Economic analysis of carbon sequestration and bioenergy production under catastrophic risk and price uncertainty

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Selected Paper prepared for presentation at the Southern Agricultural Economics Association
(SAEA) Annual Meeting, Dallas, Texas, 1-4 February 2014

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Abstract

This paper investigates how payments for carbon offsets and bioenergy impact the optimal management of hardwood forests under conditions of risk and price uncertainty that represents by the use of an E-V model. The results show that higher carbon price increases LEV and rotation age; fire risk decreases LEV and rotation age.

Key words: carbon sequestration; catastrophic risk; price uncertainty; Hartman model

Introduction

It is widely recognized that forests play an important role in the global carbon cycle by sequestering and storing carbon (Brand, 1998). Thus carbon should be considered as a product in forest management. Also, catastrophic events and price uncertainty can influence rotation age and land expectation value. Catastrophic events can be fire, insect outbreaks, or severe weather. In this paper a modified Hartman model along with an EV model that takes into account carbon sequestration, catastrophic risk, and price uncertainty will be presented.

This model will be used to investigate how payments for carbon offsets and bioenergy impact the optimal management of hardwood forests in Kentucky under conditions of risk and price uncertainty. Identifying the optimal rotation age and highest expected value of land can help guide forestry owners' decisions about management with timber and carbon both as forest products.

Literature review

1. Impact of catastrophic events

Several studies have analyzed the impact of catastrophic events like fire on the land expectation value (LEV) and the optimal rotation age. Most papers extended the Faustmann model to incorporate fire into the model. All previous studies show that catastrophic events have significant influence on LEV and optimal rotation age. For instance LEV decreases when considering fire risk and the optimal rotation age tends to be shorter with fire risk.

Martell (1979) described a stochastic model that could be used to determine the optimal rotation age for flammable forest stands using an extension of the Markov decision model proposed by Wagner (1969). The risk rate of fire was estimated by probabilistic dynamic programming and the rate was set from 0.00 to 0.05. Results showed that the optimal rotation age decreased as the conditional annual fire probability increases. Routledge (1980) introduced the idea of an extended Faustmann model since the traditional Faustmann model did not cover the effect of potential catastrophes. In this paper, the Faustmann model incorporated estimates of the likelihood of catastrophes. Also, the effect of ignoring the effect of catastrophes was presented. Results showed that impact of considering catastrophes depended on growth rates, hazard rates, and the expected salvage portion.

Reed (1984) further investigated the effects of risk of fires or other unpredictable catastrophe on the optimal rotation age using extended Faustmann. One scenario of the model was that when fire occurred, it caused total destruction. In this scenairo the effect of fire risk was equal to adding a premium to the discount rate, which implied that risk of fire would decrease optimal rotation age. The risk rates investigated were 0, 0.01, 0.02, and 0.05. Two other scenarios were presented: one was that the destruction through fire or other catastrophe was only partial, which was more realistic; the other one was that the probability of fire depended on the age of the stand following a Poisson process.

Yin and Newman (1996) analyzed the effect of catastrophic risk on forest investment using a forest-level neoclassical profit function of timber production. Unlike the traditional Faustmann model, price and cost changed over time under this model: prices and growth in the profit function followed a geometric Brownian motion; cost rose deterministically at an instantaneous rate. Property tax was set at $2.5/ac/year, and the mean rate of the catastrophic event was set at 0.008. Results showed that catastrophic risk decreased the value of an investment project, and increased the threshold of forest investment.

Englin, Boxall, and Hauer (2000) explored the joint effect of fire risks and amenities on timber harvesting using a Faustmann framework. Amenities were represented by wilderness recreation, which was estimated using a linear damage function. Martell (1994) determined the actual risk of fire in the Canadian Shield was about 1.5%. Therefore, they used a fire risk from 0.0% to 4.0% with an interval of 0.5%. Results showed that the rotation age declined as fire risk increased. But the inclusion of amenities in the model increased the rotation age at every level of risk. This implied that delaying harvesting may be substantial for many forests in the Canadian Shield.

