Farm Subsidy Incidence in the Presence of Bertrand Competitors of Complementary Factors of Production: A Theoretical Approach

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1. **Introduction**

The estimated cost of the 2008 farm bill’s commodity program in subsidy payments to farmers is 43.3 billion dollars (Goodwin et al, 2011). An (un)stated goal of the US farm commodity program is to promote financial stability, hence improving growth in the farm sector. Farm subsidy programs have become a topic of much debate among policy makers and economists alike. The central issues are the farmers’ ability to retain subsidy dollars; the degree non-retained subsidy dollars distort output and factor markets; and the identification of factors contributing to the farmers’ non-retention of subsidy dollars.

Empirical research has found that farm subsidies are heavily chaptalized into land rental rates (Bhaskar and Beghin, 2009; Kirwin, 2009; Kirwan and Roberts, 2010). More recently, theoretical and experimental research have demonstrated that disparities in market power and negotiation behavior influence the ability of the farmer to retain subsidy dollars (Russo et al., 2011; Nagler et al., 2013). To date, economists have yet to develop a widely accepted economic theory pertaining to farm subsidy incidence. This is primarily due to the complexities of agriculture’s multistage production system and various competitive relationships within and across production stages.

Our contribution to the subsidy incidence literature is that we acknowledge that i) factors of production in agriculture are largely complementary, and ii) input suppliers in technically separate markets are able to compete for the extraction of the marginal subsidy dollar from the farmer. Our method of analyzing subsidy incidence is an innovative theoretical approach that partially extends the market power work of Russo et al. (2011). Our fundamental model is a partial equilibrium two-stage dynamic game between a price-taking buyer and two price setting input suppliers. In this market setting, we identify the impacts of i) input supplier market power
and competitive interdependencies, ii) various degrees of substitutability between the input products, and iii) no, fixed and coupled subsidy policies.

Consistent with past research we find that a coupled subsidy policy results in a higher input price for the coupled factor, hence subsidy incidence. As such, the farmer is unable to retain the entire subsidy payment. However, we also observe several new outcomes only alluded to in past research. First, we find that price also rises in the non-affiliated input market. Though the subsidy incidence applies to all inputs, we find nonaffiliated input markets capture the subsidy dollar to a lesser degree as technical substitution of inputs rises. Second, aggregate subsidy incidence diminishes, as the input products become closer substitutes. The combination of Bertrand competition between the input suppliers and the degree of substitutability between technical inputs drives these results.

The details of the manuscript are as follows. We first provide a brief review of empirical, theoretical and experimental subsidy literature. Second, we discuss the meaning of subsidy incidence. Third, we develop the theoretical model and discuss in detail various aspects of the model. Fourth, we provide analytical solutions for a hypothetical farmer purchasing near perfect complement input goods, such as land and seed. Finally, we provide our conclusions, limitations, and suggestions for further research.

2. Literature

Theoretical work dates back to Floyd (1965). Floyd’s neoclassical subsidy theory focuses on the impacts of price elasticities. Hypothesized is that land being the most inelastic factor of production, will capitalize most if not all of the marginal subsidy dollar. However, the prediction is appropriate if, and only if, all input markets are perfectly competitive. Therefore, previous work that has assumed full incidence to land owners are limited based upon the realities of
imperfect competition (Kirwan, 2009). Accordingly, Myers, Sexton and Tomek (2010) suggest assuming perfect competition will lead to misleading results when an oligopoly, oligopsony or both characterize the true institution (i.e. market structure).

Currently, focus is moving towards relaxing the assumption of perfect competition. Saitone et al (2008) spawned the discussion of market power in the context of agricultural subsidies. The analytical model raised the question of market power — and states presence in the factor market will alter the subsidy distribution. Consequently, Saitone and his co-authors question the validity of previous works that assume a perfectly competitive market. An oversight was the lack of generality of the model as the Leontief function is rather restrictive. However, the conclusion put the literature on the correct path, as the results suggest larger distributional impact when market power is addressed. Thus, the welfare impactions with respect to equity (i.e. benefiting those intended) maybe lower than once expected.

