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FLOW OF WATER AROUND 180-DEGREE BENDS
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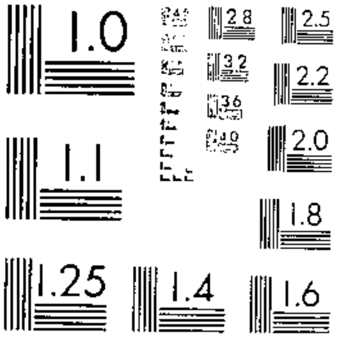
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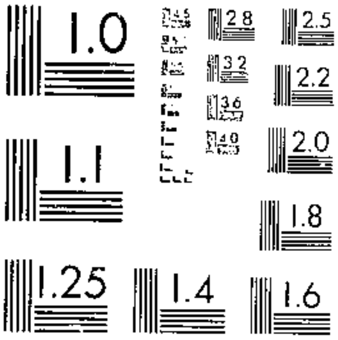
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By

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UNITED STATES DEPARTMENT OF AGRICULTURE, WASHINGTON, D.C.

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FLOW OF WATER AROUND 180-DEGREE BENDS

By DAVID L. YARNELL, *senior drainage engineer, Division of Drainage, Bureau of Agricultural Engineering*, and SHERMAN M. WOODWARD, *professor of mechanics and hydraulics, University of Iowa*

(The Bureau of Agricultural Engineering in cooperation with The University of Iowa College of Engineering)

CONTENTS

INTRODUCTION

The results of a series of experiments on the effects of 180° bends of square and rectangular cross section on the flow of water are presented in this bulletin. The various tests included some cases with uniform and other cases with nonuniform velocity distribution in the channel approaching the bend. Tests were made on channels flowing full under a slight pressure as well as on partly full channels. The experiments were made by the Bureau of Agricultural Engineering, United States Department of Agriculture, in cooperation with the University of Iowa.[1]

The investigation was undertaken for the purpose of exploring the laws governing the changes in pressure and velocity in different parts of the flowing stream, as the moving water undergoes the transition from motion along a straight path to motion around a curve, and again as it undergoes the opposite transition back to a final straight-line motion. This condition of transitional flow may be considered as representative of what always exists whenever water flows in a

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[1] The investigation was carried on at the laboratory of hydraulic research of the university, Frederick Theodore Mavis associate director in charge.

crooked channel, or whenever moving water meets a bridge pier or any other form of obstruction. A knowledge of these laws is fundamental to an understanding of the effects, harmful or otherwise, of crooked and obstructed channels upon the flow of water, and to a determination of the best means for alleviating or relieving any objectionable or troublesome effects. A novel feature of the study was the use of transparent material for the four walls of the apparatus.

REVIEW OF THE LITERATURE

During the past 150 years, numerous investigators have conducted experiments on the flow of water around bends. Among the earliest investigators, Dubuat, (*7*)[2] published in 1786 the results of his experiments on 1-inch and 2-inch pipe bends. After Dubuat came Weisbach (*17*) whose experiments were performed on pipes about 1 inch in diameter with low velocities. Using his own experiments together with those performed by Dubuat, Weisbach derived his formula for loss in bends which has since been quoted in many books on hydraulics. Williams, Hubbell, and Fenkell (*18*) carried on, in 1900, experiments on the effect of curvature upon the flow of water in pipes. These tests were notable in that they were on unusually large pipes. About this time Alexander (*1*) and Brightmore (*5*) carried on, separately, experiments which added further to the knowledge of the subject. In 1907 Schoder (*14*) made a series of tests on various elbows and bends. His work was followed by that of Davis (*6*) and Balch (*2*). Practically all of these men determined merely the loss of head around bends and did not investigate conditions of flow in the bend itself.

While all of these investigations added to our fund of knowledge on the subject, they did little towards the determination of the direction of the filaments of flow around the bend. Probably the first investigator of this phase was Eustice (*8*), who made tests on bent glass tubes, using coloring matter to study the direction of the current.

In 1912 Lell (*13*) conducted tests on a 180° bend, the channel being 3.94 inches deep by 7.87 inches wide. Lell investigated pressures at many points around the bend. Kumabe (*11*), in 1921, also investigated pressures in bends. His bend was 90° with a rectangular cross section, 3 inches wide by 1½ inches deep. The bend had a glass cover so that the flow could be observed.

Dubuat attempted to apply to open channels the results he obtained in small pipe bends. Humphreys and Abbott (*10*) used Dubuat's theory on bends of the lower Mississippi River, changing the constant in Dubuat's formula so that their experiments and the formula would check. These earlier investigators hoped to discover the laws of flow around bends by using small closed models which could be easily handled. Probably the Germans were the first actually to build in miniature, open channels with bends and study the flow in such models. In this field Beyerhaus (*3*) in 1921 conducted rather extensive experiments and found that the same conditions occurred in his models that occurred in natural water

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[2] Italic numbers in parentheses refer to Literature Cited, p. 61.

courses. Although he verified certain facts, he formulated no definite laws. Gockinga (*9*) has pointed out the necessity of passing from straight sections to curved sections in a gradual manner, endeavoring thus to reduce the erosive action as much as possible.

The loss of head of water flowing around a bend depends upon the following factors: (1) The shape of the channel; (2) the hydraulic radius of the channel; (3) the curvature of the bend; (4) the length of the bend, (measured along the center line); (5) the roughness of the walls of the channel; (6) the velocity distribution in the approach channel to the bend; and (7) the density and viscosity of the fluid.

There is no formula which takes into account all of these factors and which can be used to determine accurately the losses in bends.[3]

SCOPE OF THE INVESTIGATION

Experiments were conducted on 180° bends of two sizes and two radii. The first series of tests was made on a channel whose cross section was 10 inches square with a 5-inch inner radius. The next series of tests was made on a channel 5 inches wide by 10 inches deep with a 5-inch inner radius. The third series of tests was run on a channel 5 inches wide by 10 inches deep with a 10-inch inner radius.

Tests were run with the channels flowing full and under some pressure in which the discharges ranged from 0.7 to 3.0 cubic feet per second. Tests were also made with the channels flowing partly full. Duplicate experiments for several of the quantities were run as check tests.

Experiments were made on all bends with uniform velocity distribution in the approach channel near the beginning of the bend. Additional tests were run using the smallest and largest discharges in which unequal velocity distribution was purposely created in the approach channel near the beginning of the bend. This series consisted of the following variations: (1) High velocity along the outside wall and a low velocity along the inside wall; (2) low velocity along the outside wall and a high velocity along the inside wall; (3) high velocity along the bottom and a low velocity along the top; (4) low velocity along the bottom and a high velocity along the top.

The investigation was planned to cover the measurement of the following hydraulic factors: (1) The velocity distribution in the channel, both in the two tangents and in the bend; (2) the pressure changes at various sections in the bend; (3) the energy changes as the water passes around the bend; (4) the actual head loss due to the bend; (5) the direction of the filaments of flow as the water moves around the bend; and (6) the friction losses occurring in the approach and discharge tangents.[4]

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[3] An annotated bibliography containing about 135 references to publications relating to flow of water around bends has been prepared in mimeographed form and may be obtained without charge from the Bureau of Agricultural Engineering, U. S. Department of Agriculture, Washington, D. C.

[4] The following hydraulic-engineering graduate students assisted on the tests: Glenn Cox, F. B. Smith, G. Gangadharan, Ernest Schuleen, Emil Schuleen, Frank W. Edwards, J. Stuart Meyers, R. A. Kampmeier, C. H. Morris, and R. N. Brudenell.

TEST APPARATUS

The apparatus used consisted of a 4 by 5 foot entrance tank, 5 feet deep, a straight approach channel 25 feet long, the 180° bend, and a straight discharge channel 28 feet long (fig 1). The entrance tank was equipped with the necessary baffles for removing the turbulence in the water before it passed into the approach channel. Smooth rounded corners in the tank at the outlet to the channel leading to the bend aided in securing uniform velocity distribution in this approach channel. Further improvement in velocity distribution was obtained by the oval-shaped baffles placed in and near the entrance of the approach channel. A flap gate was attached to the outlet end of the discharge tangent for regulating the depth of flow in the apparatus.

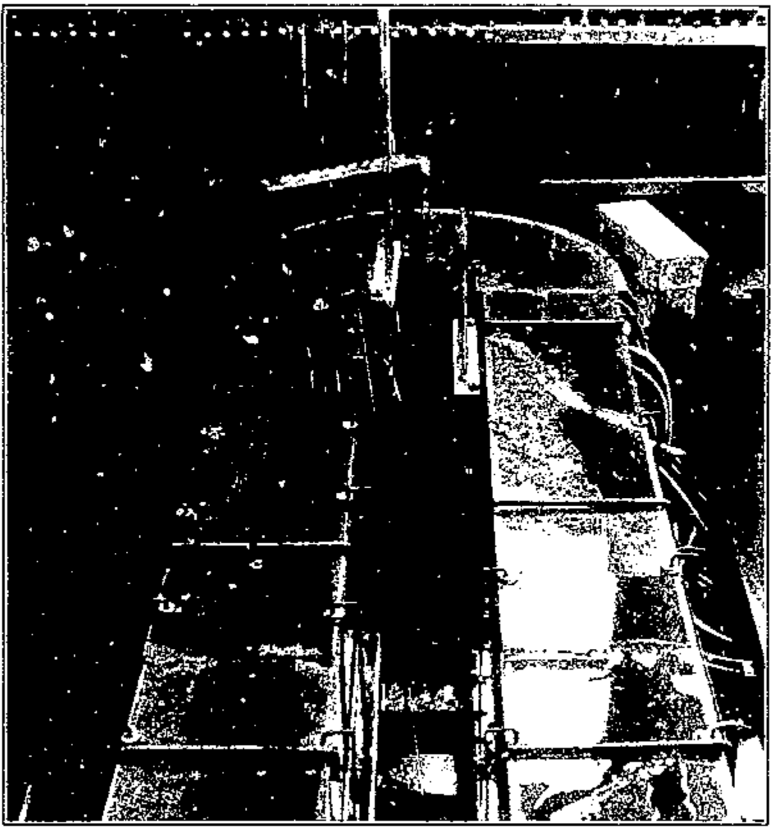

FIGURE 2.—Bend of 180° with adjacent sections of approach and discharge tangents as originally built of plate glass. Channel 10 inches square.

The bend and 8½ feet of both the approach and discharge channels adjacent to the bend were built of transparent material. The remainder of the two tangents was build of wood. The transparent portion of the bend and the two tangents were originally built of

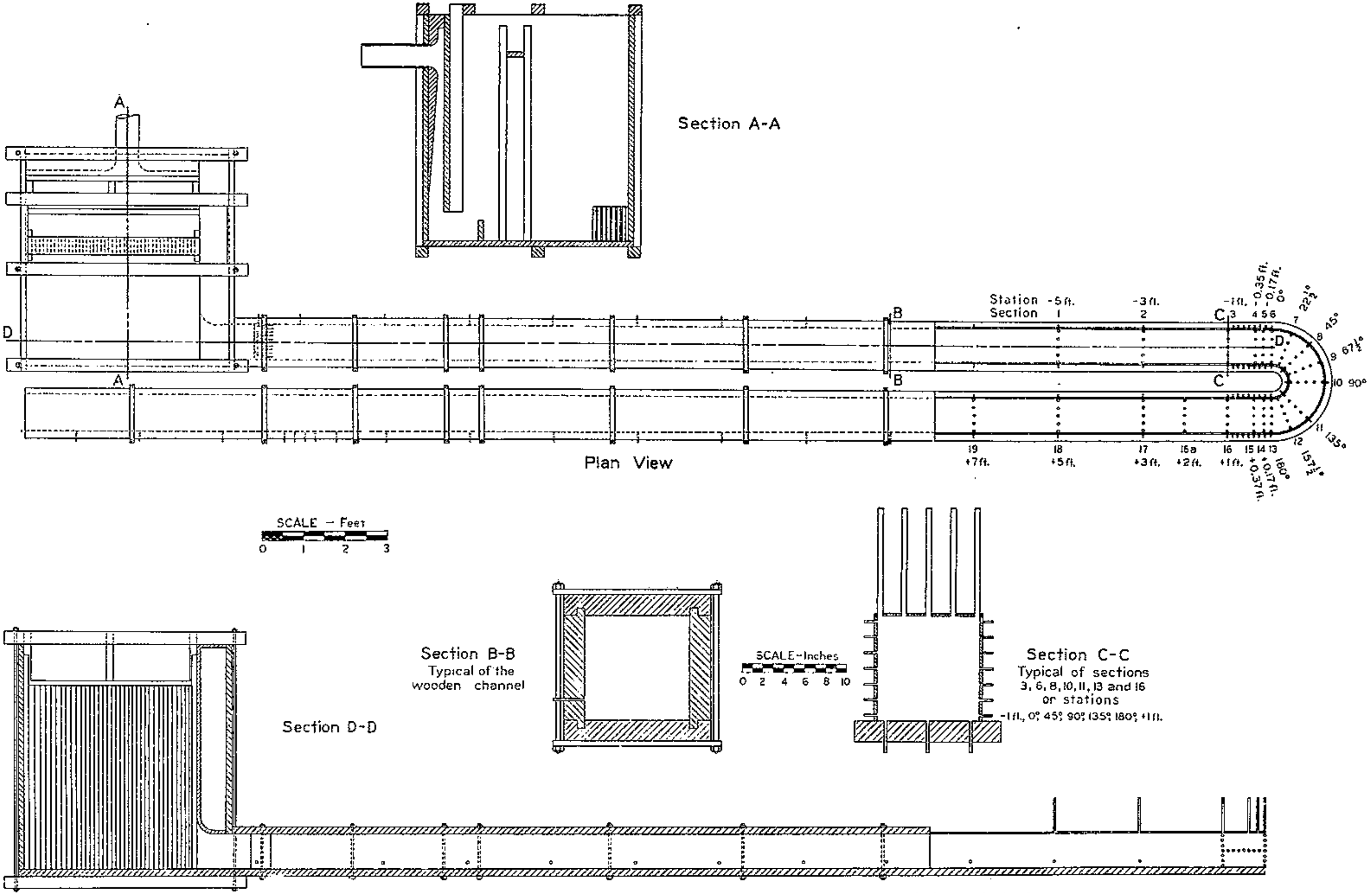

FIGURE 1.—Plan and sections of bend-test apparatus showing position of piezometers in relation to the bend.

plate glass (fig. 2). The difficulties arising from the continual cracking of the glass led to the adoption of transparent celluloid which was used in the bends tested later (fig. 3).

The bend used with the 10 by 10 inch channel was 180° with a 5-inch inner radius. For measuring the friction losses in the approach and discharge tangents, 32 piezometer openings were made in the approach tangent and 39 in the discharge tangent. For measuring the periphery pressures in and adjacent to the bend, 22 piezometer openings were made in a line passing entirely around the periphery of the channel at each of five cross sections in the bend and at two sections on the approach and discharge tangents (figs. 1 and 5). In addition, seven piezometer openings were made through a distance of 1 foot horizontally along the middle of each of the inside and outside walls of both channels adjacent to the bend for the purpose of measuring the longitudinal changes in pressure at the beginning and at the end of the bend. Thus a total of 196 piezometers were installed in a distance of 4½ feet lengthwise of the channel.

FIGURE 3.—Bend of 180° and adjacent sections of approach and discharge tangents as made of transparent celluloid. Channel 10 inches square. The celluloid being only one-eighth inch thick, a metal frame was required.

The first piezometer openings used were one-thirty-second of an inch in diameter. Later these were changed to one-eighth inch. In addition to these connections, openings were made in the cover plate at each of 10 sections on the 2 tangents (figs. 1 and 5). One-half-inch tubes were secured to the openings. These tubes served a twofold purpose in the investigations. Acting as piezometer tubes they registered the pressures against the cover plates when the channel was flowing full and under some pressure. They also afforded openings through which the Pitot tubes could be inserted for measuring the velocities.

The apparatus as originally built (fig. 2) had relatively few piezometers, but the number was increased during the progress of the tests, as it became apparent that additional information was needed, until in all there were more than 296 piezometer openings in the apparatus, 223 of which were in the transparent section. Figure 4 shows a close-up view of the piezometers in the bend.

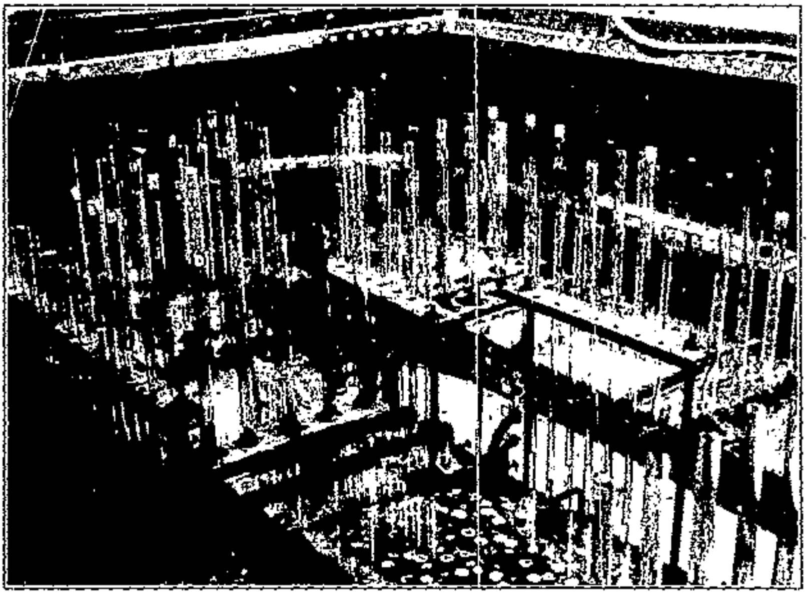

FIGURE 4.—Close-up view of piezometers in vicinity of bend. Channel of plate glass, 10 inches square with 5-inch inner radius.

The 5- by 10-inch channel was made by inserting a partition in the 10- by 10-inch channel. Only the transparent portion of the 10-inch channel was studied in these tests. The locations of the various piezometers and the sections at which the velocities were measured are shown in figure 5.

The Pitot tubes used in measuring the velocities (fig. 6) were made and calibrated at the laboratory. Figure 7 shows the 10-inch channel under test; the observers are measuring velocities with the Pitot tubes. Figure 8 is a general view of the entire bend apparatus and the weighing scales and tanks used in determining the quantity of flow.

The obstruction for creating nonuniform velocity distribution in the approach channel to the bend consisted of wire rods of various lengths set staggered in rows one-half inch apart. The downstream end of the obstruction was placed 4 inches upstream from section 3, or 16 inches from the beginning of the bend.

TEST PROCEDURE

Tests were conducted on all bend set-ups with quantities of flow ranging from 0.7 to 3.0 cubic feet per second. Each channel was tested flowing full under some pressure and also partly full. The quantity of water was determined both by a venturi meter and by weighing. Measurements of quantity were made at the beginning and end of each test.

In the velocity measurements readings were taken at 49 different points in each cross section in the 10 by 10 inch channel and at 35 different points at each measuring section by the 5 by 10 inch channel. These gave seven vertical velocity traverses at each cross section of the large channel and five at each cross section in the smaller channels. The readings were taken at seven points in each vertical as follows: Next to the top and the bottom, 1 inch and 2½ inches from the top and the bottom, and at the center or 5 inches from the top and the bottom. In the large channel the verticals were located as follows: One next to each of the side walls, one at 1 inch and one at 2½ inches from each of the two side walls, and one at the center. Similar spacing of the verticals from the side walls was used in the smaller channels.

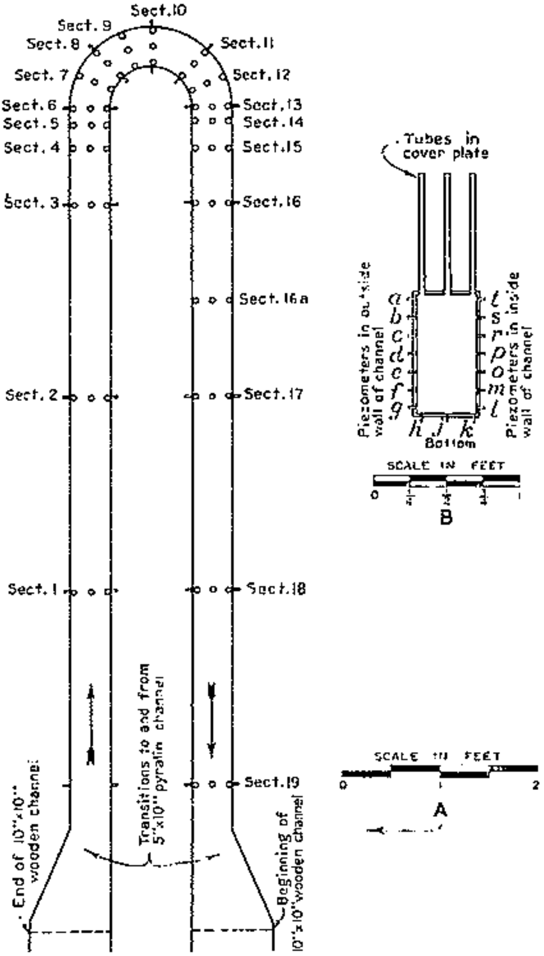

FIGURE 5.—Diagram of 5- by 10-inch bend showing location of piezometers: *A*, Plan of 180° bend with 5-inch inner radius showing sections at which velocity measurements were taken. *B*, Typical cross section for sections 3, 6, 8, 10, 11, 13, and 16, at which periphery pressures were measured. Channel with 10-inch inner radius is identical except for structure of bend.

For each run a complete set of readings was obtained on all piezometers. Much care was taken to see that all air was excluded from the various lines of hose connecting the piezometer nipples to the piezometer tubes, and also that all connections were tight. Such precautions were necessary for securing the correct pressures.

In order to compare the readings taken on the various piezometers it was necessary to refer all measurements to a common datum. This required the taking of levels on all piezometer frames, the piezometer tubes on the cover, and on the bottom of the channel.

A complete test for a single quantity of flow, comprising the recording of nearly 300 piezometer readings, the measurement of velocities at 800 different points, and the referring of all piezometers to a common datum, required about 45 man-hours.

