Bayesian Estimation of Optimal Nitrogen Rates with a Non-Normally Distributed Stochastic Plateau Function

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Abstract

Despite abundant literature on crop yield distributions, there is no strict agreement about functional form and distributional assumptions. This paper estimated the optimal nitrogen rates in wheat production assuming a stochastic plateau yield function with nonnormal random effects. The yield plateau parameter is assumed to be beta distributed. The parameters are estimated using a Bayesian estimation methods and a noninformative prior. The maximum likelihood method was also used to compare the results of the two approaches. The results indicate a slight difference in nitrogen recommendation, which can be associated with the fact that the Bayesian methods capture parameter uncertainty while the MLE does not.

Key Words: wheat, Bayesian Approach, nitrogen rates, Stochastic Plateau

Introduction

Modeling crop yield response to nitrogen has been a focus of agricultural economists for many decades. A large literature has focused on determining the optimal rate of nitrogen (a limiting nutrient). A concern is that optimal rate of nitrogen often varies depending on the functional form (Cerrato and Blackmer, 1990) and the distributional assumption being assigned to the yield data (Day, 1965). The plateau-type model has been extensively used since the extension by Day (1965) of the von Liebig limiting nutrient approach. The von Liebig law of the minimum relating crop yield to limiting resources was extended by Cate and Nelson (1965) to allow a continuous yield response to the limiting resource(s) up to the von Liebig point, where incremental change in the resource(s) has zero impact on yield. Paris (1992) developed the linear-plus-plateau model consisting of two components: a first component where the change in the crop yield with respect to a marginal change in the limiting resources is constant and positive and a second component when the marginal relative variation is zero. Berck and Helfand (1990) extended the linear plateau model to allow stochastic variation of two of the three parameters of the model.

Other extensions have allowed the parameters to vary across space and across years (Cerrato and Blackmer 1990; Sumelius 1993; Backman et al. 1997). Tembo et al. (2008) have successfully tested a similar model that allowed year-to-year variation in both the intercept and plateau, and found that it fit long-term experimental data better than the switching regression approach of Paris (1992). Makowski & Wallach (2002), Brorsen and Richter (2011), and Tumusiime et al. (2011) assigned a random error term to each of the parameters of the model to allow a year-to-year variation of crop yield. Even though past models fit the yield data well, the normality assumption that they all made might impact the accuracy of the estimation and eventually lead to biased estimates.

The assumption that field crop yield is normally distributed can be traced back to at least the middle 1950s with the work by Botts and Boles (1958). In their work on the federal crop insurance program, Botts and Boles (1958) came to the conclusion that mean crop yield on "indemnified fields" follows a normally distributed probability density function.

Since the study by Day (1965) on field crop yields in Mississippi, which found evidence against normality, the focus has shifted to alternative probability density functions, consistent with the skewness that is often found in crop yield. Day's argument was that some limiting resources, observable biological factors, and unpredictable and uncontrollable environmental factors (e.g.;

bad weather, diseases, pest invasions), may cause the standard deviation of the crop yields to vary substantially from year to year.

Alternative probability density functions including the conditional beta distribution (Nelson and Preckel, 1989; Babcock and Hennessy, 1996; Goodwin, 2009), the hyperbolic sine transformation function (Wang et al., 1998), the gamma distribution (Gallagher, 1987), and the logistic distribution (Sherrick et al., 2004) were used to fit yield data. Most of those studies however, used aggregate data and are not nitrogen response functions. In our model, we used a stochastic plateau model to depict how wheat yield responds to nitrogen fertilizer. Then, we assign a beta distribution to the plateau parameter and restrict the possibility of the plateau falling below the intercept. The choice of the beta distribution is explained by the fact that it is more flexible and can allow either a right or a left skewed distribution (Babcock and Hennessy, 1996).

Besides the data issue pointed out above, estimation methods used in some previous literature might have not been able to capture the true behavior of crop yield response to nitrogen. Most of the statistical packages used in previous studies perform maximum likelihood estimations. However, given the small sample size of crop yield data, maximum likelihood may provide biased estimates of the parameters of the model.

