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Measuring and decomposing agricultural productivity and profitability change*

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Profitability change can be decomposed into the product of a total factor productivity (TFP) index and an index measuring changes in relative prices. Many TFP indexes can be further decomposed into measures of technical change, technical efficiency change, scale efficiency change and mix efficiency change. The class of indexes that can be decomposed in this way includes the Fisher, Törnqvist and Hicks–Moorsteen TFP indexes but not the Malmquist TFP index of Caves, Christensen and Diewert (1982). This paper develops data envelopment analysis methodology for computing and decomposing the Hicks–Moorsteen index. The empirical feasibility of the methodology is demonstrated using country-level agricultural data covering the period 1970–2001. The paper explains why relatively small countries tend to be the most productive, and why favourable movements in relative prices tend to simultaneously increase net returns and decrease productivity. Australia appears to have experienced this relative price effect since at least 1970. Thus, if Australia is a price-taker in output and input markets, Australian agricultural policy-makers should not be overly concerned about the estimated 15 per cent decline in agricultural productivity that has taken place over the last three decades.

Key words: economies of scale, economies of scope, mix efficiency, scale efficiency, technical change, technical efficiency.

1. Introduction

Improvements in agricultural productivity are a fundamental precondition for sustainable economic development. When agricultural productivity increases, resources including labour and capital can be released from food production to expand the nonagricultural sectors of the economy; resources such as land and water can also be used for environmental purposes. In coming decades, if populations continue to grow and natural resource stocks continue to be depleted, growth in agricultural productivity will become increasingly important for maintaining the environment and improving standards of living.

Effective public policy in this area requires the identification of the main drivers of productivity growth. In agriculture, two important drivers are technical progress and technical efficiency improvement. Technical progress

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mainly refers to expansions in the production possibilities set that come about through increased knowledge, while technical efficiency improvement refers to increases in output–input ratios made possible by, for example, eliminating mistakes in the production process. Public policies designed to improve agricultural productivity can be targeted at these different components. For example, policies designed to improve the rate of technical progress include increased funding for scientific research and development; complementary policies designed to increase technical efficiency include education, training and extension programs. By carefully defining the various components of agricultural productivity change, this paper provides insights into the ways different public policies can promote or retard growth. Included among these are policies that affect the prices of agricultural outputs and inputs.

To empirically measure the components of productivity growth, it is first necessary to have a precise definition of productivity and then a productivity index number formula that is consistent with this definition. In the case of multiple-output multiple-input firms, total factor productivity (TFP) can be defined as the ratio of an aggregate output to an aggregate input. This definition underpins the seminal work of Jorgenson and Griliches (1967) and is the definition used in this paper. With this definition, index numbers that measure changes in TFP can be expressed as the ratio of an output quantity index to an input quantity index (i.e., a measure of output growth divided by a measure of input growth). O'Donnell (2008) uses the term *multiplicatively complete* to refer to TFP indexes constructed in this way. The class of multiplicatively complete TFP indexes includes the well-known Paasche, Laspeyres, Fisher and Törnqvist indexes. However, the popular Malmquist TFP index of Caves *et al*. (1982) is not complete, implying that it may be an unreliable measure of TFP change.

O'Donnell (2008) shows that multiplicatively complete TFP indexes play a central role in the analysis of profitability change, technical change and efficiency change. First, he shows that profitability change can be written as the product of a multiplicatively complete TFP index and an index measuring changes in the terms of trade (TT).[1] Second, he shows that *all* multiplicatively complete TFP indexes can be decomposed into an unambiguous measure of technical change and several recognizable measures of efficiency change. Among the efficiency change components are familiar measures of input- and output-oriented technical and scale efficiency change. Other less-familiar components measure the increases in TFP that are possible when technically efficient firms are permitted to vary their output and/or input mix. These

[1] 'Terms of trade' refers to the relative prices (i.e., terms) at which goods and services are exchanged (i.e., traded). The terminology is usually used in an international trade context. However, Coelli *et al*. (2005, p. 63) also use it in the general context of individual firms (output price divided by input price), while ABARE (2007) use it with specific reference to the farming sector (index of prices received to prices paid). Alternative names for the terms of trade index include price recovery index and price performance index (Balk 2003, p. 22).

measures of so-called mix efficiency change are closely related to well-known measures of allocative efficiency change. However, the two sets of measures are generally only equivalent if aggregate quantities are constructed using particular aggregator functions.