Russo et al. (2011) also demonstrate that fallacies exist in the assumption of perfect competition. Using a three-stage production model where the hypothetical farmer buys from and sells to a monopolist and monopsonist Russo et al. (2011) demonstrate by means of simulations that the distribution of the marginal dollar intended for famers is highly dependent upon the relational market power between the farmer’s upstream and downstream markets. Most recently, Nagler et al. (2013) demonstrate by means of laboratory experiments that the retention of subsidy dollars is highly dependent upon negotiation behavior and to some extent the competition between hypothetical farmers and landowners.

3. Defining and Measuring Subsidy Incidence

The economic incidence, as referred by Kirwin (2007), is a more interesting matter [then statutory incidence] on the account of economic incidence being the examination and analysis of
behavioral changes, rather than legal rights. In the context of farm subsidies, the economic incidence examines to what degree the intended beneficiary (the farmer) captures or to what extent the upstream firms extract the marginal subsidy dollar. Otherwise stated, for every additional dollar the farmer receives from a subsidy payment, how much are the landowner and seed manufacture simultaneously able to extract, when exerting market power through setting higher prices.

The distribution and extent to which farmers, the intended beneficiaries, actually benefit is critical to understanding the effectiveness of subsidy policy. First, as one of Harberger’s three postulates suggest, the benefits and cost for the participants within this market will be added without regard to whom they occur, across policy treatments for the ‘sector’ welfare (efficiency) analysis (Alston and James, 2002). Subsequently removing the price and quantity effect from a policy change will offer insights into the subsidy incidence and welfare implications of the program.

4. Theoretical Modeling

We develop a partial equilibrium dynamic game between a hypothetical farmer, seed producer (generalizable to any input that is complimentary to land and possessing market power due to patents or other causes) and landowner under three alternative subsidy policies. The subsidy policies we investigate are i) no subsidy, ii) fixed subsidy (independent of farmer’s production decisions), and iii) coupled subsidy (tied to the amount land put under production). The model permits the input suppliers to simultaneously exert market power and thus compete for the marginal subsidy dollar. Finally, the modeling approach allows for inferences as to the distribution and extent a hypothetical farmer benefits from various forms of subsidy payments under the various subsidy policies.
4.1 Competitive Relationships and Market Structure

In the model, we maintain the assumption of perfect knowledge as Alston and James (2002). However, we relax the assumptions of a perfectly competitive and static market, as technological developments in the most recent decades have been significantly influential in altering the competitive structure of the farm sector.

First, technological advances in the biotechnical industry have fueled a rapid consolidation. These consolidations lead to concentration, which reduced market competition. However, consolidation is not the only variation that has modified the seed sector. From 1996 to 2008 the usage of in acres of genetically modified (GM) seeds increased from 4.2 to 309 million acres planted (Stiegert, Shi and Chavas, 2010). Farmers highly value the traits of GM seeds however; the improved attributes do come at a monetary cost to the farmers (Moschini, 2010).

Farmers and Policy-makers similarly scrutinize bio-tech firms—who commercialized patented crop seed—in regards to the firm’s monopoly in the widely adopted Roundup Ready soybean and corn market. For example, in the soybean industry, Monsanto’s dominance is prevalent, the firm accounts for just approximately 90% of the production acres planted (Moschini, 2010). Moss (2009) indicates Monsanto’s may exercise market power adversely affects prices, quality and choices available to farmers, which Fulton and Giannkas (2001) suggest have distributional impacts that are relevant public policy considerations.

Furthermore, in a given geographical location, there are typically few buyers and sellers in the agricultural production land rental market Raup (2003). Although, we recognize that even in localized markets, input suppliers may not be monopolists, and the farmer may not be monopsonist, we ignore the impacts of bilateral negotiations analyzed in Nagler et al. (2013) and leave it as an extension to the current model. Because it is the case that farmers are price takers
for some inputs and negotiate over price for others, we assume, the upstream input providers are
more akin to Bertrand competitors.