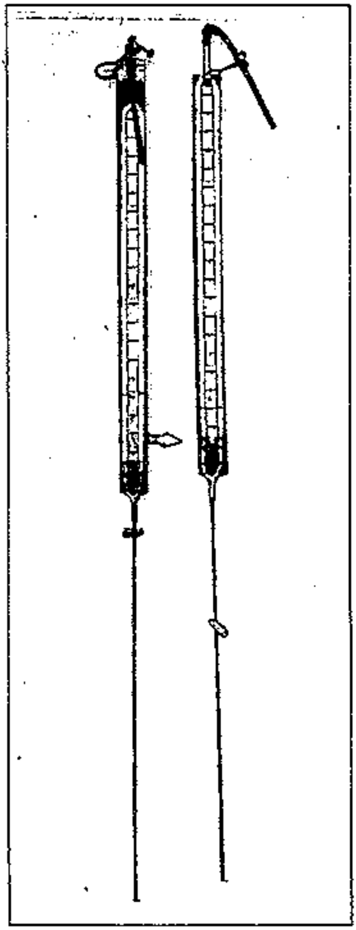
FIGURE 6.—Pitot tubes used in the investigation.

The water in the various piezometer tubes oscillated considerably during a test, frequently as much as 0.05 foot or more. These oscillations, caused probably by the turbulent flow, made the reading of correct pressures difficult. These excessive oscillations were reduced by contracting the hose connections to the piezometer tubes, and also by wedging into the piezometer tubes thin slices of cork with small one-eighth-inch holes drilled in the center of each slice. Even with these precautions, some small oscillations occurred and the mean of several oscillations in a piezometer tube was taken as the correct pressure for the piezometer. It is believed that errors in pressure determinations, with few exceptions, were not greater than 0.01 foot.

In measuring velocities on the tangents of the experimental apparatus care was taken to see that the velocity orifice of each Pitot tube was pointed directly upstream. In the bend, however, the velocity orifice was held normal to each cross section at which measurements were being taken. The turbulent flow as well as the secondary currents in the bend act on the pressure orifices of the Pitot tube (of the combined type) in such a manner that a differential greater or less than the true one may be measured, thus giving the point at which the velocity is being measured a velocity greater or less than the true one. While this condition did not exist at every point at which velocity measurements were taken, some velocity readings showed that such conditions actually occurred. These probable occasional errors in the velocities taken on the bend undoubtedly account in part for the failure of the computations on secondary currents (described in a later section) to check more closely. While it is believed that a type of Pitot tube could be made which would register correct velocities in a bend, the small Pitot tubes de-

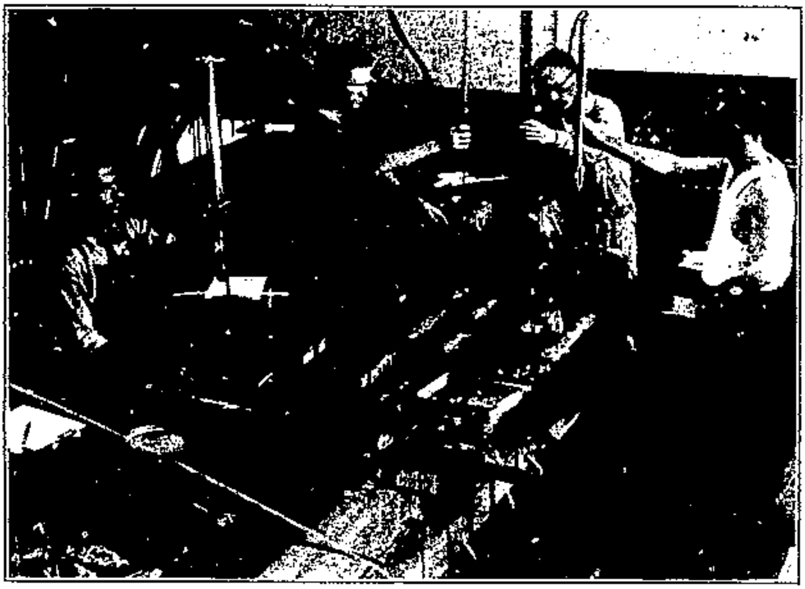

FIGURE 7.—The plate-glass 180° bend and 10-inch channel under test. Observers taking velocity measurements.

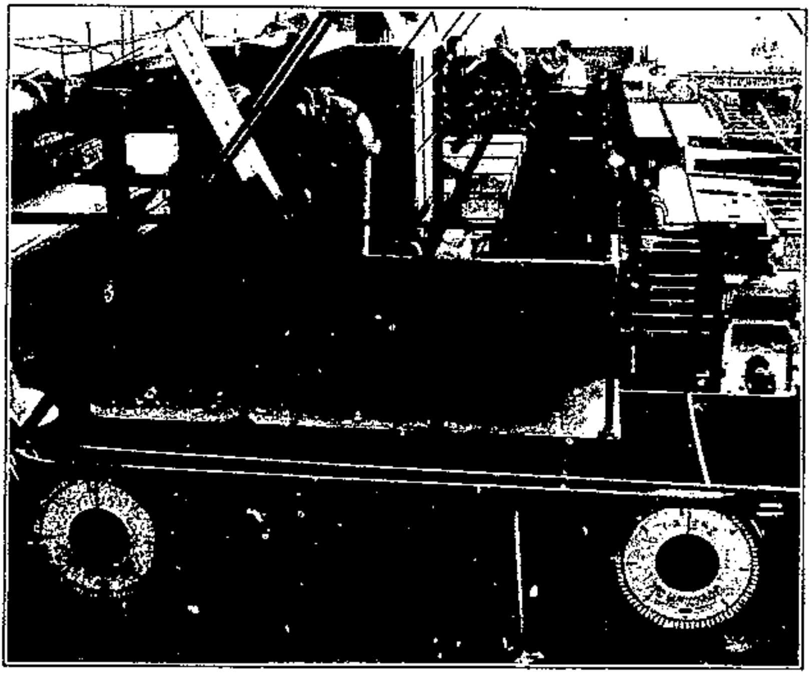

FIGURE 8.—General view of entire bend apparatus and weighing tanks used in determining the quantity of flow.

veloped had to be used because of lack of time and funds to develop the proper kind.

In figures 9 to 32, outside wall of bend is at left and inside wall at right of each section; bottom of channel was level; transverse profiles of water pressure (water surface if channel flowed partly full) are shown by solid lines, and mean velocities by broken lines; stars represent computed transverse water surfaces; side-wall piezometers are represented by triangles, bottom piezometers by squares.

VELOCITY DISTRIBUTION AROUND THE BEND

UNIFORM VELOCITY DISTRIBUTION IN THE APPROACH CHANNEL

CHANNEL FLOWING FULL

Although 10 tests were run on the 10- by 10-inch bend with quantities of flow ranging from 1 to 3 cubic feet per second, for economy in printing the results for only 2 of these tests are presented. The locations, with respect to the bend, of the cross sections traversed for velocity distribution are shown in figure 1.

The velocities obtained at the various cross sections in the 10 by 10 inch channel with a discharge of 2.67 cubic feet per second (mean velocity 3.85 feet per second) are shown in figure 9 by means of 1-foot contours. It is apparent that fairly uniform velocity distribution prevails in the approach channel at sections 1 to 4, inclusive. As the bend is approached, however, the water next to the inside wall speeds up, while the water adjacent to the outside wall slows down. The difference becomes greatest at section 9, where the water next to the inside wall has a maximum velocity of over 7 feet per second while the water next to the outside wall has a velocity of less than 2 feet per second.

After passing the halfway point on the bend, section 10, the velocity distribution gradually becomes more uniform until at the end of the bend, section 13, it is nearly uniform throughout the cross section, with the slowest velocity, however, next the inner side of the bend. After leaving the bend the smallest velocity continues close to the inner wall for some distance. At section 16, 1 foot beyond the bend, the variation is much less, and at section 18, 3 feet beyond the bend, the velocity is substantially uniform again. As the water flows around the bend, the thread of maximum velocity tends to move toward the outside wall of the bend. This is quite apparent at section 15.

Attention is called to the "well" or "depression" along the inside wall midway between the top and bottom at sections 12, 13, 14, and 15. This depression or spot of low velocity is typical of 180° bends of short radius when uniform velocity prevails in the approach channel.

A mean velocity curve is drawn above each cross section in figure 9. The values which were used in plotting these mean velocity curves were obtained in the following manner: The velocity measured at each point was given a weight proportional to the area of cross section represented by that point. The velocities near the top and the bottom of the channel were each given a weight of 1, the velocities 1 inch from the top and the bottom were each given a weight of 2, the velocities 2½ inches from the top and the bottom were each given a weight of 3, and the velocity at the center of the channel was given a weight of 4. Thus the total sum of the weighted velocities, divided by 16, gave the mean velocity for the vertical.

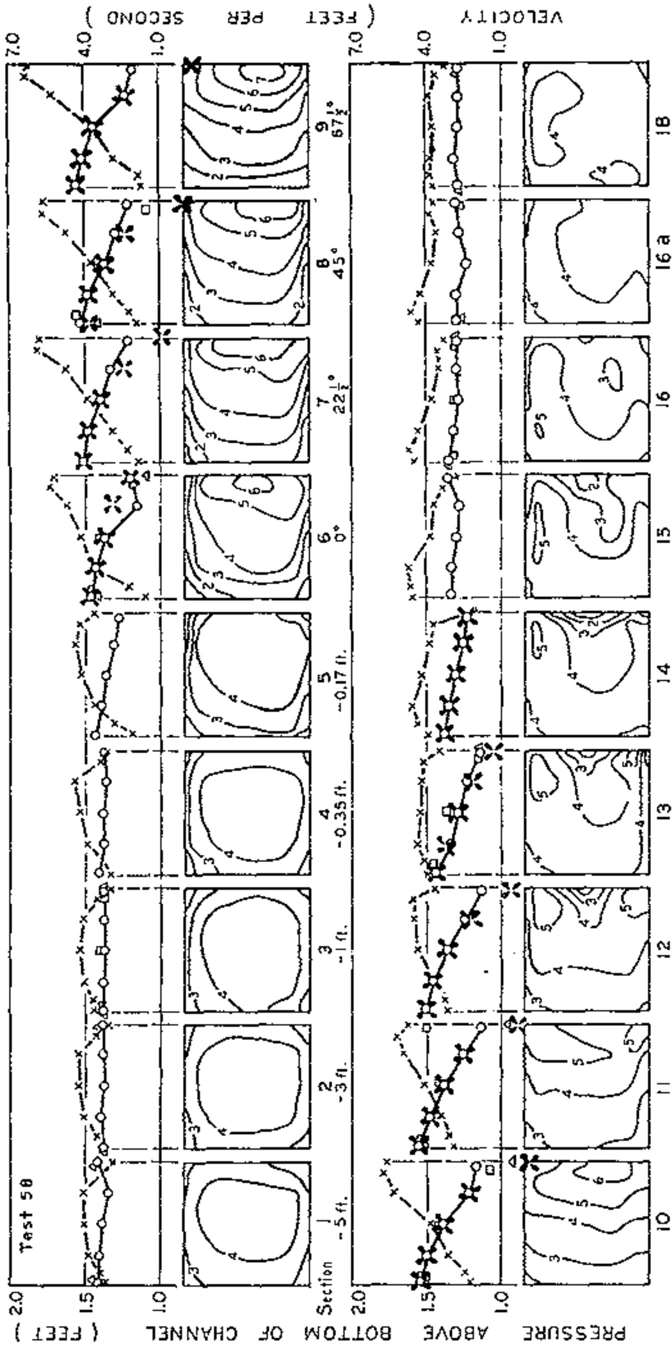

FIGURE 9.—Velocity distribution in 10- by 10-inch channel flowing full; discharge 2.67 cubic feet per second, inner radius of bend 5 inches, approximately uniform velocity distribution in approach channel.

The velocity distribution for a discharge of 2.99 cubic feet per second (mean velocity 4.31 feet per second) in the 10- by 10-inch channel is shown in figure 10. As may be seen, it is similar to the velocity distribution shown in figure 9.

For the 5- by 10-inch channel with a 5-inch inner radius, the velocity distribution for two quantities of flow are shown in figure 11 for a discharge of 1.71 cubic feet per second (mean velocity 4.93 feet per second) and figure 12 for a discharge of 1.88 cubic feet per second (mean velocity 5.41 feet per second). Attention is called to the striking similarity of the velocity distribution in figures 11 and 12 to the distribution in figures 9 and 10.

The velocity distribution for the 5- by 10-inch channel with a 10-inch inner radius is not unlike that for the bends with the 5-inch inner radius (figs. 13 and 14). In this set-up, however, the spot of low velocity along the inside wall is not so marked as in the bends with the smaller radii.

A comparison of the results shown in the above figures indicates that, for the channels having the same inner radius of curvature, there is no conspicuous difference due to a change in the width of the channel. In each case the disturbance of velocity distribution begins nearly 5 inches before the beginning of the bend, and continues for a distance of at least 2 feet following the end of the bend. Similarly an increased radius of curvature produces less violent changes but otherwise similar effects. For the wider channel and for the sharper curvature the disturbance of velocities extends a slightly longer distance upstream from the beginning of the bend. The data are not sufficient to furnish a definite rule, but it may be said that the disturbance above the beginning of the curve does not extend for a distance greater than the width of the stream, while downstream from the bend it persists for a distance of several times the width of the stream.

CHANNELS FLOWING PARTLY FULL

In the bend with uniform velocity distribution in the approach tangent and with the channel flowing partly full, the velocity distribution at the various sections on the bend is similar to that existing in the same channel when flowing full. Figure 15 shows the velocity distribution in the 10- by 10-inch channel when flowing partly full. It will be noted that as the water moves forward around the bend, the filaments of flow next to the inside wall of the bend have a higher velocity than the filaments next to the outside wall. Near the end of the bend, however, the velocity next to the outside wall is somewhat greater than that next to the inner wall. Figure 16 shows the velocity distribution in the 5- by 10-inch channel with a 5-inch inner radius while figure 17 shows the velocity distribution in the 5- by 10-inch channel with a 10-inch radius. The velocity distribution in each of these two channels is somewhat similar to that in the 10- by 10-inch channel although towards the end of the bend the velocity next to the inside wall becomes quite low. In fact, the spot of low velocity next to the inside wall begins at a point close to the end of the bend and persists to a point even beyond the end of the bend.

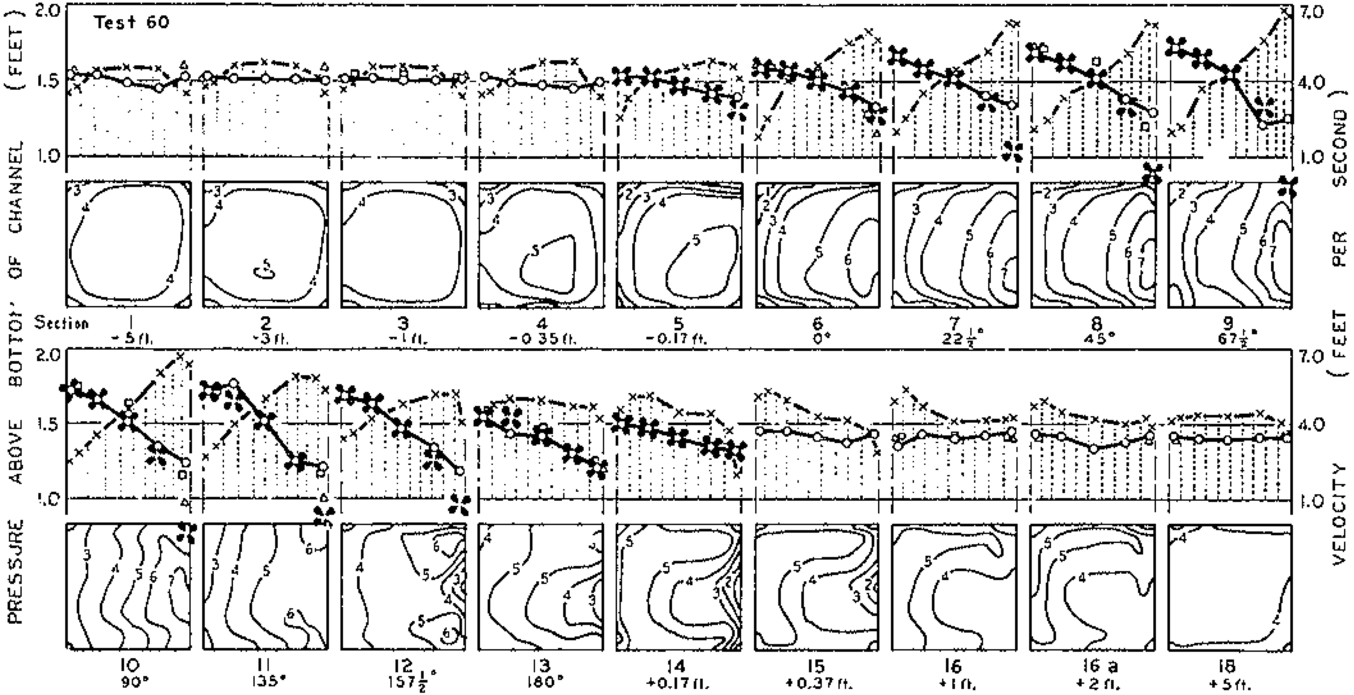

FIGURE 10. Velocity distribution in 10- by 10-inch channel flowing full; discharge 2.99 cubic feet per second, inner radius of bend 5 inches, approximately uniform velocity distribution in approach channel.

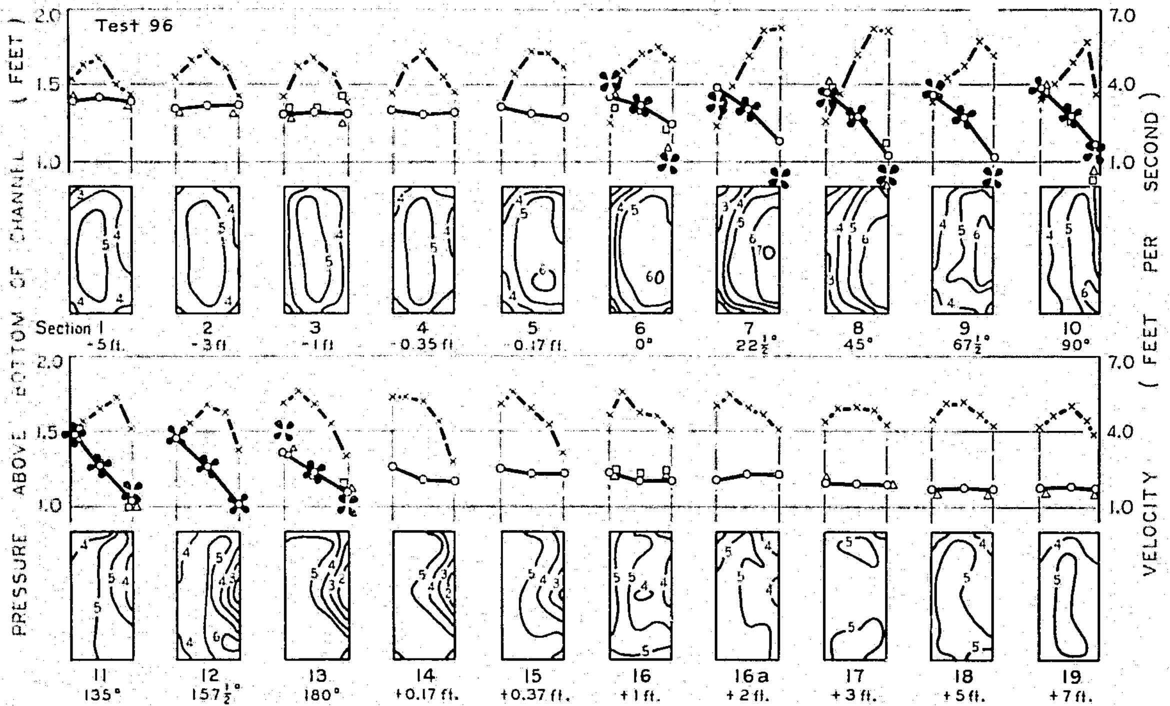

FIGURE 11.—Velocity distribution in 5- by 10-inch channel flowing full; discharge 1.71 cubic feet per second, inner radius of bend 5 inches, approximately uniform velocity distribution in approach channel.

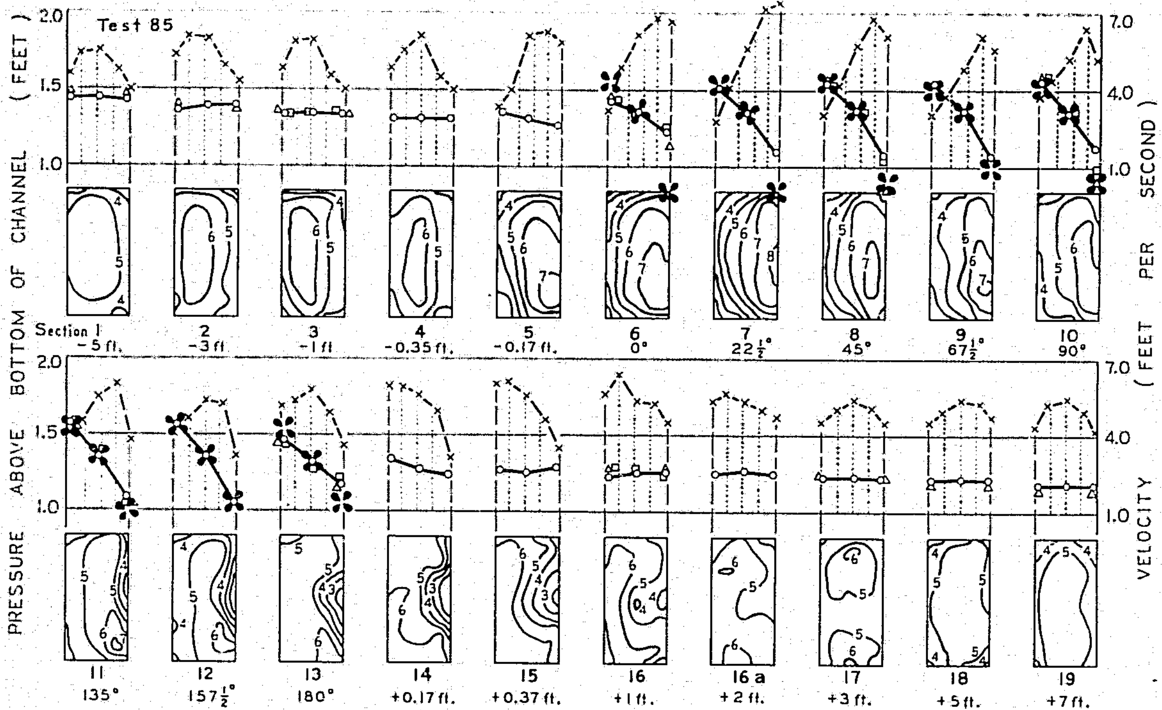

FIGURE 12.—Velocity distribution in 5- by 10-inch channel flowing full; discharge 1.88 cubic feet per second, inner radius of bend 5 inches, approximately uniform velocity distribution in approach channel.

