AJAE appendix for 'Credit Market Imperfections and the Distribution of Policy Rents'

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Note: The material contained herein is supplementary to the article named in the title and published in the American Journal of Agricultural Economics (AJAE).

A1. Proof of Proposition 1

To show: $\frac{dr}{ds} > 1$ with $\alpha_s > 0$.

We show the case when farms remain credit constrained with the subsidy.[1]

With area payments the farm credit constraint is given by $kK \leq S = \alpha_W W + \alpha_s sA$. In equilibrium the following conditions must be satisfied:[2]

(A1.1) $$pf_A + pf_K \frac{\alpha_s s}{k} - r + (1-\alpha_s)s = 0$$

(A1.2) $$A = A^T.$$

Totally differentiating (A1.1) and (A1.2) yields:

(A1.3) $$MdA + Rds - dr = 0$$

(A1.4) $$dA = 0$$

where

(A1.5) $$R = \frac{\alpha_s A^T}{k}\left(pf_{AK} + pf_{KK}\frac{\alpha_s s}{k}\right) + pf_K \frac{\alpha_s}{k} + (1-\alpha_s) \geq 1$$

(A1.6) $$M = \left(pf_{AA} + pf_{AK}^C \frac{\alpha_s s}{k} + pf_{KA}\frac{\alpha_s s}{k} + pf_{KK}\frac{\alpha_s{}^2 s^2}{k^2}\right) < 0.$$

Solving for $\frac{dr}{ds}$ yields:

(A1.7) $$\frac{dr}{ds} = \frac{\alpha_s A^T}{k}\left(pf_{AK} + pf_{KK}\frac{\alpha_s s}{k}\right) + pf_K \frac{\alpha_s}{k} + (1-\alpha_s) \geq 1.$$

With constant returns to scale in the production function it follows that $\frac{A}{K} f_{AK} = -f_{KK}$, which implies that $\frac{\alpha_s A^T}{k}\left(pf_{AK} - pf_{AK} \frac{\alpha_s s A^T}{kK} \right) \geq 0$.[3] From the first order condition with credit constraint binding ($pf_K - k(1+\lambda) = 0$) it follows that $pf_K \frac{1}{k} \geq 1$. Hence:

1. if $\alpha_s = 0$ then $\frac{dr}{ds} = 1$
2. if $\alpha_s > 0$ then $\frac{dr}{ds} > 1$.

Q.E.D.

A2. Proof of Proposition 2

To show: $\frac{d\prod}{ds} < 0$ with $\alpha_s > 0$.

We show the case when farm remains credit constrained with the subsidy.[4]

Farm profits are: $\prod = pf(A,K) - (r-s)A - kK$. It follows that:

(A2.1) $$\frac{d\prod}{ds} = -A^T \frac{\alpha_s A^T}{k}\left(pf_{AK} + pf_{KK} \frac{\alpha_s s}{k} \right) \leq 0.$$

With $pf_{AK} + pf_{KK} \frac{\alpha_s s}{k} > 0$ (see proposition 1) it follows that $\frac{d\prod}{ds} < 0$ if $\alpha_s > 0$. If $\alpha_s = 0$ then $\frac{d\prod}{ds} = 0$.

Q.E.D.

A3. Proof of Proposition 3

To show: $\frac{dU}{ds} > 0$ with $\alpha_s > 0$.

We show the case when farm remains credit constrained with the subsidy.[5]

Total welfare (U) is the sum of farm profits (Π), landowners total rents ($\Pi^L = rA^T$), and minus taxpayers costs sA^T, i.e. $U = \Pi + \Pi^L - sA^T$. The effect of subsidies on welfare is then:

$$\text{(A3.1)} \qquad \frac{dU}{ds} = \frac{d\Pi}{ds} + \frac{d\Pi^L}{ds} - A^T.$$

Using (A2.1), (A1.7) and the effect of subsidies on landowners' rent: $\frac{d\Pi^L}{ds} = A^T \frac{dr}{ds}$, it follows that:

$$\text{(A3.2)} \qquad \frac{dU}{ds} = A^T pf_K \frac{\alpha_s}{k} - A^T \alpha_s = \frac{\alpha_s A^T}{k}\left(pf_K - k\right) > 0.$$

Welfare increases with $\alpha_s > 0$, otherwise if $\alpha_s = 0$, $\frac{dU}{ds} = 0$.

