AJAE Appendix for "Determinants of Income Growth in Metropolitan and Non-metropolitan Labor Markets"

November 2007

George W. Hammond
Associate Director, BBER
Associate Professor of Economics
Bureau of Business and Economic Research and Department of Economics
West Virginia University
Phone: (304) 293-7876
Fax: (304) 293-7061
Email: ghammond@wvu.edu

Eric C. Thompson
Director, BBR
Associate Professor of Economics
Department of Economics and Bureau of Business Research
University of Nebraska-Lincoln
Phone: (402) 472-3188
Fax: (402) 472-9700
Email: ethompson2@unl.edu

Note: The material contained herein is supplementary to the article named in the title and published in the American Journal of Agricultural Economics (AJAE).

AJAE Appendix for "Determinants of Income Growth in Metropolitan and Non-metropolitan Labor Markets"

Solow CES Model Development

In order to investigate these issues of metropolitan/non-metropolitan growth and convergence, we start with a model that describes a one-sector economy and which generates its good using a CES production function. We include four inputs in the production function: labor, private physical capital, public infrastructure capital, and human capital. We depart from earlier work by employing a CES production function, which provides added flexibility in the modeling of production technology by allowing the elasticity of substitution to differ from one. CES production functions are becoming increasingly popular in the empirical literature on international growth and convergence (Masanjala and Papageorgiou (2004), Duffy and Papageorgiou (2000)). They are attractive in this context because they allow us to investigate the role of the elasticity of substitution in the growth process and because they encompass the CD specification.

Following Masanjala and Papageorgiou (2004) we specify a CES production function with labor augmenting technological progress:

(1) $$\mathrm{Y} = [\alpha K^{\rho} + \beta H^{\rho} + \gamma Z^{\rho} + (1-\alpha-\beta-\gamma)(AL)^{\rho}]^{\frac{1}{\rho}}$$

where A is exogenous technology which grows at rate g, Y is real output, K is the private physical capital stock, Z is the stock of public capital, H is the stock of human capital, and L is the labor force which grows at rate n (we suppress time subscripts). We expand on the work of Masanjala and Papageorgiou (2004) through our inclusion of public capital stock as an input. The parameters α, β, γ are distribution parameters. The elasticity of substitution (σ≥0) is defined as 1/(1-ρ). In this four factor case, we focus on the Allen

Partial Elasticity of Substitution (Allen, 1938, pp. 503-509), which states that for production functions of the form in equation (1) the elasticity of substitution $\sigma = \sigma_{ij}$ where i,j=(K, H, Z, AL) and $i \neq j$. Uzawa (1962) contains a full discussion. If $\rho=0$ ($\sigma=1$), the CES production function reduces to the CD case. On the other end of the spectrum, if $\rho=1$ ($\sigma=\infty$), we have the perfect substitution case. Finally, if $\rho=-\infty$ ($\sigma=0$) we have the fixed proportions case.

We re-write (1) in intensive form:

(2) $$y = [\alpha k^{\rho} + \beta h^{\rho} + \gamma z^{\rho} + (1-\alpha-\beta-\gamma)]^{\frac{1}{\rho}}$$

where lower case variables are expressed per unit of effective labor. The stocks of the three forms of capital evolve over time according to the following relationships:

(3) $$\dot{K} = S_k - \delta K$$

(4) $$\dot{H} = S_h - \delta H$$

(5) $$\dot{Z} = S_z - \delta Z$$

where S_k, S_h, and S_z are shares of output invested in each form of capital. We make the standard assumption that all forms of capital depreciate at the same rate (δ). We use these equations in the standard way to solve for steady-state output:

(6) $$y^* = \left\{ \frac{1}{(1-\alpha-\beta-\gamma)} - \frac{\alpha}{(1-\alpha-\beta-\gamma)}\left(\frac{S_k}{(\delta+n+g)}\right)^{\rho} - \frac{\beta}{(1-\alpha-\beta-\gamma)}\left(\frac{S_h}{(\delta+n+g)}\right)^{\rho} - \frac{\gamma}{(1-\alpha-\beta-\gamma)}\left(\frac{Sz}{(\delta+n+g)}\right)^{\rho} \right\}^{-\frac{1}{\rho}}$$