Kuhner (2006) pointed out that the accuracy of the Bayesian estimates relies upon the appropriateness of the priors, but insisted on the fact that when relevant information are available, the Bayesian method is the most efficient "especially in cases where one or more parameters are close to zero." (p.68). Banerjee et al. (2004) claimed that the Bayesian approach is more appropriate for modeling events temporally or/and spatially related, which is the case of crop yield response to nitrogen, and Bornn and Zidek (2012) argued that it should be considered a best candidate due mainly to the fact that a good likelihood representation eliminates most of the uncertainty associated with the parameters. In addition, the frequentist estimation may not be suitable to nonlinear models such as the plateau type functions because of the possibility of non unique optimal point (Brorsen, 2013). New statistical packages provide tools for Monte Carlo simulation methods, which is easy to use and do not suffer from the convergence problems associated with maximum likelihood estimation. Further, Tembo et al (2008) used the plug-in approach, which does not consider parameter uncertainty (also known as estimation risk), which will lead to biases in nonlinear models as shown by Pope and Ziemer (1984).

The literature of Bayesian estimation of yield response to nitrogen is quite recent and relatively rare in being used to determine optimal allocation of limiting resources in production economics. Holloway (2005) used the Bayesian approach to estimate the parameters of the stochastic plateau model but did not infer about the optimal level of inputs. However, Holloway's article provides the fundamentals of the estimation algorithm necessary to understand the Bayesian approach. Brorsen (2013) estimated the optimal level of nitrogen that maximizes cotton yield using Bayesian method with a stochastic plateau model similar to that used by Tembo et al. (2008). Holloway treats the error disturbance terms associated with the yield potential as being normally distributed and has also been followed by Tembo et al. (2008). This paper uses the beta distribution for the plateau error term as an alternative to the normal distribution. The beta distribution allows modeling so that the plateau cannot be below the intercept as it can when a normal distribution is assumed. None of the previous work has considered this eventuality, which contributes to strengthen the contribution of this paper. Moreover, our approach differs from those previous articles regarding the functional form of the stochastic plateau model employed. The stochastic plateau yield is set to be always greater than or equal to the intercept.

This article determines the optimal level of nitrogen for wheat production using the Bayesian approach under nonnormality of the plateau error term. We then estimate the parameters of the model using the maximum likelihood method and compare how close the two methods are in determining the optimum point.

The von Liebig Algorithm Adapted to Wheat Production

Nitrogen (N) is one of the most limiting resources in wheat production. Along with water, phosphorus (P), potassium (K), and sulphur (S), nitrogen constitutes the core nutrients required for wheat to grow normally and to yield high quality wheat grain (Stark, Mahler, and Tindall. 2004). For the simplicity of the estimation model, only nitrogen and water are limited resources.

Now consider that there exists a threshold level that keeps the yield plateau from going below. Furthermore, let's assume that this point corresponds to the intercept, which represents the yield with no nitrogen. The equation can be written as:

(1) $$y_{it} = \beta_0 + \min(\beta_1 N_i,\ \theta v_t) + u_t + \varepsilon_{it}$$

where y_{it} is the wheat yield at time t in the i^{th} plot, N is the amount of nitrogen to be applied, β_i and θ are the parameters of the yield function to be estimated, $u_t \sim N(0, \sigma_u^2)$ is an intercept shifter representing all random variables, which can cause the intercept to move up or down, v_t is a random parameter assumed to be beta distributed, θ is a scale parameter, and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ is the traditional stochastic error term.

A risk neutral wheat producer who seeks to apply an economically efficient level of nitrogen in order to maximize expected net returns will optimize the following equation:

(2) $$\max_N E(R_t|N) = pE[y_t] - rN$$

$$\text{s.t.} \quad y_t = \beta_0 + \min[\beta_1 N, \theta v] + u_t + \varepsilon_t \text{ and } N \geq 0.$$

where R_t is the producer's net return at time t, $E(y_t)$ is the expected wheat yield, and p is the price of a bushel of wheat. Yield is function of amount of nitrogen, which is under the control of the farmer, but also function of unobservable and uncontrolled factors such as weather, disease, and pests. The farmer objective is to decide how much nitrogen to apply to reach the highest expected net returns given wheat and nitrogen price and under the hypothesis that some of those factors listed above can occur.

