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AN ALINEMENT-CHART METHOD FOR PREPARING FOREST-TREE VOLUME TABLES

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INTRODUCTION

A volume table is a tabular statement of the average volume of trees of given sizes. The size classification is usually both by diameter (d. b. h. or diameter breast high, outside the bark) and height (either total or merchantable). This form of presentation implies that tree volume is correlated with both diameter and height.

If graphs are plotted showing the relation between volume and diameter for the several height classes and similarly between volume and height for the several diameter classes, curves rather than straight lines will result. This indicates that the preparation of a volume table is a problem in curvilinear multiple correlation.

A method of solving problems in curvilinear multiple correlation is described in a previous publication.[1] The technic is briefly as follows:

1. A multiple-regression equation is calculated, which is considered as a first approximation to the desired result.

2. This equation is converted into an alinement chart for ease in handling the subsequent steps. The alinement chart, being based on an equation of the first degree, has straight parallel axes and uniformly graduated scales.

3. Values are read from this chart corresponding to the data. These are called first estimates, and the differences between these

[1] BRUCE, D., and REINEKE, L. H. CORRELATION ALINEMENT CHARTS IN FOREST RESEARCH. U. S. Dept. Agr. Tech. Bul. 210, 88 p., illus. 1931.

estimates and the corresponding measured values of the data are called first residuals.

4. The first residuals are plotted over the independent variables, one at a time, and curves are fitted.

5. These curves are used to regraduate the chart axes for these independent variables wherever changes are seen to be desirable.

6. Second estimates are read from the revised chart which represents a second approximation.

7. The measured values are then plotted over these second estimates, and a curve is fitted.

8. By means of this curve the graduations of the axis representing the dependent variable are relocated if necessary, thus producing a third-approximation chart.

9. These steps—4 to 5, and 6 to 8—are then alternately repeated as many times as may be necessary until a satisfactory result is obtained.

While volume-table construction is obviously a problem in curvilinear multiple correlation, it is a type of problem which requires a specialized form of the general technic.[2] The specialized technic, which brings speed and accuracy, will be described in the following pages.

The multiple-regression equation used in the general technic can be shown on theoretical grounds to be a poor first approximation in the case of a volume table. In the first place we know that the curves of volume over diameter and volume over height are not straight lines, so that varying spacing of the graduations of the diameter and height axes are to be expected. In the second place, from the familiar formula

$$V = FHB$$

where V is the volume in cubic feet, F the form factor, H the height in feet, B the basal area in square feet, we would expect to need a chart involving multiplication rather than the addition characteristic of a multiple-regression equation. Obviously, for rapid definition of the true relationships between volume, diameter, and height, a first-approximation chart which is much closer to the final result than one based on the multiple-regression equation is needed.

The best form of base chart or first-approximation chart is that which gives the volumes of a geometric solid most closely approaching tree form. For tables giving volume in cubic feet this condition is satisfied by a very simple chart, with three straight, parallel axes with logarithmically spaced graduations. The use of the board-foot unit of volume, however, makes necessary far more complex forms with curved axes. These forms differ with the log rule used, with the height index adopted (merchantable or total), and with the standard of top utilization. After the proper base chart has been prepared or selected, however, the subsequent procedure is essentially the same for all volume tables. It will therefore be possible to describe in detail the method for the cubic-foot tables, and then merely to refer to a few minor variations in connection with the other types of tables.

[2] Examples worked out by the general technic resulted in distorted tables only half as accurate as those prepared by the more usual methods.

THE BASE CHART FOR VOLUME TABLES OF ENTIRE STEM IN CUBIC FEET

Reference has already been made to the familiar formula for the volume of a tree in cubic feet

$$V=FHB=\frac{\pi FHD^2}{4\times 144}$$

The corresponding formula for a cylinder having the same basal diameter and height is of course

$$V=\frac{\pi HD^2}{4\times 144}$$

One familiar with the construction of alinement charts will at once think of more than one form of chart by means of which this equation might be solved.[3]

The simplest is one having three parallel straight axes, with the central axis halfway between the outside axes. All the scales are logarithmic to provide for the fact that the variables in the equation are to be multiplied together instead of added. One of the outer axes, then, is graduated with the values of log H, the other with values of log $\frac{\pi D^2}{4\times 144}$, while the central is graduated with values of log V, using one-half of the scale used for the outer axes.

Such a chart will give volumes of cylinders of sizes corresponding to the trees for which data are at hand. Were the form factor a uniform factor for each species, all that would remain would be to raise the central or volume axis by an amount equal to the logarithm of this form factor. This follows from the fact that a bodily raising of the axis is equivalent to moving the slide in a slide rule. Unfortunately, however, the form factor varies with both diameter and height and that in a rather complex fashion. This type of adjustment of the central axis, therefore, while useful, would by no means be final, and would merely leave us with a first-approximation chart which was numerically closer than that for the cylinder, but no better in relation to variation in form factor. A cylinder chart, then, except for the larger numbers involved, is just as useful as one corrected for average form factor for the species. However, to gain some advantage in reduction in size of numbers, the cylinder chart used as the standard base chart is modified for a form factor of 0.4, which is a rough approximation to the average of all species.

This standard base chart used by the Forest Service is illustrated in Figure 1. The diameter axis is at the left, the height axis at the right, and the volume axis in the middle. To secure height and diameter axes of approximately the same length different scales have been used on these two axes, and as a result the volume axis is to the left of the central position.

The series of diagonal curves crossing the chart need explanation. These are the graduating curves used in constructing the chart, which are retained because of their usefulness in subsequent readjustments. They consist of four series, identified by the initials

[3] See the following publication: BRUCE, D., ALINEMENT CHARTS IN FOREST MENSURATION, Jour. Forestry 17: 773-801, illus., 1917; or such texts as LIPKA, J., GRAPHICAL AND MECHANICAL COMPUTATION, 264 p., illus., New York, 1918.

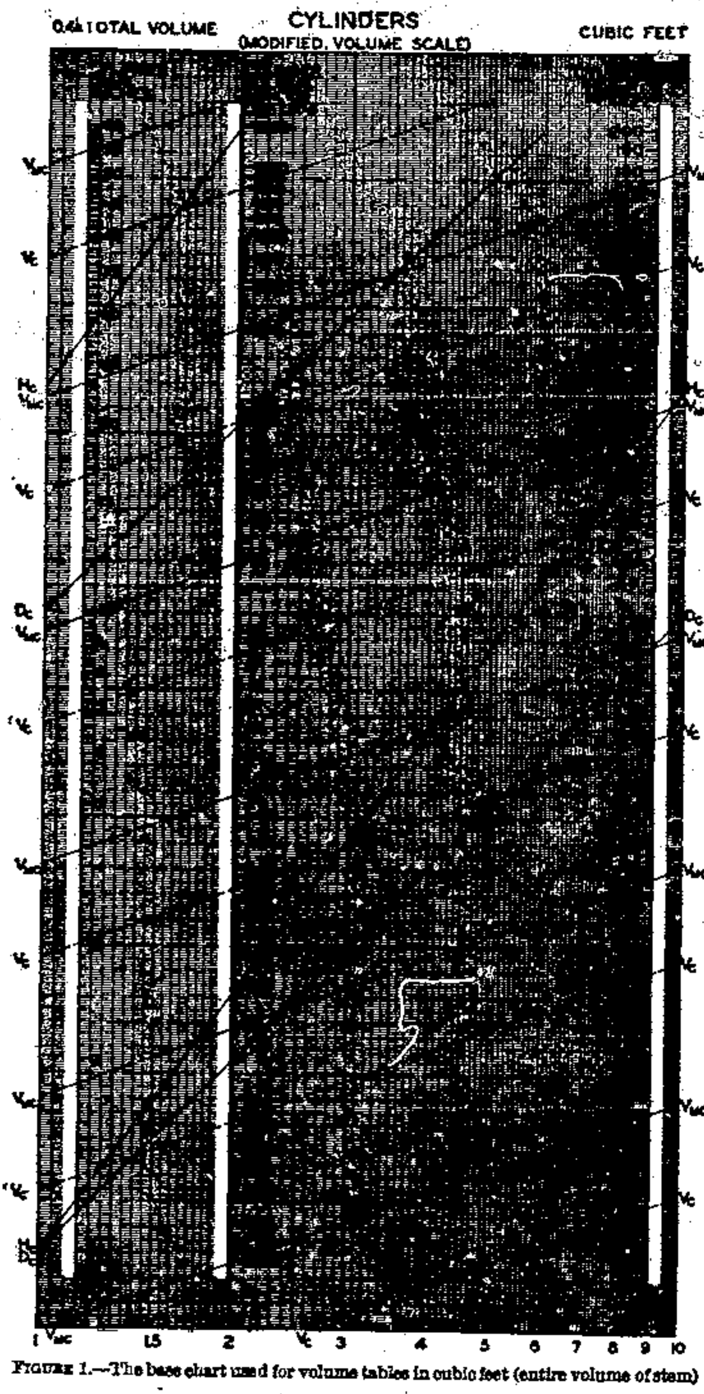

FIGURE 1.—The base chart used for volume tables in cubic feet (entire volume of stem)

D_c, H_c, V_c, and V_{mc}—standing, respectively, for diameter of cylinder, height of cylinder, volume of cylinder, and volume of modified cylinder.

They can best be understood if one series, for example, the D_c series, be carefully examined. It will be noted that the entire alinement chart is superposed on a sheet of semilogarithmic coordinate paper and that the vertical lines of this paper are given a scale across the base of 1 to 10. This scale is used in connection with all the graduating curves. Like all logarithmic scales, it applies equally well to values from 1 to 10, 10 to 100 or 100 to 1,000, etc. The semilogarithmic paper is used because on it the graduating curves will have very little curvature.

If the eye follows up the vertical line marked 1 (the left-hand edge of the chart) until it intersects for the first time the D_c curve, it will be noted that this intersection is horizontally opposite the 1 graduation on the diameter axis. Similarly, if the eye follows the 1.5 vertical to its intersection with the D_c curve, this intersection will be found opposite the 1.5 diameter graduation. The same will be found to be true for all possible values from 1 to 10. At 10 the D_c curve runs off the right-hand side of the chart. It reappears, however, opposite this point on the left-hand side, and this portion will be found to apply to diameter values of 10 or larger.