A4. Proof of Proposition 4

We analyze the general case when both farms are and remain credit constrained (and $\alpha_s^1 = \alpha_s^2 > 0$).[6]

To show: $\frac{d\Pi^1}{ds} < 0$ and $\frac{d\Pi^2}{ds} \leq 0$ or > 0 if farm 2 is more credit constrained than farm 1, (and vice versa).

Profit of farm i is $\Pi^i = pf^i(A^i, K^i) - (r - s)A^i - kK^i$. Then it follows that:

(A4.1) $$\frac{d\prod^{i}}{ds} = \frac{\alpha_s^i A^i}{k}\left(pf_K^i - k\right) - A^i \frac{dr}{ds} + A^i .$$

With area payments, farm i's credit constraint is as follows:

(A4.2) $$kK^i \le S^i(W^i) + \alpha_s^i s A^i .$$

In equilibrium the following condition must be satisfied:

(A4.3) $$pf_A^i + pf_K^i \frac{\alpha_s^i s}{k} - r + \left(1 - \alpha_s^i\right)s = 0 \text{ and } \sum_{i=1}^{2} A^i = A^T .$$

Totally differentiating (A4.3) yields:

(A4.4) $$M^i dA^i + R^i ds - dr = 0 \text{ and } \sum_{i=1}^{2} dA^i = 0$$

where

(A4.5) $$R^i = \frac{\alpha_s^i A^i}{k}\left(pf_{AK}^i + pf_{KK}^i \frac{\alpha_s^i s}{k}\right) + pf_K^i \frac{\alpha_s^i}{k} + \left(1 - \alpha_s^i\right) \ge 1$$

(A4.6) $$M^i = \left(pf_{AA}^i + pf_{AK}^i \frac{\alpha_s^i s}{k} + pf_{KA}^i \frac{\alpha_s^i s}{k} + pf_{KK}^i \frac{\alpha_s^{i\,2} s^2}{k^2}\right) < 0 .$$

Using (A4.4) it follows that:

(A4.7) $$\frac{dr}{ds} = \frac{R^1 M^2 + R^2 M^1}{M^1 + M^2} \ge 1 .$$

A necessary condition for maximum profit is that $\Pi_{AA}^i < 0$, implying that $M^i < 0$. With credit constraints it holds that $pf_K^i - k > 0$ and that $\Pi_{AK}^i = pf_{AK}^i + pf_{KK}^i \frac{\alpha_s^i s}{k} > 0$ [7] implying that $R^i \ge 1$, hence $\frac{dr}{ds} \ge 1$.

The more farm i is credit constrained the less fertilizers it can use, implying (a) that the higher is the increase in land marginal productivity, $pf_{AK}^{i} + pf_{KK}^{i}\frac{\alpha_s^i s}{k}$ when adding additional fertilizers, and (b) the higher is the difference between fertilizer marginal value product and fertilizer price, $pf_K^i - k$. Hence, for a given $\alpha_s^i > 0$, R^i is higher the more farm i is credit constrained.

Then it follows that for $\alpha_s^1 = \alpha_s^2$: if $R^2 > R^1$ (if farm 2 is more credit constrained than farm 1) then $\frac{d\prod^1}{ds} < 0$, $\frac{d\prod^2}{ds} \leq 0$ or > 0.

Q.E.D.

A5. Proof of Proposition 5

To show:

a. $\frac{dr}{ds} > 1$ with ($\alpha_s > 0,\ \alpha_W > 0$)

b. $\frac{d\prod}{ds} < 0 \geq 0$ with ($\alpha_s > 0, \alpha_W > 0$).

We show the case when farm remains credit constrained with the subsidy.[8]

Case a:

If farm credit is based on gross profitability and subsidies, in equilibrium conditions (A1.2) must be satisfied, as well as:

(A5.1) $pf_A + pf_K \frac{\alpha_s s + \alpha_W \overline{\pi}^G}{k} - r + (1-\alpha_s)s - \alpha_{\overline{\pi}^G}\overline{\pi}^G = 0$.

