calibrated to $\eta = 0.4$. This yields an optimal consumption level under the base case parameter

[3]The physical share of desalination, θ_d, that is equivalent to the announced contribution from desalination to the total water stock, $\theta_d^c = 0.33$, is calculated using equation (30) with price and costs as given in Table 3. Based on these calculations, values for the risk aversion parameter range from $14 < \gamma < 18$, throughout the year.

Table 3:

Base case parameter values for Melbourne.

Parameter	Value	Unit	Description
K_r	1290000	ML	Total reservoir capacity
K_h	1650	ML	Harvested stormwater capacity
K_d	150000	ML	Desalination plant capacity
κ_r	0.3		Proportion of fixed reservoir capacity
κ_h	0.3		Proportion of harvested stormwater capacity
	628	ha	Average stormwater catchment area per site
	0.15		Stormwater harvesting rate
μ_r	584089	ML	Mean reservoir inflow
μ_h	167	ML	Mean harvested stormwater
μ_d	0	ML	Mean flow from desalination
σ_r	196029	ML	Standard deviation of reservoir inflows
σ_h	113	ML	Standard deviation of harvested volume
σ_d	0	ML	Standard deviation of desalination flow
ρ	0.475		Correlation coefficient
c	2.47	$\$/kL$	Wholesale price of water
c_r	1.39	$\$/kL$	Total average cost of reservoir water
c_h	2.69	$\$/kL$	Total average cost of harvested stormwater
c_d	3.00	$\$/kL$	Total average cost of desalinated water
ξ	0.02		Population growth rate
δ	0.01		Rate of discount
γ	15		Implied constant parameter of risk aversion
η	0.4		Implied level parameter on consumption

values equal to 400000 ML, which corresponds to the level of consumption in 2006/2007 (MW, 2012).

4.2 Analysis

The parameter values described above are now used to compute the optimal water shares for Melbourne. Optimal water shares are calculated for three water supply situations: The first case corresponds to the crisis situation in 2007 when water stocks were at an all time low, the second case is representative of the long run average water supply scenario and the third case characterizes the situation in the beginning of 2013 when reservoirs were 80% full. Both physical shares based on equations (23), (24) and (7), and economic shares based on (30), are presented in the result Tables 4 to 6.

4.2.1 System in Crisis

Table 4 shows the resulting optimal physical and economic shares for the case in 2007 when water stocks were at an all-time low at 30% in both $(\kappa_r = \kappa_h = 0.3)$. Ignoring supply costs the optimal share of desalination is 59%, with reservoirs contributing only 35% and harvested stormwater just 6% for the year.

Within a year the share of harvested stormwater increases to up to 19% during the winter months, which is a reflection that these are characterized by high mean rainfall and low rainfall volatility. Taking the relatively high supply costs for stormwater harvesting into account reduces the stormwater economic shares to a maximum of 12% in July and an average of 4% over the year, which is still significant given the negligible contribution of stormwater harvesting to Melbourne's water stock.

A comparison of the physical and economic shares for desalination reveals that desalination costs of $\$3.00/kL$ yield optimal economic shares for desalination of 33%, which translates into the plant's current production capacity of around 33% or $150000ML$ that was announced at the time. Inspecting the economic shares for reservoirs shows that in times of crisis reservoirs can only be expected to contribute about 65% to total water stock due to their low stocks, despite their relatively lower supply costs.

4.2.2 Average System

Table 5 shows the physical and economic shares of the three water supply assets that would be optimal under normal conditions when reservoirs are about 65%

Table 4:

Optimal contributions to total water stock from reservoirs, stormwater harvesting and desalination technologies when water stocks are *low* (30% for reservoirs and 30% for stormwater).