The D_c graduating curve, then, registers exactly the positions of all the graduations on the diameter axis. Were these graduations obliterated they could be accurately restored by means of the curve. Furthermore, if finer graduations should be needed for any purpose, they could easily and accurately be inserted by using this curve.

In a similar manner the H_c curve registers the graduations on the height axis. The lower portion of this curve, however, applies to heights of from 10 to 100, and the upper to heights above 100. The V_{mc} curve applies in the same way to the volume axis, but this is in six sections because of the large range of volume involved. The lowest section applies to volumes from 0.01 to 0.10 cubic foot and the highest to 1,000 cubic feet or more.

The V_c curves apply to nothing on the alinement chart but can be used to graduate the volume axis so that it will give cylinder volumes, if a cylinder-volume chart is desired.

It will also be noted that all coordinate lines have been removed from the chart in narrow strips along each of the axes. This is to permit the entry of new graduations as the work progresses, for it will be possible to make one copy of the chart serve throughout the entire process of the preparation of one volume table.

The charts actually used are considerably larger than Figure 1. The precise size is, of course, immaterial, but those used by the Forest Service are approximately 10 by 20 inches.

ASSEMBLY OF THE DATA

With one of these charts at hand, the first step in the actual preparation of a volume table is to assemble the existing data, which it is assumed have been collected in the field in the usual way. The tree sheets should be sorted into 1-inch (or sometimes 2-inch) diameter classes, and then subsorted into 10-foot (or sometimes 5-foot or 20-foot) height classes.[4] A convenient form for tabular assembly is shown in Table 1, which is but the first sheet of several which are required. Each tree occupies a line, with the original data in the first four columns. It will be noted that three spaces are left after each d. b. h.-height class, two for totals and averages and one to separate the classes. By measured volume is meant the volume obtained from the field measurements by scaling, by computation, or by planimetering taper graphs.

THE FIRST CORRECTION OF THE CHART

The next step is to fill column 5 of Table 1 by use of the base alinement chart. For each tree a first-estimate volume is obtained

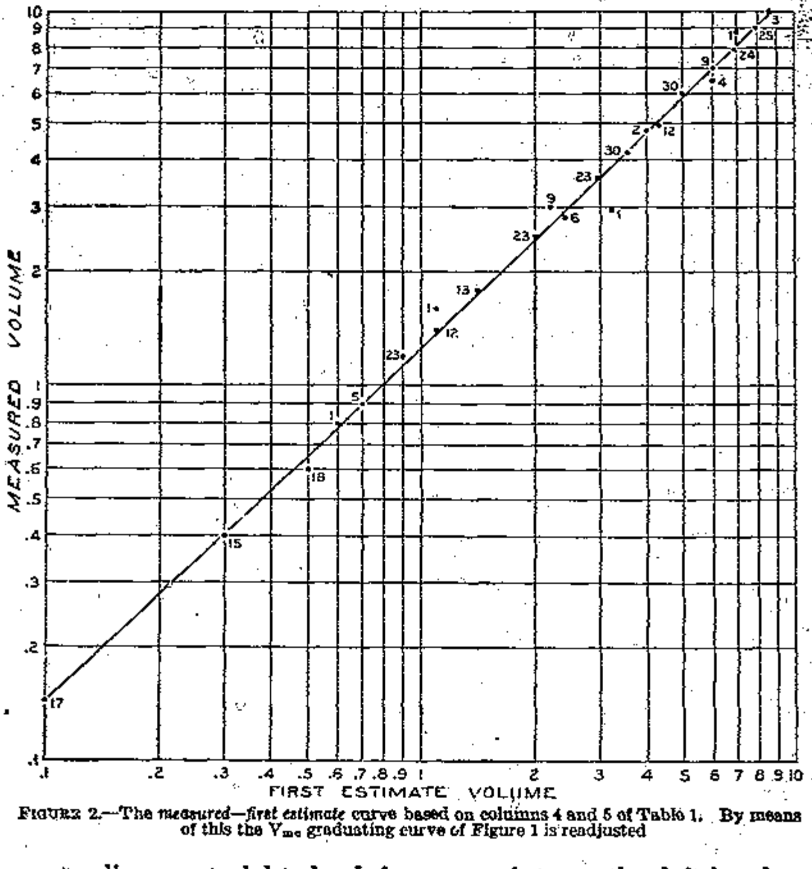

FIGURE 2.—The *measured—first estimate* curve based on columns 4 and 5 of Table 1. By means of this the V_{mc} graduating curve of Figure 1 is readjusted

by extending a straightedge[5] from a point on the left-hand axis corresponding to the diameter to a point on the right-hand axis

[4] Two methods of sorting are almost equally usable; the 8-inch diameter class, for example, may include trees from 7.5 to 8.4 inches or trees from 8.0 to 8.9 inches. The average diameter in the first case will be close to 8.0 and in the second close to 8.5. In the final volume table, volumes will be entered for 8, rather than 8.5, but it does not follow from this fact that there is any advantage in the first system of sorting. In plotting, the actual average diameters can and should be used, and the final results may be read from curves or alinement charts for even-inch diameters. When hand sorting by the second method, tenths of inches are ignored, thus eliminating errors in classifying, and increasing speed. Where punched cards are used in connection with tabulating machinery, sorting is somewhat simplified by the 8.0-8.9 classification, and this has therefore been made standard by the Forest Service.

[5] A stiff strip of transparent celluloid with a fine straight line etched on it is preferable to a straightedge because shadows are eliminated.

corresponding to the height, and reading its intersection with the central axis. When all volumes have been so obtained, both measured and first-estimate volumes are totaled and averaged for each d. b. h.-height class. The averages [6] are then plotted on logarithmic cross-section paper, with measured values on the vertical and first-estimates on the horizontal scale, and a curve is fitted. A characteristic result is shown in Figure 2. This is not the entire curve. The lowest point

TABLE 1.—*Chestnut volume table computation form*

Tree No.	Diameter breast high	Total height	Volume							Deviation (per cent of last estimate)
			Measured	Estimate						
				First	Second	Third	Fourth	Fifth		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)		(10)
	Inches	*Feet*	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*		
16	1.0	15	0.05	0.03	0.04	0.04				25
17	1.0	16	.05	.04	.05	.05				0
18	1.0	17	.05	.04	.05	.05				0
19	1.0	18	.06	.04	.05	.05				20
20	1.1	15	.07	.04	.05	.05				40
21	1.1	16	.07	.04	.06	.06				17
22	1.1	16	.07	.04	.06	.06				17
23	1.2	16	.07	.05	.07	.07				0
24	1.2	16	.07	.05	.07	.07				0
25	1.2	15	.08	.05	.07	.07				14
26	1.2	16	.08	.05	.07	.07				14
28	1.2	16	.09	.05	.07	.07				29
30	1.2	19	.10	.06	.09	.09				11
27	1.3	16	.10	.06	.09	.09				11
29	1.3	18	.10	.07	.10	.10				0
31	1.4	16	.08	.07	.10	.10				20
33	1.4	16	.09	.07	.10	.10				10
32	1.4	18	.10	.08	.11	.11				9
35	1.4	18	.11	.08	.11	.11				0
36	1.4	19	.11	.08	.12	.12				8
34	1.5	18	.12	.09	.13	.13				8
37	1.6	16	.12	.09	.13	.13				8
54	1.7	16	.17	.10	.15	.15				13
53	1.8	18	.17	.13	.18	.18				6
55	1.9	17	.19	.13	.19	.19				0
Total [a]	32.6	417	2.37	1.63	2.31	2.31				
Average	1.3	17	.09	.07	.09	.09				
38	1.1	20	.08	.05	.07	.07				14
39	1.1	20	.08	.05	.07	.07				14
40	1.3	20	.10	.07	.11	[illegible]				9
41	1.3	20	.11	.07	.11	.11				0
42	1.3	23	.13	.08	.12	.12				8
46	1.4	20	.13	.08	.13	.13				0
44	1.4	21	.13	.09	.13	.13				0
47	1.4	21	.14	.09	.13	.13				8
48	1.4	23	.14	.10	.14	.14				0
45	1.4	23	.15	.10	.14	.14				7
49	1.4	25	.16	.10	.15	.15				7
50	1.6	22	.19	.12	.18	.18				5
51	1.6	24	.16	.13	.19	.19				10
52	1.7	20	.18	.13	.18	.18				0
56	1.8	22	.24	.16	.22	.22				9
57	1.8	23	.24	.16	.23	.23				4
58	1.9	21	.23	.16	.24	.24				4
Total [b]	24.9	368	2.59	1.74	2.54	2.54				
Average	1.5	22	.15	.10	.15	.15				

[a] Total, 25 trees.
[b] Total, 17 trees.

[6] The individual values instead of the averages could, of course, be plotted, but the labor would be great and the curve fitting more difficult. There is, moreover, a theoretical advantage in resorting the values, both measured and estimated, on the basis of the first estimates, before averaging and plotting, but in most cases the increased accuracy of the result is not enough to justify the additional labor. The use of d. b. h.-height classes is equivalent to a rough sorting by first estimates, in which adjacent groups more or less overlap, but appreciable errors can hardly be caused by this rough sorting, because the resulting curves will be nearly straight lines.

on the curve, however, may be identified as that based on the second group in Table 1. It will be seen that the curve is very nearly a straight line and that the definition is excellent. The logarithmic paper is used merely because of the difficulty of plotting and reading the smaller values on ordinary paper; the shape of the curve would be the same on regular coordinate paper.

Were the curve a 45° straight line passing through the points (1, 1) (10, 10), etc., this would indicate that the first estimates agreed on the whole with the measured values and that the original chart was satisfactory without alteration. Usually this is not the case, and the curve, instead of confirming the chart, becomes a means of correcting it.