This paper briefly summarizes the conceptual framework of O'Donnell (2008) and develops data envelopment analysis (DEA) methodology for computing and decomposing the multiplicatively complete Hicks–Moorsteen TFP index. This new methodology is then applied to country-level agricultural data covering the period 1970–2001.

The structure of the paper is as follows. Section 2 outlines the mathematical relationship between profitability, TFP and the TT. Underpinning this relationship is the definition of TFP as the ratio of an aggregate output to an aggregate input. Section 3 defines well-known measures of technical and scale efficiency in terms of these aggregates. It also explains O'Donnell's (2008) concepts of mix and TFP efficiency. Section 4 shows how any multiplicatively complete TFP index can be decomposed into a potentially infinite number of efficiency and technical change components. Attention is then concentrated on a handful of exhaustive decompositions that are economically meaningful. Section 5 explains that if Shephard (1953) distance functions are used to aggregate inputs and outputs, then the resulting quantity indexes are the Malmquist indexes of Caves *et al.* (1982). The Hicks–Moorsteen TFP index is the ratio of these Malmquist indexes and can therefore be expressed in terms of quantity aggregates. Section 6 develops DEA methodology for computing and decomposing the Hicks–Moorsteen index. More familiar DEA linear programs (LPs) for decomposing Malmquist TFP indexes are also motivated and presented. Section 7 applies the methodology to FAO data on the agricultural inputs and outputs of 88 countries. The discussion in this section centres on the components of agricultural TFP change in Australia, New Zealand and the United States. The paper is concluded in Section 8.

2. Productivity and profitability change

Some measures of economic and business performance are well defined and understood. In the case of a single-input single-output firm, for example, economists have well-defined measures of profit (revenue minus cost), profitability (revenue–cost ratio),[2] productivity (output–input ratio) and the TT (relative prices at which goods and services are traded). Even in the case of multiple-input multiple-output firms, the concepts of productivity, efficiency and profitability are conceptually and mathematically well defined. This has important implications for the measurement and decomposition of productivity and profitability change. It means, for example, that a measure of the

[2] For this definition, see Althin *et al.* (1996), Balk (2003) and Coelli *et al.* (2005, p. 63).

change in the TT cannot be computed without defining, implicitly or explicitly, a measure of the change in TFP.

Let $q_{nt} \in \mathbb{R}_{+}^{J}$ and $x_{nt} \in \mathbb{R}_{+}^{K}$ denote the observed output and input vectors of firm n in period t, and let $p_{nt} \in \mathbb{R}_{+}^{J}$ and $w_{nt} \in \mathbb{R}_{+}^{K}$ denote the associated price vectors. In this paper, TFP is defined as (e.g., Jorgenson and Griliches 1967; O'Donnell 2008):

$$TFP_{nt} \equiv \frac{Q_{nt}}{X_{nt}} \tag{2.1}$$

where $Q_{nt} = Q(q_{nt})$ is an aggregate output, $X_{nt} = X(x_{nt})$ is an aggregate input, and $Q(.)$ and $X(.)$ are nondecreasing linearly homogeneous aggregator functions. With this definition, the index number that compares the TFP of firm n in period t with the TFP of firm m in period s is

$$TFP_{ms.nt} \equiv \frac{TFP_{nt}}{TFP_{ms}} = \frac{Q_{nt}/X_{nt}}{Q_{ms}/X_{ms}} = \frac{Q_{ms,nt}}{X_{ms,nt}} \tag{2.2}$$

where $Q_{ms,nt} \equiv Q_{nt}/Q_{ms}$ is an output quantity index and $X_{ms,nt} \equiv X_{nt}/X_{ms}$ is an input quantity index. Equation (2.2) demonstrates that TFP change can be written as an index of output growth divided by an index of input growth.