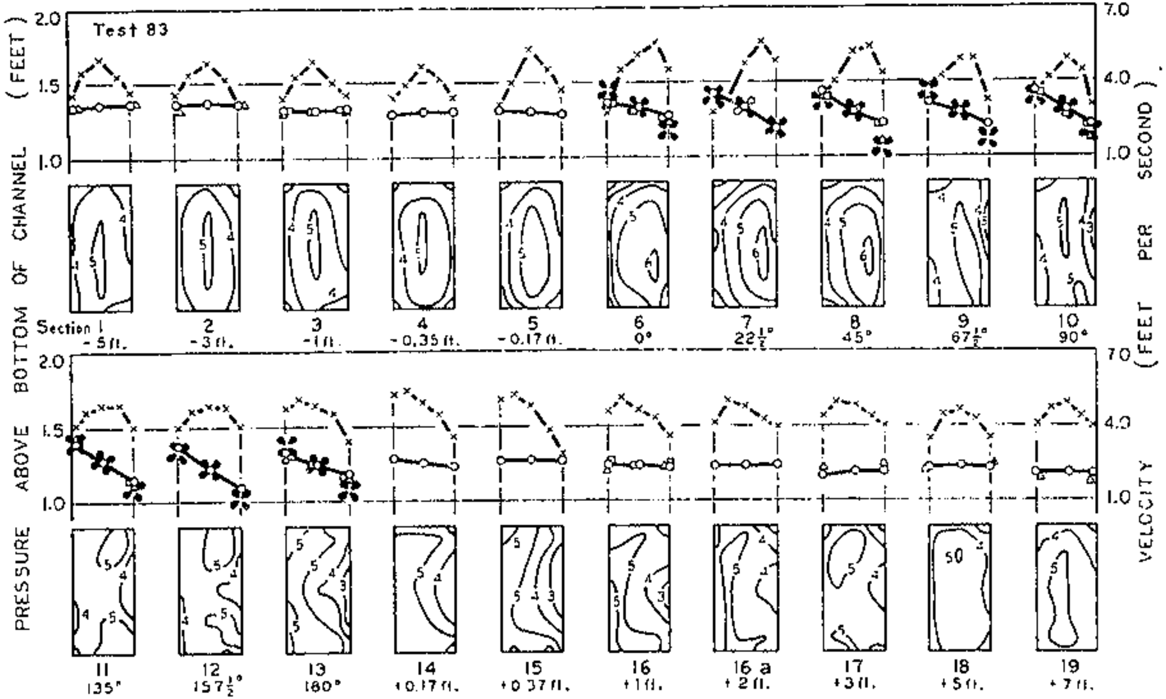

FIGURE 13.—Velocity distribution in 5- by 10-inch channel flowing full; discharge 1.67 cubic feet per second, inner radius of bend 10 inches, approximately uniform velocity distribution in approach channel.

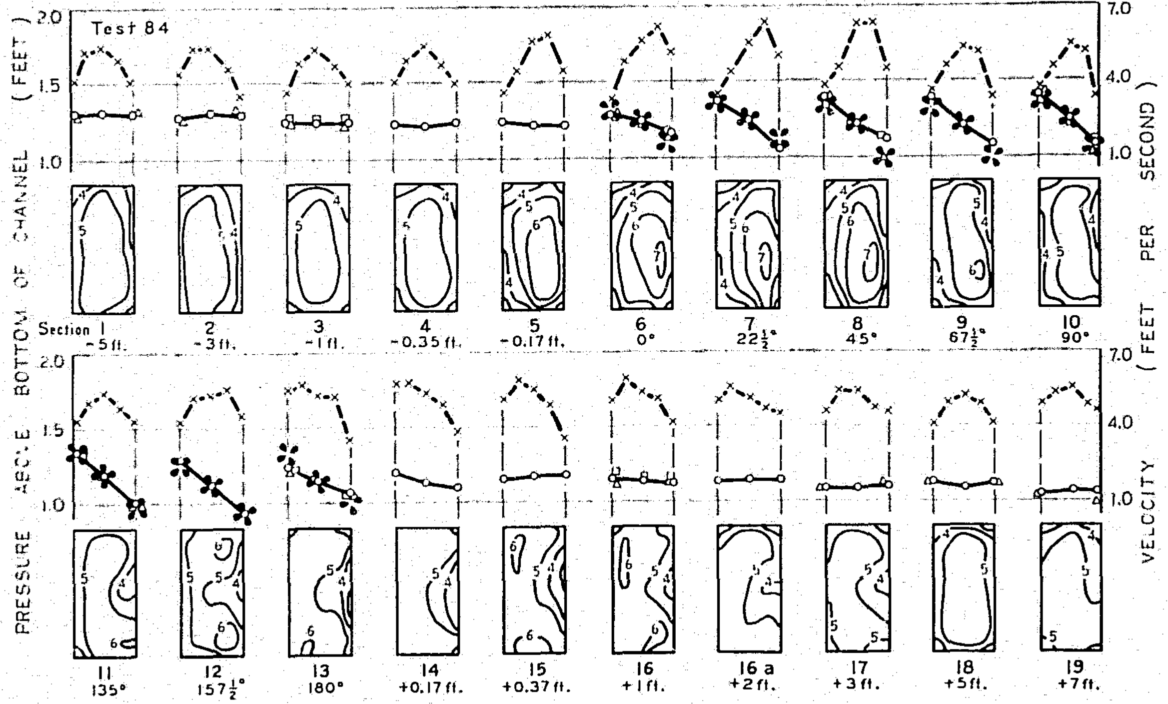

FIGURE 14.—Velocity distribution in 5- by 10-inch channel flowing full; discharge 1.81 cubic feet per second, inner radius of bend 10 inches, approximately uniform velocity distribution in approach channel.

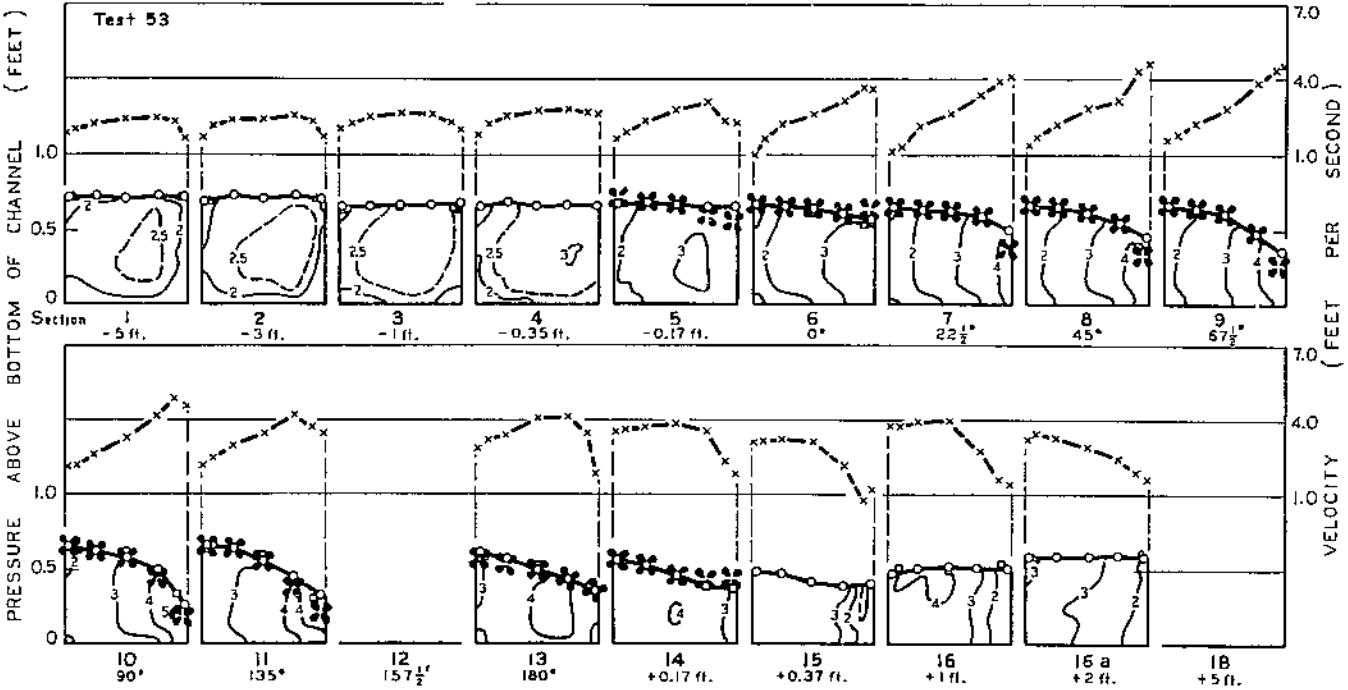

FIGURE 15.—Velocity distribution in 10- by 10-inch channel flowing partly full; discharge 1.38 cubic feet per second, inner radius of bend 5 inches, approximately uniform velocity distribution in approach channel.

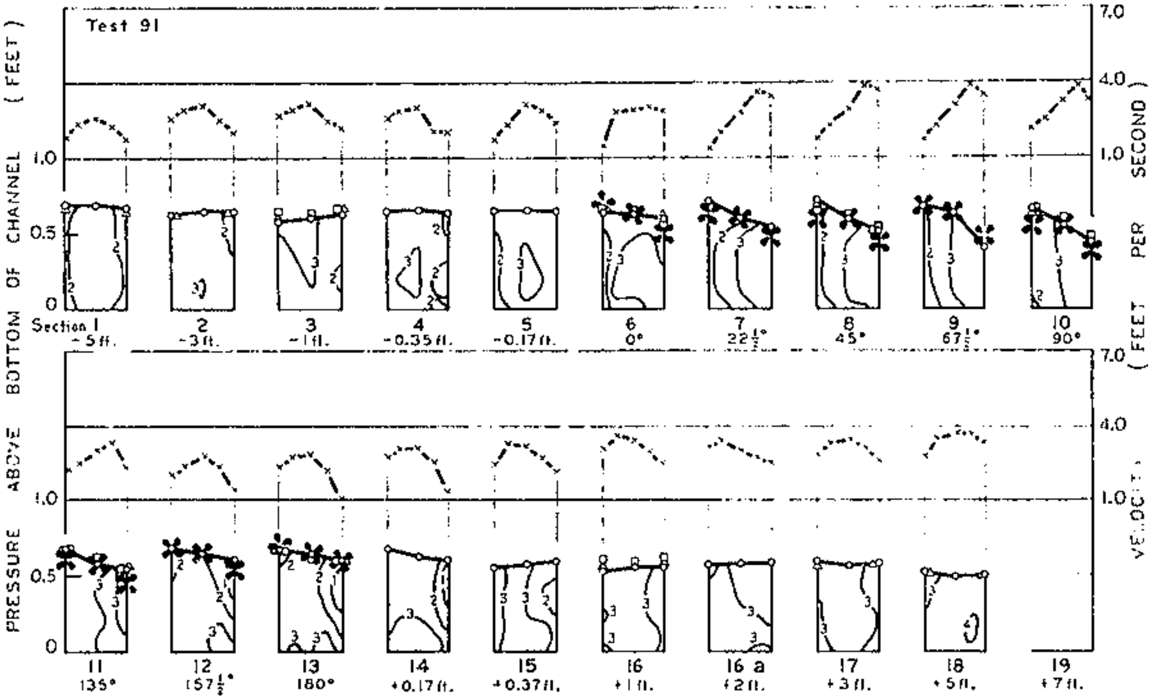

FIGURE 16.—Velocity distribution in 5- by 10-inch channel flowing partly full; discharge 0.71 cubic foot per second, inner radius of bend 5 inches, approximately uniform velocity distribution in approach channel.

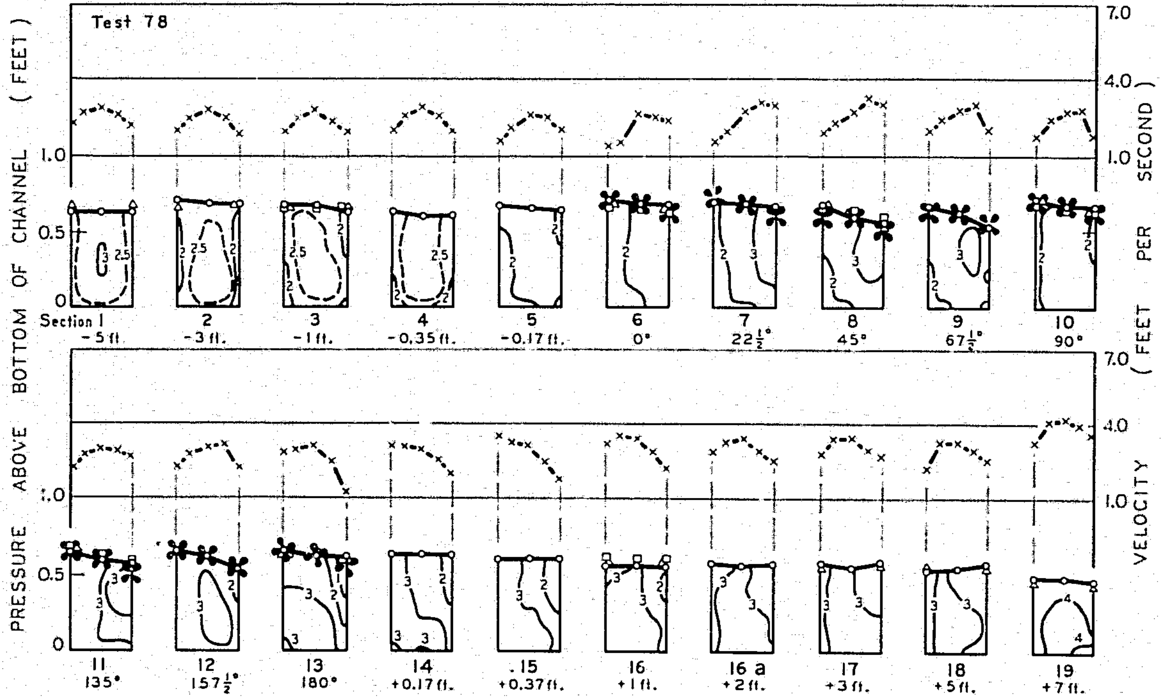

Figure 17.—Velocity distribution of 5- by 10-inch channel flowing partly full; discharge 0.71 cubic foot per second, inner radius of bend 10 inches, approximately uniform velocity distribution in approach channel.

NONUNIFORM VELOCITY DISTRIBUTION IN APPROACH CHANNEL

CHANNEL FLOWING FULL

Let us consider the case where the velocity at the beginning of the bend is higher along the outer wall than along the inner wall. As the water in the 10- by 10-inch channel moves around the bend (fig. 18) the velocity distribution tends to become uniform. Then near the end of the bend the velocity next to the inner wall slows up very materially while that next to the outer wall becomes very high as may be seen in sections 11 to 16 in figure 18. In the 5- by 10-inch channel with the 5-inch inner radius (fig. 19), the velocity distribution is similar except that the sections of fairly uniform velocity distribution are further around the bend. For nearly the entire length of the bend the velocity along the outer wall is considerably higher than along the inner. The velocity distribution in the bend of the 5- by 10-inch channel with the 10-inch inner radius (fig. 20) is similar to that in the other channels. It should be noted, however, that in this set-up the difference between the velocities at the outer and inner walls is not as great as it is for the other bends. This decrease is due no doubt to the larger radius.

Different velocity distribution prevails in the bend when the velocity in the approach channel to the bend is higher along the inner wall than along the outer wall. This kind of approach velocity distribution tends to increase the velocities which normally exist in a bend. The velocities along the inner wall are materially increased, as may be seen by comparing the velocities in figure 21 with the velocities in figure 9. There is a spot of low velocity along the inner wall from sections 11 to 14. The thread of maximum velocity, however, shifts to the outer wall at the end of the bend (fig. 21). A similar velocity distribution prevails in the 5- by 10-inch channel with a 5-inch inner radius (fig. 22). In this channel no well of low velocity appears along the inner wall and quite uniform velocity distribution takes place near the end of the bend. In the 5- by 10-inch channel with a 10-inch inner radius (fig. 23) the thread of maximum velocity shifts to the outer wall even before the end of the bend is reached. There are no real low spots of low velocity in this set-up.

When a high velocity prevails along the bottom and a low velocity along the top of the approach channel, the conditions of flow within the bend are somewhat unstable. With uniform velocity distribution in the approach channel the velocities of the filaments of flow next to the inner wall increase. This increase is more or less uniform in any one vertical. But when a portion of the water next to the inner wall has a high velocity and a portion has a low velocity, unstable equilibrium exists. This condition of flow may be seen in figure 24, which shows the velocity distribution in the 10- by 10-inch bend with a 5-inch inner radius. Nonuniform velocity distribution exists entirely around the bend and even into the discharge tangent for a considerable distance from the bend. Figure 25 shows the velocity distribution in the 5- by 10-inch channel with a 5-inch inner radius. As may be seen, the velocity distribution in this set-up is not unlike that in the 10- by 10-inch bend (fig. 24), but attention should be called to the fact that the spot of low velocity seems to occur sooner. The velocity distribution for the 5- by 10-inch bend with a 10-inch inner radius is shown in figure 26. This set-up shows a similar kind of flow. All three of these set-ups show a spot of low velocity along the bottom next to the inside wall.

When this velocity distribution is reversed, that is, having a high velocity along the top and a low velocity along the bottom of the approach channel, the conditions of flow in the bend are reversed from that existing in the previous set-ups. Figure 27 shows the velocity distribution in the 10- by 10-inch channel with a 5-inch inner radius. Attention is called to the unusual spot of low velocity evident in sections 10 to 16. The nonuniform velocity distribution prevails for some distance into the discharge tangent. Figure 28 shows the velocity distribution in the 5- by 10-inch channel with a 5-inch inner radius. This velocity distribution is similar to that in figure 27 but the velocities in the well of low velocity are not as low as in the 10 by 10 inch channel. Figure 29 shows the velocity distribution in the 5- by 10-inch channel with the 10-inch inner radius. In this set-up there is not as great a variation in the velocities in any one section as there was in the other two set-ups.

It may be observed that, when at the entrance to the bend the velocity is greatest at the top and lowest at the bottom of the channel, by the time the water has passed around the bend the areas of high and low velocity have substantially exchanged positions.

CHANNELS FLOWING PARTLY FULL

When the channel is flowing partly full and with a low velocity at the bottom and a high velocity at the top in the approach channel, the velocity along the inner wall is high up to a point about midway around the bend. Then the thread of maximum velocity shifts to the outer bank. This type of flow is shown in figure 30 which is the 10- by 10-inch channel with a 5-inch inner radius. Similar velocity distribution exists in the 5- by 10-inch channel with a 5-inch inner radius (fig. 31).

In the 5- by 10-inch channel with a 10-inch inner radius (fig. 32) the velocity distribution is somewhat different. In this set-up the difference between the velocities at the inner and outer walls near the beginning of the bend is not so great. However, as the water moves around the bend, the thread of maximum velocity shifts to the outer wall (secs. 14 to 16, fig. 32).

The discrepancy in velocity distribution between sections 3 and 4 in figures 30, 31, and 32 is due to the fact that section 3 is located only 4 inches downstream from the downstream edge of the obstruction which was 5 inches long. There is some turbulence in the water at section 3 and more stable conditions of flow do not prevail until section 4 and even section 5 is reached.

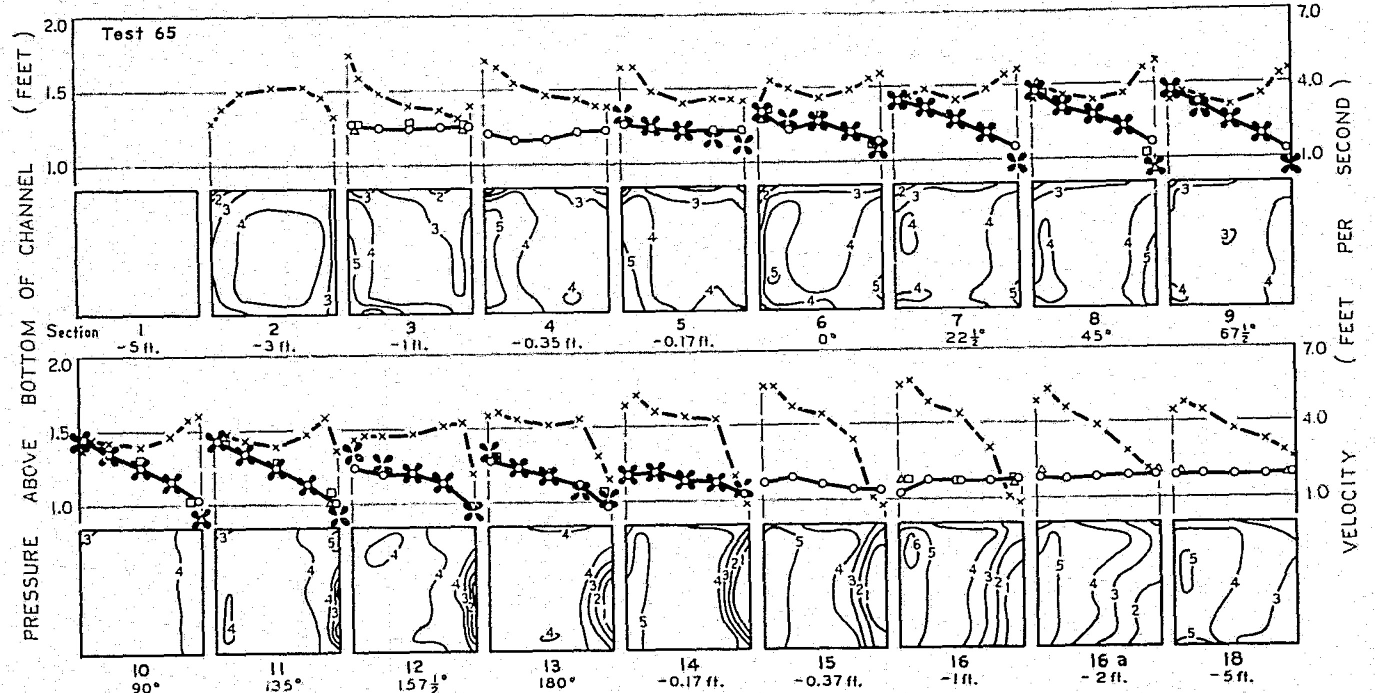

FIGURE 18.—Velocity distribution in 10- by 10-inch channel flowing full; discharge 2.74 cubic feet per second, inner radius of bend 5 inches, high velocity at outside and low velocity at inside of approach channel.