The simplest way to perform this correction is to revise the graduating curve V_{mc} first and then to use this revised curve to regraduate the volume axis. The method is as follows:

From Figure 2 it can be seen that for a first estimate of 0.1 cubic foot the corresponding measured (or "true") volume is 0.146.[7] The 0.1 point on the V_{mc} curve (fig. 3), which is its intersection with the vertical representing 0.1, must therefore be shifted horizontally to the vertical representing 0.146. Similarly, the point where it intersects the 0.2 vertical must be shifted to the vertical representing 0.28, etc. When enough of the points on the curve have been thus shifted, a revised graduating curve can be drawn through their new positions. By using this graduating curve, a new set of graduations can be entered on the blank or left side of the volume axis. The new graduating curve can be seen in Figure 3, marked V_1,[8] as well as the new volume graduations which correspond to it.

It is advisable to enter these new graduations in soft pencil so they may be erased if further revision is later found necessary. The graduating curve itself should therefore be inked in for permanent record. To avoid confusion between the various graduating curves which will eventually appear on the one chart, various colors are almost indispensable.

THE SECOND CORRECTION OF THE CHART

A set of second estimates of volume are next read from the revised chart and entered in column 6 of Table 1. These should, on an average, be much closer to the measured volumes than the first estimates.

Since a change in the volume scale has resulted in a closer agreement with the basic data, it is obvious that further improvement might be obtained by changes in either the diameter scale, the height scale, or both. The first of these possibilities is investigated by summarizing by diameter classes, disregarding height, the height-diameter class totals of certain items. These may appear in the form shown in Table 2, which like Table 1 is incomplete. Table 2 is summarized in complete form, however, in Table 3.

[7] So accurate a reading is of course impossible in Figure 2 as here shown. This figure is greatly reduced in scale from that actually used, and this reduction has made it necessary to omit most of the vertical and horizontal rulings.

[8] Subscripts denote the estimate from which the revised graduation curve was derived.

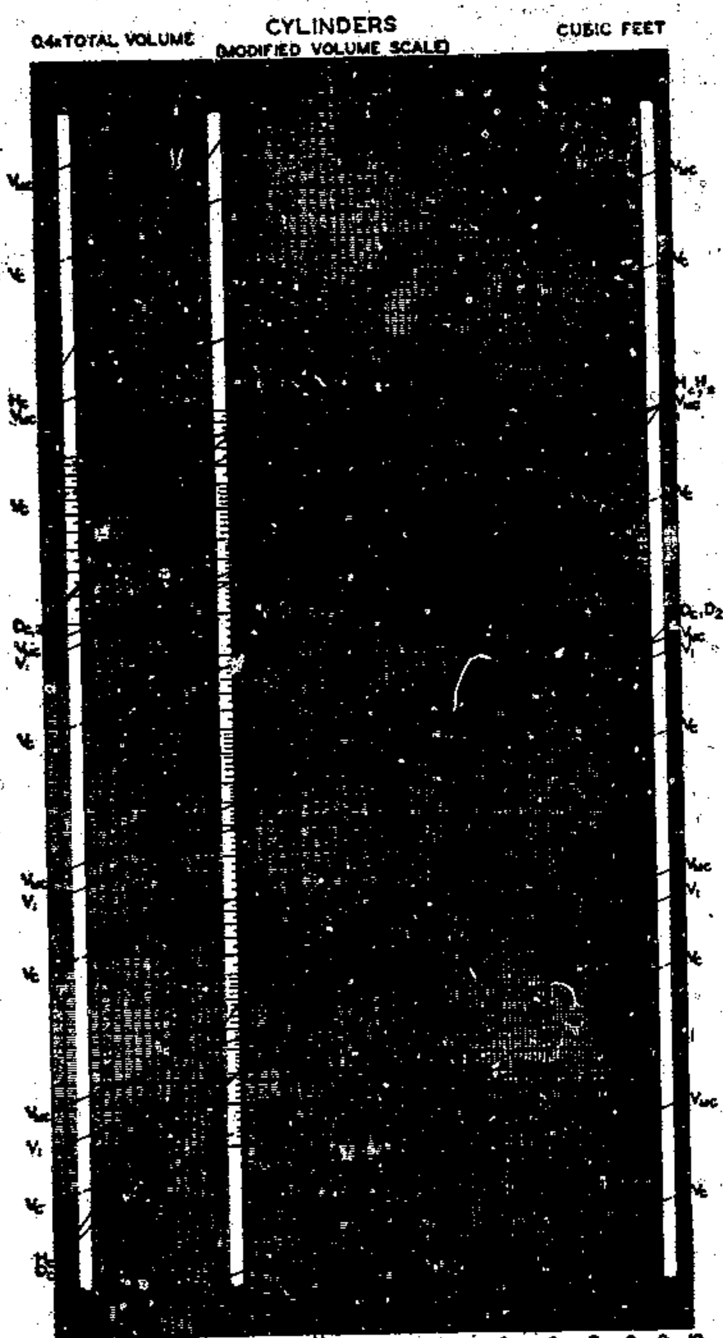

FIGURE 3.—The final appearance of the base chart

TABLE 2.—*Recapitulation by diameter classes after second estimate*

Diameter breast high class (inches)	Total height class	Trees	Totals of— Diameter breast high	Totals of— Second estimate volume	Totals of— Measured volume	Difference of second estimate	New graduation distance
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Feet	*Number*	*Inches*	*Cu. ft.*	*Cu. ft.*	*Per cent*	
1.0-1.9	10-19	25	32.6	2.31	2.37		
	20-29	17	24.9	2.54	2.59		
Total		42	57.5	4.85	4.96	+2.3	
Average			1.4	1.15	1.18		1.4
2.0-2.9	20-29	15	35.0	6.3	6.6		
	30-39	18	46.5	11.7	11.4		
	40-49	1	2.7	.8	.8		
Total		34	84.2	18.8	18.8	0.0	
Average			2.5	.553	.553		2.5
3.0-3.9	20-29	5	17.2	4.4	4.7		
	30-39	23	80.8	27.0	27.0		
	40-49	12	41.7	17.3	17.2		
Total		40	139.7	48.7	48.9	+0.4	
Average			3.5	1.22	1.22		3.5
4.0-4.9	20-20	1	4.5	1.4	1.6		
	30-39	13	56.3	23.0	23.3		
	40-49	23	103.9	55.4	56.5		
	50-59	9	39.9	23.9	26.9		
	60-69	1	4.9	3.8	2.9		
Total		47	209.5	107.5	111.2	+3.4	
Average			4.5	2.29	2.37		4.6
5.0-5.9	30-39	6	31.8	17.4	17.1		
	40-49	23	122.4	79.5	82.5		
	50-59	30	165.8	128.4	126.9		
	60-69	2	10.9	9.4	9.5		
Total		61	330.9	234.7	236.0	+0.6	
Average			5.4	3.85	3.87		5.4
6.0-6.9	40-49	12	77.3	60.8	58.6		
	50-59	30	194.4	178.3	178.6		
	60-69	9	58.6	63.1	62.7		
	70-79	1	6.5	8.1	8.8		
Total		523	36.8	310.3	308.7	−0.5	
Average			6.5	5.97	5.94		6.5

TABLE 3.—*Summary of Table 2 for all diameter classes* [1]

Trees (number)	Aggregate diameter breast high	Volume — Aggregate — Second estimate	Volume — Aggregate — Measured	Volume — Average — Second estimate	Volume — Average — Measured	Difference,[2] measured — second estimate	Diameter breast high — Average	Diameter breast high — New graduation distance
	Inches	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*	*Cu. ft.*	*Per cent*	*Inches*	*Inches*
42	57.5	4.85	4.96	0.115	0.118	+2.3	1.4	1.4
34	84.2	18.8	18.8	.553	.553	0.0	2.5	2.5
40	139.7	48.7	48.9	1.22	1.22	+.4	3.5	3.5
47	209.5	107.5	111.2	2.29	2.37	+3.4	4.5	4.6
61	330.9	234.7	236.0	3.85	3.87	+.6	5.4	5.4
52	336.8	310.3	308.7	5.97	5.94	−.5	6.5	6.5
59	442.1	498.4	488.9	8.45	8.29	−1.9	7.5	7.4
64	543.6	727.2	729.8	11.4	11.4	+.4	8.5	8.5
59	554.3	867.6	881.3	14.7	14.9	+1.6	9.4	9.5
58	604.5	1,068.4	1,077.2	18.4	18.6	+.8	10.4	10.5
63	719.6	1,415.4	1,453.4	22.5	23.1	+2.7	11.4	11.6
33	408.4	847.8	857.0	25.7	26.0	+1.1	12.4	12.5
28	377.0	834.6	866.0	29.8	30.9	+3.8	13.5	13.7
21	302.0	744.3	753.1	35.4	35.9	+1.2	14.4	14.5
23	355.9	934.6	939.0	40.6	40.8	+.5	15.5	15.5
15	246.7	716.4	690.9	47.8	46.1	−3.6	16.4	16.1
3	52.3	142.1	128.6	47.4	42.9	−9.5	17.4	16.4
2	37.1	120.4	111.2	60.2	55.6	−7.6	18.6	17.8
704		9,642.05	9,704.96					

[1] This condensed form is the one actually used, but Table 2 illustrates better how the data are combined.
[2] Computed from aggregate rather than average volumes.

Following the total-height classes under each height-diameter class in Table 2, three lines are left for totals, averages, and to separate the d. b. h. classes. When each height-diameter class has been entered, the diameter-class totals and averages are calculated and entered. The percentage differences between the measured and estimated volumes, using the estimates as the base, are then computed and entered in column 7.

The purpose of this step is to discover whether there is any residual correlation between the volume errors of the revised chart and diameter. If there were none, the percentages in column 7 would either all be approximately zero, or, what is more normally the case where the number of trees used is limited, would at least have no well-defined trend away from zero. In some cases this may be judged by inspection, but it will usually be necessary to plot the percentages of column 7 over the average diameters of column 4. A typical result is illustrated in Figure 4. It will be seen that there is in this case a fairly well-defined trend, which shows that the chart in its present form tends in particular to overestimate the volume of trees above 16 inches in diameter.