Month	Physical Shares[a]			Economic Shares[b]		
	θ_r	θ_h	θ_d	θ_r^c	θ_h^c	θ_d^c
Jan.	0.39	0.01	0.60	0.67	0.00	0.33
Feb.	0.39	0.00	0.61	0.67	0.00	0.33
March	0.39	0.00	0.61	0.67	0.00	0.33
April	0.38	0.01	0.61	0.66	0.01	0.33
May	0.35	0.06	0.59	0.64	0.04	0.32
June	0.32	0.14	0.54	0.61	0.09	0.30
July	0.29	0.19	0.52	0.60	0.12	0.28
Aug.	0.31	0.12	0.57	0.62	0.07	0.31
Sept.	0.31	0.13	0.56	0.62	0.08	0.30
Oct.	0.35	0.05	0.60	0.64	0.03	0.33
Nov.	0.36	0.04	0.60	0.65	0.02	0.33
Dec.	0.38	0.02	0.60	0.66	0.01	0.33
Average[(c)]	0.35	0.06	0.59	0.64	0.04	0.32

(a) Physical shares (θ_i) based on (23), (24) and (7).

(b) Economic shares (θ_i^c) are based on (30).

(c) Based on monthly averages in respective columns.

full and stormwater harvesting systems can be expected to hold sufficient water 80% of the time during a year. It is shown that, for the same level of risk aversion of $\gamma = 15$, the optimal economic share of desalination is only about 4% on average, as opposed to the 18% its production capacity actually represents in the case of average water stocks prior to the building of the desalination plant, which is computed as

$$\frac{150000}{0.65 \times 1290000 + 0.8 \times 1650} = 0.18. \tag{32}$$

This result suggests that there was an over-investment in desalination technology when viewed against long run averages. However, this analysis does not take a changing climate into account, under which inflows are expected to decline significantly, making lower average reservoir stocks and even crisis level stocks more likely and therefore desalination technology more attractive.

Inspection of the economic shares for desalination in Table 5 suggests that a desalination plant may optimally provide around 5% to 8% of the water stock for part of the year (October to May), while being switched off for the remainder of the year, when supply from stormwater harvesting is sufficiently large and secure to make an important contribution to total water stock. This is also the time when reservoir stocks should be conserved for the dryer summer months by taking advantage of available stormwater. To fulfill this role stormwater harvesting systems would need to have a total annual capacity of around $((0.65 \times 1290000) + (0.8 \times 1650)) \times 0.28 \approx 235000$, where 0.28 is the optimal economic share for stormwater for August in Table 5. From the last row of Table 5 corresponding to θ_h^c, stormwater harvesting could on average represent about 10% of total water stock. Both of these values suggest that future investments in water supply augmentations should target stormwater harvesting projects ahead of desalination technology. However, this conclusion may change if augmentation investments are specifically made to overcome seasonal

Table 5:

Optimal contributions to total water stock from reservoirs, stormwater harvesting and desalination technologies for *average* water stocks (65% for reservoirs and 80% for stormwater).

Month	Physical Shares[a]			Economic Shares[b]		
	θ_r	θ_h	θ_d	θ_r^c	θ_h^c	θ_d^c
Jan.	0.84	0.02	0.14	0.92	0.01	0.07
Feb.	0.83	0.00	0.17	0.92	0.00	0.08
March	0.85	0.00	0.15	0.93	0.00	0.07
April	0.83	0.03	0.14	0.91	0.02	0.07
May	0.77	0.14	0.09	0.87	0.08	0.05
June	0.62	0.38	0.00	0.79	0.21	0.00
July	0.50	0.50	0.00	0.72	0.28	0.00
Aug.	0.68	0.32	0.00	0.82	0.18	0.00
Sept.	0.66	0.34	0.00	0.81	0.19	0.00
Oct.	0.75	0.15	0.10	0.87	0.08	0.05
Nov.	0.78	0.11	0.11	0.88	0.06	0.06
Dec.	0.82	0.05	0.13	0.91	0.03	0.06
Average[(c)]	0.74	0.17	0.09	0.86	0.10	0.04

(a) Physical shares (θ_i) based on (23), (24) and (7).

(b) Economic shares (θ_i^c) are based on (30).

(c) Based on monthly averages in respective columns.

shortages.