O'Donnell (2008) uses the term multiplicatively complete to describe TFP indexes that can be expanded in the form of Equation (2.2). He also demonstrates that all such TFP indexes can be decomposed into a measure of technical change and several measures of efficiency change. His demonstration is aided greatly by the ability to depict the TFP of a multiple-input multiple-output firm in two-dimensional aggregate quantity space. The basic idea is illustrated in Figure 1. In this figure, the TFP of firm n in period t is given

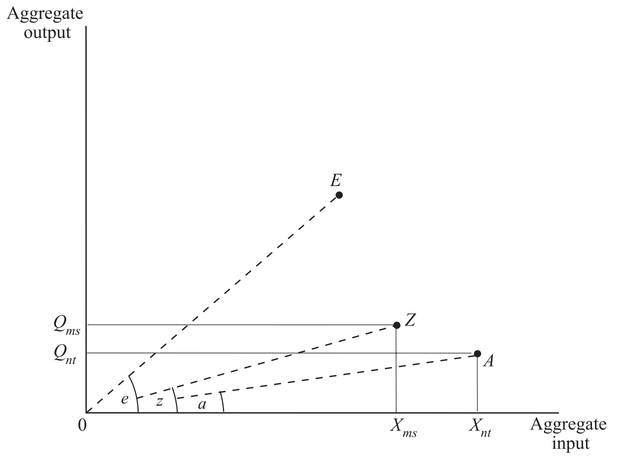

Figure 1 Total factor productivity change.

by the slope of the ray passing through the origin and point A, while the TFP of firm m in period s is given by the slope of the ray passing through the origin and point Z. Let lower case a and z denote the angles between the horizontal axis and the rays passing through points A and Z. Then, the TFP index that measures the change in TFP between the two firms can be compactly written $TFP_{ms,nt} = \tan a/\tan z$. This ability to write a multiplicatively complete TFP index as the ratio of (tangent) functions of angles in aggregate quantity space is used by O'Donnell (2008) to conceptualize several alternative decompositions of TFP change. For example, let e denote the angle between the horizontal axis and the ray passing through the origin and *any* non-negative point E. Then, it is clear from Figure 1 that the difference in TFP between the two firms can be decomposed as $TFP_{ms,nt} = \tan a/\tan z = (\tan a/\tan e)(\tan e/\tan z)$. An infinite number of points E can be used to effect such a decomposition. Section 3 considers the points that feature in common definitions of input- and output-oriented measures of efficiency.

Associated with the aggregate quantities Q_{nt} and X_{nt} are aggregate prices P_{nt} and W_{nt} with the properties $P_{nt}Q_{nt} = p'_{nt}q_{nt}$ and $W_{nt}X_{nt} = w'_{nt}x_{nt}$. These properties are known as *product rules* and are trivially satisfied. The existence of these price aggregates means that profitability (the revenue–cost ratio) can be written as

$$PROF_{nt} \equiv \frac{p'_{nt}q_{nt}}{w'_{nt}x_{nt}} = \frac{P_{nt}Q_{nt}}{W_{nt}X_{nt}}. \tag{2.3}$$

Moreover, the index number that compares the profitability of firm n in period t with the profitability of firm m in period s is

$$PROF_{ms,nt} \equiv \frac{PROF_{nt}}{PROF_{ms}} = \frac{P_{ms,nt}}{W_{ms,nt}}\frac{Q_{ms,nt}}{X_{ms,nt}} = TT_{ms,nt} \times TFP_{ms,nt} \tag{2.4}$$

where $P_{ms,nt} \equiv P_{nt}/P_{ms}$ is an output price index, $W_{ms,nt} \equiv W_{nt}/W_{ms}$ is an input price index, and $TT_{ms,nt} \equiv P_{ms,nt}/W_{ms,nt}$ is a terms of trade index measuring the growth in output prices relative to the growth in input prices. Thus, profitability change can be written as the product of a multiplicatively complete TFP index and an index measuring the change in the TT.