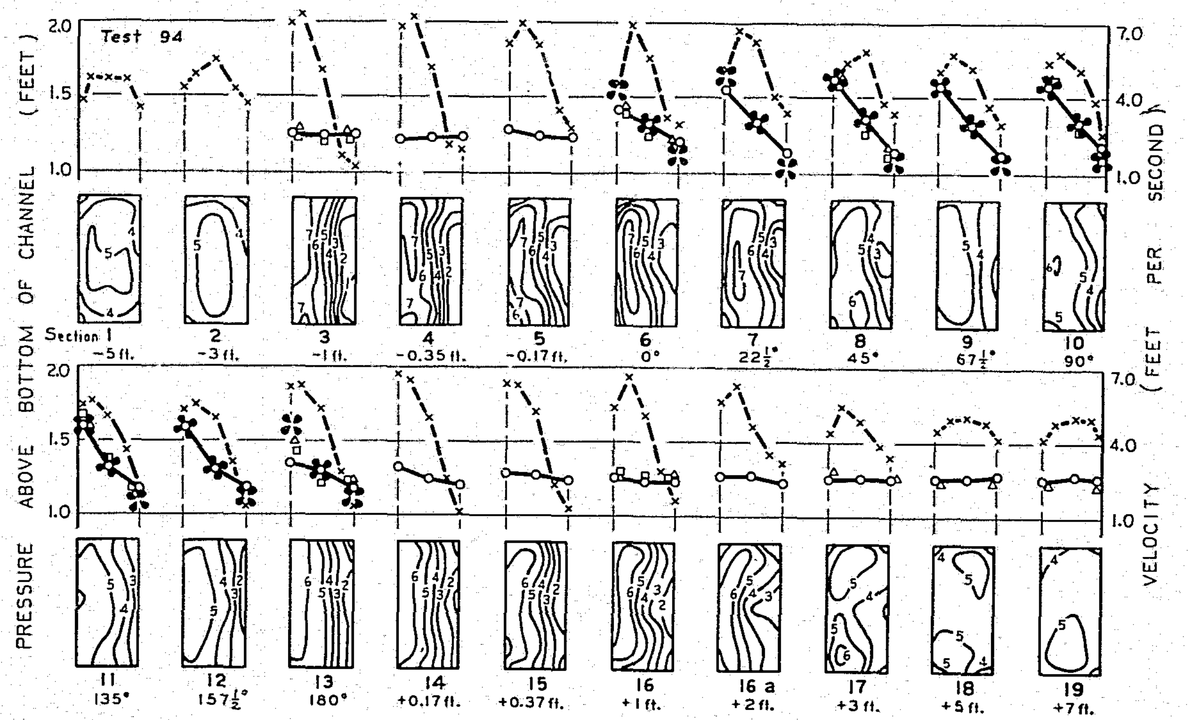

FIGURE 19.—Velocity distribution in 5- by 10-inch channel flowing full; discharge 1.68 cubic feet per second, inner radius of bend 5 inches, high velocity at outside and low velocity at inside of approach channel.

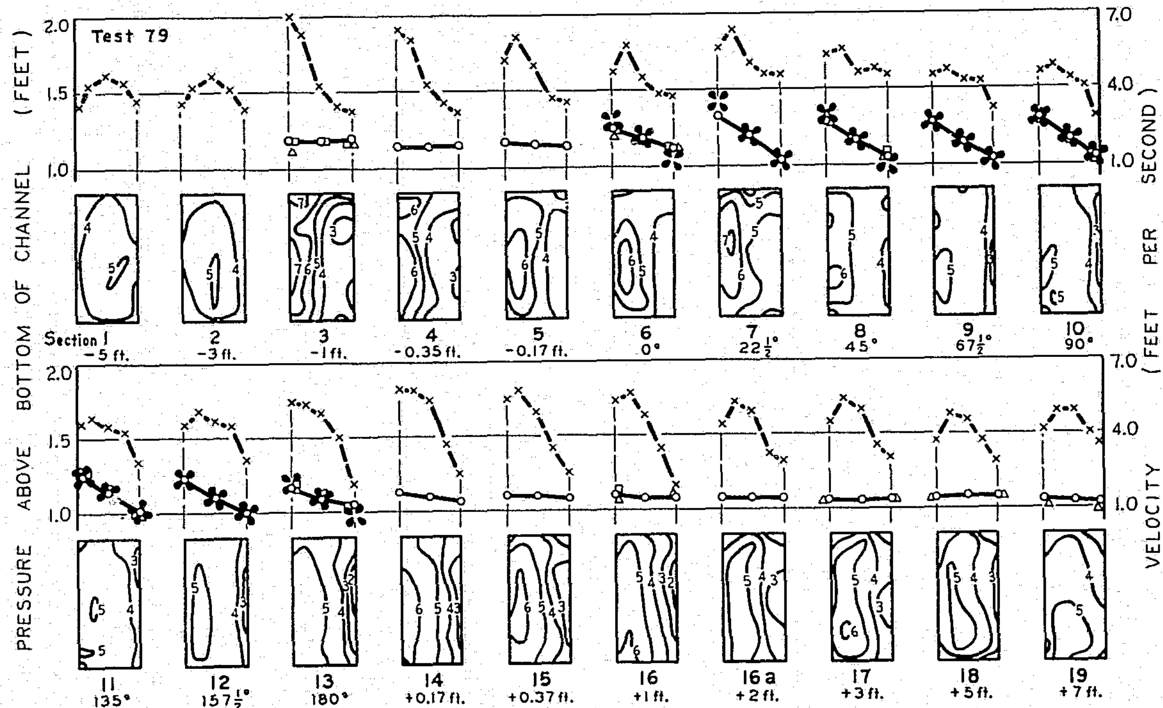

FIGURE 20.—Velocity distribution of 5- by 10-inch channel flowing full; discharge 1.62 cubic feet per second, inner radius of bend 10 inches, high velocity on outside and low velocity on inside of approach channel.

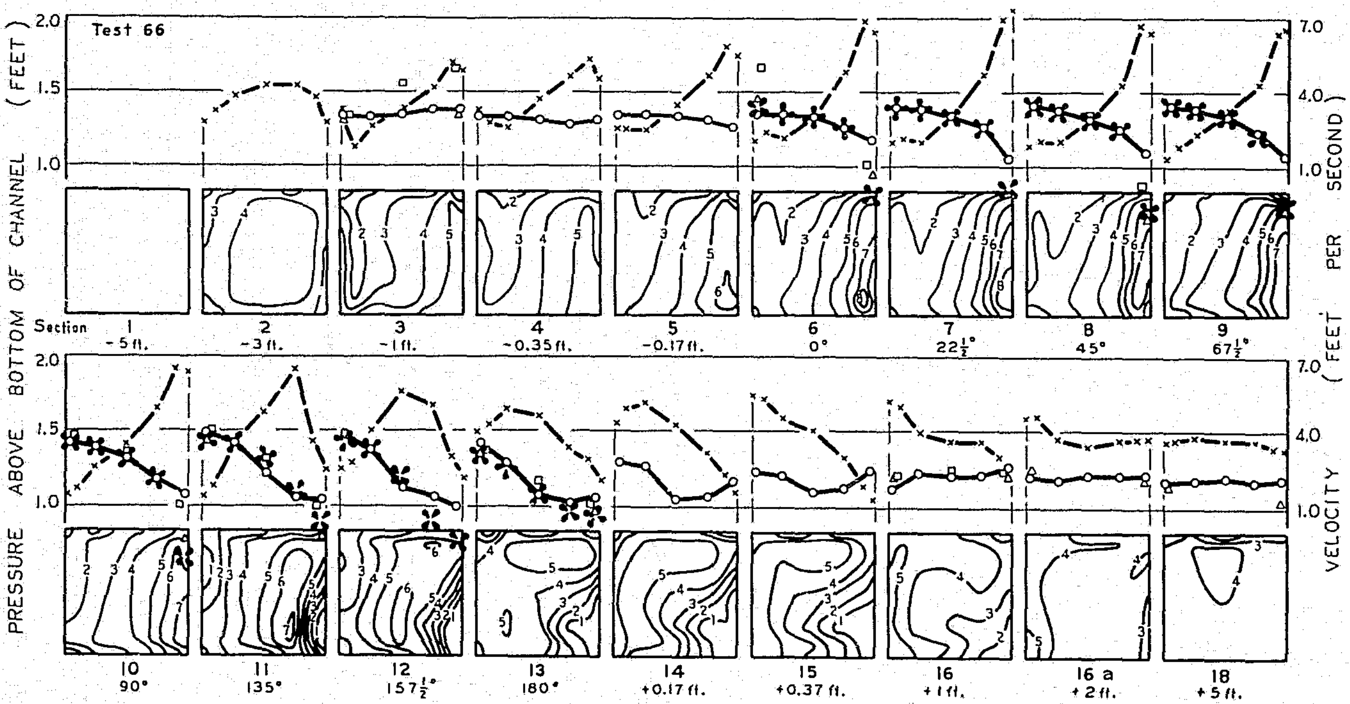

FIGURE 21.- Velocity distribution in 10- by 10-inch channel flowing full; discharge 2.74 cubic feet per second, inner radius of bend 5 inches, high velocity at inside and low velocity at outside of approach channel.

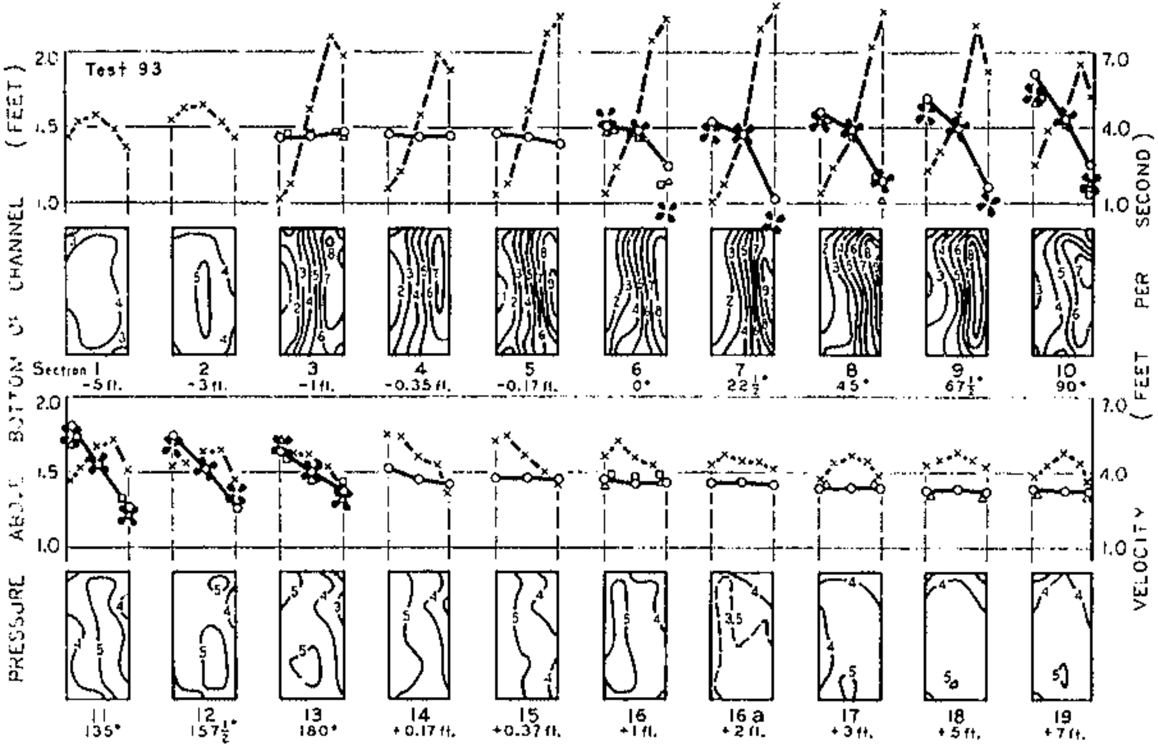

FIGURE 22. Velocity distribution in 5- by 10-inch channel flowing full; discharge 1.63 cubic feet per second, inner radius of bend 5 inches, high velocity at inside and low velocity at outside of approach channel

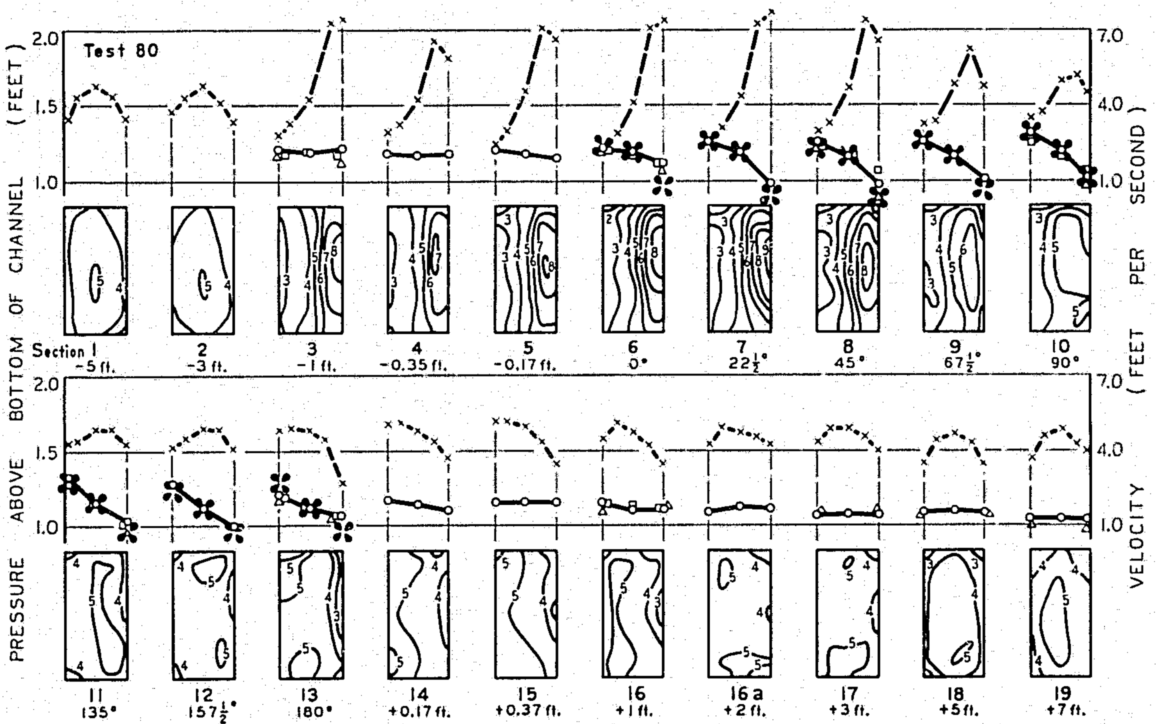

FIGURE 23.—Velocity distribution in 5- by 10-inch channel flowing full; discharge 1.63 cubic feet per second, inner radius of bend 10 inches, high velocity at inside and low velocity at outside of approach channel.

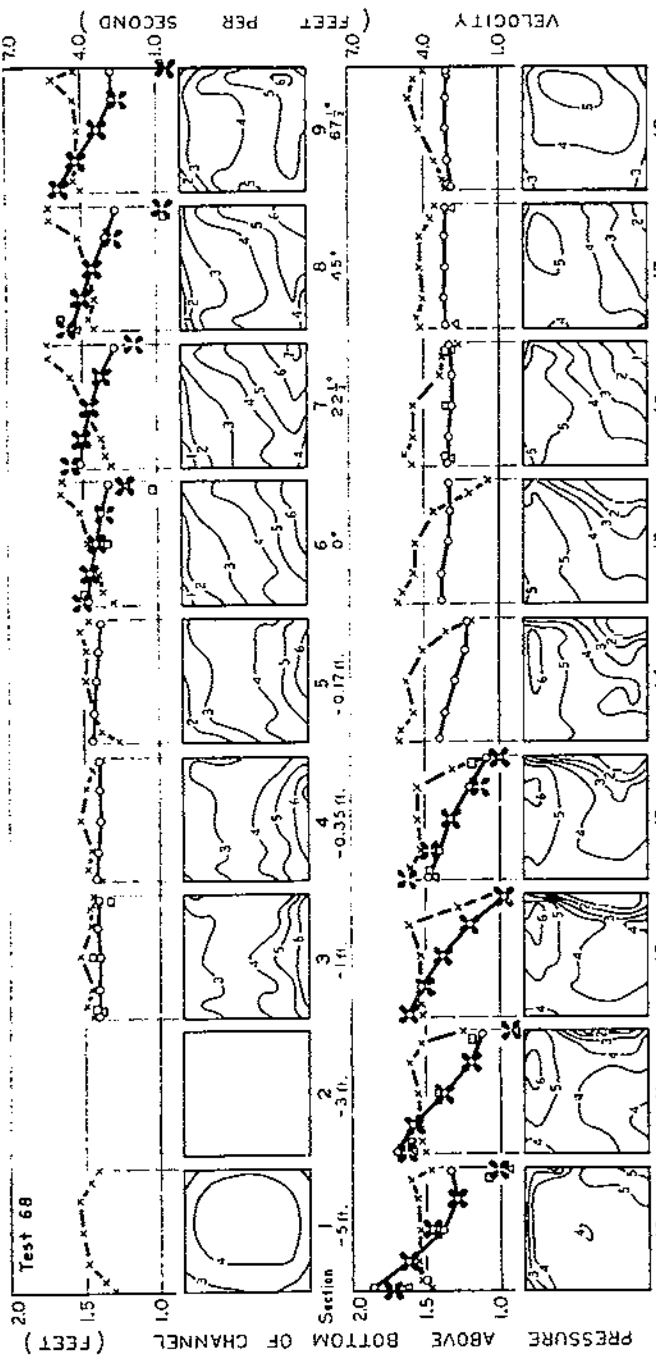

FIGURE 24.—Velocity distribution in 10- by 10-inch channel flowing full; discharge 2.83 cubic feet per second, inner radius of bend 5 inches, high velocity at bottom and low velocity at top of approach channel.

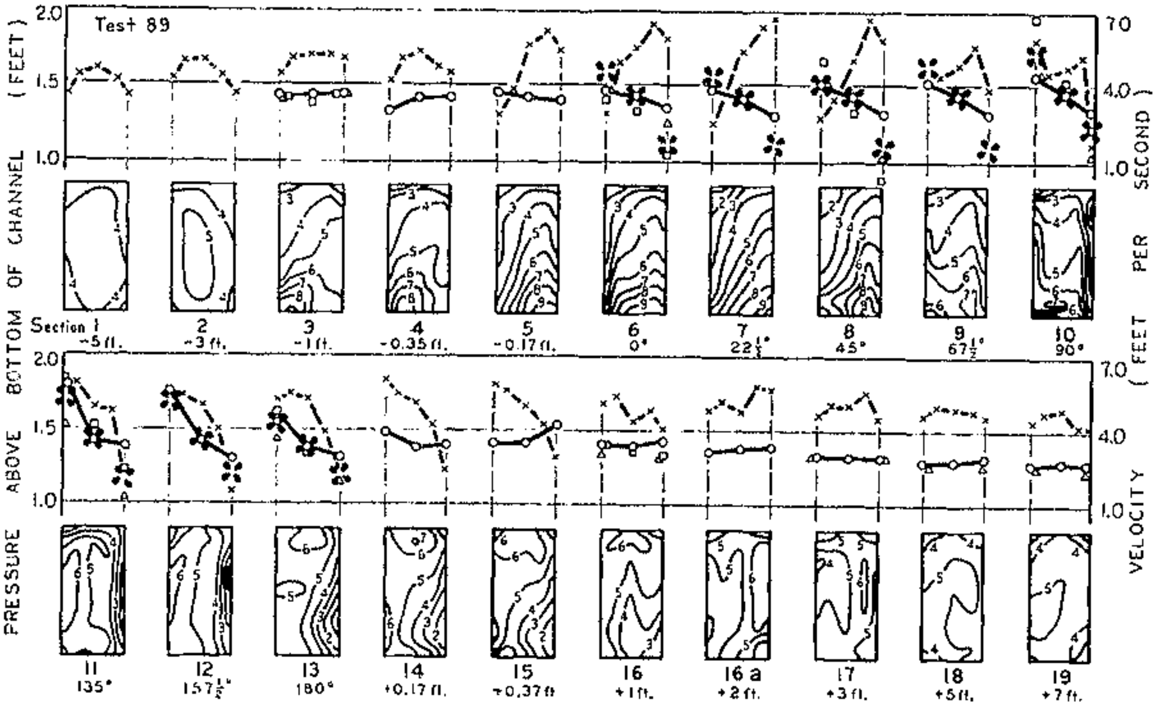

FIGURE 25. Velocity distribution in 5 by 10-inch channel flowing full; discharge 1.71 cubic feet per second, inner radius of bend 5 inches, high velocity at bottom and low velocity at top of approach channel.

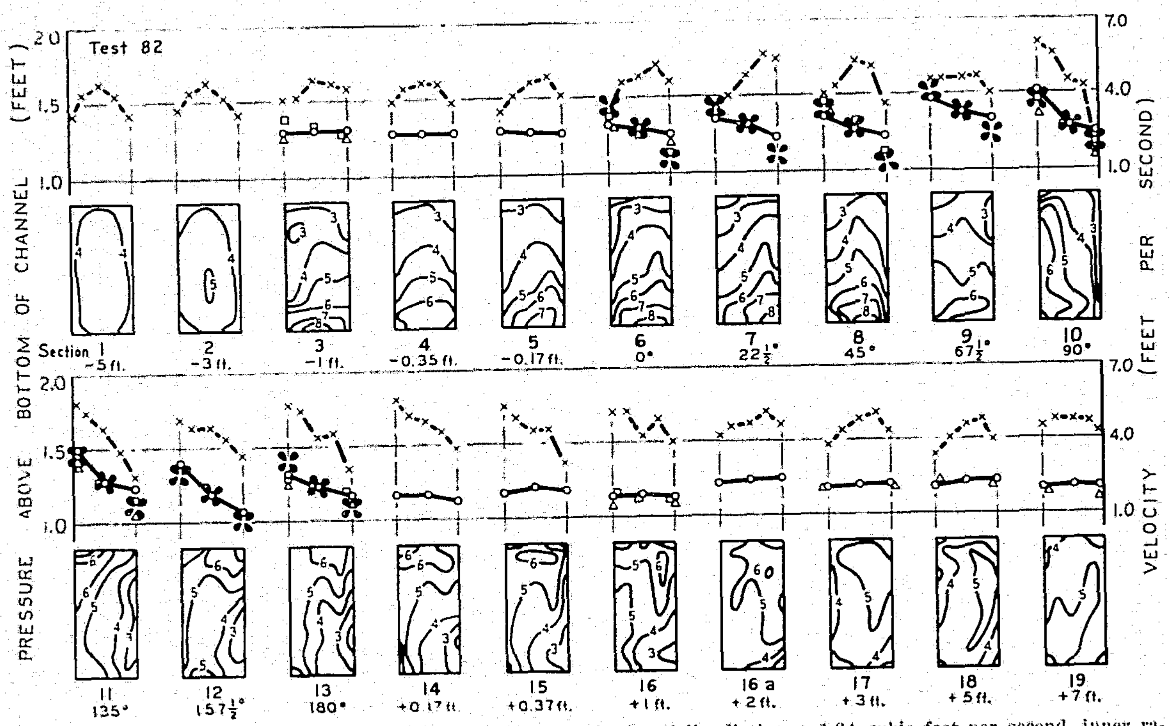

FIGURE 26.—Velocity distribution in 5- by 10-inch channel flowing full; discharge 1.64 cubic feet per second, inner radius of bend 10 inches, high velocity at bottom and low velocity at top of approach channel.