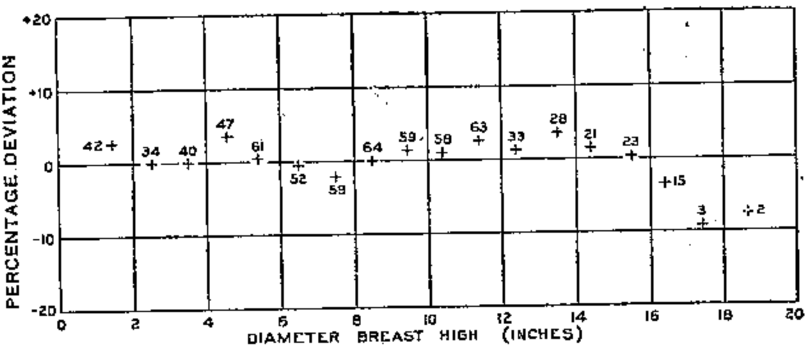

FIGURE 4.—Percentage deviations between measured and estimated (second) volumes over d. b. h. A correction of the diameter scale, particularly at the upper end, is indicated

If a change is necessary, the desired correction is accomplished by shifting each graduation on the diameter axis to such a position that the measured or "true" volumes will be read on the volume axis instead of the corresponding second estimates. This is done as follows:

For each of the diameter-class averages in Table 2 pass a line through the average d. b. h. (as shown in column 4 of the table) and the average second-estimate volume and hold its intersection with the height axis (this is in a sense the average height for this diameter class); then pivoting on this point, shift the line so that it passes through the average measured volume and note the diameter reading at which it intersects the diameter axis. This process is illustrated in Figure 5, which shows the procedure for the class averaging 18.6 inches, which has an average second-estimate volume of 60.2 and an average measured volume of 55.6. This gives the new position of the particular d. b. h. used, the graduation distance being expressed in terms of the original scale. The value noted, 17.8 in the instance illustrated, is entered in the last column of Table 2 or Table 3. Table 3 illustrates the form actually employed. Table 2 illustrates more completely the manner of combining the d. b. h.-height classes.

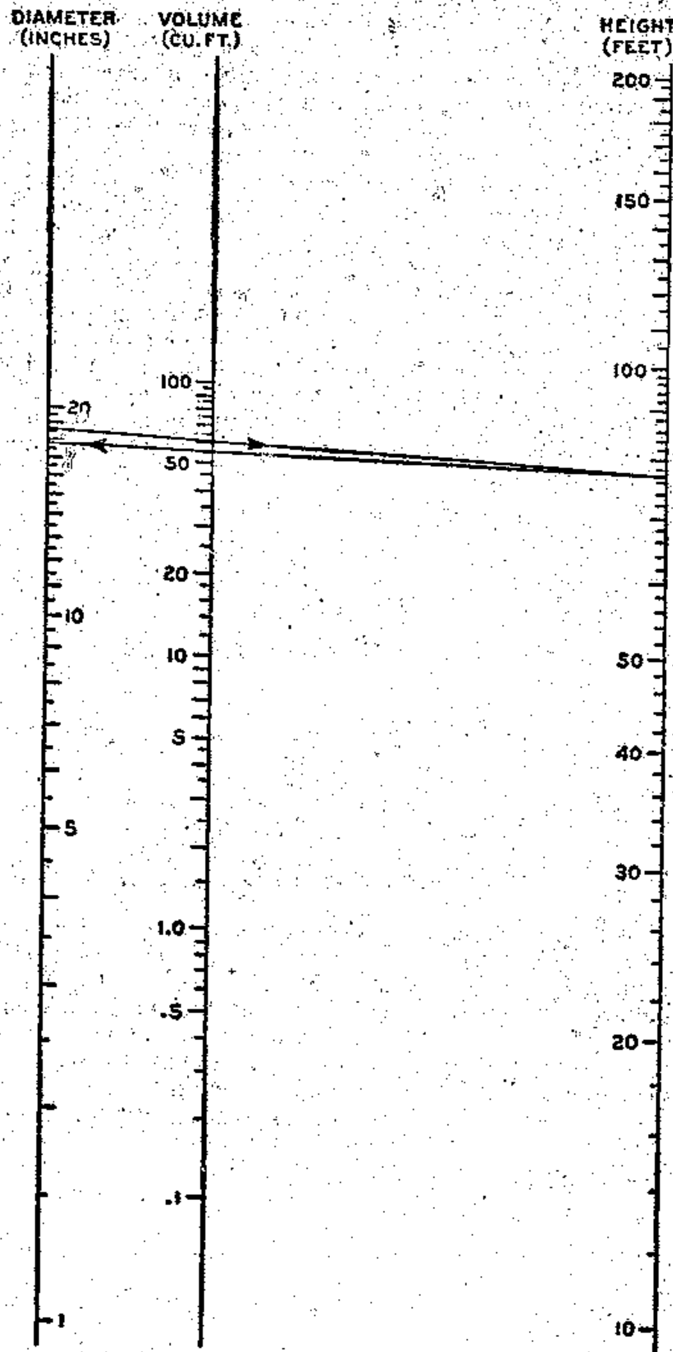

FIGURE 5.—Method of determining new graduation distances on diameter axis

The graduation distances thus determined are then plotted over the average d. b. h. of the class on regular coordinate paper, and a curve is fitted, as in Figure 6.

This curve may then be used to regraduate the diameter axis. For example, in Figure 6 the graduation distance for a d. b. h. of 18 inches is seen to be 17.3. The new position of the 18-inch graduation is therefore opposite 17.3 on the old scale, etc. This altered series of graduations should be entered in pencil, but may well be permanently recorded by means of a revised graduating curve in ink, labeled D_2.

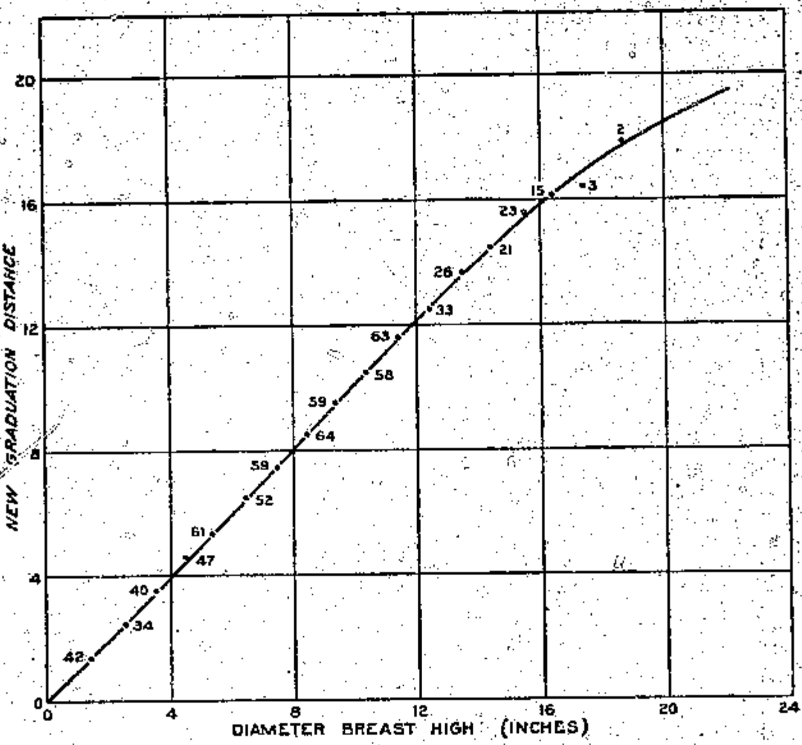

FIGURE 6.—Curve fitted to graduation distances derived by means of Figure 5

The curves of Figures 4 and 6 are merely two different expressions of the same facts. In the former a horizontal straight line indicates that no change need be made; in the latter a 45° straight line means the same thing. It is therefore desirable to omit Figure 4, and with it column 7 of Table 2, since the curve of Figure 6 not only indicates the need for change but also provides the mechanism for making it.

Before reading a new series of estimates, a similar process should be performed for height. The same information from the second estimate should be summarized by height classes, regardless of diameter. If any readjustments of the height scale are found to be necessary, they should be made exactly as for diameter. The process illustrated in Figure 5 is, however, now exactly reversed; a line is passed through each average height and average second-estimate volume to an intersection with the diameter axis, which is held as a pivot, etc., etc. The resulting chart, which may have all three axes now modified, is considered a third approximation, and a series of third estimates is read and entered in column 7 of the original table. For the example at hand, no correction for the H axis was indicated. H_3 is, therefore, identical with H_c.

THE THIRD CORRECTION OF THE CHART

When column 7 has been totaled and averaged by height-diameter classes, a third correction may be made which is in every way similar to the first. The adjustment of the height and diameter axes may

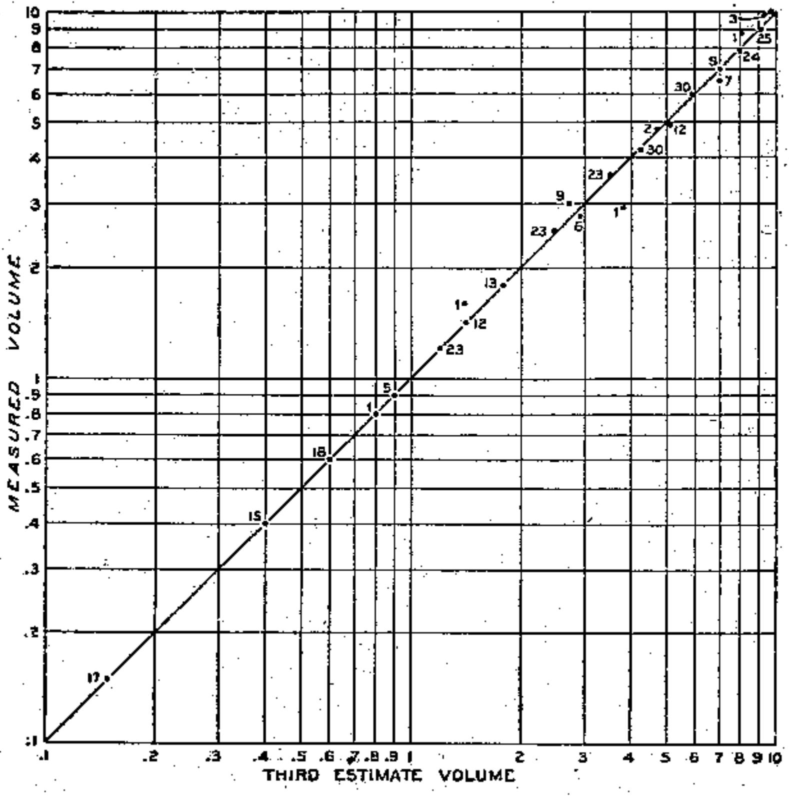

FIGURE 7.—The *measured—third estimate* curve based on columns 4 and 7 of Table 1. The 45° straight line indicates that no further revision is necessary

have thrown the volume axis out of adjustment, and this must be tested. Average measured volumes are therefore now plotted over average third estimates. The curve will resemble in general Figure 2, but will approach more nearly a 45° straight line. In the case which is being illustrated it is, indeed, exactly a 45° straight line, and this shows that no further improvement can be made. (Fig. 7.)