4.2 Farmer Technology

We begin by stipulating the production technology of the farmer, as it is pivotal in the derivation
of the differentiated factor demands faced by the input suppliers. As discussed by Saitone et al
(2008), whom employ a Leontief production function, a relatively inelastic demand exists
between land and seed. However, the functional form fails to provide negatively sloped
differentiated factor demand curves—a requirement for input suppliers to exert market power.
Therefore, the choice of production technology (i.e. the functional form) for agricultural inputs
must be flexible enough while still allowing for near fixed proportion technology.

We initially considered a fully flexible and the well-known CES production function. However, we are able to demonstrate there exist limitations of this functional form in the
presence of Bertrand market power. Primarily, regardless of product differentiation, Bertrand
competition results in input suppliers employing marginal cost pricing. Alternatively, we utilize
a quadratic production technology that follows from the well-known quadratic utility function
derived by Bowley, (1924). The Bowley model has been used extensively in modeling
differentiated product in Bertrand and Cournot competition (e.g. Dixit, 1979: Singh and Vives,
1984; Häckner 2000; Symeonidis, 2002). By rephrasing the parameters of the Bowley model,
the farmer’s (F) technology can be written as

\[ U(q_A, q_B) = a(q_A + q_B) - \frac{1}{2} b(q_A^2 + 2\theta q_A q_B + q_B^2) + m \]

The quantities demanded of the two differentiated products in the market are \( q_A \) and \( q_B \), \( m \) represents all other goods with price normalized to 1, and \( \theta \) represents the degree of substitution of the two products.
\[ Y = a(L + S) - \frac{1}{2} b(L^2 + 2\sigma LS + S^2) + m, \]

where land \((L)\) and seed \((S)\) are the two representative inputs used by the farmer to produce output of commodity \((Y)\). The intercept for the aggregate demand of inputs, \(a\), and the slope of the aggregate demand, \(b\), are positive parameters; \(\sigma\) represents the degree of substitutability between the two inputs \(-1 \leq \sigma \leq 1\), where a value of -1 denotes perfect technical complements, 0 technical independence, and 1 perfect technical substitutes. Finally, \(m\) represents all other inputs with price normalized to 1.

4.3 No Subsidy Policy \((NS)\)

The two stage dynamic game depicted in Figure 1 is as follows. In stage two, the buyer (farmer) chooses quantity subject to the prices set by the suppliers (landowner and seed manufacturer). In stage one, the suppliers simultaneously choose prices accounting for i) horizontal competition, and ii) by backward induction the buyer’s technology and complementarity and of the products they supply the buyer.

In stage two, the general form of the farmer’s objective function is

\[ \text{Max } \pi_F = PY(L, S) - C(L, S). \quad (2) \]

We assume the farmer takes output price \(P\) and input costs \(C\) as given. Substituting equation (1) into (2) results in the farmer’s objective function

\[ \text{Max } \pi_F^{NS} = P\left( a(L + S) - \frac{1}{2} b(L^2 + 2\sigma LS + S^2) \right) - w_L L - w_S S \quad (3) \]

where \(w_L\) and \(w_S\) represent the price of land and seed respectively. Taking the first order necessary conditions of equation (3) results in

\[ \frac{\partial \pi_F^{NS}}{\partial L} = P\left( a - \frac{1}{2} b(2L + 2S\sigma) \right) - w_L = 0 \quad (4) \]
and
\[
\frac{\partial \pi_{FS}^N}{\partial S} = P \left( a - \frac{1}{2} b(2S + 2L \sigma) \right) - w_S = 0. \tag{5}
\]

Solving (4) and (5) results in the following factor demands for land and seed
\[
L_{NS}^N = -aP(1-\sigma) - \sigma w_L + w_S \frac{b}{bP(\sigma^2 - 1)} > 0 \ \forall \ P > 0, \ -1 < \sigma < 1, \ a > 0, \text{ and } b > 0, \tag{6}
\]
and
\[
S_{NS}^N = -aP(1-\sigma) - w_L \sigma + w_S \frac{b}{bP(\sigma^2 - 1)} > 0 \ \forall \ P > 0, \ -1 < \sigma < 1, \ a > 0, \text{ and } b > 0. \tag{7}
\]

Because the farmer is a price taker, equations (6) and (7) represent the farmer's second stage best response functions to input prices. The second order sufficient conditions of (3) result in
\[
\frac{\partial^2 \pi_F^N}{\partial L^2} = \frac{\partial^2 \pi_F^N}{\partial S^2} = -bP < 0 \ \forall \ P > 0 \text{ and } b > 0, \tag{8}
\]
which confirms that the derived factor demands are the farmer's best response functions to input prices.