Stainback and Alavalapati (2004) extended Reed (1984)'s model by including carbon into the analysis. In the paper, fire risk was set from 0.0% to 4.0%. The range was based on Runkle (1985), and Hooper and McAdie (1995). The portion of the stand that is salvageable after a catastrophic event was set for 0% and 70%. Results showed that risk of catastrophic mortality decreased the land value and rotation age for all carbon prices; these decreases were greater for higher carbon price.

Susaeta, Alavalapati, and Carter (2009) extended the Hartman model by incorporating the probability of a stand of being affected by catastrophic events and then combined it with the Black-Scholes formula to investigate the impact of price uncertainty and catastrophic disturbance.

They modeled two scenarios: a no thinning scenario and a thinning scenario. Results showed LEV increased when risk rate decreased in both scenarios.

2. Impact of price uncertainty

A few studies have analyzed the importance of taking price uncertainty into account in forest management. Norstorm (1975) applied the Markov Decision Process to estimate the optimal rotation age. A comparison between policies that considered price uncertainty and the others that didn't was made to investigate the importance of price fluctuations. The results showed that on average, individual forest owners were better off with fluctuations in prices than with a constant price equal to the long-run average of actual prices. This indicated the importance of taking price fluctuations into account in the determination of harvesting.

Ismail Kaya (1987) looked at how to determine economic management strategies for uneven-aged stands using a Markov Decision Process that takes into account the uncertainty of future product prices and stand growth. This method needs to define different state and transition probabilities. The transition probability matrix for the stand was computed by simulation, using a stochastic model of stand growth for northern hardwoods. Then a method of successive approximations was used to find the management policy that would maximize the expected net discounted value of the returns from the stand. Results showed that on average, the expected rotation age was 8.4 years and the expected yield was 2.52 ft^2/ac/yr.

Brazee and Mendelsohn (1988) applied an asset sale model to estimate the optimal rotation age with unpredictable price fluctuations. The basic idea of this method was to compare the current price of stumpage and the reservation price which was the present value of the maximum expected timber value: if the current price exceeded the reservation price, the forestry owner could harvest; if the current price was lower than the reservation price, it was rational delay harvest for at least another year. The paper presented two examples: Douglas fir and loblolly pine. Results showed that the expected age of harvest was slightly longer than the Faustmann rotation: the optimal rotation age was about two years longer than the Faustmann age for Douglas-fir and one year longer for loblolly pine. The Faustmann harvest age was the optimal rotation length with zero price variation.

Haight and Smith (1991) analyzed the effects of stochastic stumpage prices on the economically optimal thinning and rotation ages for loblolly pine plantations in the Piedmont region of North Caroline using dynamic programming. Two standard deviations of sawtimber price were presented: 17.19 and 34.38. Results showed that with a higher level of price variation, the optimal rotation age would decrease and the expected value of plantation management would increase as the price variation increased.

Brazee and Bulte (2000) used a flexible management model to incorporate thinning decisions into optimal harvesting models with fluctuating stumpage prices. The reservation stumpage prices were estimated with a random draw stumpage price model. Results showed that the land expectation value increased with the spread of the stumpage price distribution. Also the expected thinning age decreased sharply while expected harvest age increased under flexible management compared to Faustmann management due to an increase in the precommercial incentives to thin from an increase in net present value of older stands.

Buongiorno (2001) presented how to apply the Markov decision process to decide the optimal rotation age. Both growth and stumpage prices were assumed to be stochastic. This paper clarified that the optimal harvesting policy was related to the current state of forestry instead of the initial condition. This paper also showed that the Faustmann formula was a special case of a Markov decision process model, in which the transition probabilities were unity or zero.

Susaeta, Alavalapati, and Carter (2009) developed an integrated Black-Scholes and modified Hartman model to analyze the impacts of price uncertainty on nonindustrial private forest management in the southeastern United States. Black-Scholes was widely used in finance and adopted in this paper to calculate the volatility of stumpage price. Results showed that since the stochastic price variation could offset the cost of performing silvicultural activities like thinning, it is profitable when pulpwood or forest biomass was incorporated in the model.