While this is a normal result, it sometimes happens that some curve or some line is defined which is not a 45° line passing through the (1, 1) point. In this case the volume scale must be recorrected, using exactly the same process[9] as was used in making the first correction. This new graduation curve is labeled V_3. The first penciled corrections must be erased from the volume axis and new ones entered by means of the graduating curve V_3, which should be in ink as before.

The revision of the chart permits a series of fourth estimates to be read.

FOURTH AND SUBSEQUENT CORRECTIONS OF THE CHART

If the previous step has resulted in an alteration of the chart—other than that indicated by a 45° straight line, not through (1, 1) (10, 10) which merely raises or lowers the entire scale—a fourth correction should be made. This is similar in every way to the second correction, already described, except that the fourth estimates are used. Probably neither the height nor diameter axis can be further improved, but if any improvement is possible it should, of course, be made.[10] In this case a fifth correction similar to the first and third should be attempted.

In other words, the two types of correction described should be alternately applied until a stage is reached where no further correction is indicated. When this result has been attained the volume table is considered complete except for the translation of the chart into a table of the customary form.

The final appearance of the base chart in the case which has been illustrated is shown in Figure 3. The revised graduating curves for volume (V_1) and for diameter (D_2) will be seen, with the corresponding new scales on the volume and diameter axes.

THE NUMBER OF APPROXIMATIONS NECESSARY

Fortunately the technic described rarely requires many repetitions. In some of the cases which have been worked out by the Forest Service a single correction of the volume axis has been all that has been necessary, the attempt to make second corrections merely establishing this fact. It is rare to have to make more than three corrections. The process, then, is by no means laborious. The last estimate indicating completion of the chart is not wasted, since these values are needed for the checks to be made.

FINAL PREPARATION OF THE VOLUME TABLE

The final graduations of the chart are inked in, and if a volume table in alinement-chart form is desired, a tracing is made of the three axes only with their final scales. This may be reproduced photographically, in reduced size if desired. A typical result is shown in Figure 8. (The two diameter scales will be discussed later.)

A final table may also be read from the chart. The conventional tabular form is more convenient for most purposes, but the chart form is very useful in any work where interpolations in the table are necessary.

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[9] Graduation curve V_1, rather than V_{mc}, is, of course, used in making this correction.

[10] The new graduation distances should be obtained from the diameter and height scales from the D_3 and H_3 graduating curves.

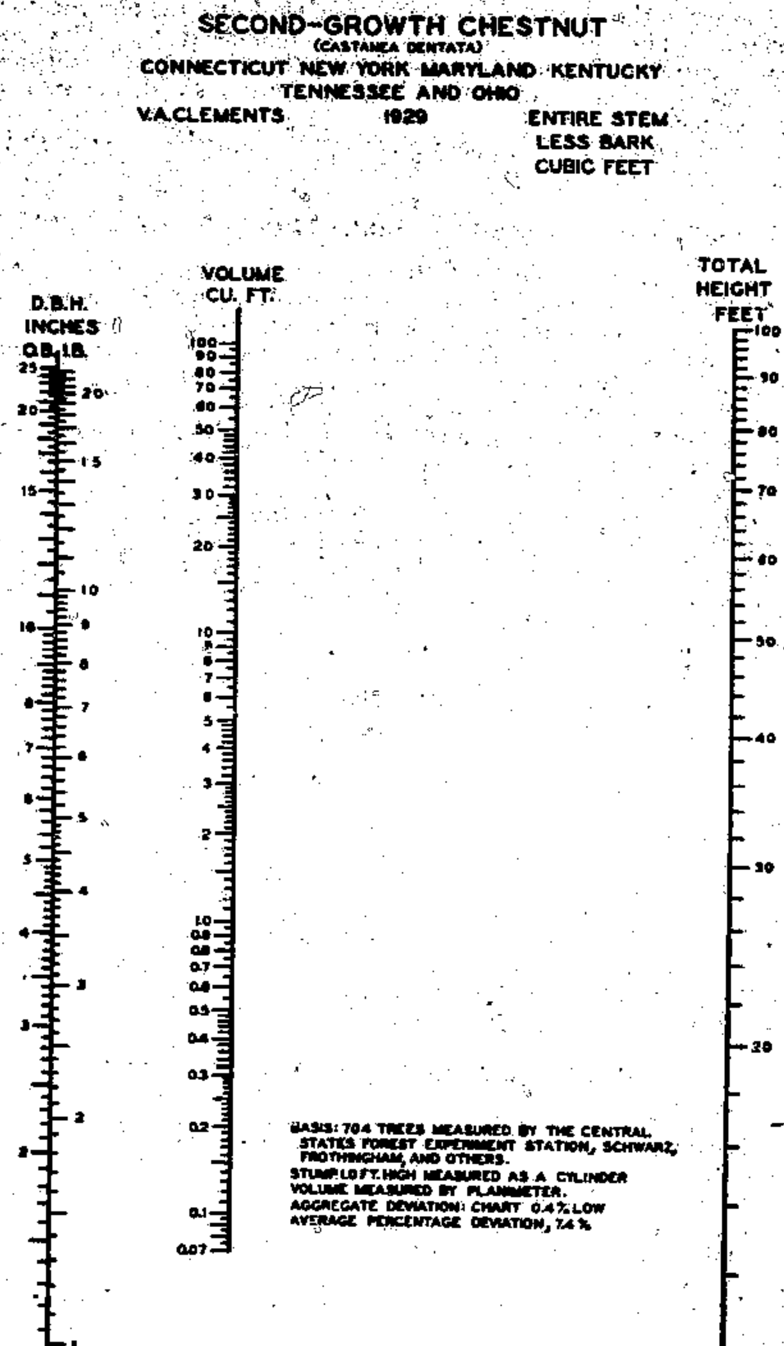

FIGURE 8.—The final volume table in alinement-chart form

MEASURES OF ACCURACY OF THE FINAL TABLE

It is customary to calculate two measures of accuracy for any volume table. These are the aggregate deviation and the average percentage deviation.

The aggregate deviation is merely the difference between the grand totals of the measured and last-estimated volumes (columns 4 and 7, Table 1) and is expressed as a percentage of the grand total of the measured volumes. It can be most clearly presented as follows:

Aggregate deviation: Chart ------ per cent high (or low).

This serves merely as a check on the accuracy with which the curves have been balanced and does not check the form of the curves, which depend on good technic in fitting the *measured-estimate* curves and the correction curves for height and diameter. If the aggregate deviation is less than 0.5 per cent for important tables and less than 1 per cent for secondary tables, the result is considered satisfactory.[11]

The average percentage deviation is computed from the measured and last-estimated volumes of the individual trees. The difference between the measured and last-estimated volume is expressed as a percentage of the estimated volume. Slide-rule accuracy is sufficient. An alinement chart which gives the percentage deviation directly from a setting of measured and estimated volumns is shown in Figure 9. The percentages may be entered in a final column of Table 1. Signs are disregarded. The percentages are totaled and averaged, and the result may be presented as follows:

Average percentage deviation, ------ per cent.

A PERMISSIBLE SHORT CUT IN READING THE SECOND ESTIMATES

In the foregoing pages it was stated that the second estimate should be read from the corrected base chart. It is somewhat easier to obtain these values indirectly from the first estimates by means of the measured-over-first-estimate curve. (Fig. 2.) For example, tree No. 54 in Table 1 has a first-estimate volume of 0.10. On Figure 2 it will be seen that the curve above 0.10 reads 0.15, and this can be entered as the second estimate. The results should be approximately identical with estimates read by means of d. b. h. and height from the corrected alinement chart, but being based on a single value (the first estimate) instead of on two values (the diameter and the height) they can be more quickly obtained. It is well to check a few values by both methods to insure against blunders in correcting the chart.

Similarly, fourth estimates may be read from the *measured—third estimate* curve.

THE ORDER IN WHICH THE CORRECTIONS ARE MADE

It may have been noted that in the procedure described, the order of the operations has been reversed from that which is normal in the more general method of curvilinear multiple correlation. The reason for this is that form factor usually is more closely correlated with volume than with either diameter or height. If the height and diameter axes are corrected first, curvature may be thrown into one

[11] SOCIETY OF AMERICAN FORESTERS (Committee on standardization of volume and yield tables). METHODS OF PREPARING VOLUME AND YIELD TABLES. Jour. Forestry 21:653, Item 37 (a). 1924.