An important implication of Equation (2.4) is that changes in the TT can be expected to induce changes in productivity, at least in the case of firms who have access to a variable returns to scale (VRS) production technology and whose preferences are strictly increasing in net returns.[3] Such a case is

[3] The argument presented in this paragraph is still valid even if the technology exhibits constant returns to scale (CRS), provided it exhibits variable returns to scope – the production frontier will also be strictly concave in this case. For details concerning the characteristics of production frontiers represented in aggregate quantity space, see O'Donnell (2008).

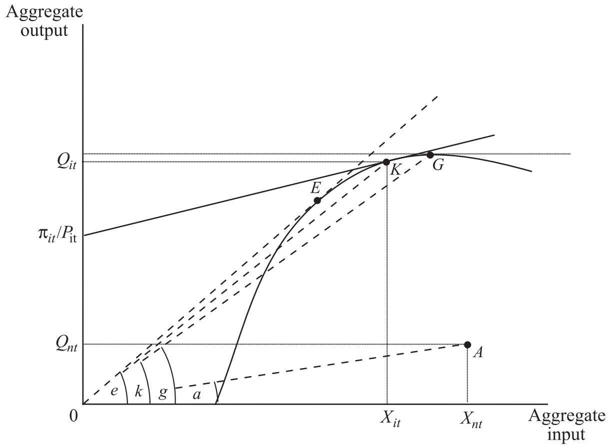

Figure 2 Productivity, profitability and the terms of trade.

illustrated in Figure 2, where the curved line passing through points E, K and G is a VRS production frontier. Point E is the aggregate input–output combination that maximizes TFP but not necessarily the combination that maximizes profit. To see this, suppose firm *i* in period *t* faces aggregate output and input prices P_{it} and W_{it}. For this firm, profit is $\pi_{it} = P_{it}Q_{it} - W_{it}X_{it}$ and isoprofit equations take the form $Q_{it} = (\pi_{it}/P_{it}) + (W_{it}/P_{it})X_{it}$. It is apparent from Figure 2 that if the maximum TFP possible using the production technology (the slope of the ray through point E) equals the inverse of the TT (the slope of the isoprofit line), then the TFP-maximizing and profit-maximizing points will coincide at point E and profits will be zero. It is also clear that if the firm has a benefit function that is increasing in profits, then an improvement in the TT will draw the optimizing firm away from the point of maximum productivity to a point where profits are strictly positive – the isoprofit line passing through point K, for example, has slope $W_{it}/P_{it} < \tan e$ and intercept $\pi_{it}/P_{it} > 0$. Further improvements in the TT will lead to further reductions in productivity until the firm reaches point G. At this limiting point, the slope of the isoprofit line (the normalized input price) is zero and aggregate output is maximized. The curved segment between points E and G is the region of (locally) nonincreasing returns to scale (NIRS). It is apparent from Figure 2 that productivity will be maximized by the smallest efficient firm that operates in this region.[4]

[4] Productivity could also be maximized by the largest efficient firm operating in the region of strictly increasing returns to scale (IRS). Such firms are ignored in this paper because rational efficient profit-maximizing firms cannot generally operate in the region of IRS without incurring a loss.

3. Measures of efficiency

O'Donnell (2008) measures the overall productive efficiency of a firm as the ratio of observed TFP to the maximum TFP possible using the available technology.[5] Mathematically, the so-called TFP efficiency of firm n in period t is

$$TFPE_{nt} = \frac{TFP_{nt}}{TFP_t^*} = \frac{Q_{nt}/X_{nt}}{Q_{nt}^*/X_{nt}^*} \qquad \text{(TFP efficiency)} \tag{3.1}$$

where TFP_t^* denotes the maximum TFP possible using the period-t technology, and Q_t^* and X_t^* denote the aggregate output and aggregate input at the TFP-maximizing point (point E in Figure 2). O'Donnell (2008) also defines several other measures of efficiency in terms of aggregate quantities. For example, the efficiency measures that feature in an output-oriented decomposition of TFP change are