Taking the expectation of the yield function specified leads to:

$$(3) \qquad E(y_t) = \beta_0 + E[\min(\beta_1 N, \theta . v)]$$

Under our assumption that the plateau shifter (θ) is beta distributed, we can define a probability density function (f) and a cumulative distribution function (F) associate with it. The expected yield function given the amount of nitrogen to be applied can be expressed as:

$$(4) \qquad E(y|N) = \beta_0 + \min\left\{\beta_1 N[1 - F(\beta_1 N)] + \int_{-\infty}^{\beta_1 N} \theta f(\theta) d\theta\right\}$$

Given the uncertainty associated with reaching a high yield in a given year, a producer choice problem will consist on maximizing expected utility of profit, which can be mathematically expressed as follows:

$$(5) \qquad \max_N E(\pi|N) = prob[\beta_0 + \min\left\{\beta_1 N[1 - F(\beta_1 N)] + \int_{-\infty}^{\beta_1 N} \theta f(\theta) d\theta\right\}] - rN$$

where $prob$ is the probability of the desired outcome. However, this might be used in further research since it is not the focus in this paper. Our optimization problem will consist of finding the optimal level of nitrogen that maximizes equation (2).

Bayesian Estimation of the Profit Maximizing Nitrogen for Wheat

The Bayesian approach is used to estimate the economically nitrogen level for crop production. We use a non-informative prior with an inequality constraint to determine the optimal level of nitrogen. All the difficulty associated with the Bayesian approach as opposed to the frequentist method is that it requires the user to specify a probability model for the data and to decide on the prior distribution for each of the parameters involved in the model (Glickman and van Dyk, 2003). Fortunately, with the new development in computer-based estimation methods, modeling even more complicated distributions have become less challenging. The SAS PROC MCMC provides an easy way to model any probability distributions using the prior statement to assign a specific distribution to the parameter and the model statement for associating a specific likelihood function to the data. Prior distributions for variances, expectations, and model coefficients are based on experiences or repeated observations.

Having defined the optimization problem as given in equation (4), we need to define the prior distribution of the parameters of the mean equation ($\beta_0, \beta_1, \theta, \sigma_u^2, \text{and } \sigma_\varepsilon^2$) and the parameters of the beta distribution ($\alpha \text{ and } \gamma$) before we can proceed in the MCMC modeling in SAS (the SAS code is attached in appendix 1). This approach based on prior distributions enables researcher to make inferences using parameters estimated using prior distributions from data instead of simple subjective knowledge about the parameters (Gelman, 2006). Several noninformative prior distributions were used in the literature including an improper uniform density, inverse-gamma distribution, and distributions that depend on the variance of the data used. In this paper, we use mixed distributions, with inverse-gamma distribution for the two variance parameters, a uniform distribution for the beta parameters, and a normal distribution for the intercept and the slope coefficient in the mean equation.

Given that the error term $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, we can express the yield distribution and the set of noninformative prior distribution can be expressed as:

(6) $\quad y_t \sim N(\beta_0 + \min[\beta_1 N, \theta v] + u_t)$

$\beta_0 \sim N(\mu_0, \sigma_0^2)$and $\beta_1 \sim N(\mu_1, \sigma_1^2)$

$\theta \sim igamma(0.01, scale), \sigma_u^2 \sim igamma(0.0001, scale), \sigma_\varepsilon^2 \sim igamma(0.0001, scale)$

$\alpha \sim uniform(0{,}50)$and $\gamma \sim uniform(0{,}50)$

with α and γ being the parameter of the distribution of v.