To test whether the assumed quadratic technology results in appropriate the derived factor demands, we take the following comparative statics. The comparative static with respect to \( P \) results in
\[
\frac{\partial L_{NS}^N}{\partial P} = \frac{\partial S_{NS}^N}{\partial P} = \frac{a \sigma - a}{bP(\sigma^2 - 1)} > 0 \ \forall \ P > 0, \ -1 < \sigma < 1, a > 0, \text{ and } b > 0. \tag{9}
\]

Thus, increasing output price increases the amount of land and seed the farmer is willing to purchase. Taking the comparative static with respect to input prices results in
\[
\frac{\partial L_{NS}^N}{\partial w_L} = \frac{\partial S_{NS}^N}{\partial w_L} = \frac{1}{bP(\sigma^2 - 1)} < 0 \ \forall \ P(-1 + \sigma^2) < 0 \text{ and } b > 0. \tag{10}
\]

Therefore, the factor demands are downward sloping allowing for input provider market power. Finally, taking the cross price comparative static results in a negative relationship
when the input factors are technical compliments and positive when inputs are technical substitutes. Therefore the quadratic production technology factor demands have the appropriate characteristics for further analysis.

Now that the optimal reaction of the farmer in stage two has been derived, we turn our attention to the competition between input suppliers in stage one. In stage one, the seed firm and landowner simultaneously set price while accounting for the farmer’s substitutability of their respective products. Without loss of generality, we let the representative input suppliers have constant marginal cost (set equal to 0), no fixed cost, and no capacity constraints (Dixit, 1979). Given each input provider face their respective factor demand, by backward induction the landowner’s and seed manufacture’s objective functions are

\[
\text{Max}_{w_L} \pi^{LS}_L = w_L \left( L^{NS} \right) \quad (12)
\]

and

\[
\text{Max}_{w_S} \pi^{NS}_S = w_S \left( S^{NS} \right). \quad (13)
\]

After substitution of equations (6) and (7) into (12) and (13), the first order necessary conditions, with respect to own price are

\[
\frac{\partial \pi^{LS}_L}{\partial w_L} = \frac{aP(1-\sigma) + \sigma w_S - 2w_L}{bP(\sigma^2 - 1)} = 0 \quad (14)
\]

and

\[
\frac{\partial \pi^{NS}_S}{\partial w_S} = \frac{aP(1-\sigma) + w_S \sigma - 2w_S}{bP(\sigma^2 - 1)} = 0. \quad (15)
\]

Solving the first order conditions with respect to own price, the landowner’s and seed manufacture’s Bertrand reaction functions are
\[ w^\text{NS}_i = \frac{1}{2} (aP - aP + w_s \sigma) > 0 \quad \forall \ w_s > 0, \ P > 0 \quad \text{and} \quad a > \frac{\sigma w_s}{P(\sigma - 1)}. \] (16)

and

\[ w^\text{NS}_s = \frac{1}{2} (aP - aP + w_s \sigma) > 0 \quad \forall \ w_s > 0, \ P > 0 \quad \text{and} \quad a > \frac{\sigma w_s}{P(\sigma - 1)}. \] (17)

It is easily shown that both reaction functions are downward sloping. Simultaneous solution of equations (16) and (17) result in the Bertrand no subsidy equilibrium (NSE) prices of land (rental rate) and seed of

\[ (1 ) = > 0  \quad P>0, \quad -1 1, \quad \text{and} \quad a > 0 \quad \text{NSE NSE} \]

\[ \sigma \leq \sigma - 2.\] (18)

Therefore if the suppliers are acting optimally, they will exert market power by setting prices above marginal cost. It is interesting to note that for all previous restrictions of parameter values. Therefore, market power is a function of substitutability. For instance, as \( \sigma \rightarrow 1 \), equilibrium prices revert to the purely competitive equilibrium, and market power is maximized as \( \sigma \rightarrow -1 \). The Bertrand Equilibrium prices in relation to the substitutability, ranging from perfect complements at -1 to perfect substitutes at 1 are depicted graphically using parameters assumptions \( P=1 \) and \( a=2 \) in Figure 2.