$$OTE_{nt} = \frac{Q_{nt}}{\bar{Q}_{nt}}, \qquad \text{(output–oriented technical efficiency)} \tag{3.2}$$

$$OSE_{nt} = \frac{\bar{Q}_{nt}/X_{nt}}{\tilde{Q}_{nt}/\tilde{X}_{nt}}, \qquad \text{(output–oriented scale efficiency)} \tag{3.3}$$

$$OME_{nt} = \frac{\bar{Q}_{nt}}{\hat{Q}_{nt}}, \qquad \text{(output–oriented mix efficiency)} \tag{3.4}$$

$$ROSE_{nt} = \frac{\hat{Q}_{nt}/X_{nt}}{Q_t^*/X_t^*} \qquad \text{(residual output–oriented scale efficiency) and} \tag{3.5}$$

$$RME_{nt} = \frac{\tilde{Q}_{nt}/\tilde{X}_{nt}}{Q_{nt}^*/X_{nt}^*} \qquad \text{(residual mix efficiency)} \tag{3.6}$$

where $\bar{Q}_{nt}$ is the maximum aggregate output that is technically feasible when x_{nt} is used to produce a scalar multiple of q_{nt}; $\hat{Q}_{nt}$ is the maximum aggregate output that is feasible when using x_{nt} to produce any output vector; and $\tilde{Q}_t$ and $\tilde{X}_t$ are the aggregate output and input obtained when TFP is maximized subject to the constraint that the output and input vectors are scalar multiples of q_{nt} and x_{nt}, respectively.

The OTE measure (3.2) is the measure proposed by Farrell (1957), while the scale efficiency measure (3.3) is that discussed by, for example, Balk (2001). The OME measure (3.4) is not well known, although it is closely

[5] An input–output combination that maximizes productivity will exist if, for example, the production technology is nondecreasing in inputs and at least one input is nonzero for any level of output. If a point of maximum productivity does not exist then, of all the efficiency measures discussed in this section, only the measure of TFP efficiency will be undefined.

related to a measure of revenue-allocative efficiency first proposed by Farrell (1957). O'Donnell (2008) motivates this unfamiliar measure by considering a two-output case where the aggregator function is linear: $Q(q_{nt}) = \alpha_1 q_{1nt} + \alpha_2 q_{2nt}$. Figure 3 depicts this special case in output space. In this figure, the curve passing through points V and C is the familiar production frontier representing all technically efficient output combinations that can be produced using x_{nt}. The dashed line passing through point A is an isooutput line that maps all output combinations that have the same aggregate output as at point A. If the output mix and the input vector are held fixed, then aggregate output and TFP are maximized by radially expanding outputs to point C. However, if restrictions on the output mix are relaxed, aggregate output and TFP are maximized by moving around the frontier to point V. The ratio of the distance 0A to the distance 0C in Figure 3 is the output-oriented measure of technical efficiency defined by Equation (3.2): $OTE_{nt} = Q_{nt}/\bar{Q}_{nt} = \|0A\|/\|0C\|$. The ratio of the distance 0H to the distance 0V is the output-oriented measure of mix efficiency defined by (3.4): $OME_{nt} = \bar{Q}_{nt}/\hat{Q}_{nt} = \|0H\|/\|0V\|$. Thus, the OME of a technically efficient firm is simply a measure of the increase in TFP that comes about by holding inputs fixed and relaxing restrictions on output mix. In the same way that scale efficiency is a measure of the potential productivity gains that can be achieved through economies of scale, mix efficiency is a measure of the potential gains that can be achieved through economies of scope.