To facilitate estimation, we follow Masanjala and Papageorgiou (2004) and Kmenta (1967) by computing a linearized version of the steady-state solution via a second-order Taylor series expansion of (6) around $\rho=0$. This linearization yields:

$$
(7)\qquad \begin{aligned}
\ln\left(\frac{Y}{L}\right)_t^* &= \ln A(0) + gt + \frac{\alpha}{1-\alpha-\beta-\gamma}\ln\left(\frac{S_k}{\delta+n+g}\right) + \frac{\beta}{1-\alpha-\beta-\gamma}\ln\left(\frac{S_h}{\delta+n+g}\right) + \frac{\gamma}{1-\alpha-\beta-\gamma}\ln\left(\frac{S_z}{\delta+n+g}\right) \\
&+ \frac{\rho}{2}\frac{1}{(1-\alpha-\beta-\gamma)^2}\left[\begin{array}{l}\alpha\left(\ln\left(\frac{S_k}{\delta+n+g}\right)\right)^2 + \beta\left(\ln\left(\frac{S_h}{\delta+n+g}\right)\right)^2 + \gamma\left(\ln\left(\frac{S_z}{\delta+n+g}\right)\right)^2 \\ -\alpha\beta\left(\ln\left(\frac{S_k}{S_h}\right)\right)^2 - \beta\gamma\left(\ln\left(\frac{S_h}{S_z}\right)\right)^2 - \alpha\gamma\left(\ln\left(\frac{S_k}{S_z}\right)\right)^2\end{array}\right]
\end{aligned}
$$

Note that if ρ=0 (σ=1) equation (7) reverts to the CD solution. This will facilitate a test for mis-specification in research that has assumed a CD production function. That is, if ρ=0 the squared terms on the second line of equation (7) will be jointly insignificantly different from zero.

Since regional economies may not be at their steady-states at all times, we follow Crihfield and Panggabean (1995) and account for the adjustment to steady state, using:

$$
(8)\qquad \ln\left(\frac{Y}{L}\right)_t - \ln\left(\frac{Y}{L}\right)_0 = (1-\pi)\left[\ln\left(\frac{Y}{L}\right)_t^* - \ln\left(\frac{Y}{L}\right)_0\right]
$$

Substituting equation (7) into equation (8) we have:

$$
(9)\qquad \begin{aligned}
\ln\left(\frac{Y}{L}\right)_t - \ln\left(\frac{Y}{L}\right)_0 &= (1-\pi)\ln A(0) + (1-\pi)gt - \frac{(1-\pi)(\alpha+\beta+\gamma)}{(1-\alpha-\beta-\gamma)}\ln(\delta+n+g) + \frac{(1-\pi)\alpha}{(1-\alpha-\beta-\gamma)}\ln(S_k) \\
&+ \frac{(1-\pi)\beta}{(1-\alpha-\beta-\gamma)}\ln(S_h) + \frac{(1-\pi)\gamma}{(1-\alpha-\beta-\gamma)}\ln(Sz) \\
&+ \frac{\rho}{2}\frac{(1-\pi)}{(1-\alpha-\beta-\gamma)^2}\left[\begin{array}{l}\alpha\left(\ln\left(\frac{S_k}{\delta+n+g}\right)\right)^2 + \beta\left(\ln\left(\frac{S_h}{\delta+n+g}\right)\right)^2 + \gamma\left(\ln\left(\frac{S_z}{\delta+n+g}\right)\right)^2 \\ -\alpha\beta\left(\ln\left(\frac{S_k}{S_h}\right)\right)^2 - \beta\gamma\left(\ln\left(\frac{S_h}{S_z}\right)\right)^2 - \alpha\gamma\left(\ln\left(\frac{S_k}{S_z}\right)\right)^2\end{array}\right] \\
&- (1-\pi)\ln\left(\frac{Y}{L}\right)_0
\end{aligned}
$$

This is the form of the equation which we estimate in the following section, after converting to annual rates. It allows us to test for the relative influence of each form of investment on growth and to identify possible asymmetries in the impact of investment (manufacturing plant and equipment, human capital, and public capital) and the elasticity of substitution across metropolitan and non-metropolitan regions.

Data Definitions and Sources

Real Per Capita Personal Income Growth

Personal income and population data come from the U.S. Bureau of Economic Analysis, *Regional Economic Analysis System CD-Rom,* May 2001. Personal income includes earnings from work, asset income, and transfer payments. Population estimates reflect residents in the county on July 1 of the year. The growth rate is the compound average annualized rate of growth.

Manufacturing Capital Investment and Depreciation

Private manufacturing new capital expenditures by county come from the Census of Manufacturers, *Geographic Area Series,* for 1972, 1977, 1982, 1987, and 1992. Data from 1977-1992 come from the *USA Counties CD-ROM*. Data for 1972 was compiled by hand by the authors. New capital expenditures include permanent additions and major alterations to manufacturing establishments and machinery and equipment. The investment rate is computed by summing county-level new capital expenditures to labor market areas and then dividing by personal income less transfers. In those cases in which no counties reported data, due to disclosure requirements, we substitute the state value.