With the equilibrium price solutions from stage one, equilibrium quantities can be derived. Substituting equation (18) into (6) and (7) results in the Bertrand equilibrium quantity of land and seed

\[ S^\text{NSE} = L^\text{NSE} = -\frac{a}{b(\sigma - 2)(1+\sigma)} > 0 \quad \forall \ a > 0, \ b>0, \ \text{and} \quad -1<\sigma < 2. \] (19)

As can be seen in Figure 3 under the parameter assumptions \( a=2 \) and \( b=1 \), the farmer’s input product substitutability and equilibrium quantity have an inverse relation.
The farmer’s equilibrium profit is

\[ \pi_f^{NSE} = \frac{a^2 P}{b(\sigma-2)^2(1+\sigma)} > 0 \quad \forall \quad P > 0, \quad -1 < \sigma < 2, \quad a > 0, \text{ and } b > 0, \]

(20)

and is strictly increasing in theta as \( \frac{d\pi_f^{NSE}}{d\sigma} = -\frac{3a^2 P \sigma}{b(\sigma-2)^3(1+\sigma)^2} > 0 \) for all conventional parameter restrictions. The equilibrium profit of the seed and land suppliers are

\[ \pi_s^{NSE} = \pi_l^{NSE} = -\frac{a^2 P(\sigma-1)}{b(\sigma-2)^2(1+\sigma)} > 0 \quad \forall \quad P > 0, \quad -1 < \sigma < 1, \quad b > 0, \text{ and } a > 0, \]

(21)

and are strictly decreasing in theta as \( \frac{d\pi_s^{NSE}}{d\sigma} = \frac{2a^2 P(1+\sigma + \sigma^2)}{b(\sigma-2)^3(1+\sigma)^2} > 0 \) for all conventional parameter restrictions.

Finally, we analyze the distribution of the system’s total surplus (sector surplus).

\[ s_i^{NSE} = \frac{\pi_i}{\sum_{i=1}^{\pi_n} \pi_i}. \]

(22)

The farmer’s equilibrium share of sector surplus is \( s_f^{NSE} = \frac{1}{3-2\sigma} \). The seed firm and landowner’s equilibrium shares are \( s_{s,l}^{NSE} = \frac{\sigma-1}{2\sigma-3} \). As demonstrated in Figure 4, technical input substitutability plays a significant role in the welfare distribution along the supply chain. Input markets of near perfect complements, such as seed and land, significantly lower the farmer’s surplus distribution, hence calling into question the importance of market power’s overall influence of subsidy incidence.
4.4 **Fixed Payment Subsidy Policy (FS)**

As defined by Bhaskar and Beghin (2009) decoupled payments are “not tied to current production, and/or factor use or prices,” therefore causing little market distortion. To begin with, the fixed “lump-sum” subsidy policy, we define the farmer’s objective function as

\[
\max_{L,S} \pi^F_{FS} = P \left(a(L + S) - \frac{1}{2} b(L^2 + 2\sigma LS + S^2)\right) - w_L L - w_S S + \text{FIX}. \tag{23}
\]

Taking the derivative of the farmer’s profit will result in the same Bertrand equilibriums as in the no subsidy policy. This is due to the additive nature of the payment system.²

4.5 **Coupled Payment Subsidy Policy (CS)**

The coupled subsidy enters the farmer’s objective function as essentially an input price support of land. The payment mechanism is a proxy for base acreage payments determined by historic acreage and yield (Peckham and Kropp, 2012). Base payments are tied to the land, thus whoever farms the land receives payments based upon the acreage’s base. Again, we initiate with stage two where the farmer’s objective function is now