O'Donnell (2008) provides further insights into the relationships between aggregate quantities and measures of efficiency by mapping multiple-input multiple-output production points into aggregate quantity space. Figure 4 presents such a mapping for the input–output combinations represented by points A, C and V in Figure 3. In Figure 4, the curve passing through point C represents the frontier of a *restricted* production possibilities set. The set is

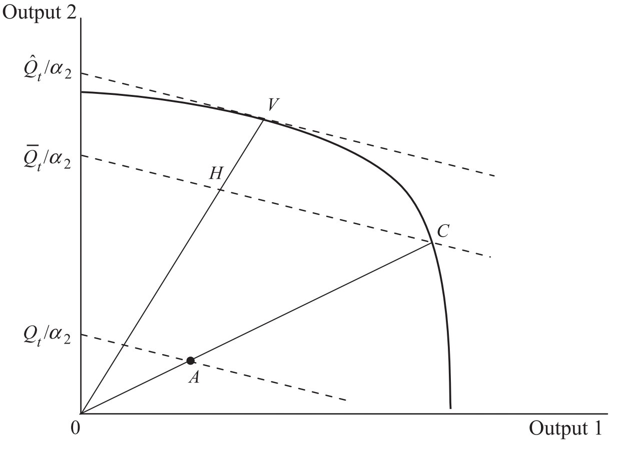

Figure 3 Output-oriented mix efficiency for a two-output firm.

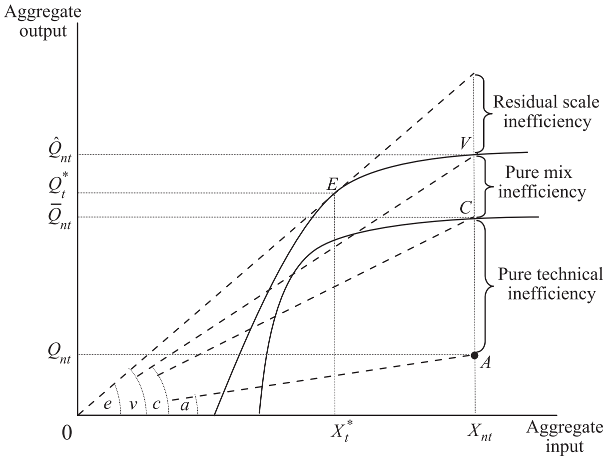

Figure 4 Output-oriented measures of efficiency.

restricted in the sense that it only contains input and output vectors that can be written as scalar multiples of x_{nt} and q_{nt}. When these mix restrictions are relaxed, the firm has access to the expanded production possibilities set bounded by the curve passing through points V and E. OTE measures the proportionate increase in TFP that occurs as the firm moves from point A to point C on the restricted frontier; OME measures the increase in TFP as the firm moves from C to V on the unrestricted frontier; and the measure of ROSE defined by (3.5) measures the increase in TFP as the firm moves around the unrestricted frontier from V to E. O'Donnell (2008) uses the term *scale* for this last measure because 'any movement around an unrestricted production frontier is a movement from one mix-efficient point to another, so any improvement in TFP is essentially a scale effect … [He also uses] the term *residual* because, even though all the points on the unrestricted frontier are mix-efficient, they may nevertheless have different input and output mixes … Thus, what is essentially a measure of scale efficiency may contain a residual mix effect' (p. 15).

Two related output-oriented measures of efficiency are depicted in Figure 5. In this figure, point D represents the input–output combination that maximizes TFP when the input and output mixes of the firm operating at point A are held fixed. For this reason, point D is known as the point of *mix-invariant optimal scale* (MIOS) (for the firm operating at point A). The measure of residual mix efficiency defined by (3.6) is a measure of the proportionate increase in TFP that occurs as the firm moves from point D to point E. The term residual is also used here because, although the move from point D on the mix-restricted frontier to point E on the unrestricted frontier is primarily a mix effect, it may also involve a change in scale. Residual mix efficiency can also be viewed as the component of TFP

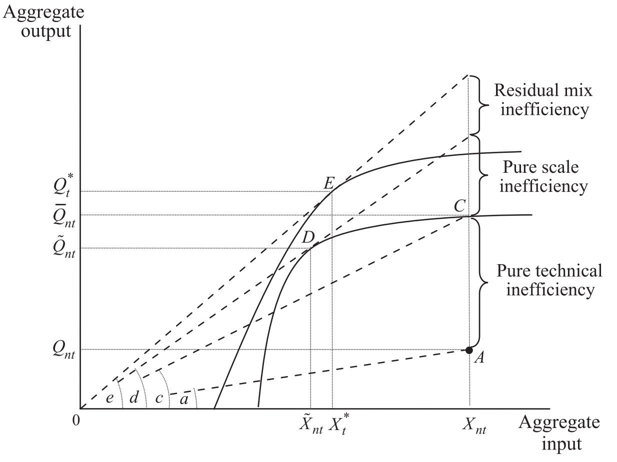

Figure 5 Output-oriented measures of efficiency.

change that remains after accounting for pure technical efficiency and pure scale efficiency effects.