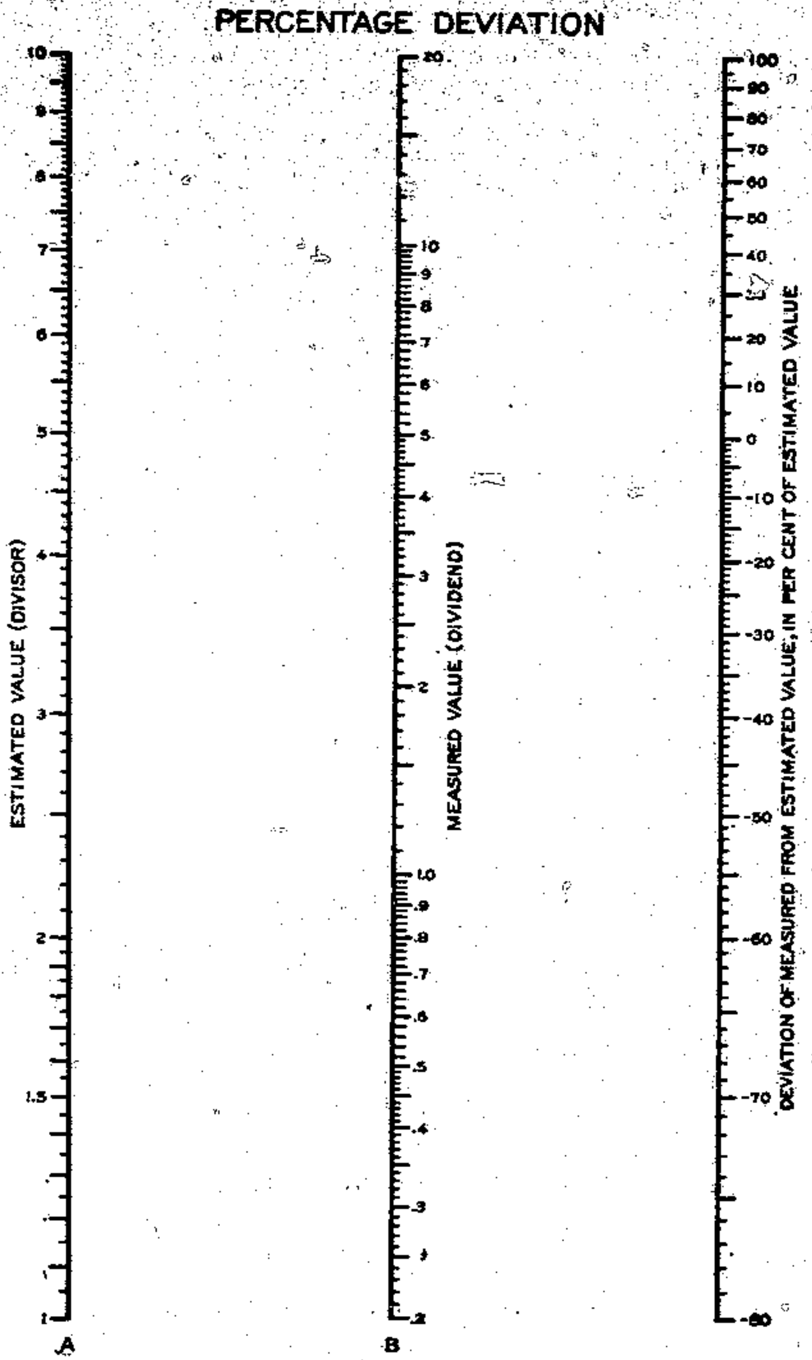

FIGURE 9.—This chart gives directly the percentage deviation of measured from estimated volumes without computation of differences

of their graduating curves which belongs in that for the volume axis. If this happens it may require several corrections to transfer it to the volume axis. In actual experience it has been found that several more estimates are usually required if the height and diameter axes are corrected first.

TABLES OF VOLUME IN CUBIC FEET OF ENTIRE STEM, INCLUDING BARK

If it is desired to make a volume table in which the stated volume is to include the bark the same base chart may be used. Since the geometrical solid on which the form factor is based is the same in either case, i. e., a cylinder having a diameter equal to the d. b. h. outside the bark, it is obvious that the form factor for the volume including bark will be higher than for that omitting it. It might theoretically be preferable, therefore, to use a slightly altered chart in which the "modified cylinder" volumes were those of a cylinder multiplied by some factor such as 0.45 instead of 0.40. In practice, however, the difference would be very slight and would appear merely as a slight horizontal displacement of the curve such as is illustrated in Figure 1, since the estimates would all be increased by a constant percentage. The difference will therefore completely disappear in the second estimate.

TABLES BASED ON D. B. H. INSIDE OF BARK

There is a strong possibility that in some cases a volume table made in a certain locality will not be usable in a different locality because of a difference in the average thickness of the bark. This is particularly true when the volume given is inside the bark while the diameter measurement, the d. b. h., is outside the bark. There are, therefore, some advantages in preparing the alinement chart in such a form that d. b. h. inside of bark may be used as an alternative. If this is done the correction of the table to apply to a new locality may be simplified.

It is a simple matter to prepare a curve showing inside bark d. b. h. over outside bark d. b. h. for the basic data of the table. The readings of this curve can then be transferred to the diameter axis of the finished chart, marking d. b. h. inside of bark opposite the corresponding d. b. h. outside of bark on the final scale. This has been done in Figure 8.

It is equally possible to work from the beginning with diameters inside of bark, and then later enter additional graduations applying to diameters outside of bark. Somewhat better definition of the various curves is obtained if diameters inside bark are used. As a matter of fact, the chestnut table illustrated in Figure 3 was thus prepared, and the diameter graduations there given are for d. b. h. inside of bark. The d. b. h. graduating curve for outside of bark has been omitted for simplicity.

THE FORM OF BASE CHART FOR MERCHANTABLE VOLUME IN CUBIC FEET

It would be possible to use the base charts already described for volume tables giving the merchantable volume in cubic feet to some fixed standard of top utilization, such as 4 inches. It is preferable, however, to use a slightly different chart giving first-estimate volumes which are a little closer to the actual measured volumes of the trees, and diameter-correction curves (such as those in Figure 6) which are more nearly straight lines.

The chart adopted by the Forest Service is based on the paraboloid. This is the simplest geometrical solid which is closely similar to a tree. The formula for the volume of a paraboloid is

$$V = \frac{BH}{2}$$

where V is the volume in cubic feet, B the area of the base in square feet, and H the height in feet. Allowing a stump height of 1 foot, the volume of the paraboloid above this is

$$V = \frac{BH'}{2}$$

$$\text{where } H' = H - 1$$

If the unmerchantable portion of the tip of such a paraboloid is truncated, this volume will be reduced by

$$v = \frac{bh}{2}$$

where v is the volume of the unmerchantable tip, b the area of the tip at the point of truncation and h the length of the tip. The merchantable volume of the paraboloid is, therefore,

$$V - v = \frac{BH'}{2} - \frac{bh}{2}.$$

In a paraboloid, however, the taper in area is porportional to the length. It follows therefore that

$$\frac{h}{H'} = \frac{b}{B}$$

and that

$$h = \frac{bH'}{B}.$$

Substituting this value in the previous equation, we have

$$V - v = \frac{1}{2}\left(BH' - \frac{b^2H'}{B}\right)$$

or simplifying,

$$V - v = \frac{H'}{2}\,\frac{(B^2 - b^2)}{B}.$$

This may now be expressed logarithmically as follows:

$$\log\,(V - v) = \log\frac{H'}{2} + \log\left(\frac{B^2 - b^2}{B}\right)$$

$$\text{or } \log\,(V - v) = \log\left(\frac{H - 1}{2}\right) + \log\left(\frac{B^2 - b^2}{B}\right).$$

For any desired standard of top utilization, b, and therefore b^2, is a constant. It is a simple matter to prepare an alinement chart similar in form to that already described by means of which this formula may be solved. An example is shown in Figure 10. The graduating curve for V is a straight line, that for H is not quite straight because of the correction for stump height, while that for diameter is considerably lowered for the small diameters, because of the small differences between B^2 and b^2.

The base charts of this type, as used by the Forest Service, are for top utilization to 2, 3, 4, 5, and 6 inches.

It should be noted that there would have been no similar advantage in the use of the paraboloid rather than the cylinder in the case of the previous base chart for the volume of the entire stem. The volume of a paraboloid is one-half that of the cylinder of the same dimensions. The cylinder volumes multiplied by the form factor 0.40 would therefore be identical with the paraboloid volumes multiplied by 0.80.

It will be noted that the three sets of graduating curves on Figure 10 are designated D_p, H_p, and V_p, respectively. These are abbreviations for diameter of paraboloid, height of paraboloid, and volume of paraboloid.

After the proper base chart has been selected the procedure followed in preparing the volume table is in all respects identical with that already described.

THE FORM OF BASE CHART FOR MERCHANTABLE VOLUME IN BOARD FEET

The formula for the volume of a geometrical solid which resembles a tree in form and which is scaled in board feet by a log rule, is unfortunately very complex and not such as can be expressed accurately by an alinement chart. It has been found, however, that a fairly close approximation can be worked out. The process of preparing such an approximation chart is both tedious and involved and is difficult to describe. It consists in general of the following steps:

A series of values for volume are computed for some assumed solid, for a range of values of diameter and height. After experiments with several alternatives, the solid preferred is the cone frustum for tables to be related to merchantable height and the paraboloid for those related to total height. The solid is divided into 16-foot logs and scaled by whatever log rule is desired.