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² Though fixed subsidies are theoretically independent of the farmer’s output decisions, the experimental literature suggests payments of this form may not be as decoupled as assumed by theory due to human behavior (Bhaskar and Beghin, 2009). Bhaskar and Beghin (2009) indicate that decoupled payment mechanisms can enter a farmers production function through risk (perception and/or preferences), and expectations. However, to what extent decoupled-payments modify commodity production, and subsequently the subsidy incidence, is an empirical question and is outside the scope of this analysis.
\[ \text{Max } \pi_{L,S}^{CS} = P \left( a(L + S) - \frac{1}{2} b(L^2 + 2 \sigma LS + S^2) \right) - (w_L - \alpha)L - w_S S, \]  

where \( \alpha \) is the subsidy payment per acre of land. The subsidy is coupled because the total payment received by the farmer is directly tied to the acres of land the farmer chooses to purchase. This payment mechanism effectively reduces the rental rate per acre of land, thus decreases the marginal factor cost of land to the farmer.

By taking the first order necessary condition of equation (24) and simultaneously solving for the optimal input quantities subject to input prices results in the coupled subsidy factor demands

\[ L_{CS} = \frac{-aP(1 - \sigma) - \sigma w_S - \alpha + w_L}{bP(\sigma^2 - 1)}, \]  

and

\[ S_{CS} = \frac{-aP(1 - \sigma) - w_L \sigma + \alpha \sigma + w_S}{bP(-1 + \sigma^2)}. \]

Notice, with the addition of the coupled subsidy policy, the factor demands are no longer symmetric as was the case in the no subsidy policy. It can be confirmed that the impact of the coupled subsidy increases the demand for both input factors. However, the demand for land increases more so than the demand for seed.

Now that the factor demands (optimal reaction to input prices) of the farmer in stage two has been derived, we turn our attention to the competition between input suppliers in stage one. Following the same solution process outlined in section 4.3, we find the Bertrand reaction functions in the presence of a coupled subsidy are

\[ w_{L}^{CS} = \frac{1}{2} (aP - aP \sigma + \alpha + w_S \sigma), \]

and
Simultaneous solution of equations (27) and (28) result in the coupled subsidy equilibrium (CSE) price of land (rental rate)

$$w^CSE_L = \frac{1}{2} \left( aP - aP\sigma - a\sigma + w_L\sigma \right) .$$

(28)

and price of seed

$$w^CSE_S = \frac{\alpha \sigma + aP(-2 + \sigma + \sigma^2)}{\sigma^2 - 4} .$$

(29)

The comparative statics $\frac{dw^CSE_L}{d\alpha} = \frac{(-2 + \sigma^2)}{\sigma^2 - 4} > 0$ and $\frac{dw^CSE_S}{d\alpha} = \frac{-\sigma}{\sigma^2 - 4} > 0$ demonstrates that increasing subsidies increase prices in both sectors for all conventional restrictions on substitutability. Just as in the no subsidy policy, prices are above marginal cost, thus the firms are able to exert market power.

It can be shown that $w^CSE_L > w^CSE_S$ for all conventional restrictions on the parameters. The gap between land and seed prices diminishes and approaches zero as technical substitutability $\sigma \to -1$. The pricing differentials are demonstrated by graphically depiction of equations (29) and (30) in Figure 5. The parameter assumptions in figure 5 are $P=1$, $a=2$, and $\alpha=1$ and the prices range over the degree of technical substitutability of input factors.

Given the optimal prices set by the input suppliers, we consider the input quantity decision by the farmer in the second stage. Given the input providers’ equilibrium prices, the farmers’ factor demands and the resulting Bertrand equilibrium quantities are

$$L^CSE = \frac{\alpha(-2 + \sigma^2) + aP(-2 + \sigma + \sigma^2)}{bP(4 - 5\sigma^2 + \sigma^4)} > 0 \forall -1 < \sigma < 2$$

(31)

and
for land and seed and are plotted in Figure 6 under the parameter assumptions \(a=2, b=1, P=1\) and \(\alpha=1\). The graph presents how optimal factor quantities change in regards to technical substitutability. The introduction of the subsidy depicts a different relationship than in the prior non-subsidy case. For land, any movement away from the origin (independent factors of production) increases optimal factor quantity. However, the optimal factor quantity of seed decreases in substitutability.