Depreciation data are available only for states for 1977, 1982, 1987, and 1992. State depreciation rates are assigned to labor market areas based on the state containing the largest county within the region. Annual investment and depreciation rates for local labor market areas are then averaged across years.

Public Capital Investment

Local government capital outlay data were hand-compiled by the authors from the Census of Government, *Compendium of Government Finances,* for 1972, 1977, 1982, 1987, and 1992. Capital outlays include direct expenditures for construction of buildings, acquisition of land, and purchases of equipment. The investment rate is computed by summing county-level public capital outlays to local labor market areas and then dividing by personal income. Annual investment rates are then averaged across years.

Human Capital Investment

Human capital investment is the increase in average years of schooling from 1970 to 2000. Years of schooling in a county in each year is calculated based on high school and college attainment rates from the Census of Population. In particular, years of schooling is computed by multiplying the share of the population (age 25 and older) with a given level of educational attainment by the assigned years of schooling. College graduates or higher are assigned 17 years of schooling, while high school graduates who did not complete college were assigned 13 years of schooling, and persons who did not complete high school were 10 years of schooling. These weighted years of schooling are then summed for the region. High school and college attainment data for 1970 was collected by hand by the authors and data for 2000 was extracted from the U.S. Bureau of the Census website.

Tax Variables

The local government tax revenue data come from the Census of Government, *Compendium of Government Finances*, for 1972, 1977, 1982, 1987 and 1992. Data was extracted from the *USA Counties 1998 CD-Rom* for the 1977-1992 period. Data for 1972 were hand compiled by the authors. Tax rates are computed relative to personal income.

Energy Price Variables

Electricity and natural gas rates for industrial customers are based on average annual data from the U.S. Department of Energy, Energy Information Administration, (ftp://ftp.eia.doe.gov/pub/state.prices/html/icwy.htm) by state from 1970 to 1999. State data is assigned to labor market regions based on the state containing the largest county within the region. Labor market data are averaged across years.

Unionization Rates

Data on the share of the workforce which is unionized are based on state-level annual averages from the years 1970, 1972, 1978, 1980, 1982, 1983, 1994, which are taken from the *Statistical Abstract of the United States*. State data is assigned to labor market regions based on the state containing the largest county within the region. Labor market data are averaged across years.

Topography

The topography scale is from McGranahan (1999), who mapped topographic information from The National Atlas of the United States of America 1970 to U.S. counties. The land surface code scale (1 through 21) runs from 1 (plains) to 21 (high mountains).

Death Rate

The county-level deaths for the years 1970, 1975, 1978-1980, 1982, 1984-1994 from the Bureau of Census. The authors extracted these data from the *USA Counties 1998 CD-Rom*. Labor market area death rates are computed by dividing deaths by population in the area. Data are averaged across years.

University Count

The number of four-year colleges or universities in 1980 for counties in each labor market area was downloaded from the National Center for Education Statistics website (http://nces.ed.gov/ipedspas/). Institutions are initially geo-located by ZIP codes, which are then assigned to counties using a ZIP-to-county correspondence purchased from zipinfo.com.

Temperature and Water Surface Area

The mean January temperature, mean July temperature, and water access variables in each labor market area were developed by the U.S. Department of Agriculture, Economic Research Services, see McGranahan (1999) for a complete description of the data. County data were aggregated into labor market area based on surface area. Temperature data were annual averages for 1941 through 1970.

References

Allen, R.G.D. 1938. *Mathematical Analysis for Economists.* London: Macmillan.

Crihfield, J.B. and M.P.H. Panggabean. 1995. "Growth and Convergence in U.S. Cities." *Journal of Urban Economics*, 38, 138-165.

Duffy, J. and C. Papageorgiou. 2000. "A Cross-Country Empirical Investigation of the Aggregate Production Function Specification." *Journal of Economic Growth* 5: 87-120.

Kmenta, J. 1967. "On Estimation of the CES Production Function." *International Economic Review* 8: 180-189.

Masanjala, W.H. and C. Papageorgiou. 2004. "The Solow Model with CES Technology: Nonlinearities and Parameter heterogeneity." *Journal of Applied Econometrics*, 19: 171-201.

McGranahan, D.A. 1999. "Natural Amenities Drive Rural Population Change," Agricultural Economic Report #781, USDA Economic Research Service, Food and Rural Economics Division.

Uzawa, H. 1962. "Production Functions with Constant Elasticities of Substitution." *The Review of Economic Studies* 29(4): 291-299.