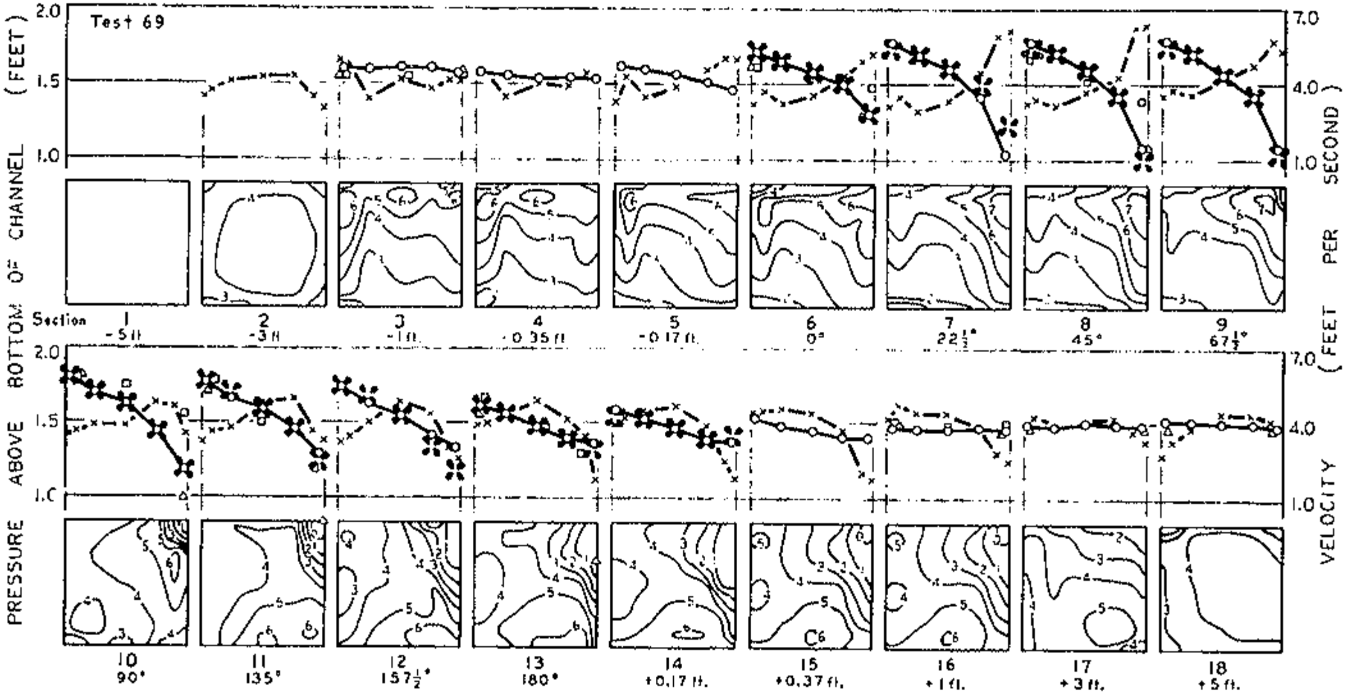

FIGURE 27.—Velocity distribution in 10- by 10-inch channel flowing full; discharge 3.10 cubic feet per second, inner radius of bend 5 inches, high velocity at top and low velocity at bottom of approach channel.

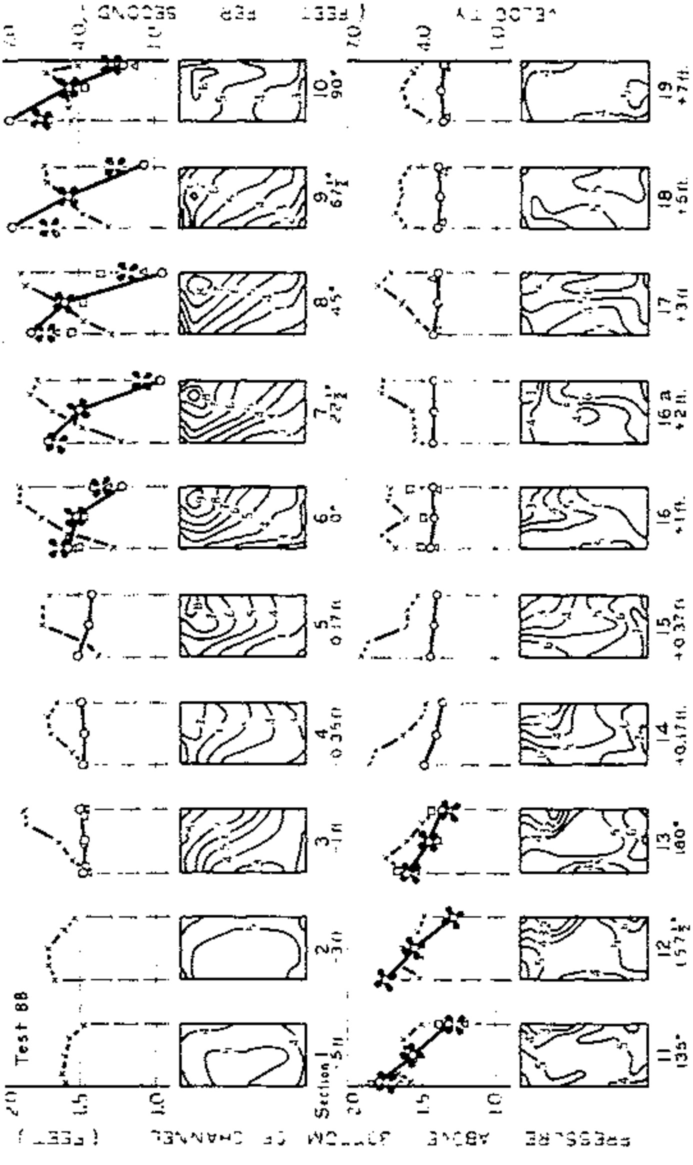

FIGURE 28. Velocity distribution in 5- by 10-inch channel flowing full; discharge 1.07 cubic feet per second, inner radius of bend 3 inches, high velocity at top and low velocity at bottom of approach channel.

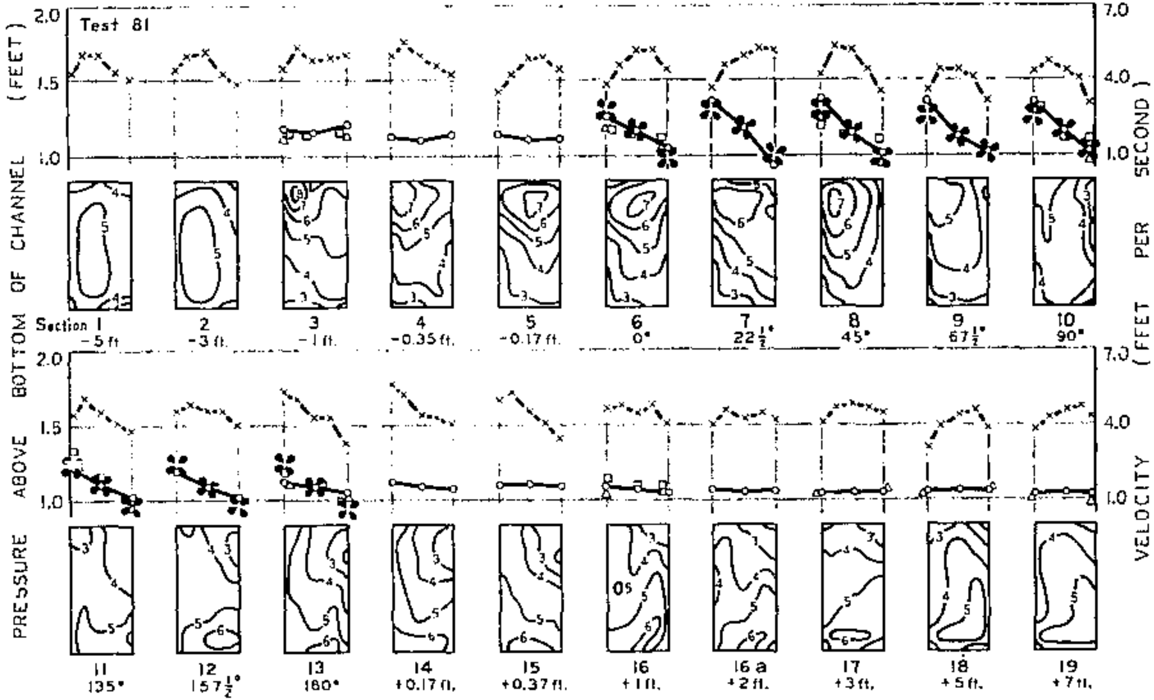

FIGURE 20.—Velocity distribution in 5- by 10-inch channel flowing full; discharge 1.59 cubic feet per second, inner radius of bend 10 inches, high velocity at top and low velocity at bottom of approach channel.

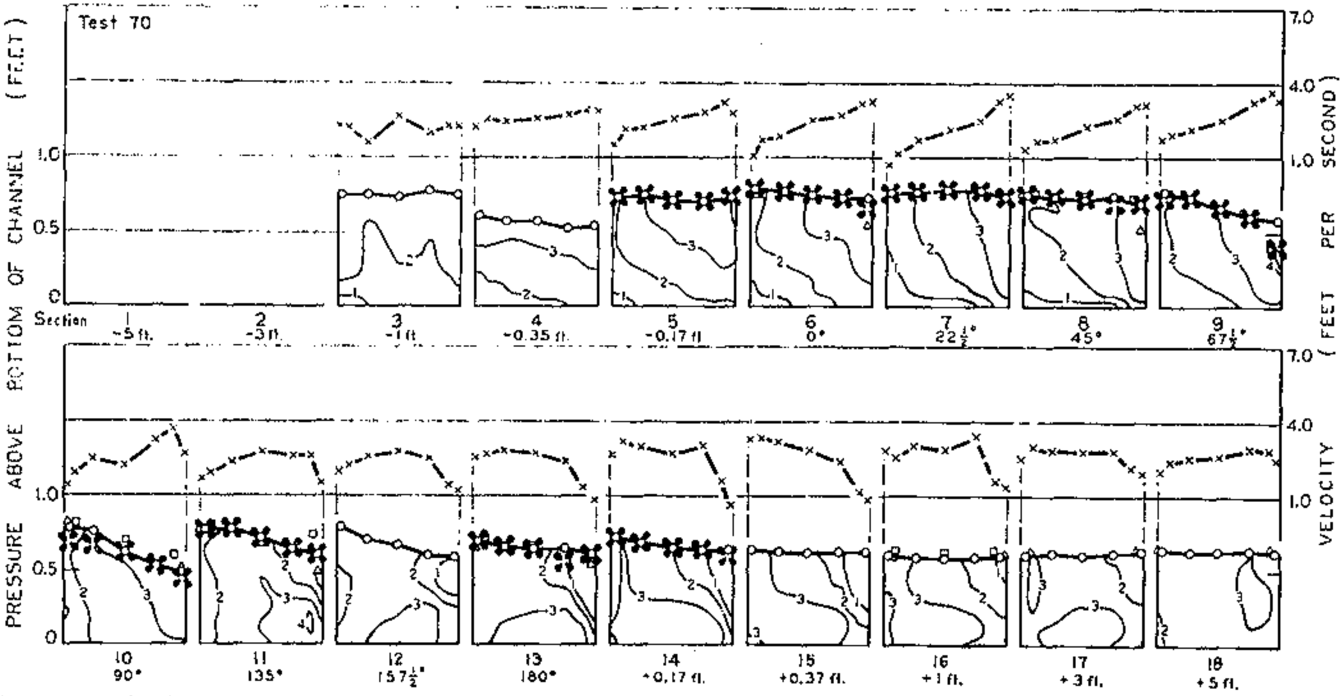

FIGURE 30.—Velocity distribution in 10- by 10-inch channel flowing partly full; discharge 1.40 cubic feet per second, inner radius of bend 5 inches, high velocity at top and low velocity at bottom of approach channel.

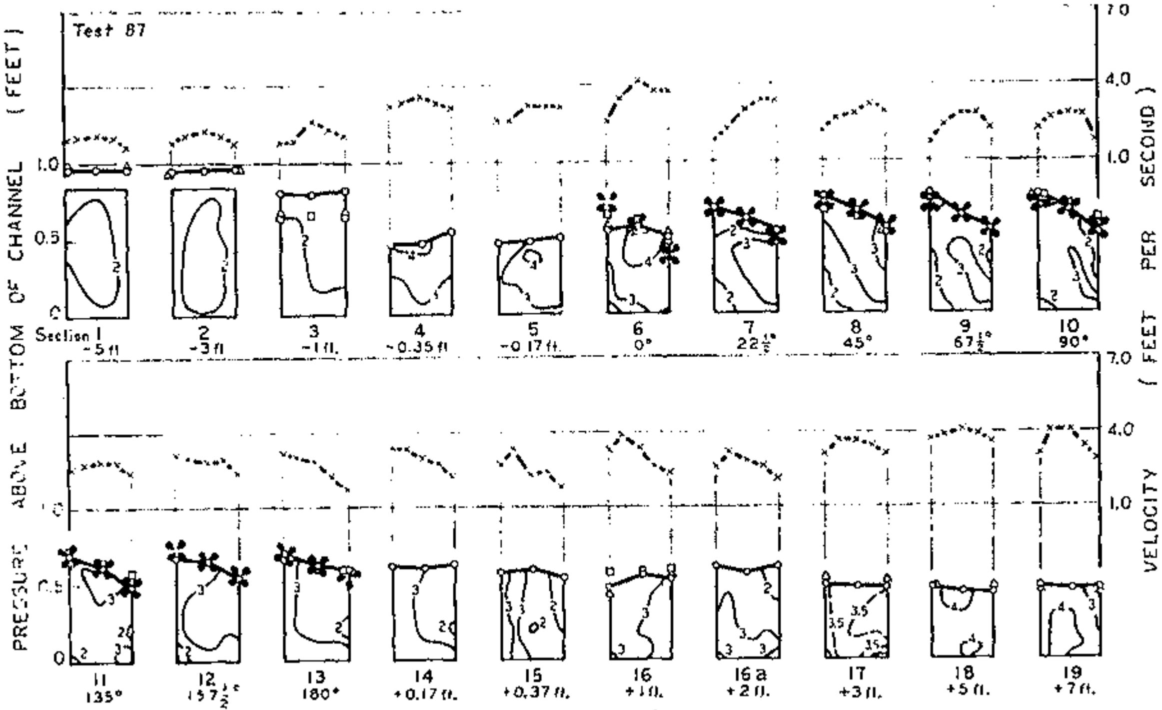

FIGURE 31.—Velocity distribution in 5- by 10-inch channel flowing partly full; discharge 0.72 cubic foot per second, inner radius of bend 5 inches, high velocity at top and low velocity at bottom of approach channel.

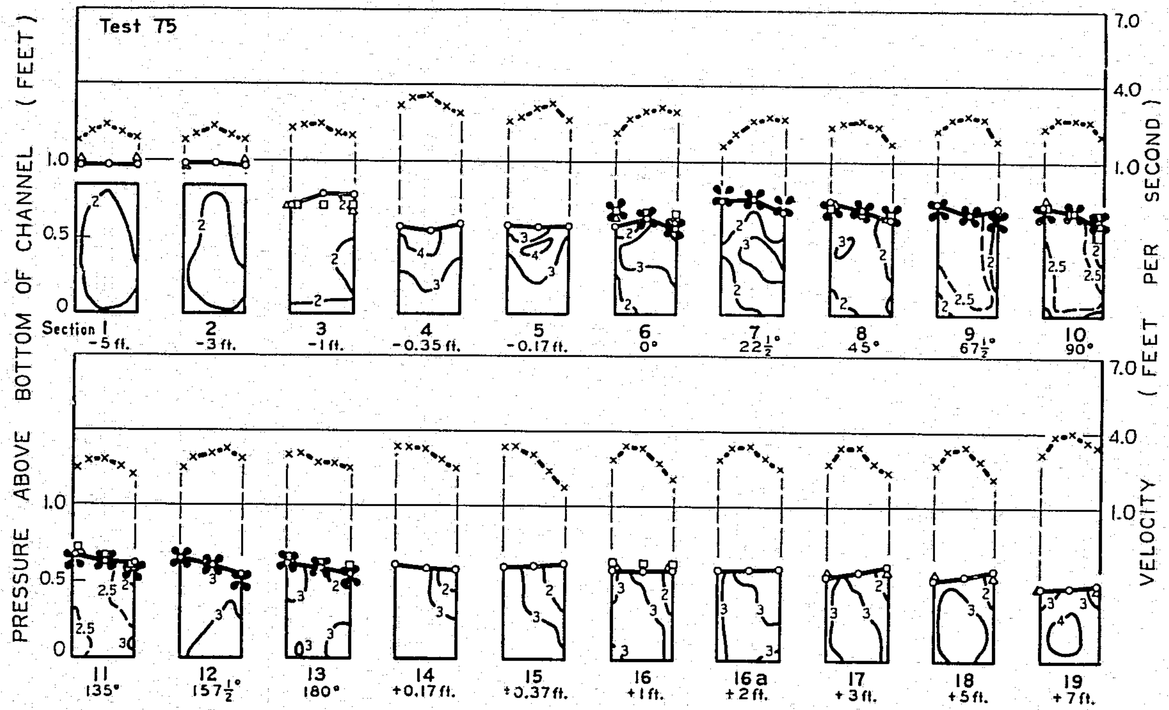

FIGURE 32.—Velocity distribution in 5- by 10-inch channel flowing partly full; discharge 0.74 cubic foot per second, inner radius of bend 10 inches, high velocity at top and low velocity at bottom of approach channel.

PRESSURE CHANGES IN THE BEND

UNIFORM VELOCITY DISTRIBUTION IN THE APPROACH CHANNEL

When a flowing stream of water meets an obstruction so that the various particles of water are diverted from motion along a straight line to motion in a crooked or curved path, the speed of the various particles of water must change, and change by unequal amounts. That is, in general, some of the water must have its velocity increased, and other parts of it must be decreased. Accompanying these changes of velocity are related changes of pressure.

A bend may be considered as an obsruction to the flow of water, and causes related changes of velocity and pressure. The pressures against the top of the channel shown by the elevation of the water in the glass tubes in the cover have been plotted for each section for the various tests as illustrated in figures 9 to 14 inclusive. The amounts of these pressures against the top of the channel are shown by the solid lines for the various sections in figures 9 to 14, inclusive.

Uniform pressure against the four sides of the straight channel usually occurs when there is uniform velocity distribution in the channel. As the velocity distribution changes, the pressure likewise changes, but inversely as the square of the velocity. The higher the velocity, the lower the pressure will be. This is clearly brought out by the mean velocity and pressure curves shown in figures 9 to 17, inclusive.

The relation of pressure to velocity can best be studied by comparing the pressure and velocity changes in a filament of flow at the various sections as the filament of water moves around the bend. Bernoulli's theorem will apply to a particle of water or a unit volume of water in its travel around the bend. The areas under the mean velocity curves for the various sections in test 60, figure 10, have each been divided into 10 filaments of equal area. A unit volume of water in the filament along the outside wall of the channel for test 60 will be studied. There is comparatively little difference in the velocity of this unit volume as it passes sections 1, 2, and 3. This unit volume of water has a velocity of 4.0 feet per second at section 3 and a velocity of about 3.35 feet per second at section 6.

Bernoulli's theorem states that the pressure plus the elevation (above a common datum) plus the velocity head of a unit volume of water at one location in the channel is equal to the pressure plus the elevation plus the velocity head of this same unit volume in its new position at a point downstream from the first position. Since the floor of the channel was level, the elevation factor need not be considered. The pressure of the unit filament at section 3 is 1.50 as shown in figure 10. The mean velocity of the unit particle at this section is 4 feet per second for which the velocity head is 0.249 foot. The pressure exerted by this unit particle of water at section 6 (fig. 10) is 1.565. The mean velocity for the unit particle at section 6 is 3.35 feet per second (taken from the mean velocity curve) for which the velocity head is 0.174 foot. Applying Bernoulli's theorem we get

$$1.50+0.249=1.565+0.174$$

which is within the usual limits of error in taking measurements during a test.

The pressure and velocity changes in the filaments along the inside wall of the channel do not appear to have any direct relation. This apparent discrepancy is due to the difficulty of securing the correct pressures of the inside filaments. It will be noted in figure 10 that for sections 3, 6, 8, 10, 11, 13, and 16, pressure readings were obtained for piezometers located in the floor and both side walls of the channel. The floor readings are designated by a square while the side-wall readings are indicated by a triangle. The pressures against the top of the channel as measured in the glass tubes in the cover plate are shown by circles connected by straight lines. It will be noted that for sections 3 and 16 located on the two tangents, the side wall and floor pressures check the pressures recorded against the top. For the sections on the bend the side wall and floor pressures for the outside of the bend check quite closely the pressures obtained for the top. But the side wall and floor pressures for the inside of the bend do not agree with the pressures against the top for this part of the bend. Invariably the side wall and floor pressures are lower than the pressures against the top.

Knowing the velocity distribution at each section and the radii of curvature which the various particles of water take in going around the bend, it is possible to compute the elevations to which the water surface must rise in the glass tubes on the cover. For convenience in computations each section as shown in figure 10 has been divided into four units of equal width. The mean velocity for each of these units was taken from the velocity curve. The width of the unit in this case would be 2½ inches, one-fourth of the width of the channel. The radius for the center of each unit was determined from the curves shown in figure 33. Substituting these known values in formula 4 (p. 63), the difference of elevation between the two sides of each unit was obtained. These differences plotted as computed pressure points at the various sections are designated by stars in figures 9 to 17.

Attention is called to the remarkable agreement of the computed transverse pressures with the observed pressures for the various tests shown in figures 9 to 17, inclusive. The points of greatest variation are along the inside of the bend. As shown in figure 10 the side-wall pressures for the inside of the bend agree fairly closely with the computed pressures for this part of the bend but do not check the pressures against the top as represented by circles.