4.2.3 Abundant Water

Since the desalination plant was completed in 2012 it has not added any water to Melbourne's supply system, due to high levels in reservoirs at around 80% of capacity. Setting $\kappa_r = 0.8$ and assuming that stormwater harvesting systems always have sufficient water stocks, $\kappa_h = 1.0$, Table 6 shows that the optimal contribution from desalination is indeed zero for every month of the year. On average, the optimal contribution by reservoirs is 88% to total water stock with the remainder supplied through harvested stormwater. The optimal stormwater capacity in abundant water systems is greater than under average conditions as it should be able to deliver up to 1/3 of total supply in some months thereby reducing the demand on reservoir water.

5 Concluding Remarks

Worldwide, cities increasingly struggle to meet the demand for water from conventional sources as urban populations continue to grow and inflows into reservoirs decline. In this environment, one may think of a portfolio of water supply assets to augment future water supply. Investment options include large centralized infrastructure projects, capable of producing water at any time and decentralized infrastructures that capitalize on greater predicted future variation in rainfall.

In this paper, a general framework was developed to determine the optimal water consumption and contribution to total water supply from conventional and alternative sources by using a continuous-time dynamic model of long-term optimal portfolio allocation across alternative water assets, consisting of reservoirs, harvested stormwater and desalination. Water sourced from water reservoirs and harvested stormwater are subjected to inflow and precipitation volatility, whereas the flow from desalination is not weather related and thereby considered as riskfree. Important features of the framework consist of the specification for reservoir inflows and rainfall as gamma distributions, and allowance for cost heterogeneity across all water assets. A closed-from solution of the theoretical model was shown to exist for optimal consumption and optimal individual contributions from the three types of water supply assets to total water stock investigated.

The properties of the model were investigated using calibration methods

Table 6:

Optimal contributions to total water stock from reservoirs, stormwater harvesting and desalination technologies when water stocks are *high* (80% for reservoirs and 100% for stormwater).

Month	Physical Shares^{a,c}			Economic Shares^{b,c}		
	θ_r	θ_h	θ_d	θ_r^c	θ_h^c	θ_d^c
Jan.	0.97	0.03	0.00	0.99	0.01	0.00
Feb.	1.00	0.00	0.00	1.00	0.00	0.00
March	1.00	0.00	0.00	1.00	0.00	0.00
April	0.96	0.04	0.00	0.98	0.02	0.00
May	0.82	0.18	0.00	0.90	0.10	0.00
June	0.53	0.47	0.00	0.73	0.27	0.00
July	0.38	0.62	0.00	0.66	0.34	0.00
Aug.	0.60	0.40	0.00	0.78	0.22	0.00
Sept.	0.57	0.43	0.00	0.76	0.24	0.00
Oct.	0.82	0.18	0.00	0.90	0.10	0.00
Nov.	0.86	0.14	0.00	0.92	0.08	0.00
Dec.	0.94	0.06	0.00	0.97	0.03	0.00
Average$^{(d)}$	0.79	0.21	0.00	0.88	0.12	0.00

(a) Physical shares (θ_i) based on (23), (24) and (7).

(b) Economic shares (θ_i^c) are based on (30).

(c) A non-negativity restriction was imposed on some shares that are reported as being zero.

(d) Based on monthly averages in respective columns.

based on parameter values characterizing the Melbourne water supply system, which has recently come under pressure due to a major drought that led to the decision to invest in a large desalination plant to boost Melbourne's water supply. By accounting for different costs of supply, the results showed how optimal contributions of each asset to the total water stock depend on existing water stocks, thereby providing insights into the appropriateness of historical water augmentation decisions. When assessed against average water stocks, rainfall and inflows, it was shown that the optimal contribution by reservoirs is close to current levels, whereas harvested stormwater should optimally constitute a larger share in the portfolio than is currently the case.

Finally, the analysis provided insights into the importance of various water supply assets throughout the year, with desalination technology being the preferred supplement to reservoir water during the summer months when mean rainfall is low and subject to high variation. During winter it is harvested stormwater that is the preferred alternative supply as it would optimally provide a significant share of the total water stock, thereby helping to conserve reservoir water for the dryer summer months. However, the current capacity of Melbourne's stormwater harvesting systems is insufficient to fulfill this role.

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