Substituting equilibrium prices and quantities results in the farmer’s equilibrium profit of

\[
\pi_f^{CSE} = \frac{2a^2P (3 + 2P(\sigma - 1) - 2\sigma(-1 + \sigma)(2 + \sigma)^2 + 2aP(\sigma - 1)(1 + (P - 1)\sigma + (2P - 1)\alpha^2(3\sigma^2 - 4))))}{2bP(\sigma^2 - 4)^2(\sigma^2 - 1)}.
\]

It can be shown that equation (33) is strictly convex. The Bertrand equilibrium profits of the landowner and seed firm are now

\[
\pi_L^{CSE} = -\frac{(\alpha(2 + \sigma^2) + aP(\sigma(1 + \sigma) - 2))^2}{bP(\sigma^2 - 4)^2(\sigma^2 - 1)}
\]

and

\[
\pi_S^{CSE} = -\frac{(\alpha\sigma + aP(-2 + \sigma + \sigma^2))^2}{bP(\sigma^2 - 4)^2(\sigma^2 - 1)}.
\]

Both equilibrium prices increases as the goods move towards a stronger complementary.

### 4.6 Summary of Sector Welfare Distribution Per Subsidy Policy

To analyze the sector welfare distribution for the two subsidy policies (fixed and coupled), we utilize the relative profit share method depicted in equation (22). For the fixed subsidy policy,
the farmers aggregate profit share is $s_{F}^{\text{FSE}} = \frac{1 + FIX}{(3 - 2\sigma) + FIX}$ and for the landowner and seed firm

\[ s_{L}^{\text{FSE}} = s_{S}^{\text{FSE}} = \frac{\sigma - 1}{2\sigma - 3 + FIX}. \]

For the coupled subsidy policy, the profit share is now a more complex function of parameters $a$ and $P$ (not shown). For simplicity we set $a=2$ and $P=1$ resulting in an equilibrium profit share for the farmer of $s_{F}^{\text{CSE}} = \frac{(8 + \alpha(4 + \alpha))(4 - 3\sigma^2) - 4(2 + \alpha)\sigma^2}{(8 + \alpha(4 + \alpha))(12 - 9\sigma^2 + 2\sigma^4) + (2 + \alpha)(12\sigma^3 - 32)}$.

However, due to the impacts of reducing the net cost of land to the farmer in the coupled subsidy policy, the profit shares of the landowner and seed firm are no longer symmetric. As such, the profit share of the landowner is now

\[ s_{L}^{\text{CSE}} = \frac{2((\sigma^2 - 2)(2 + \alpha) + 2\sigma^2)}{(8 + \alpha(4 + \alpha))(12 - 9\sigma^2 + 2\sigma^4) + (2 + \alpha)(12\sigma^3 - 32\sigma)} \]

and for the seed manufacturer

\[ s_{S}^{\text{CSE}} = \frac{2(\alpha\sigma + 2(-2 + \sigma + \sigma^2))^2}{(8 + \alpha(4 + \alpha))(12 - 9\sigma^2 + 2\sigma^4) + (2 + \alpha)(12\sigma^3 - 32\sigma)}. \]

Figure 7 provides a graphical summary of the percent distributions of total sector surplus for the farmer over the range of input substitutability for the three subsidy policies. As can be seen, the farmer realizes the greatest allocation in profit share under the fixed subsidy policy. Interestingly, the farmer’s coupled subsidy profit share is not a large improvement in relation to no subsidy policy. However, this relationship (i.e. absence of improvement) does not hold when inputs are moderate to nearly perfect substitutes. In fact, we observe a large reduction in profit share, due to the coupled policy, when the goods are nearly perfect substitutes. Though unrealistic, this is primarily due to the seed firm setting a negative price, and hence paying the farmer to take their product. Further analysis into the region of discontinuity is required.
Figure 8 provides a graphical summary of the percent distribution of total sector surplus for the landowner and seed firm. As can be seen, the input suppliers realize the lowest profit share in the presence of the fixed subsidy policy; as the profits of the landowner and seed firm are left unchanged with the addition of the fixed subsidy payment to the sector. The landowner and seed firm have nearly symmetric profit shares when the inputs are nearly perfect complements in the absence of a subsidy policy and under the coupled subsidy policy. However, under the coupled subsidy policy, the landowner’s profit surplus in relation to the seed firm increases as the goods approach nearly perfect substitutes.