The computed transverse water-surface profiles for the 5- by 10-inch channels check fairly close with the observed values, figures 11 to 14, inclusive. Even when the channels were flowing partly full, the computed transverse profiles check the measured values remarkably close as may be seen in figures 15 to 17, inclusive.

Since the pressures as indicated by the various piezometers along the inside of the channel did not agree as they seemingly should, an investigation was made of the differences in pressure entirely around the periphery of the channel at each of seven different sections. In the 10- by 10-inch channel, 22 piezometer connections were made at each section, 3 in the floor, 5 in the cover plate, and 7 on each of the

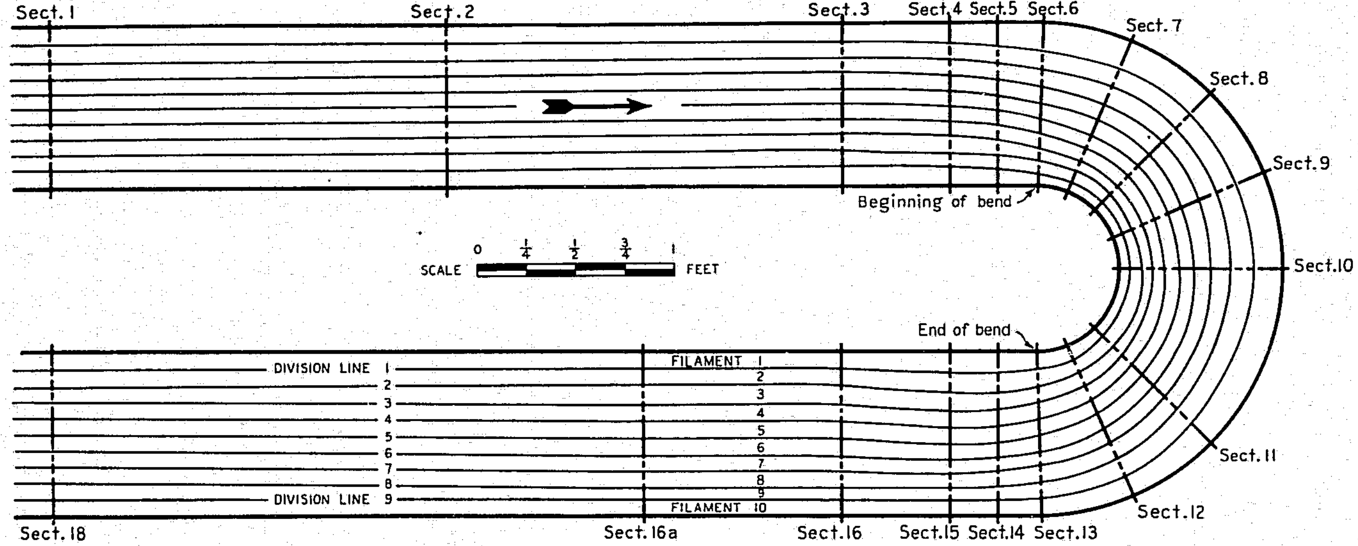

FIGURE 33.—Plan of 180° bend with 5-inch inner radius, channel 10 inches square, showing equal filaments of flow with a total discharge of 2.99 cubic feet per second.

2 side walls as shown in figure 1. The pressures as obtained for the various piezometers for each of seven sections with several different discharges are given in figures 34 and 35. As shown in figure 1, section 3 is located 1 foot upstream from the beginning of the bend, section 6 is at the beginning of the bend, section 8 is 45° or one-fourth of the distance around the bend, section 10 is halfway around the bend, section 11 is three-fourths of the distance around the bend, section 13 is at the end of the bend, and section 16 is 1 foot downstream from the end of the bend. As may be seen in figure 5, piezometers *a* to *g* are located on the outside wall of the channel, piezometers *h*, *j*, and *k* are in the floor, and piezometers *l* to *t* are located on the inside wall. The pressures in the piezometers on the cover directly over the piezometers in the floor have been represented by open circles in figures 34 and 35.

It will be noted that for the smaller discharges the piezometer readings for the sections on the tangents and at the beginning and the end of the bend are practically level, showing that at each section the pressure is approximately uniform on the four sides of the channel. However, for the sections on the bend, namely, 8, 10, and 11, the piezometers along the outside wall denote uniform pressure whereas the piezometers along the inner wall show unequal pressure. Not only are the pressures on the inner wall unequal at each of sections 8, 10, and 11, but a definite relation appears to exist between certain piezometers, particularly for sections 10 and 11, regardless of the quantity of flow and the material of which the channel was built. The piezometer readings for the smaller discharges were taken on the glass channel and the readings for the larger discharges were taken on the celluloid channel.

A study of the graphs in figures 34 and 35 shows that for sections 10 and 11 piezometers *l* and *t* (fig. 5) generally read higher than *o*, *p*, and *r*, except for the highest quantities, and also that *o*, *p*, and *r* read higher than *m* and *s*. It will be noted that *m* and *s* usually read lower than any of the piezometers on the inside wall at sections 10 and 11. The piezometers along the inside wall for section 8 show practically the same variation for all discharges. In this section piezometers *l* and *t* read higher than the other piezometers. Since these piezometers consistently register approximately the same differences regardless of the quantity of flow, an attempt will be made later to explain the reason for the variation in pressure.

The 5- by 10-inch channels had only three piezometer connections in the cover plate, thus they had only 20 piezometers for each set of periphery readings. The periphery pressures for these two channels with uniform velocity distribution in the approach channels are shown in figures 36 and 37. It will be noted in these charts that while the pressures against the outside wall are quite uniform, those against the inside wall have the same irregularities as existed in the 10- by 10-inch channel.

NONUNIFORM VELOCITY DISTRIBUTION IN THE APPROACH CHANNEL

In the tests with nonuniform velocity distribution in the approach channel the computed transverse water surface profiles did not check the observed water surfaces as closely as the data with uni-

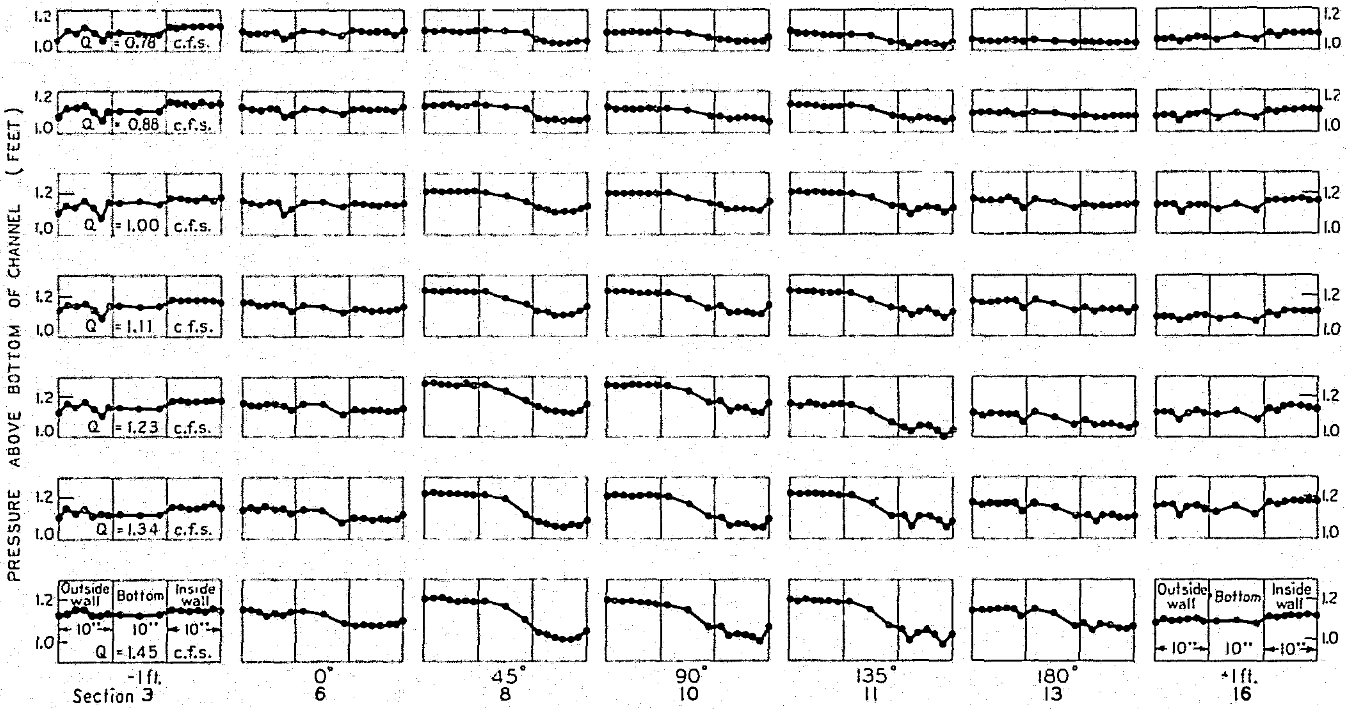

FIGURE 34.—Periphery pressures in 10- by 10-inch channel for discharges from 0.78 to 1.45 cubic feet per second. Uniform velocity distribution in approach channel.

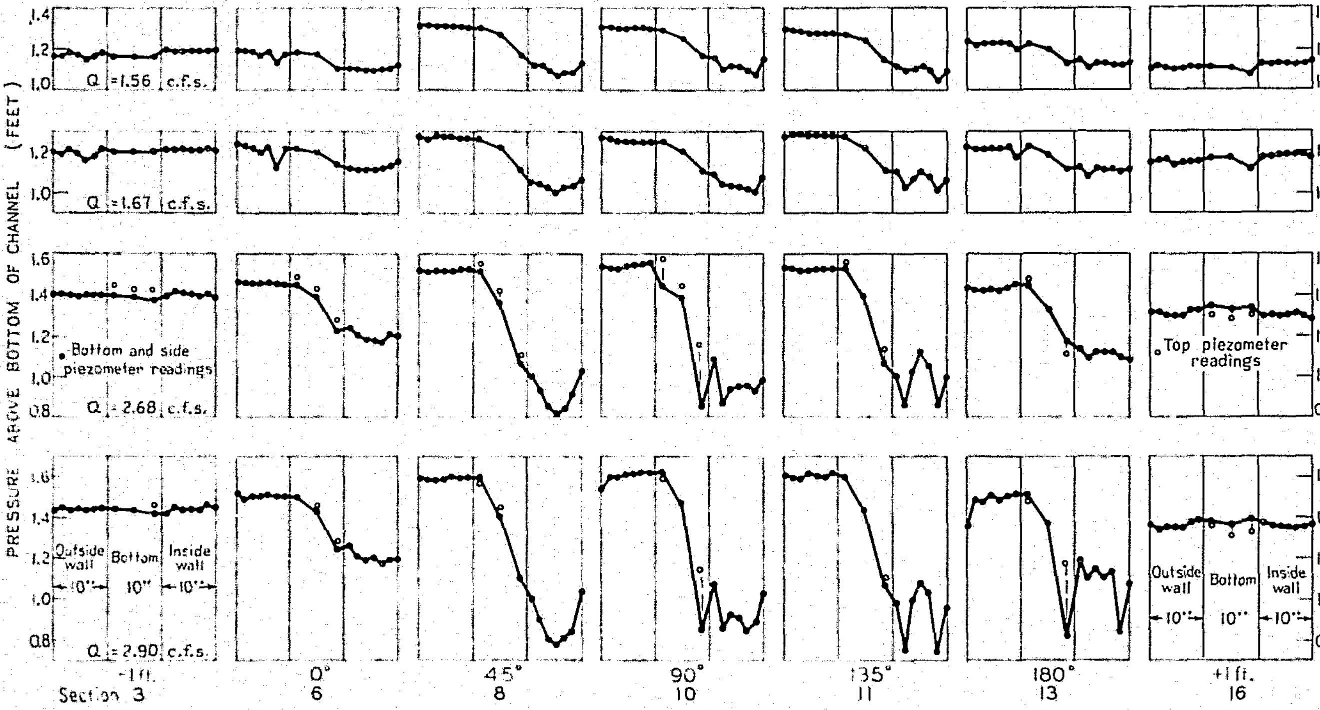

FIGURE 35.—Periphery pressures in 10- by 10-inch channel for discharges from 1.56 to 2.9 cubic feet per second. Uniform velocity distribution in approach channel.

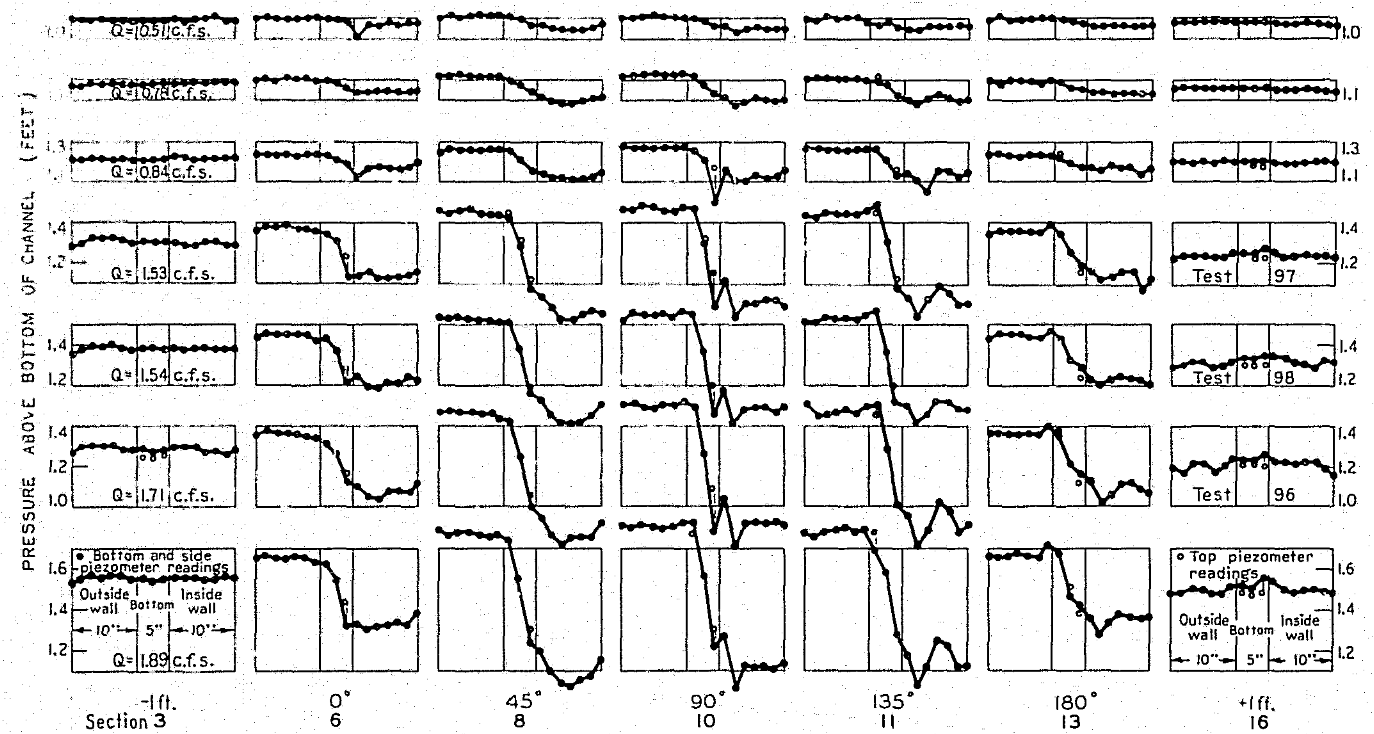

FIGURE 36.—Periphery pressures in 5- by 10-inch channel with 5-inch inner radius for discharges from 0.51 to 1.89 cubic feet per second. Uniform velocity in approach channel.

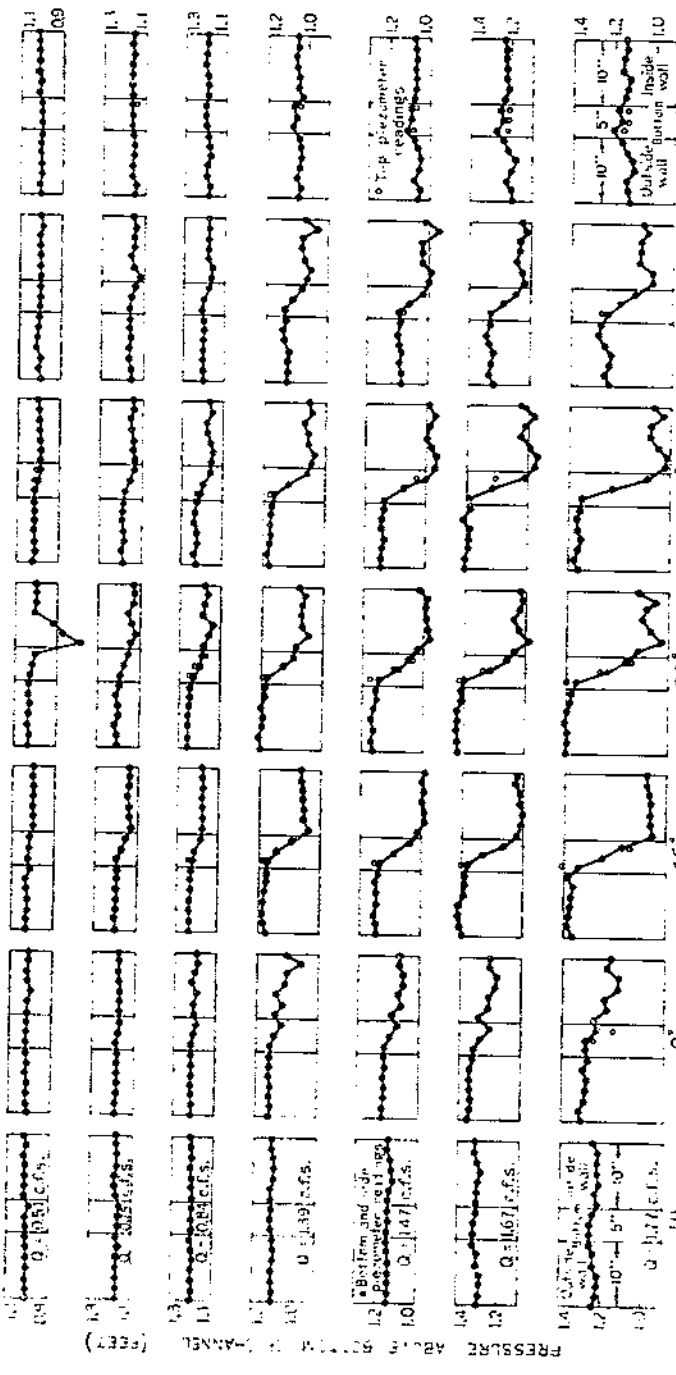

FIGURE 37.—Periphery pressures in 5 by 10 inch channel with 10-inch inner radius for discharges from 0.51 to 1.77 cubic feet per second. Uniform velocity distribution in approach channel.

form distribution. There are, however, no great differences as may be seen in figures 18 to 29, inclusive. These figures cover the three sizes of channels with various types of velocity distribution in the approach tangents.

The data for these three channels when flowing partly full are given in figures 30 to 32, inclusive. It will be seen that the computed transverse water surface profiles check the actual water surfaces fairly well.

Periphery pressure readings were taken only on the 5- by 10-inch channel with a 5-inch inner radius. Various types of velocity distribution were used. The data for these tests, shown in figures 38 and 39 are worthy of considerable study. The great differences in periphery pressures at single sections show the need for several piezometric connections when pressures at any section are desired. These excessive differences are caused in part by the variations in velocity distribution at successive cross sections as the water flows around the bend. In addition, the strength and direction of the internal secondary currents within the mass of forward-flowing water, the diameter of the piezometer openings in relation to the diameter of the piezometer nipples, the length of hose connecting the piezometer nipple to the piezometer tube, and the diameter of the piezometer tube, all affect to a greater or less degree the amount of oscillation within the piezometer tube, and hence the accuracy of a pressure reading at any point on the periphery of the bend. Further investigation is needed to determine to what extent each one of these factors affects the pressure.

ENERGY CHANGES WITHIN THE BEND

When water flows around a bend it follows different laws from those it follows in straight channels. Usually parallel filaments of flow prevail in a straight channel which ordinarily means uniform velocities and pressures. But when water flows around a bend it is acted upon by centrifugal force and it follows the laws of free vortex motion. This centrifugal force causes unequal velocity and pressure distribution within the bend, hence we have the highest velocities along the inside of the bend and the greatest pressures along the outside of the bend. In an open channel differences of pressure may be indicated by an inclination of the water surface in the transverse section of the channel; that is, the water along the outer edge of the bend may be higher than the water surface along the inner edge of the bend. Usually at the beginning of a bend in an open channel the longitudinal slope along the outer bank becomes flat, frequently level for a short distance, while there is a noticeable drop in the water surface along the inner bank. In a closed channel such freedom of movement does not exist, hence we have high pressures along the outside wall of the bend and low pressures along the inside wall.

Even though unequal velocity distribution and unequal pressures exist at the bend, the total energy consumed by all of the various filaments of water in flowing around the bend aside from the loss caused by the bend is about the same.