A base chart is then laid out with tentative parallel straight-line axes for diameter, height, and volume. The height and diameter axes are not graduated; that for volume is graduated logarithmically for several inches at the top. The graduations for the maximum diameter used and the maximum height are placed horizontally opposite that for the maximum volume. Lines through this maximum-diameter graduation and a series of three or four corresponding volumes intersect the height axis at corresponding heights, and fine pencil lines are drawn to record these indications of the positions of these height graduations. Similarly, starting from the maximum height, indications of two or three diameter graduations are obtained. Intersecting back from the second and third of these tentative diameter graduations will give a second and third series of intersections on the height axis, which should agree closely with the first. If they do,

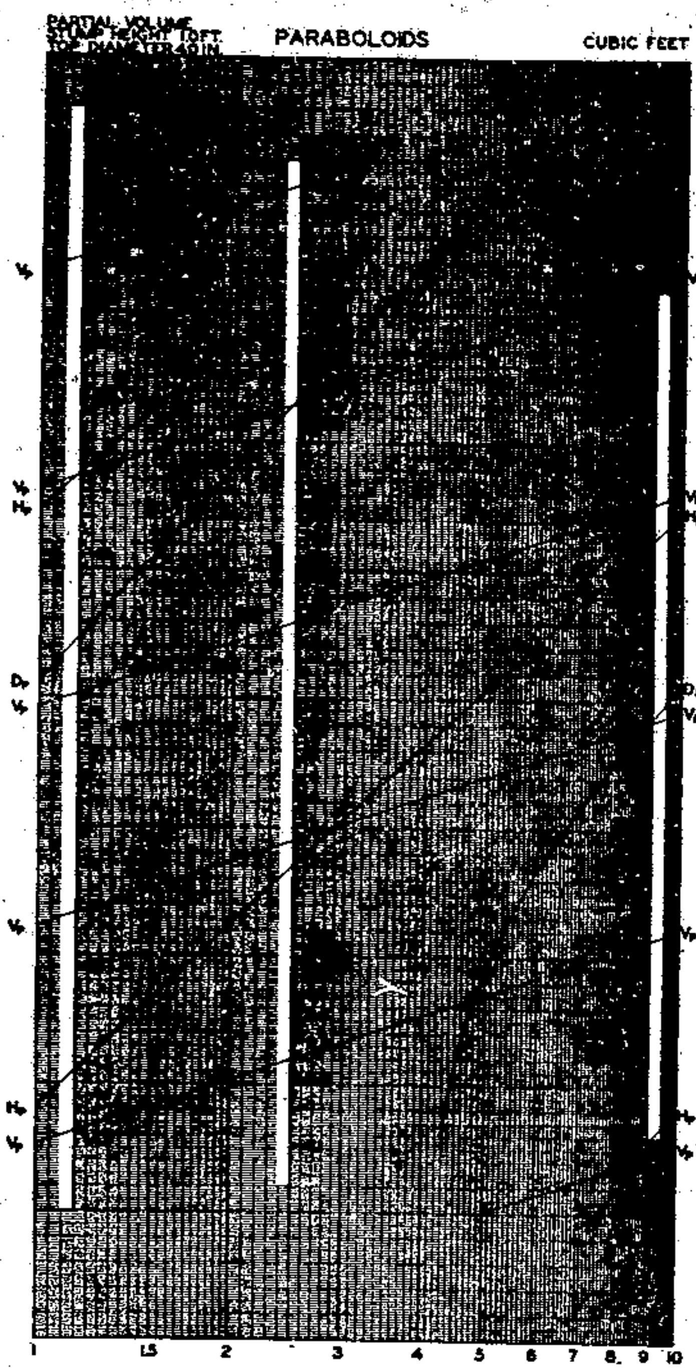

FIGURE 10.—Form of base chart used for volume tables giving merchantable volume to a 4-inch top, in cubic feet, based on diameter and total height

these height graduations are accepted. Reversing the process, the tentative graduations of the diameter axis are checked by other intersections. When this diameter axis has been carried a few inches, attention is again turned to the height axis, and so on alternately.

The intersections of lines from the larger diameters and heights will usually be well defined. Later, as smaller values of height are intersected back to the larger diameters, they may be found to fall above or below the previously established points. Similarly, lines from the smaller diameters may consistently intersect high or low. This is an indication that the lower part of the volume scale needs expansion (when the intersections are above) or contraction.

The correction of the volume scale is accomplished by locating new points on the volume graduating curve, which is done by laying the straightedge on the diameter and height and noting the intersection on the volume axis. This intersection is then projected horizontally to the ordinate representing the corresponding volume. For this correction the diameter and height points used are those established by the upper values, which have been confirmed by good intersections.

Sooner or later, the intersections may not fall on the original straight diameter and height axes and a curve must be substituted. If the intersections are well defined, the axes alone should be curved. If, however, the intersections fall, not around a point but along a line, such a multiple intersection usually indicates a simultaneous adjustment of volume scale and of one or both of the diameter and height axes.

The graduating curves are carried along simultaneously for all three axes. Each intersection located will fix a point on one of these graduating curves, but since the result being attained is only an approximation chart the points will not lie exactly on the curve, and a smoothing process has to be resorted to. The same thing is true of the axes themselves.

An example of the sort of chart finally produced by this process of trial and error is shown in Figure 11. It must be admitted that there is a large personal equation in the preparation of such a chart. Different workers might produce charts materially different in appearance. The test of their adequacy is whether the original basic values mentioned in step 1 can be reproduced with sufficient accuracy by means of the chart. This is true of the charts presented here.

A peculiarity of the chart illustrated in Figure 11 is the intersection of the lower ends of the volume and diameter axes. This is to be expected as a result of the scaling practice which neglects taper in 16-foot logs. All trees which are exactly one log in height to a 6-inch top will therefore have exactly the same scale regardless of their d. b. h. This condition can be met only by an alinement chart which has a common point for 16.3 feet on the height scale and 20 (the volume of a 6-inch, 16-foot log) on the volume scale.

It has been found, however, that this condition can not be met in all of the cone-frusta charts, particularly those based on the Scribner rule. The difficulty appears to be associated in part with the eccentricities of the rule itself, which does not follow any mathematical law. Accurate results at this one fixed point can only be obtained at the expense of material errors in adjacent diameter classes. An example of the sort of compromise chart which has been adopted is illustrated in Figure 12.[12]

In base charts for board-foot volumes related to total height the peculiarity noted above (fig. 11) does not exist because merchantable length is not rigidly correlated with total height. The characteristic form of these total-height base charts is shown in Figure 13.

THE TECHNIC IN THE CASE OF THE BOARD-FOOT TABLES

The technic for board-foot tables follows, in general, exactly the same procedure as that followed in the cubic-foot tables already described, and only one or two details need be mentioned.

For reasons already discussed, the measured and first-estimated volumes in the tables related to merchantable height must be identical for a 1-log tree of the minimum top diameter used. This means that this volume determines a fixed point on the *measured—first estimate* curve, and this fixed point makes the fitting of this curve easier. The same thing is not true, of course, of the tables related to total height, in which the lower end of this curve is often ill defined. Near the minimum d. b. h. many of the trees will be so rapidly tapering that they will not contain a merchantable log and so will have a volume of zero. An average of a small number of trees, some of which have a zero volume, will naturally be erratic. This difficulty can be corrected only by obtaining a relatively large number of data for these very small sizes, which may not be practicable. At all events, moderate errors in the very small trees are usually of little moment.

The small trees also cause some difficulty in connection with the average percentage deviation. Since this deviation is expressed as a percentage, there are many cases near the minimum diameter where the percentage deviation of an individual tree will be either 100 per cent, when the basic solid has a volume and the tree itself has not, or infinity when the converse is true. An average including even a single infinity is, of course, itself infinite, and highly misleading. The most practicable plan seems to be to omit from the calculation of the average percentage deviation (but not from that of the aggregate difference) the entire diameter classes for which either the estimated or any of the measured volumes are zero. The expression of average percentage deviation should then include the minimum diameter used, together with the number of trees entering into the average percentage given, as:

Average percentage deviation (494 trees, 13 inches plus): 9.8 per cent.

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[12] The series of charts in use by the Forest Service for board-foot volumes includes the following: (Based on merchantable height) Scribner rule, height and volume to 6-inch top, to 7-inch top, to 8-inch top, and to a 10-inch top; International rule, height and volume to 5-inch top, to 6-inch top, to 8-inch top, and to 10-inch top. (Based on total height) Scribner rule, volume to 6-inch top, to 8-inch top, and to 10-inch top; International rule, volume to 5-inch top, to 6-inch top, to 8-inch top, and to 10-inch top.

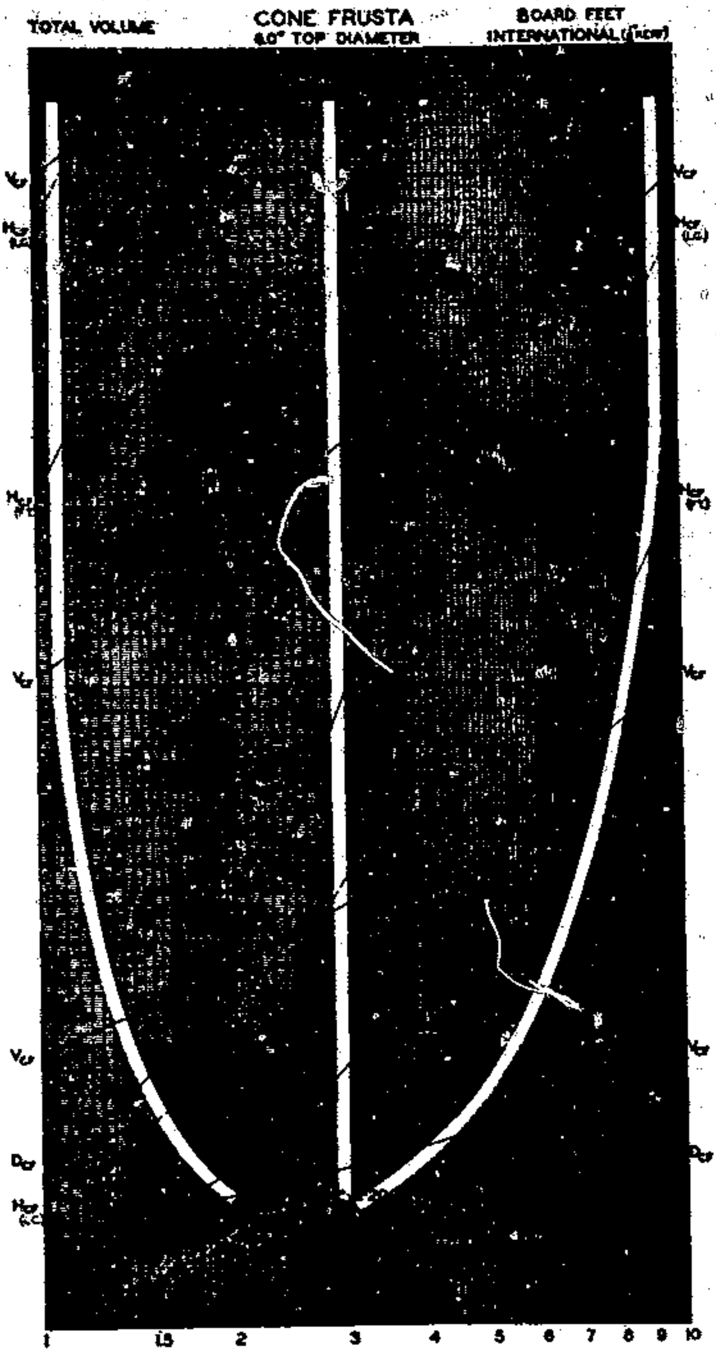

FIGURE 11.—Form of base chart used for volume tables giving total volume to a 5-inch top, in board feet by the International rule (¼-inch kerf), based on diameter and used length. The curve $H_{CF(LO)}$ may be used to graduate the height axis in terms of 16.3-foot logs