Finally, the measure of TFP efficiency defined by Equation (3.1) measures the proportionate increase in TFP as the firm moves all the way from point A to point E. Figures 4 and 5 illustrate two of many possible pathways from A to E and therefore illustrate two of many possible decompositions of TFP efficiency: in terms of angles,

$$TFPE_{nt} = \frac{\tan a}{\tan e} = \left(\frac{\tan a}{\tan c} \times \frac{\tan c}{\tan v} \times \frac{\tan v}{\tan e}\right) = \left(\frac{\tan a}{\tan c} \times \frac{\tan c}{\tan d} \times \frac{\tan d}{\tan e}\right); \quad (3.7)$$

in terms of aggregate quantities,

$$TFPE_{nt} = \frac{Q_{nt}/X_{nt}}{Q^*_{nt}/X^*_{nt}} = \left(\frac{Q_{nt}}{\bar{Q}_{nt}} \times \frac{\bar{Q}_{nt}}{\hat{Q}_{nt}} \times \frac{\hat{Q}_{nt}/X_{nt}}{Q^*_t/X^*_t}\right) = \left(\frac{Q_{nt}}{\bar{Q}_{nt}} \times \frac{\bar{Q}_{nt}/X_{nt}}{\tilde{Q}_{nt}/\tilde{X}_{nt}} \times \frac{\tilde{Q}_{nt}/\tilde{X}_{nt}}{Q^*_{nt}/X^*_{nt}}\right); \quad (3.8)$$

and in terms of measures of efficiency,

$$\begin{aligned} TFPE_{nt} &= \frac{TFP_{nt}}{TFP^*_t} = (OTE_{nt} \times OME_{nt} \times ROSE_{nt}) \\ &= (OTE_{nt} \times OSE_{nt} \times RME_{nt}). \end{aligned} \quad (3.9)$$

4. The components of TFP change

The measures of efficiency defined in Section 3 provide a basis for an output-oriented decomposition of a multiplicatively complete TFP index. An easy way to see this is to rewrite Equation (3.9) as

$$\begin{aligned} TFP_{nt} &= TFP_t^* \times (OTE_{nt} \times OME_{nt} \times ROSE_{nt}) \\ &= TFP_t^* \times (OTE_{nt} \times OSE_{nt} \times RME_{nt}). \end{aligned} \tag{4.1}$$

A similar equation holds for firm m in period s. It follows that the index that compares the TFP of firm n in period t with the TFP of firm m in period s can be written

$$\begin{aligned} TFP_{ms,nt} = \frac{TFP_{nt}}{TFP_{ms}} &= \left(\frac{TFP_t^*}{TFP_s^*}\right) \times \left(\frac{OTE_{nt}}{OTE_{ms}} \times \frac{OME_{nt}}{OME_{ms}} \times \frac{ROSE_{nt}}{ROSE_{ms}}\right) \\ &= \left(\frac{TFP_t^*}{TFP_s^*}\right) \times \left(\frac{OTE_{nt}}{OTE_{ms}} \times \frac{OSE_{nt}}{OSE_{ms}} \times \frac{RME_{nt}}{RME_{ms}}\right). \end{aligned} \tag{4.2}$$

The first term in parentheses on the right-hand side of Equation (4.2) is a measure of technical change – it measures the difference between the maximum TFP possible using the period-t technology and the maximum TFP possible using the period-s technology. This can be seen in Figure 6 where, in terms of angles, $TFP_t^*/TFP_s^* = \tan e/\tan r$. The economy/industry experiences technical progress or regress as TFP_t^*/TFP_s^* is greater than or less than one. The other ratios on the right-hand side of Equation (4.2) are obvious measures of technical efficiency change, (residual) mix efficiency change and (residual) scale efficiency change.