5. Analytical Results of Farm Subsidies: Market Outcomes and Sector Welfare

In this section, we provide analytical results to gain insights into the strategic interactions of the firms, surplus distributions, subsidy payments that are directly a result of the quantity of land put under production, and sector welfare. Presented in Table 1 are the equilibrium market outcomes for the no subsidy, fix subsidy and coupled subsidy policies. All results are derived from the following parameter assumptions: (1) $\sigma = -.99$, (2) $P = 1$, (3) $a = 2$, (4) $b = 1$, and (5) $\alpha = 1$ and associated equations. The most important assumption we make is the near perfect technical complementarity between land and seed inputs.

The results demonstrate that only the farm’s surplus rises with a fixed payment. However, under a coupled policy input prices, quantities, profits per firm, sector surplus (welfare) rise. Interestingly, with nearly perfect technical complement inputs, the subsidy payment under a coupled policy is less than the increase in sector welfare; whereas, the increase in welfare is equal to the subsidy payment in the fixed subsidy policy. Thus, only the coupled subsidy payment results in a multiplier effect.
6. Conclusion and Discussion

As we stated in the introduction, “An (un)stated goal of the US farm commodity program is to promote financial stability, hence [improving growth] in the farm sector.” We also identify one of the central issues in the debate is “the farmers’ ability to [retain] subsidy dollars.” Given the results of our analysis, we find a natural tension in the farm subsidy debate between the distribution of the subsidy dollar and sector growth.

We find, theoretically, that only the fixed subsidy policy results in full retention of the subsidy payment. Consistent in principal with past empirical research, we find that under a coupled payment policy the farmer experiences the least improvement in sector welfare. However, we find that a fixed subsidy policy also results in less sector growth as compared to the coupled subsidy policy. The higher growth in welfare is due to the multiplier effect.

Past literature has focused almost exclusively on the subsidy incidence in land prices. Interestingly, our modeling approach identifies that non-land rivals of differentiated products also have a stake subsidy payments. The size of the stake increases as the complementarity of the inputs increases. For instance, if prices were to rise in land, so to would the price of seed due to the technical complementarity of the products. This requires no additional market power exerted by the seed manufacturer to increase profits. By extension, the ‘high’ prices of seed (and other complementary inputs) typically associated with market power may not be entirely due to market power cited by past research, but rather by both complementarity of inputs and a coupled farm subsidy as well.

Future research should include bilateral negotiations between the farmer and landowner as analyzed in Nagler et al. (2013). Intuitively from our current results, if the farmer were to gain some degree of pricing power, the input price of land would decline. The increase in the
market power of the farmer would thus result in less subsidy incidence, potentially at the cost of sector growth.
References


Figure 2: The Equilibrium Bertrand Output Prices Over the Degree of Technical Input Substitutability

Figure 3: The Equilibrium Bertrand Quantities Over the Degree of Technical Input Substitutability
Figure 4: Relative Optimal Profit Share of Farmer and Bertrand Competitors

Figure 5: The Bertrand Prices Over the Degree of Substitutability In the Presence of an Input Subsidy Tied to Land
Figure 6: Optimal Factor Quantities of Farmer Facing Bertrand Competitors Ranging From Perfect Compliments to Perfect Substitutes When a Subsidy is Provided for Land

Figure 7: Optimal Relative Profit Shares of Farmer
Figure 8: Optimal Relative Profit Shares of Landowner and Seed Firm
Table 1 Analytical Results: Market Outcomes and Sector Welfare

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<th>Subsidy Situation</th>
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<th>Π_S</th>
<th>Payment</th>
<th>Welfare</th>
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<td>66.90</td>
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* The fixed payment is set equal to 84 for comparative purposes.