Data and scenarios

The study area of this paper is upland oak dominated mixed hardwood forests in the Central Hardwood Forest Region (CHFR) that includes Kentucky. Site index 65 was chosen, because it is an average site index[1] for this area.

1. Data

Yield data was obtained from Gingrich (1971) for upland oak stands. The growth and yield gave the volume of sawtimber and pulpwood. Volume of bioenergy and carbon were derived from the volume of sawtimber and pulpwood using factors obtained from Birdsey (1996) and will be given in the next section.

Timber prices were from Timber Market South. The data set included quarterly stumpage price of sawtimber in Kentucky from the second quarter in 1980 to the second quarter in 1994 for a total of

[1] Site index is commonly measured by tree height: if the average height of tallest trees at age 50 on that site is 55 feet, then the site index is defined as 55

57 quarters'. After 1994, Timber Market South stopped collecting timber price data from Kentucky. Prices were converted to 2013 dollars using the Consumer Price Index (CPI) provided by the Bureau of Labor Statistics (2013) and are listed in Table 1. Overall, stumpage price increased from 1980 to 1994: the lowest price was $10.38 per ton occurred in 1985; the highest price was $30.37 occurred in 1994.

Carbon price and bioenergy price were based on existing carbon markets and previous studies. There are two types of carbon price sources: carbon market and studies about social cost of carbon emissions. Carbon price varies substantially across markets and studies. Carbon markets includes regulated markets and voluntary markets. California Cap and Trade is one of the regulated carbon markets who auctions carbon permits. The price usually depends on the quantity of permits available in the market. The auction price of California Cap and Trade in February 2013 was $13.62 per metric ton (Point Carbon, 2013). Mountain Association for Community Economic Development (MACED) is one of the voluntary carbon markets. The price depends on carbon offsets and volume of timber the landowners sell. The price ranges from $5.05 per ton to $15 per ton (Prativa, 2013). In general, social cost of carbon emission estimated by previous studies is higher than current market prices. Richard S.J. Tol (2008) conducted a meta-analysis about 211 estimates of the social cost of carbon: the mean estimates of social cost of carbon is $127/tC using Fisher-Tippett kernel; the mean estimates of social cost of carbon were $88/tC using Gauss kernel. A sensitive analysis of carbon price from $1 per metric ton to $25 per metric ton was used for this paper.

2. Scenarios

The base scenario is when only sawtimber and bioenergy are sold as forest products. Since the pulpwood market is limited in Kentucky, it is assumed that pulpwood is sold as woodenergy. The optimal rotation age of the base scenario reflects the optimal decision when there is no carbon market. Landowners are assumed to pay a penalty for carbon emissions as carbon sequestered in forest products decay. In the carbon scenario, forestry products include sawtimber, bioenergy, and carbon. It is assumed that forest owners get paid annually for each year's carbon dioxide equivalent after the base scenario's optimal rotation age. For example, if the base scenario rotation age is 57 years, then the forestry landowners only get paid for carbon sequestrated after year 57. Since they get paid after the optimal rotation age of the base scenario, they are charged only for carbon emission after that year too. Under these two scenarios, there are two sub-scenarios: one sub-scenario models fire risk; while in the other there is price uncertainty. Therefore, there are four models under each scenario.

Methods

1. Growth and yield model

The yield data from Gingrich (1971) was fitted using nonlinear regression to equation 1 following Prativa (2013):

$$Q_y = a * t^b * e^{-ct} \qquad (1)$$

Where Q_y is the volume of wood products (either sawtimber or pulpwood), t is stand age, and a, b, c are parameters to be estimated.

Amount of bioenergy volume

Residue was assumed to be sold as bioenergy. Therefore, the amount of bioenergy volume is the difference between total aboveground tree biomass and merchantable volume. The merchantable volume was calculated by adding the volume of sawtimber and pulpwood. The ratio of above ground tree biomass to merchantable volume for hardwood in South Central was assumed to be 2.304 (Birdsey, 1996).

Amount of carbon sequestered and emitted

The amount of carbon was estimated by multiplying the total aboveground tree biomass by the conversion factor 19.76 to obtain carbon in metric tons (Birdsey, 1996). This could transfer merchantable biomass in cubic feet to carbon equivalent volume in pounds. Sequestered carbon was then multiplied by 3.67 to convert it to carbon dioxide equivalents.