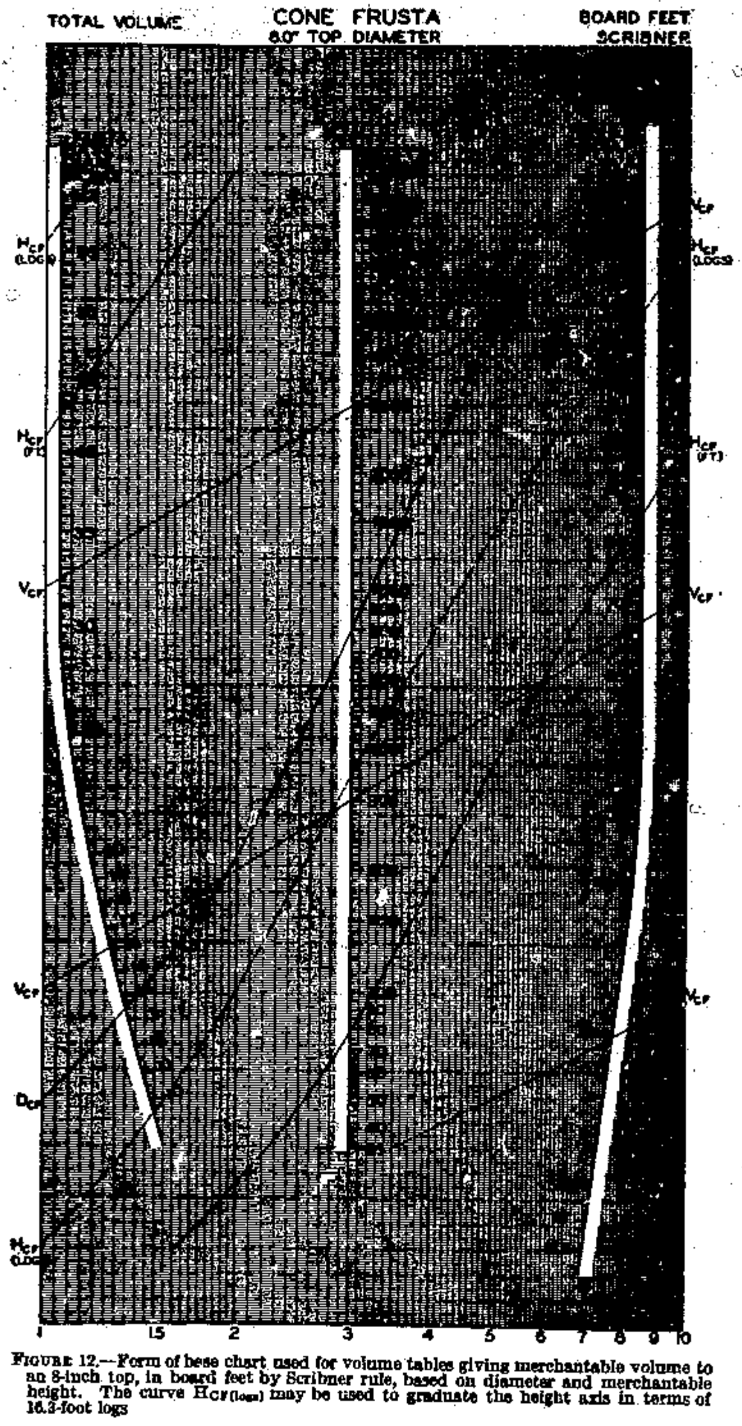

FIGURE 12.—Form of base chart used for volume tables giving merchantable volume to an 8-inch top, in board feet by Scribner rule, based on diameter and merchantable height. The curve $H_{CF(logs)}$ may be used to graduate the height axis in terms of 16.3-foot logs

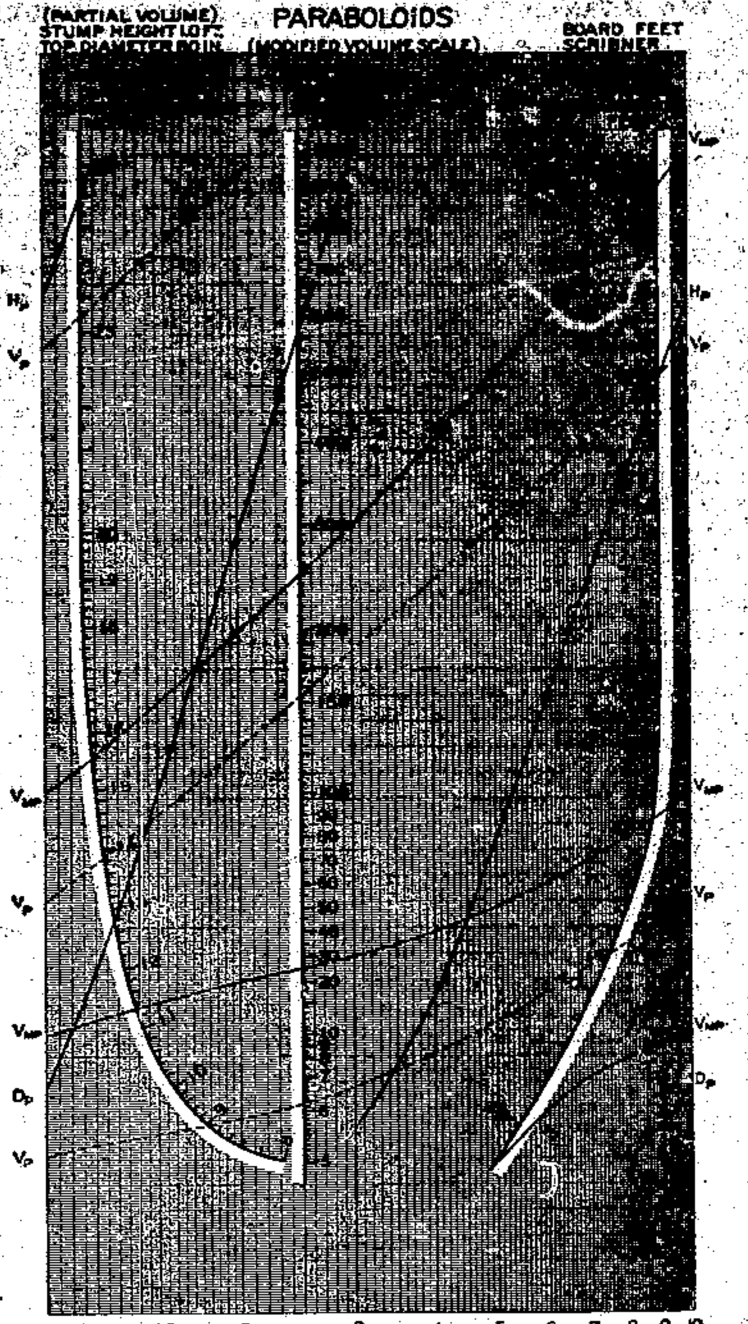

FIGURE 13.—Form of base chart used for volume tables giving merchantable volume to an 8-inch top, in board feet by Scribner rule, based on diameter and total height. The volume scale has been modified to agree more closely with the average form of trees. True paraboloid volumes may be obtained through the use of the V_p graduating curve

THE ADVANTAGES OF THE ALINEMENT-CHART METHOD

The advantages of the alinement-chart method can best be brought out by comparing it with alternative methods in use. The old-fashioned method of fitting harmonized curves was cumbersome and ineffective because the data available were distributed among a number of curves. As a result none of these curves was well defined unless an exceedingly large number of data were at hand. In contrast, each curve which must be fitted in the alinement-chart procedure is based on all the available data and is nearly straight-line in form. Furthermore, the weighting of the points in the case of the harmonized curves was exact only in the first stage; in cross curving it was customary to ignore the weights altogether, and impossible to assign weights in any satisfactory manner. Serious errors sometimes resulted. In contrast, in the new technic exact weights are available at every stage.

In comparison with the method based on frustum-form factors, the new method is more flexible in two respects. In the first place, it does not involve the implicit assumption that the frustum-form factor is correlated with diameter alone and is independent of height within a diameter class. As a matter of fact, it has been fairly well established that this assumption is false, especially in second growth; although for mature timber the errors involved in it are small. Tables produced by the alinement-chart procedure may show a partial correlation between form factor and height.

In the second place, the new procedure is applicable to any type of table, while the frustum-form factor method was restricted to tables of volume in board feet based on merchantable height.

The new method is almost self-checking. The aggregate difference is obtained by a mere addition of two columns of one of the work sheets, while the average percentage deviation involves a relatively small additional amount of labor, the measured and tabular or estimated values being already listed in parallel columns. This contrasts strongly with other methods, in which the calculation of these two measures of accuracy is excessively laborious.

As a result of these several advantages the alinement-chart method is a more efficient procedure. This efficiency may be utilized either by reducing the number of data required for a table of given accuracy or by increasing the accuracy obtained with a given number of data. In addition, time and labor will be saved through its use.

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