Totally differentiating (A1.2) and (A5.1) and solving for $\frac{dr}{ds}$ yields:

$$\text{(A5.2)}\quad \frac{dr}{ds} = \frac{\left(pf_{KK} \frac{\left(\alpha_s s + \alpha_W \bar{\pi}^G\right)A}{k^2} + pf_{AK} \frac{A}{k} + pf_K \frac{1}{k} - 1 \right)\left(1-\alpha_W\right)\alpha_s}{1 + \alpha_W \left[pf_{AK} \frac{A}{k} + pf_{KK} \frac{\left(\alpha_s s + \alpha_W \bar{\pi}^G\right)A}{k^2} - 1 \right]} + 1$$

where $\left(pf_{AK} + pf_{KK} \frac{\left(\alpha_s s + \alpha_W \bar{\pi}^G\right)}{k} \right) \geq 0$, the intuition is the same as shown in the proof of proposition 1 in Appendix A1.

In order to have a stable equilibrium situation, it must be the case that:

$$\text{(A5.3)}\quad 0 < 1 + \alpha_W \left[pf_{AK} \frac{A^T}{k} + pf_{KK} \frac{\left(\alpha_s s + \alpha_W \bar{\pi}^G\right)A^T}{k^2} - 1 \right] < 1 .$$

This implies that with $\alpha_W < 1$, $\dfrac{\left(pf_{AK} + pf_{KK} \frac{\left(\alpha_s s + \alpha_W \bar{\pi}^G\right)}{k} \right)\alpha_W \frac{A^T}{k}}{1 + \alpha_W \left[pf_{AK} \frac{A^T}{k} + pf_{KK} \frac{\left(\alpha_s s + \alpha_W \bar{\pi}^G\right)A^T}{k^2} - 1 \right]} < 1$.[9]

$pf_{KK} \frac{\left(\alpha_s s + \alpha_W \bar{\pi}^G\right)A^T}{k^2} + pf_{AK} \frac{A^T}{k} + pf_K \frac{1}{k} - 1$ is positive, hence $\frac{dr}{ds} > 1$.

The impact of subsidies on gross profit and on total credit:

$$\text{(A5.4)}\quad \frac{d\bar{\pi}^G}{ds} = \frac{-\left[pf_{KK} \frac{\left(\alpha_s s + \alpha_W \bar{\pi}^G\right)}{k} + pf_{AK} \right] \frac{\alpha_s A^T}{k}}{1 + \alpha_W \left[pf_{AK} \frac{A^T}{k} + pf_{KK} \frac{\left(\alpha_s s + \alpha_W \bar{\pi}^G\right)A^T}{k^2} - 1 \right]} < 0$$

$$\text{(A5.5)}\quad \frac{dS}{ds} = \frac{\alpha_s A^T \left(1-\alpha_W\right)}{1 + \alpha_W \left[pf_{AK} \frac{A^T}{k} + pf_{KK} \frac{\left(\alpha_s s + \alpha_W \bar{\pi}^G\right)A^T}{k^2} - 1 \right]} > 0 .$$

Gross profits decline, and with $\alpha_W < 1$ overall credit increases with subsidies.

If farm credit is based on own land assets and subsidies, in equilibrium condition (A1.2) must be satisfied, as well as:

(A5.6) $pf_A + pf_K \frac{\alpha_s s}{k} - r + (1-\alpha_s)s = 0$.

Totally differentiating (A1.2) and (A5.6) and solving for $\frac{dr}{ds}$ yields:

$$\text{(A5.7)}\quad \frac{dr}{ds} = \frac{\left(pf_{AK} + pf_{KK}\frac{\alpha_s s}{k}\right)\frac{\alpha_s A^T}{k} + pf_K \frac{\alpha_s}{k} + (1-\alpha_s)}{1 - pf_{KK}\frac{\alpha_s s \alpha_W A_o}{k^2}\frac{\partial R}{\partial r} - pf_{AK}\frac{\alpha_W A_o}{k}\frac{\partial R}{\partial r}} > 1$$

In order to have stable equilibrium, $0 < pf_{AK}\frac{\alpha_W A_o}{k}\frac{\partial R}{\partial r} + pf_{KK}\frac{\alpha_s s}{k}\frac{\alpha_W A_o}{k}\frac{\partial R}{\partial r} < 1$.

$\left(pf_{AK} + pf_{KK}\frac{\alpha_s s}{k}\right) > 0$, the intuition is the same as shown in the proof of proposition 1 in Appendix A1. This implies that land rent increases by more than the size of the subsidy.