Carbon emitted was modeled based on half-life decay function. Here the half-life is assumed to be 100 years for sawtimber and 2.6 years for pulpwood. This means that half of the carbon stored in sawtimber and pulpwood will be released in the atmosphere in 100 and 2.6 years, respectively. Carbon emissions here only account for emissions from sawtimber and pulpwood. In terms of carbon emissions from residue, it is assumed to be sold as electricity production, so it is offset by the carbon emissions avoided if that quantity of electricity was produced from coal. The decay function is given in equation 2:

$$N(t) = N_0 * e^{-\mu * t} \qquad (2)$$

Where N (t) is the current quantity, N_0 is the initial quantity, μ is the half-life, and t is current time.

2. E-V model

The divergence between observed and modeled behavior led Markowitz to include a variance term resulting in the expected value variance (E-V) model. Timber price could vary substantially in 80 years. Since it is difficult to get annual price data, it is important to take price variance into account in the analysis.

The basic idea of E-V model is that the average value or utility matrix subtracts the product of aversion to price uncertainty coefficient and variance of value or utility. Aversion to price uncertainty theory assumes that decision makers don't like risk and are willing to pay to reduce it. Generally, if the aversion to price uncertainty coefficient is higher, decision makers are more averse to price uncertainty and are willing to pay more to reduce it; if the aversion coefficient is zero, decision makers are neutral to price uncertainty. In our model, forestry owners are assumed to be averse, which is that they will choose to lengthen or shorten rotation age to decrease the impact of price uncertainty. Aversion coefficient was estimated by the McCarl and Bessler approach. The E-V model for price uncertainty is listed in equation 3:

$$P_y - \theta * V_y \qquad (3)$$

Where P_y is the price of timer, y is timber product (sawtimber or pulpwood), θ is the aversion coefficient, and V_y is the variance of timber price.

3. Modified Hartman model

The Hartman is an extension of the Faustmann model that accounts for the value of a standing forest (for example the value of carbon sequestration). Equation 4 lists the basic Faustmann model that includes only timber as products:

$$LEV(T) = \frac{P*Q*e^{-rt}}{1-e^{-rt}} \qquad (4)$$

Where LEV is the land expectation value, P is the price of timber, Q is the volume of timber at time t, r is the discounted rate, t is the current age of stand, T is the optimal rotation age. The expression e^{-rt} discounts timber value to present value. The expression $(1- e^{-rt})$ discounts timber value to eternity assuming and infinite series of rotations or harvest cycles.

Hartman (1976) analyzed the optimal rotation age of a forest when the forest provided value like the value before the trees were harveste (e.g. recreation or wildlife) in addition to the value of timber. Equation 5 presents the general form of the Harman model.

$$LEV(T) = \frac{P * Q * e^{-rt} + \int_0^t F(x) * e^{-rt}}{1 - e^{-rt}} \quad (5)$$

Where F(x) denotes the value or the standing trees of age t and (P*Q) stands for the timber value.

Since carbon is included in our model and it is paid annually, we adopted the Hartman model. By incorporating fire risk into the Hartman model we get equation 6:

$$LEV(T) = \frac{\lambda+r}{r*(1-e^{-(r+\lambda)*T})} * \left[\sum_y (P_y - \theta * V_y) * Q_y * e^{-(r+\lambda)*T} + \sum_0^T Q'_c * P_c * e^{-(r+\lambda)*T} + Q_c * P_c * e^{-(r+\lambda)*T}\right] + \frac{\lambda+r}{r*(1-e^{-(r+\lambda)*T})} * \left\{\int_0^T \left[\lambda * \left(\sum_y (P_y - \theta * V_y) * Q_y * k * e^{-(r+\lambda)*T} + \sum_0^T Q'_c * P_c * e^{-(r+\lambda)*T} + Q_c * P_c * e^{-(r+\lambda)*T}\right)\right] dt\right\} \quad (6)$$

Where y represents different timber products (sawtimber, pulpwood), Q'_c is the carbon sequestration volume at year t, Q_c is the total carbon sequestration volume prior to year t, Q'_c*P_c is the carbon benefit at year t, Q_c*P_c is the carbon penalty if the trees are harvest or a catastrophic event happens, P_c is the price for carbon, λ is the risk of fire, k is the salvageable portion.

