AJAE appendix for The Benefit Function Approach to Modeling Price-Dependent Demand Systems: An Application of Duality Theory

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Elasticity Equations for the SNAB Function

Let M_{ij} denote the Marshallian quantity elasticities for commodity i with respect to x_j, S_i the scale elasticity of commodity i, and H_{ij} the Hicksian quantity elasticities for commodity i with respect to x_j. To facilitate thinking about preferences in terms of a benefit function, the quantity and scale elasticity functions can be written in terms of **x** and u:

$$M_{ij} = \frac{\partial \log(R_i^M)}{\partial \log(x_j)} = \frac{\partial \log\left\{ B_{x_i}[\mathbf{x}, U(\mathbf{x})] / \sum_j B_{x_j}[\mathbf{x}, U(\mathbf{x})] x_j \right\}}{\partial \log(x_j)}, \quad S_i = \sum_j M_{ij}, \text{ and}$$

$$H_{ij} = \frac{\partial \log(R_i^H)}{\partial \log(x_j)} = \frac{\partial \log\left[B_{x_i}(\mathbf{x}, u) / \sum_j B_{x_j}(\mathbf{x}, u) x_j \right]}{\partial \log(x_j)}.$$

Given the functional form of the SNAB function, it follows that the quantity and scale elasticity equations are expressed as:

$$M_{ij} = -\delta_{ij} + \frac{\delta_{ij} V_i + U_{ij} - \mu_i Z_j}{B_{x_i} x_i} - \left[\frac{V_j + \sum_j U_{ij} - Z_j}{\sum_j B_{x_j} x_j} \right],$$

$$S_i = -1 + \sum_i \left(\frac{\delta_{ij} V_i + U_{ij} - \mu_i Z_j}{B_{x_i} x_i} \right) - \sum_i \left[\frac{V_j + \sum_j U_{ij} - Z_j}{\sum_j B_{x_j} x_j} \right], \text{ and}$$

$$H_{ij} = -\delta_{ij} + \frac{\delta_{ij} V_i + U_{ij} - \mu_i \mu_j \eta^2 \log(u) / X1^{\eta}}{B_{x_i} x_i} - \left[\frac{V_j - \mu_j \eta^2 \log(u) / X1^{\eta}}{\sum_j B_{x_j} x_j} \right],$$

where $V_i = \delta_i x_i + \frac{\Phi_i x_i}{x_i - \gamma_i} - \frac{\Phi_i x_i^2}{(x_i - \gamma_i)^2}$,

$$U_{ij} = \frac{x_i}{x_i - \gamma_i} \frac{\partial \Phi_i}{\partial \log(u)} \frac{\partial \log(U)}{\partial \log(x_j)} \text{ in which } \frac{\partial \log(U)}{\partial \log(x_j)} = \frac{-B_{x_j} x_j}{\sum_{j'} \frac{\partial \Phi_{j'}}{\partial \log(u)} \log(x_{j'} - \gamma_{j'}) - \frac{1}{X1^{\eta}}},$$

$$\frac{\partial \Phi_i}{\partial \log(u)} = \frac{(\beta_i - \alpha_i) u}{(1+u)^2}, \quad B_{X_i} = \partial B / \partial x_i = \delta_i + \Phi_i / (x_i - \gamma_i) + \frac{\mu_i \eta \log(u)}{x_i X1^{\eta}}, \text{ and}$$

$$Z_j = \frac{\eta}{X1^{\eta}} \left[\frac{-B_{x_j} x_j}{\sum_{j'} \frac{\partial \Phi_{j'}}{\partial \log(u)} \log(x_{j'} - \gamma_{j'}) - \frac{1}{X1^{\eta}}} - \mu_j \eta \log(u) \right].$$