Case b:

If farms credit is based on gross profitability and subsidies, total differentiating profits ($\Pi = pf(A,K) - (r-s)A - kK$) yields:

$$\text{(A5.8)}\quad \frac{d\Pi}{ds} = \left[pf_K \frac{\alpha_W A^T}{k} - \alpha_W A^T\right]\frac{d\bar{\pi}^G}{ds} + \left[pf_K \frac{\alpha_s A^T}{k} + (1-\alpha_s)A^T\right] - A^T \frac{dr}{ds}$$

where

(A5.9) $$\frac{d\overline{\pi}^G}{ds}=\frac{\left[pf_K\frac{\alpha_s}{k}+(1-\alpha_s)\right]-\frac{dr}{ds}}{\left(1-pf_K\frac{\alpha_W}{k}\right)}.$$

From equations (A5.2), (A5.8) and (A5.9) it follows that:

(A5.10) $$\frac{d\prod}{ds}=-\frac{\alpha_s A^{T2}}{k}\left[1-\frac{\alpha_W\frac{A^T}{k}\left(pf_{AK}+pf_{KK}\frac{\left(\alpha_s s+\alpha_W\overline{\pi}^G\right)}{k}\right)}{1+\alpha_W\left(pf_{AK}\frac{A^T}{k}+pf_{KK}\frac{\left(\alpha_s s+\alpha_W\overline{\pi}^G\right)A^T}{k^2}-1\right)}\right]\left(pf_{AK}+pf_{KK}\frac{\left(\alpha_s s+\alpha_W\overline{\pi}^G\right)}{k}\right)<0$$

With $\dfrac{\left(pf_{AK}+pf_{KK}\frac{\left(\alpha_s s+\alpha_W\overline{\pi}^G\right)}{k}\right)\alpha_W\frac{A^T}{k}}{1+\alpha_W\left[pf_{AK}\frac{A^T}{k}+pf_{KK}\frac{\left(\alpha_s s+\alpha_W\overline{\pi}^G\right)A^T}{k^2}-1\right]}<1$ and with $\left(pf_{AK}-pf_{AK}\frac{\left(\alpha_s s+\alpha_W\overline{\pi}^G\right)}{k}\right)\geq 0$, $\frac{d\prod}{ds}<0$.

If farm credit is based on agricultural land assets and subsidies total farm income is $\prod = pf(A,K)-(r-s)A-kK+rA_o$. Then from equations (A5.7) it follows that:

(A5.11) $$\frac{d\prod}{ds}=\frac{\alpha_W A_o\frac{\partial R}{\partial r}\left(pf_K\frac{1}{k}-1\right)\left(pf_K\frac{\alpha_s}{k}+(1-\alpha_s)\right)-\left(A\frac{\alpha_s A}{k}+A\frac{\alpha_W A_o}{k}\frac{\partial R}{\partial r}\right)\left(pf_{KK}\frac{\alpha_s s}{k}+pf_{AK}\right)}{1-pf_{KK}\frac{\alpha_s s\alpha_W A_o}{k^2}\frac{\partial R}{\partial r}-pf_{AK}\frac{\alpha_W A_o}{k}\frac{\partial R}{\partial r}}+A_o\frac{dr}{ds}<$$

or ≥ 0.

Q.E.D.

Footnotes

[1] The case when area subsidies remove the full credit constraint can be analogously derived.

[2] To simplify the derivations we assume one representative farm. This assumption does not affect the results.

[3] If the initial value of $\alpha_s s$ is zero or not large, then with decreasing return to scale it also holds that

$$\frac{\alpha_s A^T}{k}\left(pf_{AK} + pf_{KK}\frac{\alpha_s s}{k}\right) \geq 0 .$$

[4] The case when area subsidies remove all credit constraints can be analogously derived.

[5] The case when area subsidies remove all credit constraints can be analogously derived.

[6] The case when area subsidies remove all credit constraints can be analogously derived.

[7] The intuition is the same as shown in the proof of proposition 1 in Appendix A1.

[8] The case when area subsidies remove the full credit constraint can be analogously derived. To simplify the derivations we assume one representative farm. This assumption does not affect the results.

[9] We consider the case when $\alpha_W < 1$. If this is not the case then this would imply that farm is not credit constrained. Banks would be willing to give sufficient credit to farms.