4. Sensitivity analysis

A sensitivity analysis was conducted to explore the impact of different carbon prices. We use four prices in this paper: \$0, \$1, \$5, \$10, and \$25 per metric ton. In general, a higher carbon price increases LEV and rotation age. Another sensitivity analysis was conducted to see the impact of different aversion parameters to price uncertainty. Three aversion levels were selected: 0, 0.049, 0.264, and 0.644 that were calculated by the McCarl and Bessler approach, which represents the risk neutral, low, medium, and high level of aversion.

Results

1. Base scenario

The base scenario assumes that there is no carbon market and forestry products only consist of sawtimber and bioenergy. The results are showed in graph 1 and graph 2. Graph 1 illustrated the LEV for different levels of aversion and different assumptions of fire risk. It shows that fire risk could substantially reduce LEV, which holds for every level of aversion. For example, when landowners are risk neutral to price uncertainty, the land expectation value decreases from $55 to $30 per acre fire risk is included. Similarly, when landowners have a relatively high level of aversion to price uncertainty, the LEV declines to $28 from $33 per acre when fire risk is considered. This means that fire risk could affect financial returns to landowners. Also, the impact of fire risk gets smaller as aversion to price uncertainty increases. In terms of the impact of price uncertainty, high level of aversion to price uncertainty lowers LEV. For example, LEV is reduced to $46 with medium level of aversion from $55 with zero aversion and it keeps going down with higher levels of aversion in the "no fire risk" scenario. The trend is the same though smaller in the "with fire risk" scenario, which leads to the difference of LEV between those two scenarios becoming smaller.

Graph 2 reveals the relationship of rotation age, aversion to price uncertainty, and fire risk. Fire risk reduces rotation age for every level of aversion. For example, optimal rotation age is 52 years without the effect of fire risk while the optimal rotation age is 37 years with fire risk when landowners have low aversion to price uncertainty. Similarly, optimal rotation age is 33 years without the effect of fire risk while the optimal rotation age is 30 years with fire risk when they have high aversion to price uncertainty. However, the impact of fire risk gets smaller as aversion parameter gets higher. Rotation age is 17% higher than with risk for landowners neutral to price uncertainty; Rotation age is around 4% higher with risk for landowner with low and medium aversion to price uncertainty; Rotation age is 1% higher with risk for landowners with a high level of aversion. At the same time, aversion to price uncertainty influences optimal rotation age: It decreases rotation age for both "with fire risk" and "without fire risk" scenarios. For example, in the "without fire risk" scenario, rotation age drops from 53 years to 33 years; in the "with fire risk" scenario, rotation age drops from 42 years to 3 years. This means landowners tend to harvest earlier when there is risk of fire or other catastrophic event.

2. Carbon Scenario

In this scenario, four prices of carbon are modeled and they are displayed in Graph 3 and Graph 4[2] In the "without fire risk" scenario, when carbon price is relatively low, like $1 and $5 per metric ton, with higher aversion level, LEV is lower; when carbon price is relatively high, like $10 and $25 per metric, with higher aversion level, LEV is higher. This means that LEV is dominated by the impact of price uncertainty at first, but is overpowered by the impact of high carbon price. In reality, carbon price is usually under $5 per metric ton. Therefore, under current market conditions there would be a major impact to the financial return to forestry landowners. LEV exhibits an increasing trend when carbon price goes from $1 to $25 per metric ton. In the "without fire risk" scenario, with higher carbon prices, LEV increases. When carbon price is $1 per metric ton, LEV is lower with a higher aversion level; when carbon price is $5, $10 and $25 per metric ton, LEV is higher with higher aversion level. Since the optimal rotation age of the base scenario with fire risk is slightly lower than

[2] Comparison of "with risk" scenario and "without risk" scenario is not made, because both carbon benefit and penalty starts at optimal rotation age of base scenario, which is different for these two scenarios. Therefore, it is not directly comparable.

the one without risk, carbon payment starts earlier when fire risk is considered. Therefore, the influence of carbon price is stronger in this scenario while the influence of price uncertainty is weaker.