Researchers must also decide on how many burn-in samples to use to get the expected results. The number of Markov Chain Monte Carlo samples varies from one study to another. Since our model showed slow convergence we used a relatively large burn-in sample. In fact, we use 50,000 samples, which is relatively higher than the 1,000 and 10,000 samples commonly used in the literature.

Assuming our risk neutral wheat producer's objective function in (2), let's define the Bayesian estimation method necessary to obtain the optimal level of nitrogen that maximizes the objective function given our prior distributions.

From the fact that $v \sim Beta(\alpha, \gamma)$ and $E(y_i | v \geq \beta_0 + \min(\beta_1 N) = \beta_0 + \min(\beta_1 N)$ and $E(y_i | v \leq \beta_0 + \min(\beta_1 N) = \beta_0$, we can re-write equation (5) as:

(7) $\quad E(y_i) = (1 - \varphi)[\beta_0 + \min(\beta_1 N)] + \varphi \beta_0 \sigma_v^2$

where $\varphi = F(x|\alpha, \gamma) = \frac{1}{B(\alpha,\gamma)} \int_{\beta_0}^{x} t^{\alpha-1} (1-t)^{\gamma-1} dt$ is the cumulative distribution of the beta distribution characterizing the parameter v in the yield equation. Substituting (7) into (2) gives:

(8) $\quad E(R_t|N) = pE[(1 - \varphi)[\beta_0 + \min(\beta_1 N)] + \varphi \beta_0 \sigma_v^2] - rN$

To compute the optimal nitrogen level, we used a nonlinear programming method, to compute the expected profit for each year and then take the average level of nitrogen of all years. The expected profit equation used to determine the optimal level of nitrogen is given below:

(9) $E(\pi_t) = P_w\{\beta_0 + \min(\beta_1 N, \emptyset) - P_N * N\}/50000$

Where $E(\pi_t)$ is the expected profit in year t, $P_w$and P_N the price of a bushel of wheat and pound of nitrogen respectively, Ø a quantile function for the plateau parameter.

Experimental Data

Cross-sectional time series wheat data from Lahoma (OK) experiment station are used. The experiment (E502) was established in 1971 to evaluate the impacts of N, P, and K levels on winter wheat yield under conventional tillage on 4.9 by 18.3 m plots. The soil content of P and K was above the sufficiency level before the E502 was established and might have declined since then given that the experimental site has been continuously used since 1971 to now for wheat production without rotation.

Since the experiment did not show a significant yield increase due to P and, only the impacts of N will be the main focus in this analysis. The experiment was a randomized complete block design with four replications and six nitrogen levels: 0, 20, 40, 60, 80, and 100 lb/acre. Nitrogen was broadcast and incorporated into the soil before planting, which ranges from mid September to late October. Early harvests start at the beginning of June and the late harvests usually occur during the first half of July. Grains forming the center ten feet of each plot are collected using a conventional plot combine. A seeding rate of 60 pounds per acre was used and the planting and the harvesting date differ from one year to another, which might have significant impacts on wheat yield.

Eight wheat varieties were used since 1971 to 2010 at the Lahoma experimental site: *Nicoma* (1971-1974), *Triumph 64* (1975-1976 and 1978), *Osage* (1977-1979), *TAM W-101* (1980-1991), *Karl* (1993-1994), *Tonkawa* (1995-1998), *Custer* (1999-2004), and *Overley* (2005-2008). Two other varieties have been experimented since 2009 to now: *Endurance* (2009) and *Bullet* (2010-201). These varieties correspond to those grown by local farmers at that period.

One of the biggest issues remains whether or not wheat yield has exhibited a positive, negative, or a stagnant trend over the years. Ignoring such a trend if it really exists, may lead to an overestimation of the optimal nitrogen. Early study by Bennett (1938) on wheat yield trend since 1885 has clearly enumerated three regions corresponding to the three class of wheat yield trend and the U.S. hard winter regions belong to the group of no yield trend. Therefore, we will not assume yield trend in this paper as Oklahoma belongs to that part of the U.S.