To determine the energy changes within the bend and the section of the bend where the loss of head takes place, energy gradients as

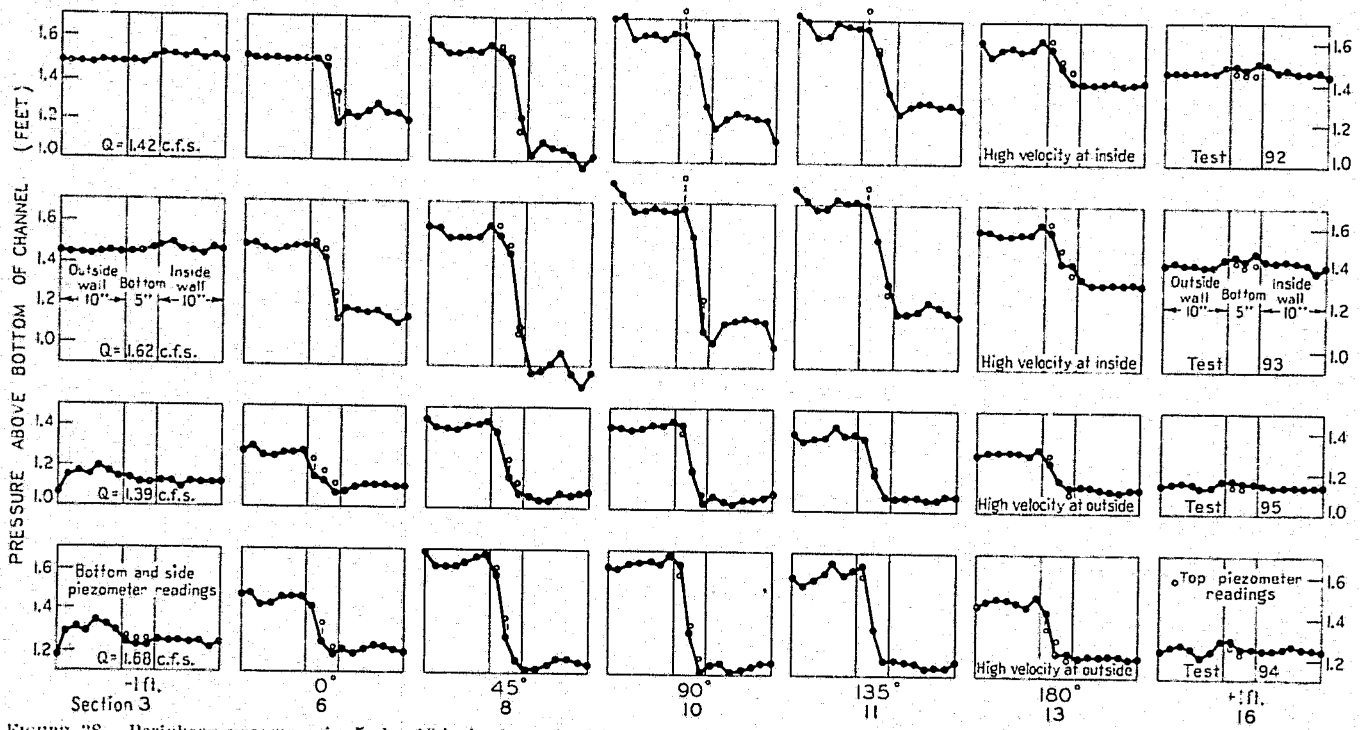

FIGURE 38. Periphery pressures in 5- by 10-inch channel with 5-inch inner radius for discharges from 1.42 to 1.68 cubic feet per second. Nonuniform velocity distribution in approach channel.

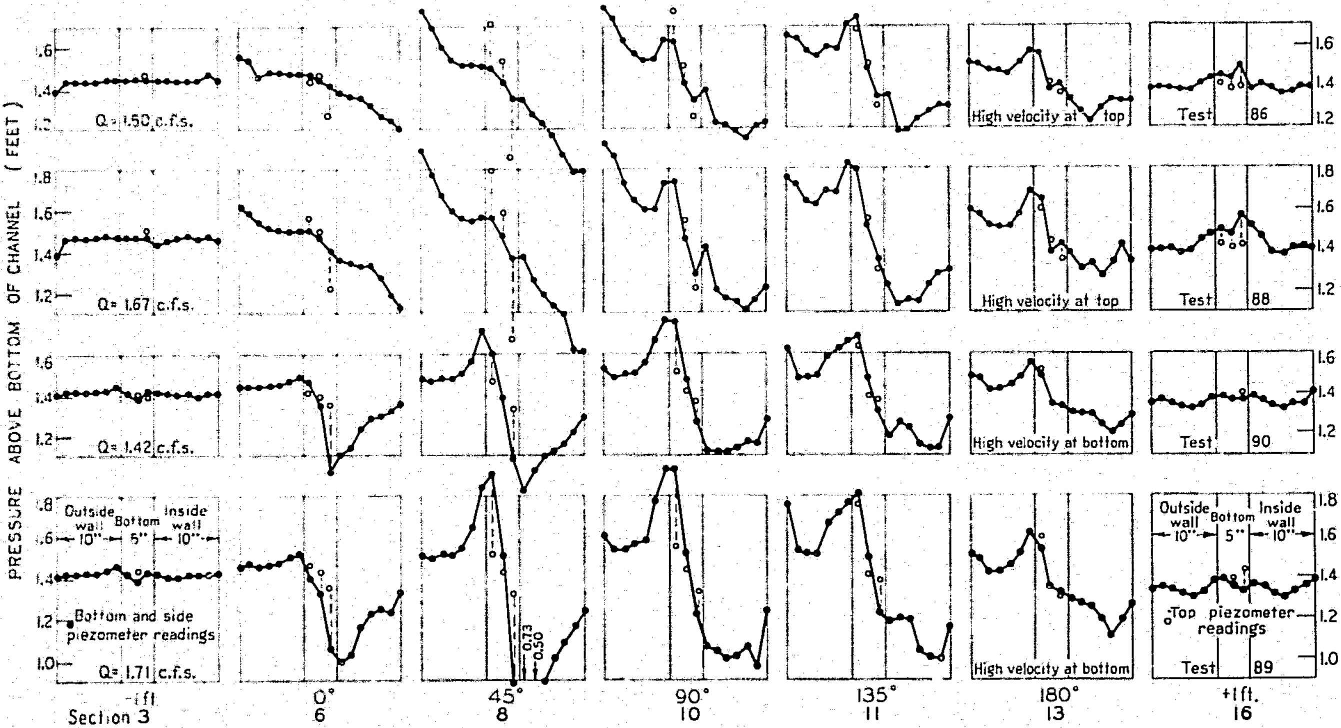

FIGURE 39.—Periphery pressures in 5- by 10-inch channel with 5-inch inner radius for discharges from 1.50 to 1.71 cubic feet per second. Nonuniform velocity in approach channel.

shown in figure 40 have been plotted for the various elementary strips. These gradients were plotted in the following manner. It will be noted in figure 10 that the areas under the mean velocity curves for the various sections have been divided into 10 equal parts.

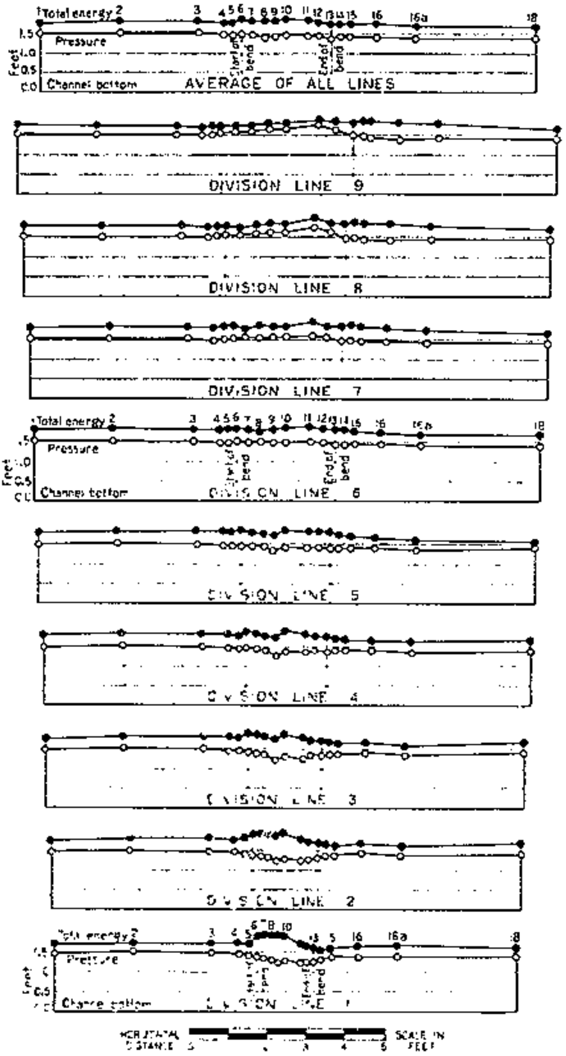

FIGURE 40.—Pressure and total energy gradients for equal filaments of flow in 180° bend with 5-inch inner radius. Channel 10 inches square. Discharge 2.89 cubic feet per second.

Since for the pipe flowing full the area under any velocity curve represents in effect the quantity of water passing that section, each one of these small unit areas represents one-tenth of the quantity of flow. The dividing lines for these elementary quantities as determined in figure 10 are shown in figure 33. For convenience in computing, the energy gradients have been drawn for the division lines between the elementary quantities. As shown in figure 40 the division lines have been numbered consecutively beginning with the division line next to the inside of the bend.

The pressure gradients were obtained by plotting as ordinates the pressures for the points represented in figure 10 by the intersection of the division lines with the pressure curves. The distances along the respective division lines between the various sections were used as abscissae. These pressure gradients are shown in figure 40. The velocities for the intersections of the division lines with the mean velocity curves were read from the diagrams in figure 10. The velocity head for each velocity was then computed and added to the pressure for the corresponding point. This sum, which is the pressure head plus the velocity head, represents the total head or energy at that point. The bottom of the channel was level for these tests. These total energy values for the various division lines have been plotted in figure 40. In addition to determining the pressure and energy gradients for the various filaments, the average pressure and energy gradients for the entire channel have also been computed. The values used in the average gradients are the average of the sum of the values for the individual filaments. These average pressure and energy gradients are also shown in figure 40.

This method has certain limitations, particularly on the bend and the tangent downstream from the bend. The mean velocity curves for the various sections on the bend may be somewhat in error due to the difficulty of measuring correct velocities on a curve. In addition, the turbulent flow in the bend causes greater oscillations in the piezometer tubes and thus greater difficulty in obtaining the correct pressures. This turbulence frequently continues downstream for some distance which naturally causes possible errors in pressures and velocities. It is believed that the measurements on the upstream tangent are the most accurate.

The movement of the individual filaments may be studied as they travel around the bend. Filament no. 1, the filament of water next to the inside of the bend, gains speed as it travels around the bend. During its travel it is imparting part of its motion to the adjacent filament and at the same time part is consumed in friction from the inside wall of the channel. At the end of the bend it has lost most of its increased velocity as may be seen in the mean velocity curve for section 13 (fig. 10). Filament no. 2 in its travel around the bend gains energy from filament no. 1 and imparts part of its speed to the filament next to it. By the time it has reached section 13 it also has lost most of its increased speed. Filament no. 3 in turn loses part of its speed in helping to pull filament no. 4 around the bend. Thus, in turn, each filament tends to pull its neighbor on the left. This transfer of energy and velocity takes place throughout the entire bend section and finally results in a higher velocity along the outside of the channel. Readjustment or uniform velocity distribution for this particular quantity (test no. 60) does not take place until a point about 5 feet downstream from the end of the bend, section 18, is reached.

This change of energy just described is illustrated graphically in figure 40. Pressure gradients as well as total energy gradients have been plotted for each filament as well as average pressure and energy gradients for the entire section. The individual gradients for the various filaments show in turn the slacking up of the velocity of each filament. This is represented by the interval between the pressure and energy gradients for each filament. For the last filament, no. 9, this interval is the smallest of any of the individual filament gradients.

The average pressure and average energy gradients are of more than usual interest in that they show not only the general balancing up of the total energy change within the bend but also show the loss of head incurred because of the bend. However, due to the great number of various factors which enter into the construction of this general curve, the loss incurred from the bend can be obtained more accurately by the hydraulic gradients for the various tests.

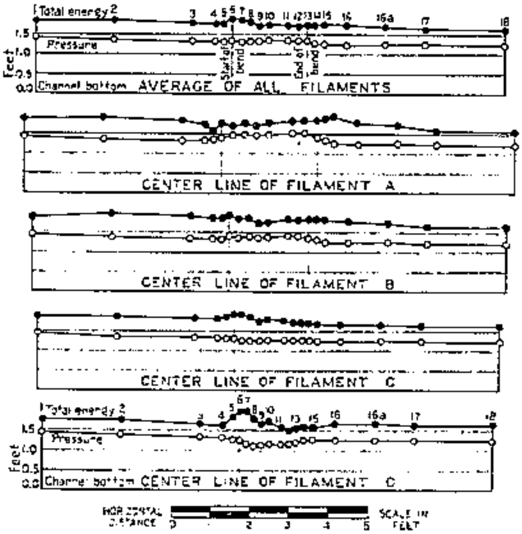

FIGURE 41.—Pressure and total energy gradients for four equal filaments of flow in a 180° bend with a 5-inch inner radius. Channel 5 inches wide by 10 inches deep. Discharge 1.88 cubic feet per second.

This same procedure was followed in test no. 85 (figure 12), which is for the 5 by 10 inch channel with a 5-inch radius and for a discharge of 1.88 cubic feet per second. For this test, however, the channel was divided into four equal quantities of flow and the pressure and energy gradient were determined for each filament, the center line of each filament being used as its base (fig. 41). The general average pressure gradient and energy gradient for the entire discharge in the channel are also shown. As may be noted these energy gradients are similar to those in figure 40.

HEAD LOST IN THE BENDS

The head lost in a bend is due to several factors. Among these are the frictional resistance offered by the walls of the channel, the unequal velocity distribution within the bend, and the troublesome vortices formed along the inside wall of the channel near the end of the bend. In short, the loss is the energy required to bring a disorderly condition of flow back to normal flow.

Investigators have used different methods for measuring this loss of head. The usual method has been to install piezometers at points above and below the bend, measure the total loss, subtract the loss for a length of straight pipe equivalent to the distance between the piezometers, and the result is the loss due to the bend.

One method for obtaining accurately the loss of head in a bend is to determine the hydraulic gradients for the channel for some distance above and below the bend. If the channel has the same characteristics throughout its entire length the two hydraulic gradients may be parallel; the gradient downstream from the bend will lie at a lower elevation than the gradient upstream from the bend. The vertical difference between these gradients is the actual loss due to the bend and does not include the friction loss in the bend. Figure 42 shows two representative examples of hydraulic gradients for two quantities of flow in the 10 by 10 inch channel. A difficulty about using this method is that the hydraulic gradient for some distance below the bend is likely to be flatter than under normal conditions, because of the great variations in velocity distribution in the cross section.

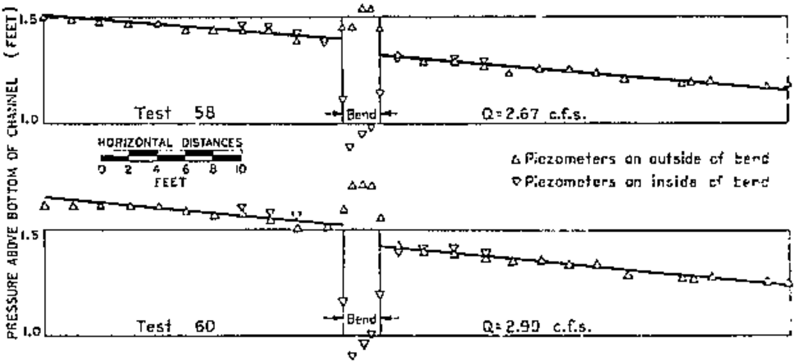

FIGURE 42.—Representative hydraulic gradients above and below the bend for two quantities of flow.

The loss of head may also be obtained by another method. The total energy gradient shown in figure 40 represents the energy expended by the water in flowing around the bend. If the general elevation of the average energy gradient below the bend is subtracted from the general elevation of the average energy gradient above the bend, the results will be the loss of head caused by the bend. The loss of head for test 60 (fig. 40) as given by the difference of the elevations of the average energy gradients above and below the bend is 0.10 foot, whereas the loss of head as determined by the difference of the hydraulic gradients above and below the bend for the same test as shown in figure 42 is 0.09 foot.

The loss due to any bend has been represented in various ways. A common method is to reduce the loss to an equivalent length of straight channel. Another is to plot the loss as ordinate and the ratio of the radius of the channel (if a pipe) to the radius of the bend as the abscissa. Loss-of-head curves are drawn for various velocities using these values as coordinates. In still another method the coefficient C is used in the formula,

$$\text{Loss due to the bend} = C \times \frac{\text{Width of channel}}{\text{Inner radius}} \times \frac{V^2}{2g}.$$

The average coefficient for 17 tests in the 10 by 10 inch channel was 0.18: for 12 tests on the 5- by 10-inch channel with a 5-inch inner radius, 0.23; and for the 5- by 10-inch channel with a 10-inch inner radius, 0.23. The average value for all the experiments was 0.21. It should be remembered that this coefficient applies only to a bend of the same radius and section as the one used.

In the experiments performed by Kumabe *(11)* the hydraulic gradients were shown. These data were treated in the same manner as the Iowa experiments. The average coefficient for the Japanese tests was found to be 0.31.

These coefficients, however, are only approximate since the loss of head in these tests was quite small; in some cases only four times the probable error in measurement.

Apparently the loss of head coefficient has no direct relation to velocity. This is verified by the results of Schoder's, Davis', and Brightmore's tests.

In general, it might be said that the greater the disturbance in flow within the bend, the greater the loss in the bend. Excessive disturbance usually is caused by high velocities. Small bodies of water, so to speak, moving within the main mass of flowing water, and possessing velocities higher than the average velocity as a whole, quickly change positions within the bend with respect to the inner and outer walls of the channel. The shifting of small moving masses of water usually small in number within the entire body, naturally means a shifting of energy from one position to another. In their movement throughout the bend, these small separate masses of water create friction with the neighboring water. These disorderly conditions make it difficult to measure the loss of head at successive points around the bend. The easier and more accurate method is to measure the total loss of head caused by the bend.

SECONDARY CURRENTS IN BENDS

An interesting phase of the investigation was the study of the secondary currents within the bend as the water flowed around it. It has long been known that such currents exist but apparently few investigators have made a detailed study of this phenomena.

Thomson *(15)* knew that when water flows around a bend of a river, the greatest velocity is along the inner bank and not along the outer bank as commonly supposed. Since erosion takes place chiefly along the outer bank and not along the inner bank where the greatest velocity occurs, he began a search for the reason why such conditions existed. In 1876 he built a small wooden open channel, 8 inches wide and 2 inches deep with a 180° bend. His research on the flow of water in this model furnished the clue to the apparent anomaly of erosion in bends. Naturally, when water flows around a bend, due to centrifugal force, the water along the outer bank stands higher than the water along the inner bank. This causes a greater hydraulic pressure on the bottom at the outer bank of the bend which starts a secondary current. To prove that secondary currents exist in bends, Thomson fastened short threads to pins which were stuck in the bottom of his model and studied the direction of the yarns while water was flowing around the bend. These threads clearly proved that transverse motion took place. He mentions that this oblique flow along the bottom towards the inner bank began even upstream from the bend (which was not apparent in the Iowa experiments).

Eustice (*8*) probably made the first observations of secondary currents in pipe bends. He experimented on circular glass tubes, 1 cm in diameter, both in 90° and 180° bends. The 90° bends were tested with eight different radii and the 180° bends with five different radii. Tests were also made on 360° and abrupt or sharp right-angled bends. The motion of the water was studied by introducing various colored liquids and Eustice definitely established the existence of secondary currents in pipe bends. He states that in open channels there is freedom in a vertical direction whereas in pipes no such freedom exists. Hence "the effect of pressure on the transfer of water from the outer to the inner curve is more pronounced in a pipe than in a channel."

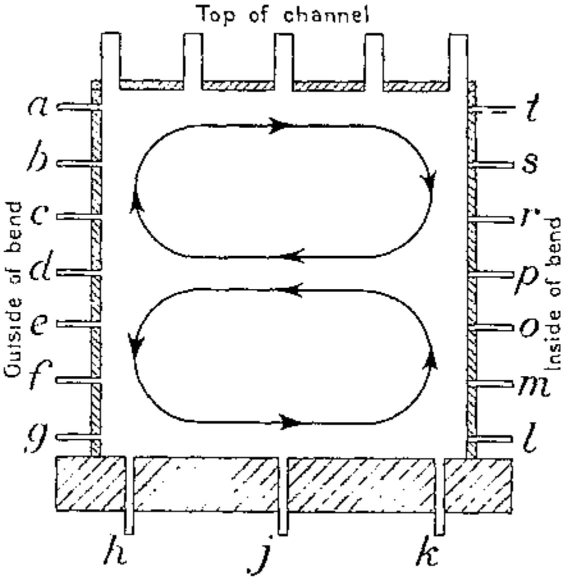

FIGURE 43.—Diagrammatic representation of two secondary currents within the bend.

These secondary currents are also mentioned by Boussinesq (*4*), Léliavski (*12*), Gockinga (*9*), and Townsend (*16*).

That secondary currents exist within the bend was demonstrated in the Iowa experiments by means of yarns attached to small rods placed in the channel. When uniform velocity distribution exists at the beginning of the bend, there are two secondary currents in the closed channel, as illustrated in figure 43. The bottom current moves toward the inside of the bend along the bottom, then upward, then back to the outside wall, and finally down to the bottom. The top current moves toward the inside of the bend along the top, then down the inside of the channel to about its center, and thence outward to the outside of the bend.

The variation in pressure between the piezometers on the inside of the bend at any one cross section is due principally to these secondary currents. From the sketch it will be seen that the water is flowing toward piezometers l and t. At o and m the flow is upward, while at s and r the flow is downward. At the upper and lower corners of the square cross section, where the transverse horizontal component of velocity is diverted into a vertical velocity, there is naturally a slight increase of pressure. Similarly at the piezometer p where the opposing vertical currents are again diverted to a horizontal direction there is a similar slight increase of pressure. These local variations in pressure are so small that careful observation is necessary to determine them with certainty. Examination of figures 34 and 35 shows that the variations are most conspicuous with the larger flows and corresponding greater velocities, as would be expected. Furthermore, these secondary currents are only gradually developed during the progress of the water around the bend, and hence the resulting variations in pressure show clearly only at cross sections 10 and 11 in the latter half of the bend. It might be expected that similar variations in pressure would show on the outer wall of the bend, but the smaller velocities existing along the outer wall rendered the local pressure variations too small to be detected.

In the Iowa experiments transparent material was used for the sides and top of the bend so that the direction of the filaments of flow around the bend could be studied. It was decided to use the largest quantity of flow, 3 cubic feet per second, in studying the movement of the water around the bend. Wires were placed in the floor at sections spaced 22½° apart at the bend and at some sections on the two tangents. A short yarn was tied to each wire and all yarns set one-fourth inch above the floor of the channel. Figure 44 shows that diagonal flow across the channel toward the inner bank exists in the bend proper but not on the approach tangent. Therefore, it was decided to study further the flow within the bend itself.

A series of photographs were taken showing the yarns placed one-fourth inch above the floor, 1 inch above the floor, and at successive increments of an inch, the last set being 9¾ inches above the floor or one-fourth inch below the top of the channel. The set of yarns placed one-fourth inch above the floor is shown in figure 44. These tests showed that the yarns placed next to the bottom and next to the top of the channel indicated diagonal flow across the channel from the outer wall to the inner wall. The yarns placed midway of the channel show the flow filaments tend to incline toward the outside wall of the bend.