Without the impact of carbon price fluctuation, rotation age should decrease as aversion to price uncertainty increases. With the impact of carbon price fluctuation, rotation age should increase as carbon price increases. Graph 5 and Graph 6 shows the joint effect of carbon price and aversion to price uncertainty on optimal rotation age. In the “without risk” scenario, rotation ages tend to increase as carbon price increase as expected. Since carbon price goes up, landowners could benefit more if they make the rotation age longer. However, with a carbon price of $1 per metric ton, rotation age decreases as landowners get increasingly averse to price uncertainty. When the carbon price is $5, $10, and $25, rotation age increases as landowners get increasingly averse. This implies that landowners who are averse to price uncertainty tend to harvest early to avoid the impacts of this uncertainty. In the “with risk” scenario, it follows the same pattern as “without risk” scenario: rotation age goes up as carbon price goes up; rotation age increases as the aversion parameter increases when carbon price is above $1 per metric ton. In this scenario, when carbon price reaches $10, the rotation age goes to 80 years or beyond. This means that carbon price can potentially significantly affect rotation age Like LEV, optimal rotation age is dominated by carbon when carbon price is high.

Conclusion

Price uncertainty to sawtimber decreases LEV and shortens rotation age and higher carbon price drives up LEV and lengthens rotation age. When it comes to the joint effect of price uncertainty and carbon price, carbon price has a larger influence. This implies a higher carbon price could be an effective way to encourage forestry landowners to lengthen the rotation age to sequester more carbon. The optimal base rotation age affects LEV and rotation age indirectly. This means that the design of a carbon market (e.g. when the payment and penalty starts) could be crucial to how landowners will respond. In general, the earlier the payment starts, the higher financial return landowners will get, and the longer the rotation age will be.

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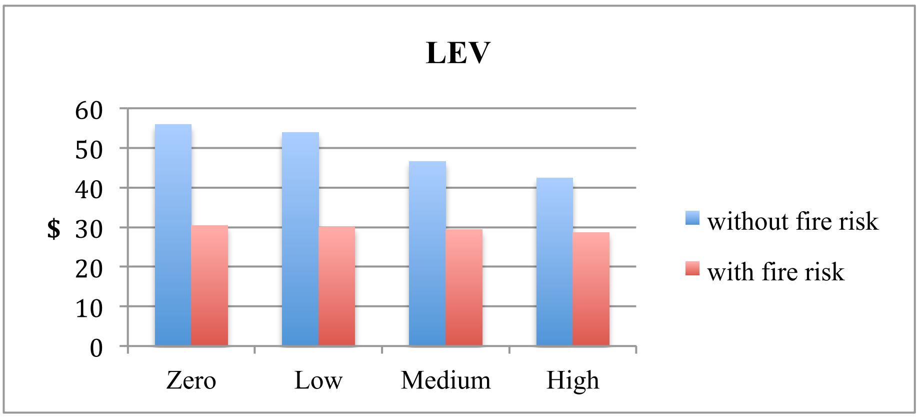
Table:

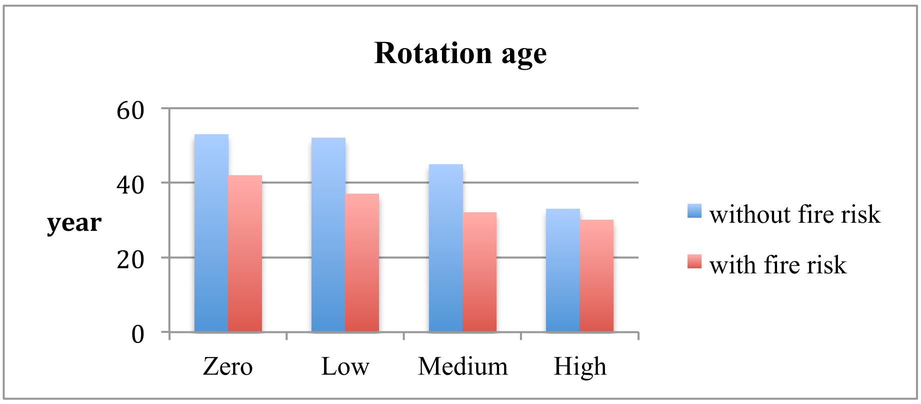
Table 1: Stumpage price data for each quarter