Results

Parameters of the stochastic plateau model are estimated using the Markov Chain Monte Carlo simulation method and the maximum likelihood method. Results are reported in table 1. All the estimates from the maximum likelihood are statistically significant at the 5% significance level. The posterior distribution of the parameters of the mean equation as well as the beta parameters are presented in appendix. Both MLE and Bayesian method lead to approximately the same optimal amount of nitrogen per acre. The optimal level using the Bayesian estimation is slightly higher than that of the MLE, but that is not surprising since the Bayesian considers parameter uncertainty. An optimal nitrogen level of 82 and 80 lb/acre was found under the Bayesian and the maximum likelihood method, respectively.

Yield potential is estimated to be around 1510 bushels per acre under the Bayesian approach, slightly lower than that obtained with the maximum likelihood estimation. Results also show a high variance of the intercept shifter indicating that there are strong random factors affecting yield. Similarly at all levels of nitrogen, a slope coefficient of more than 20 indicates how important nitrogen represents in wheat production.

Those optimal nitrogen rates and the corresponding profit maximizing yields are substantially different from those obtained by Tembo et al (2008). When Tembo et al. (2008) predict that on average an amount of nitrogen within the range of 38 to 70 lbs/acre will be necessary to reach the highest expected yield; our results indicate that up to 80 lbs/acre will be required to achieve the plateau. This discrepancy of results is merely due to the fact that Tembo et al. (2008) have assumed normally distributed random variables.

Table 1. Estimates of Wheat Yield (lbs/acre) Response to Nitrogen

Parameter	Symbol	Parameter Estimates	
		MCMC	MLE
Intercept	β_0	1510.1	1628.95***
		(28.935)	(45.7546)
Slope coefficient	β_1	24.0556	21.7644***
		(0.991)	(0.9773)
Variance of the intercept shifter	σ_u^2	346172	50626***
		(89163.7)	(5280.95)
Variance of the error term	σ_ε^2	41193.5	219748***
		(4795.5)	(14862)
Alpha	α	1.401	2.8909***
		(0.449)	(0.2757)
Gamma	ɣ	25.458	6.3431***
		(11.663)	(0.6131)
Theta	θ	25601.6	3972.05***
		(15717)	(45.7546)
Optimal N	N^*	82.183[b]	80.1673***[a]
			(3.1557)
-2 Log likelihood			3237.8

Note: Standard errors are in parentheses. All the estimations were used assuming nitrogen price of \$0.6/lb and wheat price of \$0.14/bushel.
*** means that the estimated parameter is statistically significant at 1% significance level.
[a] Optimal N is estimated using the model developed by Tembo et al. (2008) under the assumption that the plateau shifter follows a beta distribution.
[b] The nonlinear optimization algorithm is used to estimate the optimal N under the Bayesian approach.

Conclusion

Profit maximizing level of nitrogen is estimated using data from a long-term wheat experiment at Lahoma (OK). A stochastic plateau model was used, but unlike previous papers, we assumed that the plateau shifter random effect follows a beta distribution consistent with the skewness observed in wheat yield.

The Bayesian method is the main focus in this estimation, but we used also the maximum likelihood method to see how different the results from those two approaches might be. The results indicate a slight difference between the optimal nitrogen rates obtained under the Bayesian approach and that under the maximum likelihood estimation. Nitrogen rate of 80 lbs per acre under the Bayesian method is slightly higher than the 80 lbs per acre obtained using MLE. This difference is due to the fact that the Bayesian method captured additional uncertainty associated with parameter estimation.