If the secondary currents illustrated in figure 43 exist in the bend it should be possible to show them by photographing the direction of flow of the various filaments of water at any one section. This was accomplished in the following manner: All the wires were removed from the channel. Then a wire with 11 yarns attached was set in the floor of the channel at section 8, one-fourth inch from the outside wall. The yarns were so spaced that the bottom yarn was one-fourth inch above the bottom of the channel, the second yarn 1 inch above the channel bottom, followed by yarns every inch up to the eleventh yarn which was one-fourth inch from the top of the channel. The cover was replaced and the water run through the channel. A photograph was then taken of the yarns looking through the side wall of the channel. The water was then shut off, the cover removed, and the wire moved toward the inside wall until it was 1 inch from the outside wall. The cover was then replaced, the water turned on, and a photograph taken. Views were taken at successive distances of 1 inch from the outside wall. These separate photographs were then trimmed and mounted on cardboard with the view next to the outside wall placed at the right and the view adjacent to the inside wall placed at the extreme left. A photograph taken of this arrangement is shown in figure 45. This figure is extremely interesting since it is one verification of the existence of secondary currents within the bend. If two secondary currents exist within the bend, as shown in figure 43, then, of the yarns next to the outside wall, those below the center will dip downward toward the floor while those above the center will tip up toward the top of the channel. The yarns attached to the rod placed next to the inside wall will tend to dip to the center of the channel or a point midway between the top and the bottom. The yarns attached to the rod placed 5 inches from the outside wall will tend to approximate a position in which they will be all parallel. These conditions are

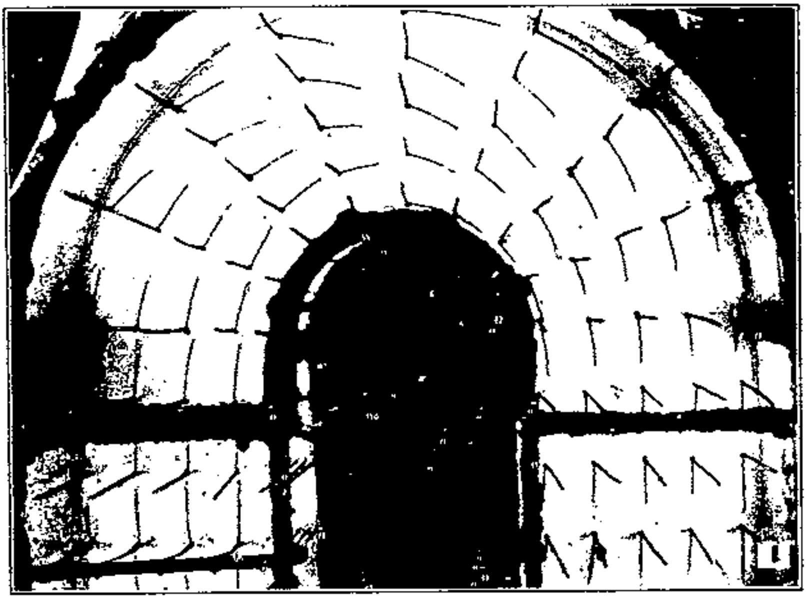

FIGURE 44.—View looking down on bend showing yarns placed one-fourth inch above the floor of the channel. Note how straight the yarns in the approach and discharge channels point downstream, while on the bend they incline toward the inside of the bend.

all met in figure 45. The existence of the secondary currents may perhaps be seen better by looking lengthwise of figure 45 from the outside to the inside of the channel.

Additional measurements[6] on the secondary currents in the bend were made with a quantity of flow of about 3 cubic feet per second and with uniform velocity in the approach channel at the beginning of the bend. This condition formed two secondary currents in the bend as shown in figure 43.

The deflection of the threads upward or downward from the horizontal was measured by means of a clinometer. From the top of the channel was read, by means of a protractor, the angle of deflection of these threads from the tangent to the circumference of the circle. With these two angles it was possible to determine exactly the direction of flow of the water particles at any point in the channel.

The velocities at definite points in the channel had previously been determined by means of Pitot tubes. The horizontal components of the velocities for seven points of three verticals for section 10, located midway around the bend, were computed. Likewise, the vertical components of the velocities for five points on each of three horizontals were determined. All components which were downward or inward towards the center of curvature of the bend were called negative whereas all velocity components which were upward or outward were called positive. At first thought it would appear that to have stable conditions at any section on the bend, the summation of the upward velocity components should balance the summation of the downward velocity components, also the summation of the outward horizontal components should balance the summation of the inward horizontal components, but due to the changes in the velocities in three dimensions, these conditions do not necessarily exist. The computed horizontal and vertical components are shown graphically in figure 46 *A* and *B*.

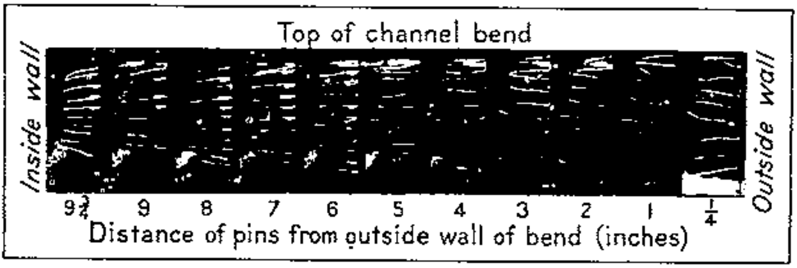

FIGURE 45.—Secondary currents in 180° bend. Yarns attached to a rod photographed at successive points across the channel. Views taken at section 8, 45° downstream from the beginning of the bend.

For convenience of analysis the cross section of the channel was divided into 16 equal areas, each one of which was called a unit (fig. 46, *C*). These units were numbered consecutively beginning at the upper left-hand corner and going from left to right. For each side of each unit the velocities were taken from the curves in figure 46.

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[6] Condensed from the following: GANGADHARAN, G. SPIRAL FLOW IN CURVED CHANNELS. Unpublished master's thesis, Univ. Iowa. 1927.

A and *B*. Each unit was 2½ inches square which gave four units along each side of the channel. Thus in figure 46, *B*, the vertical scale of the curve for the horizontal components, 2½ inches from the outside wall, was divided into four equal parts and the mean velocity under each part determined. Thus for unit no. 1. the velocity was towards the inside an amount of −1.28 feet per second. The values for the various units are given in table 1 and shown in figure 46, *C*. This figure clearly shows the existence of two secondary currents within the bend.

TABLE 1.—*Comparison of ΣLV_t and (V_2-V_1) x in formula $\Sigma LV_t=(V_2-V_1)$ x, for condition of two secondary currents*[1]

TRANSVERSE VELOCITIES AT SECTION 10 IN FEET PER SECOND (V_t)

Part of unit	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
Inside	−1.28	−1.22	−1.62	......	+0.08	+0.55	+0.06	......
Outside	......	+1.28	+1.22	+1.62	......	−.08	−.55	−0.06
Bottom	+.37	+.09	−.06	−.12	+.20	+.14	+.11	+.04
Top	......	......	......	......	−.37	−.09	+.06	+.12

SIDE LENGTHS OF UNITS MULTIPLIED BY TRANSVERSE VELOCITY (LV_t)

Part of unit	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
Inside	−2.1	−1.60	−1.59	......	+0.13	+0.72	+0.06	......
Outside	......	+2.1	+1.60	+1.59	......	−.13	−.72	−0.06
Bottom	+.66	+.13	−.07	−.10	+.30	+.20	+.12	+.03
Top	......	......	......	......	−.60	−.13	−.07	+.10
ΣLV_t	−1.44	+.63	−.06	+1.49	−.17	+.66	−.47	+.07
$(V_2-V_1)x$	−.17	+.24	+.19	+.10	+.03	+.02	+.08	−.18

TRANSVERSE VELOCITIES AT SECTION 10 IN FEET PER SECOND (V_t)

Part of unit	Unit 9	Unit 10	Unit 11	Unit 12	Unit 13	Unit 14	Unit 15	Unit 16
Inside	+0.35	+0.76	+0.51	......	−0.43	−0.86	−0.40	......
Outside	......	−.35	−.76	−0.51	......	+.43	+.86	+0.49
Bottom	−.07	−.14	+.18	+.50	......	......	......	......
Top	−.20	−.14	−.11	−.04	+.07	+.14	−.18	−.50

SIDE LENGTHS OF UNITS MULTIPLIED BY TRANSVERSE VELOCITY (LV_t)

Part of unit	Unit 9	Unit 10	Unit 11	Unit 12	Unit 13	Unit 14	Unit 15	Unit 16
Inside	+0.58	+0.99	+0.52	......	−0.71	−1.13	−0.48	......
Outside	......	−.58	−.99	−0.52	......	+.71	+1.13	+0.48
Bottom	−.13	−.21	+.20	+.11	......	......	......	......
Top	−.36	−.20	−.12	−.03	+.13	+.21	−.20	−.41
ΣLV_t	−.09	+.00	−.39	−.14	−.58	−.21	+.45	+.07
$(V_2-V_1)x$	−.04	+.05	−.09	−.34	+.13	+.18	+.13	−.05

[1] + indicates flow upward or outward with respect to center of bend; − indicates flow downward or inward with respect to center of bend.

When water flows from one cross section to another, say for example from cross section 8 to cross section 11, along one of these units, the excess of water flowing out of each unit, if any, at section 11 over that of the water into the same unit at section 8 must come from one or more of the neighboring units. In order to check the measurements of the small velocities through the sides of these units the following analysis was made: Each unit was considered between sections 8 and 11 as shown in figure 47, in which

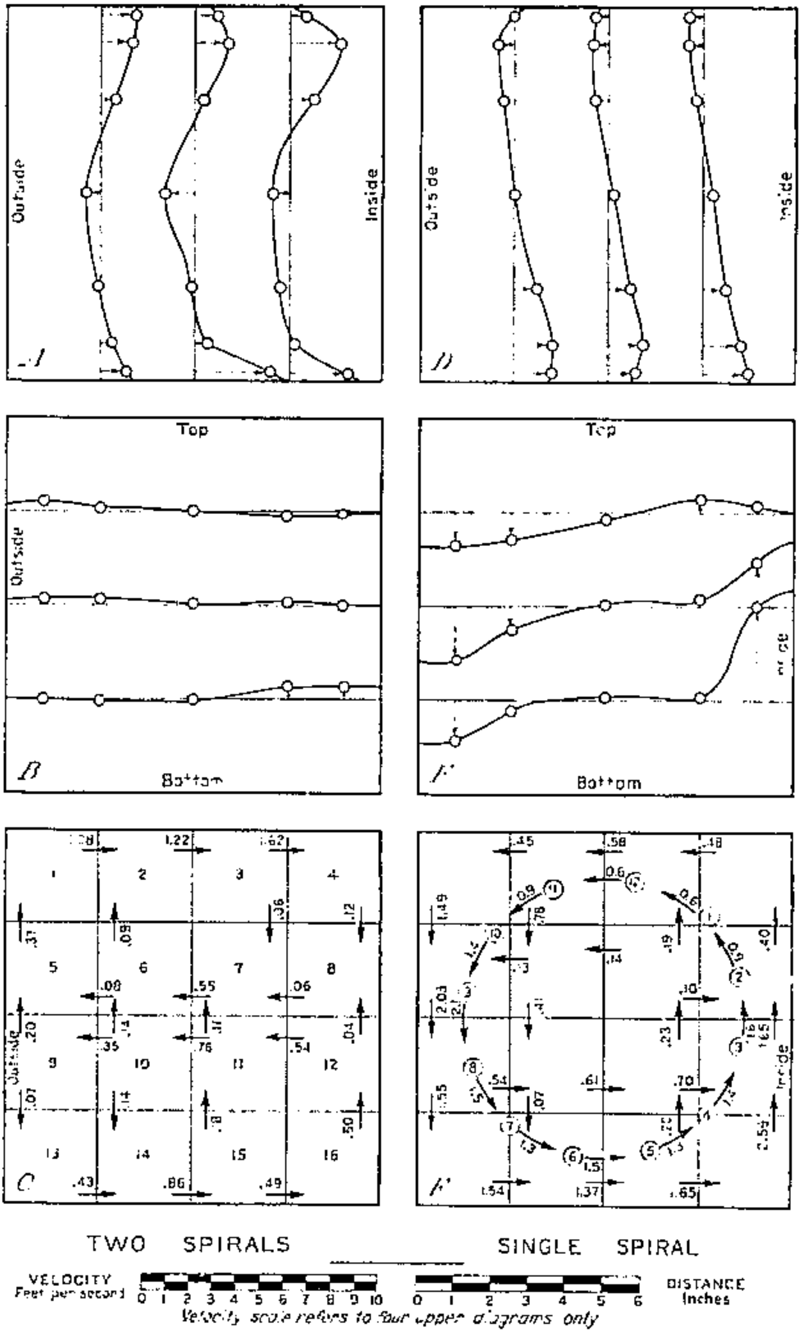

FIGURE 46.—Horizontal (*A* and *D*) and vertical (*B* and *E*) components of velocities of various filaments at section 10, in 10- by 10-inch channel both for the two secondary currents (*C*) and for the single secondary current (*F*). Section 10 of channel shown (*C* and *F*) divided into unit areas with direction of current and magnitude of horizontal and vertical components of the velocities for a discharge of 2.7 cubic feet per second.

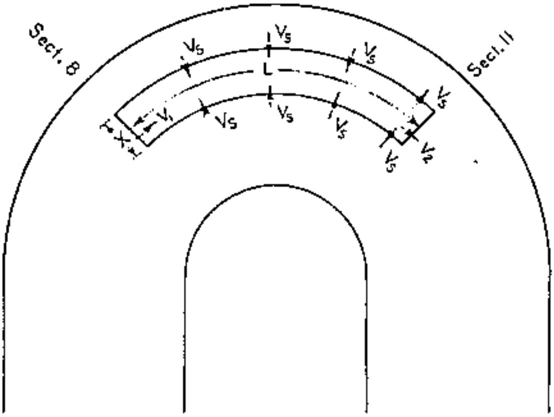

FIGURE 47.—Flow of water in and out of each unit.

x = the height and the width of each unit
V_1 = the entrance velocity into the unit at cross section 8
V_2 = the discharge velocity from the unit at cross section 11
V_s = the average velocity component entering one side of this unit whose length is L.

Then

V_1x^2 = quantity entering one end of the unit
V_2x^2 = quantity discharging from the other end of the unit
xLV_s = quantity entering one side of a unit

Then

$$V_1x^2 + \sum xLV_s = V_2x^2 \qquad 1$$

or

$$x^2(V_2 - V_1) = \sum xLV_s$$

Equation (1) was tested between sections 8 and 11. The results are shown in the last two lines of table 1, which should be identical. The results do not appear to check very closely. One reason is the difficulty in measuring correct velocities in the bend. This may be due to the low velocities obtained as well as to the inaccuracies in measuring the angles of deflection of the threads.

A similar study was made of a single secondary current within the bend. This condition was obtained by inserting an obstruction in the channel at the beginning of the bend so as to create a low velocity at the bottom of the approach channel. Thus flow conditions somewhat similar to those existing in open channels were created and a single secondary current resulted.

Since the amount of deflection of the threads in the case of two secondary currents is not as great as that in the case of a single secondary current it was thought that more accurate results would be obtained in the study of a single secondary current. The same method was followed as was used in the study of the two secondary currents.

The vertical and horizontal velocity components are shown in figure 46, *D* and *E*, while the final velocity components for each unit are given in table 2. The data as shown in table 2 were transferred to figure 46, *F*. It will be seen that the results in table 2 (those of equation 1) check more closely than those in table 1.

TABLE 2.—*Comparison of* ΣLV_s *and* $(V_2-V_1)x$ *in formula* $\Sigma LV_s=(V_2-V_1)x$, *for condition of one secondary current*[1]

TRANSVERSE VELOCITIES AT SECTION 10 IN FEET PER SECOND (V_s)

Part of unit	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
Inside	+0.15	+0.58	+0.48	....	+0.13	+0.14	−0.10	....
Outside	....	+.45	−.58	−0.48	....	−.13	−.14	+0.10
Bottom	−1.49	−.78	+.19	+.40	−2.03	−.41	+.23	+1.65
Top	....	....	....	....	+1.40	+.78	−.19	−.40

SIDE LENGTHS OF UNITS MULTIPLIED BY TRANSVERSE VELOCITY (LV_s)

Part of unit	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
Inside	+0.74	+0.76	+0.47	....	+0.21	+0.18	−0.10	....
Outside	....	−.74	−.78	−0.47	....	−.21	−.18	+0.10
Bottom	−1.68	−1.15	+.22	+.33	−3.65	−.60	+.26	+1.35
Top	....	....	....	....	+1.68	+1.15	−.22	−.33
ΣLV_s	−.94	−1.15	−.07	−.14	−1.76	+.52	−.24	+1.12
$(V_2-V_1)x$	−.06	−.14	−.23	−1.11	−.41	−.08	−.12	−.65

TRANSVERSE VELOCITIES AT SECTION 10 IN FEET PER SECOND (V_s)

Part of unit	Unit 9	Unit 10	Unit 11	Unit 12	Unit 13	Unit 14	Unit 15	Unit 16
Inside	−0.54	−0.61	−0.70	....	−1.51	−1.37	−1.65	
Outside	....	+.54	+.61	+0.70		+1.51	+1.37	[illegible]
Bottom	−1.55	−.07	−.20	+2.59				
Top	+2.03	+.41	−.21	−1.65	+1.75	+.67	−.20	[illegible]

SIDE LENGTHS OF UNITS MULTIPLIED BY TRANSVERSE VELOCITY (LV_s)

Part of unit	Unit 9	Unit 10	Unit 11	Unit 12	Unit 13	Unit 14	Unit 15	Unit 16
Inside	−0.89	−0.80	−0.69	....	−2.53	[illegible]	−1.62	....
Outside	....	+.89	+.80	+0.69	....	+2.54	+1.80	+1.62
Bottom	−1.73	−.11	−.23	+2.12	....	....	....	....
Top	+3.65	−.60	−.26	−1.85	+1.79	+.11	−.23	−2.12
ΣLV_s	+.97	−.58	+.08	+1.36	−.74	+.84	−.05	−.50
$(V_2-V_1)x$	−.05	+.26	+.27	−.12	+.31	+.60	+.67	+.39

[1] + indicates flow upward or outward with respect to center of bend; − indicates flow downward or inward with respect to center of bend.

While these calculations prove analytically the existence of one or two secondary currents in the bend as the case may be, they do not give the radius of curvature of the secondary current or spiral. Since a particle of water in its movement around the bend follows a spiral or helical path, the length of the radius of curvature of the path and the position of the center of curvature both vary.

In order to further study the path of a particle of water flowing around the bend, imagine a thread wound around a straight circular cylinder in the shape of a helix. This thread is assumed to be the path of a particle of water. Next let the cylinder be bent into the shape of an annulus and assume the path of the particle is along the thread as before. (Actually the experimental channel had a square cross section, but the spiral was assumed to be on the cylinder). The transverse velocities at section 10 are shown on the circle in figure 46, *F*. They were computed from the curves in figure 46, *D* and *E*.

The path of the particle of water passing around the bend now has two factors influencing its direction: (1) The motion due to the spiral, and (2) the motion due to the curvature of the bend. If desired, the radius of curvature of a particle of water at any point on the bend can be determined by descriptive geometry if the components of the above factors are known.

THEORY OF THE FLOW OF WATER AROUND BENDS

A common characteristic of open channels is the presence of curves or bends. When water flows in a straight channel the transverse surface profile is presumably a straight line. On a curve or bend the transverse water surface profile cannot be level. Water naturally moves in a straight line unless deflected by the action of some unbalanced force. When water moves around a bend, an unbalanced force directed toward the center of curvature acts against the water.

For convenience of illustration let us consider a plan of the channel at the bend (fig. 48).

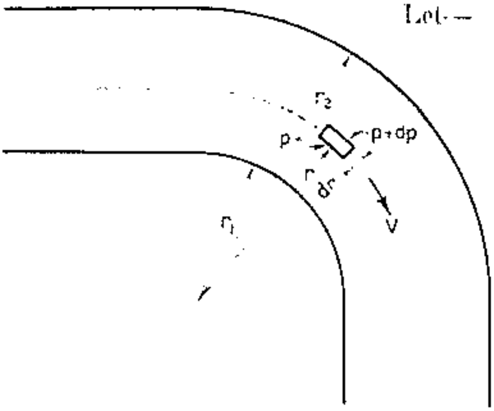

FIGURE 48.—Plan of bend showing symbols used in derivation of formulas.

Let—

r_2 inner radius of bend.
r_1 outer radius of bend.
r radius at any point in the bend.
R radius at center of channel.
V the velocity of any strip dr at radius r.
p the pressure at any point at radius r.
$\frac{dp}{dr}$ rate of radial change of pressure.
b width of channel.
y depth of strip dr.
w weight of a unit volume of water.

If the water moves with streamline motion along a circular path the centrifugal force on the particles of water will be balanced by the radial difference in pressure. If a vertical elementary slice of water has unit length along the streamline the volume of the element is ydr, its weight is $wydr$, and its mass $\frac{w}{g}ydr$. The excess pressure on the outside face of the strip dr is $wydy$.

It is known from mechanics that centrifugal force $= \frac{MV^2}{r}$. Hence

$$wydy = \frac{w}{g}ydr\frac{V^2}{r} \tag{2}$$

or

$$dy = \frac{V^2dr}{gr} \tag{3}$$

In order to integrate this equation it is necessary to determine the velocities at the several sections across the channel in terms of r. A common method (*19*) is to assume V constant as the average velocity and to assume r a constant value for the center of the channel. Then the

Difference in elevation of the water surface between the two sides of the channel

$$= \frac{V^2 b}{gR} \qquad (4)$$

The total difference in elevation of the water surface between the two sides of the channel may be obtained by formula 4. Or if desired, the difference in elevation of the water surface between the two sides of a strip of the channel may be calculated. It should be noted that the radius, r, varies depending upon the location of the strip or point in the bend transversely.

CONCLUSIONS

The following general conclusions have been drawn from the experimental data:

1. A smooth bend acts an an obstruction disturbing the distribution of pressure and velocity in the cross section for a short distance upstream from the beginning of the bend.

2. As water approaches the beginning of a smooth bend, in the outside filaments the pressure always increases and the velocity decreases. In the inside filaments these changes are reversed.

3. As water flows around a bend, there must be higher pressure on the outside.

4. With substantially uniform velocity distribution in the approach channel to the bend, the velocities of the filaments along the inside wall of the bend are increased while the velocities of the filaments along the outside wall are reduced.

5. As water flows around a bend the effect of wall friction is to produce an unstable relation between the filaments flowing at different velocities, with the result that the filaments of highest velocity tend through a secondary circulation to drift toward the outer wall of the bend.

6. At the end of the bend there is another rather abrupt rearrangement of pressures and velocities in the cross section, whose effects persist for a considerable distance downstream from the bend.

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