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Farsighted Fuzzy Coalitions in an Economy with Multilateral Negative Externalities

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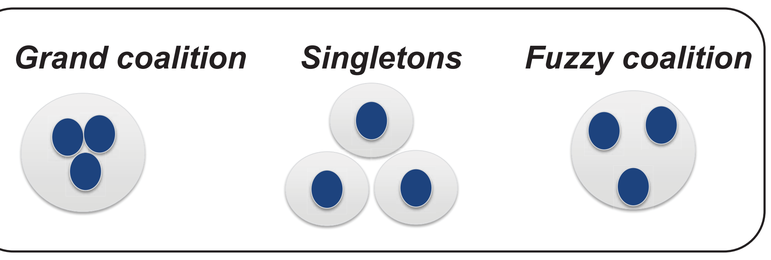
Introduction

How do coalitions form in games with externalities? This is important for understanding whether full cooperation be sustained in tackling problems such as climate change.

In traditional coalition formation games, each player makes a binary decision on whether or not to cooperate.

But what if players within coalitions choose not to cooperate fully with each other? This study looks at the coalition formation process in games with **fuzzy coalitions**.



Research Questions

1) What kind of coalition structures are stable in games with fuzzy coalitions?

2) Is efficiency enhanced or inhibited by allowing the formation of fuzzy coalitions?

Defining fuzzy coalition structures

Let N denote a set of players.

- A **fuzzy coalition** is a vector $s \in [0, 1]^n$.
- For any player i, s_i denotes its **participation level** within the coalition.
- A partition of N into fuzzy coalitions (denoted by Π) is a **fuzzy coalition structure**.
- A **fuzzy refinement** of Π (denoted by $R(\Pi)$) is a coalition structure formed by one or more players reducing their participation levels (which may involve leaving the coalition).

Examples (for 3-player game):

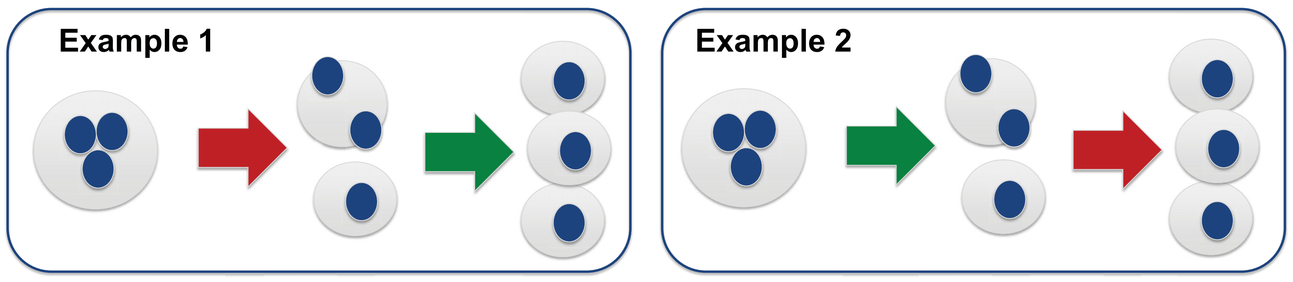
- $\Pi_1 = [\{1, 2\}, \{3\}]$ consists of the crisp coalitions {1,2} and {3}
- $\Pi_2 = [\{1^{0.5}, 2^{0.2}, 3^{0.7}\}, \{1^{0.5}\}, \{2^{0.8}\}, \{3^{0.3}\}]$ consists of the fuzzy coalition $\{1^{0.5}, 2^{0.2}, 3^{0.7}\}$ and three fuzzy singletons $\{1^{0.5}\}$, $\{2^{0.8}\}$, and $\{3^{0.3}\}$.

Stability of fuzzy coalition structures

Stability: A coalition structure Π is **stable** if there exists no $R(\Pi)$ that **blocks** Π.

Blocking: A coalition structure Π is **blocked** by a refinement $R(\Pi)$ if at least one player gains in the move from Π to $R(\Pi)$ and $R(\Pi)$ is itself **stable**.

In example 1, the grand coalition is blocked by its fuzzy refinement, which is stable.
In example 2, the grand coalition is stable because its fuzzy refinement is blocked by the set of singletons.



Equilibrium coalition structures in the emissions game

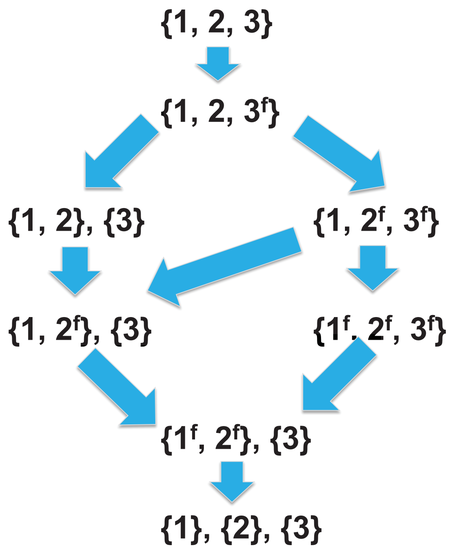
Consider an economy with 3 identical players. Each player i produces a private good y_i and in doing so, generates emissions e_i that contribute to a global public bad z.

Let $y_i = \sqrt{e_i}$, $z = \sum e_i$, and $u_i(y_i, z) = y_i - z + 1$ (1)

How does fuzzy cooperation work in this game?

- We assume that each player chooses e_i to maximize its own payoff, plus the payoff of the rest of the coalition weighted by the participation level s_i.
- To simplify the analysis, we assume that $s_i \in \{0, 0.5, 1\}$.
- If an agent i is partially cooperating in a coalition, we will represent that agent by i^f.

Stable coalition structures:



Coalition structure	u_1, u_2, u_3	Stable?	z
{1, 2, 3}	1.083, 1.083, 1.083	NO	0.083
{1, 2, 3f}	1.058, 1.058, 1.108	NO	0.143
{1, 2}, {3}	0.875, 0.875, 1.125	YES	0.375
{1, 2f, 3f}	1.024, 1.060, 1.060	YES	0.226
{1, 2f}, {3}	0.861, 0.861, 1.028	YES	0.472
{1f, 2f, 3f}	1.000, 1.000, 1.000	YES	0.333
{1f, 2f}, {3}	0.830, 0.830, 0.930	YES	0.570
{1}, {2}, {3}	0.750, 0.750, 0.750	YES	0.750

- **The grand coalition is not stable!**
- **The coarsest stable structures are {1, 2}, {3} and {1, 2f, 3f}, both of which are inefficient.**
- **But compare {1, 2}, {3} and {1, 2f, 3f}: the fuzzy coalition structure leads to lower emissions and is less inefficient.**

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Conclusions and Future Work

- The study introduces a framework for analyzing the coalition formation process in games with fuzzy coalitions.
- Applying this model to a simple 3-player emissions game, we find that inefficient coalition structures emerge in equilibrium.
- Further research will look more generally at n-player games with externalities.

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58th Annual Conference of the Australian Agricultural & Resource Economics Society, Port Macquarie , New South Wales, 4-7 February 2014