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Appendix 1 A: SAS codes for the Monte Carlo estimation

```
PARMS B0 1631;PARMS B1 15;
PARMS THETA 1042; PARMS ALPHA .2; PARMS GAMMA .1;
PARMS S2E 66000;PARMS S2U  277000;
PRIOR B0~ NORMAL(MEAN = 1500, VAR =1e3);
PRIOR B1~ NORMAL(MEAN = 13, VAR =1e2);
PRIOR THETA ~ IGAMMA(.01, SCALE=100);
PRIOR ALPHA ~ Uniform(0,50);
PRIOR GAMMA ~ Uniform(0,50);
PRIOR S2E: ~IGAMMA(.0001, SCALE=10000);
PRIOR S2U: ~IGAMMA(.0001, SCALE=10000);
P_i = CDF('NORMAL',v);
V_i= QUANTILE('BETA',p_i,ALPHA,MAX(GAMMA, 0.35),0,theta);
RANDOM U ~ NORMAL(0, VAR= S2U) SUBJECT = YEAR MONITOR = (U);
RANDOM v ~ NORMAL(0, VAR= 1) SUBJECT = YEAR MONITOR = (v);
MEAN= B0+ MIN(B1*NITROGEN, V_i)+ U;
MODEL YIELD~ NORMAL(MEAN, VAR = S2E);
RUN;
ods graphics off;
PROC NLP DATA = POSTOUT MAXITER = 2000 TECH = NEWRAP;
MAX EXPROFIT;
PARMS NSTAR = 60;
BOUNDS NSTAR >= 0; OPRICE = .14; IPRICE = .6;
if gamma < .35 then gamma=.35;
EXPROFIT1971 =(oprice*(B0+MIN(B1*NSTAR,quantile('beta',cdf('normal',V_1971),
alpha,gamma,0,theta)))-iprice*nstar)/50000;
(…)
EXPROFIT2008 =(oprice*(B0+MIN(B1*NSTAR,quantile('beta',cdf('normal',V_2008),
alpha,gamma,0,theta)))-iprice*nstar)/50000;
EXPROFIT = (EXPROFIT1971 + EXPROFIT1972  + EXPROFIT1974 + EXPROFIT1975 +
EXPROFIT1976 + EXPROFIT1977 + EXPROFIT1978 + EXPROFIT1979 + EXPROFIT1980 +
EXPROFIT1981 + EXPROFIT1982 + EXPROFIT1983 + EXPROFIT1984 + EXPROFIT1985 +
EXPROFIT1986 + EXPROFIT1987 + EXPROFIT1988 + EXPROFIT1989 + EXPROFIT1990 +
EXPROFIT1991 + EXPROFIT1992 + EXPROFIT1993 + EXPROFIT1994 + EXPROFIT1995 +
EXPROFIT1996 + EXPROFIT1997 +EXPROFIT1998 + EXPROFIT1999 + EXPROFIT2000 +
EXPROFIT2001 + EXPROFIT2002 + EXPROFIT2003 + EXPROFIT2004 +
EXPROFIT2005 + EXPROFIT2006 + EXPROFIT2007 + EXPROFIT2008)/37;

RUN;
```

Appendix 1 B: SAS codes for the MLE

```
PROC NLMIXED DATA = ONE  COV HESS TECH = trureg FCONV = .000000000001
optcheck=1 GCONV = .0000000000001 HESCAL = 2 NOAD QMAX = 299 QTOL = .01
MAXITER = 3000 METHOD = gauss;
PARMS B0 1650 B1 17.31 S2U100=250  S2E100=48  ALPHA10 .5 GAMMA10 .8 ;
s2u=s2u100*1000;s2e=s2e100*1000;GAMMA=GAMMA10/1;ALPHA=ALPHA10/1;
theta = 5601-b0;P_i = CDF('NORMAL',v_i,0,1);
V= QUANTILE('BETA',p_i,ALPHA,GAMMA,0,1);
MEAN= B0+ MIN(B1*N, theta* V)+U;
MODEL YIELD~ NORMAL(MEAN, S2E); ;
RANDOM U v_i~ normal([0, 0] ,[ s2u,0,1]) subject=YEAR;
ESTIMATE 'OPTIMAL N' (1/b1)*theta*Quantile('Beta',1-
.6/(.14*b1),alpha,gamma,0,1);
Estimate "s2e" s2e100*1000;
Estimate "s2u" s2u100*1000;
Estimate  "theta" theta;
RUN;
```

Appendix 2: Diagnostics for the parameters of the model