	Price for each quarter ($/ton)			
Year	I	II	III	IV
1980		16.46	18.07	18.92
1981	19.89	22.53	22.43	17.36
1982	17.17	17.27	13.60	14.61
1983	15.30	14.95	13.53	15.40
1984	15.95	13.21	13.06	14.33
1985	10.88	10.38	11.12	12.11
1986	15.05	15.46	14.71	23.69
1987	16.08	17.33	23.80	18.97
1988	17.31	18.88	14.83	18.67
1989	18.22	21.45	22.52	22.52
1990	23.80	24.02	20.76	18.52
1991	16.40	14.85	13.86	19.53
1992	20.09	18.58	22.18	25.77
1993	30.37	28.71	29.63	30.00
1994	28.00	29.06		

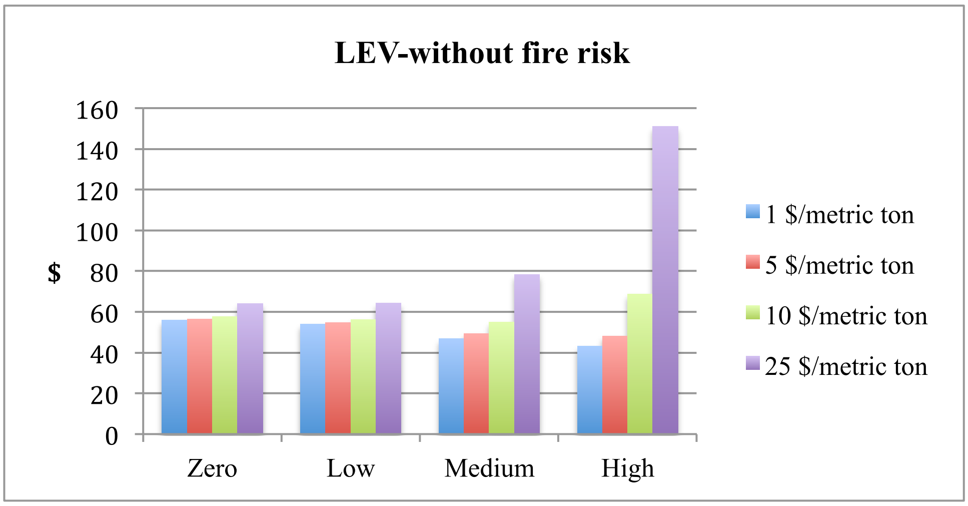
Graphs:



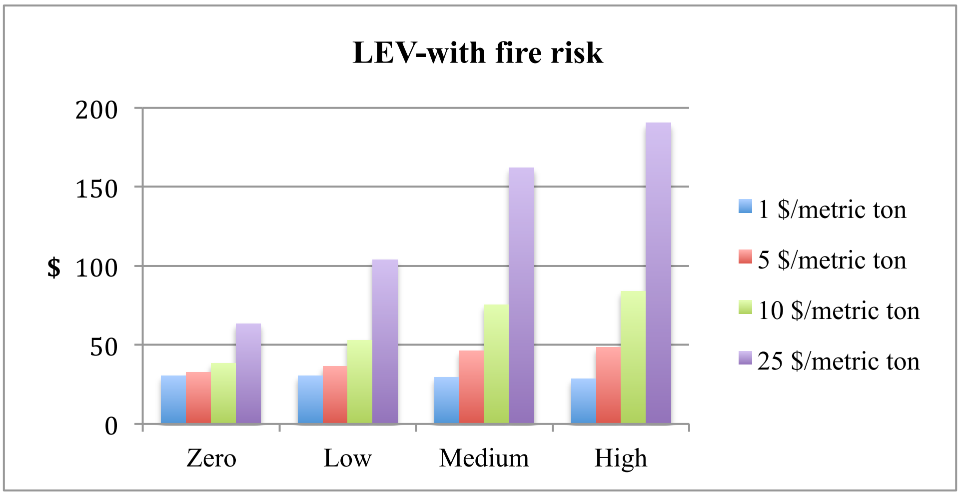
Graph 1. LEV in base scenario



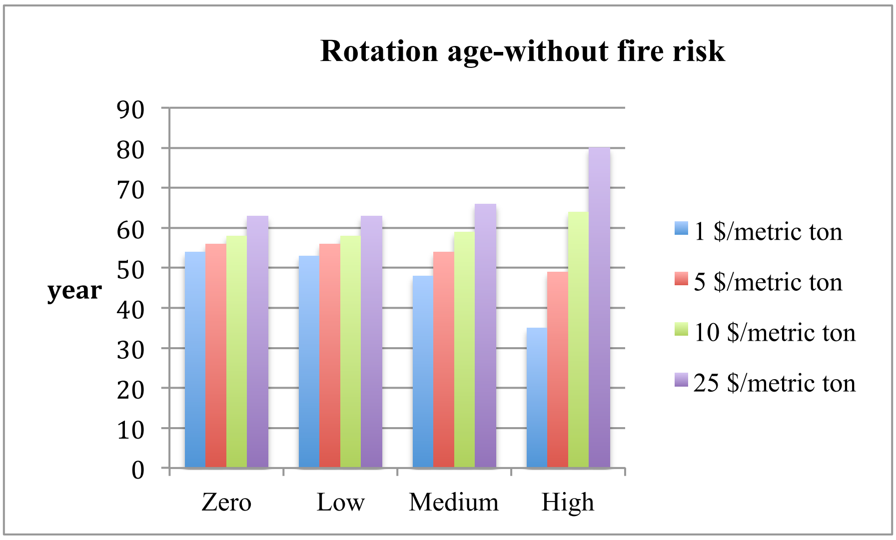
Graph 2. Rotation age in base scenario



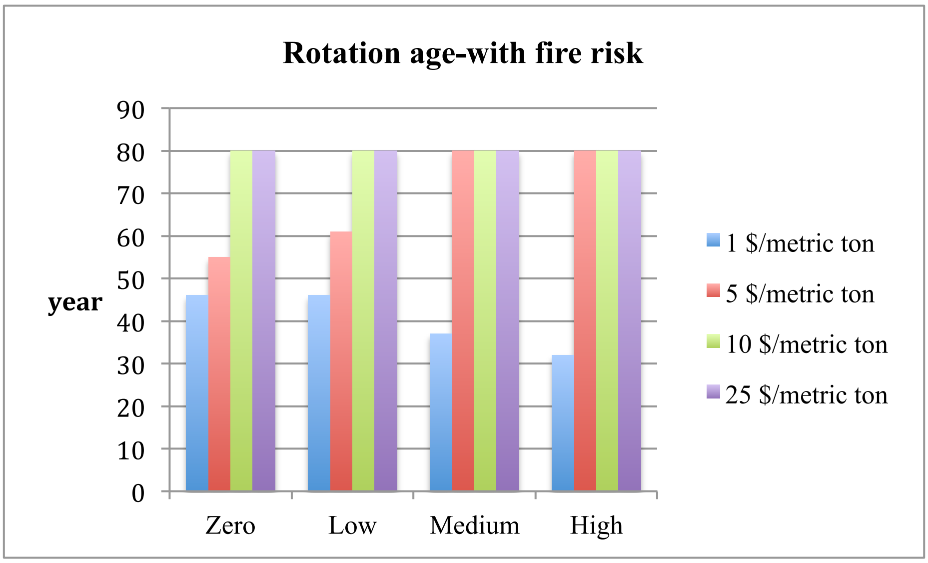
Graph 3. LEV without fire risk in carbon scenario



Graph 4. LEV with fire risk in carbon scenario



Graph 5. Rotation age without fire risk in carbon scenario



Graph 6. Rotation age with fire risk in carbon scenario