O'Donnell (2008) derives the input-oriented counterparts to Equations (4.1) and (4.2) and demonstrates that the input- and output-oriented measures of technical change are plausibly identical. For the sake of completeness, the input-oriented analogue of Equation (4.2) is

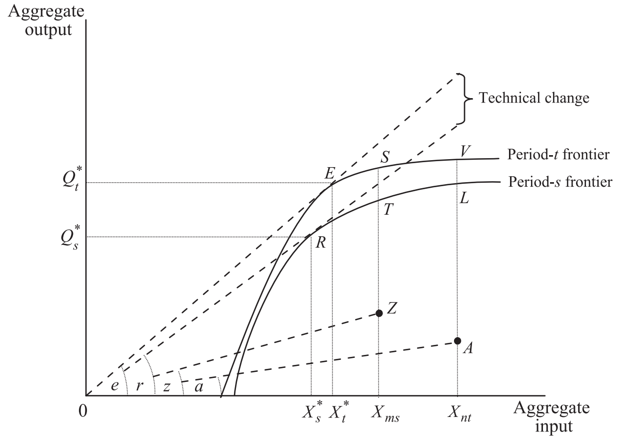

Figure 6 Technical change.

$$
\begin{aligned}
TFP_{ms,nt} = \frac{TFP_{nt}}{TFP_{ms}} &= \left(\frac{TFP_t^*}{TFP_s^*}\right) \times \left(\frac{ITE_{nt}}{ITE_{ms}} \times \frac{IME_{nt}}{IME_{ms}} \times \frac{RISE_{nt}}{RISE_{ms}}\right) \\
&= \left(\frac{TFP_t^*}{TFP_s^*}\right) \times \left(\frac{ITE_{nt}}{ITE_{ms}} \times \frac{ISE_{nt}}{ISE_{ms}} \times \frac{RME_{nt}}{RME_{ms}}\right)
\end{aligned} \tag{4.3}
$$

where

$$ITE_{nt} = \frac{\bar{X}_{nt}}{X_{nt}} \quad \text{(input−oriented technical efficiency)}, \tag{4.4}$$

$$ISE_{nt} = \frac{Q_{nt}/\bar{X}_{nt}}{\tilde{Q}_{nt}/\tilde{X}_{nt}} \quad \text{(input−oriented scale efficiency)}, \tag{4.5}$$

$$IME_{nt} = \frac{\hat{X}_{nt}}{\bar{X}_{nt}} \quad \text{(input−oriented mix efficiency)}, \tag{4.6}$$

$$RISE_{nt} = \frac{Q_{nt}/\hat{X}_{nt}}{Q_{nt}^*/X_{nt}^*} \quad \text{(residual input−oriented scale efficiency)} \tag{4.7}$$

and RME_{nt} is the measure of residual mix efficiency defined by Equation (3.6). In Equations (4.4) to (4.7), $\bar{X}_{nt}$ is the minimum aggregate input that is possible when using a scalar multiple of x_{nt} to produce q_{nt}, while $\hat{X}_{nt}$ is the minimum aggregate input possible when using *any* input vector to produce q_{nt}.

Finally, it is important to remember that the decompositions given by Equations (4.2) and (4.3) are only four of a potentially infinite number of exhaustive decompositions of a multiplicatively complete TFP index. These particular decompositions have been chosen because they can be expressed in terms of well-known measures of output- and input-oriented technical and scale efficiency, as well as some unfamiliar measures of mix and residual (scale and mix) efficiency. Other exhaustive decompositions of TFP change may also involve new definitions of efficiency. For example, one local measure of technical change is the difference in TFP at points V and L in Figure 6, and another is the difference in TFP at points S and T. If the geometric mean of these two local measures is used to decompose the difference in TFP at points A and Z, then, in terms of angles,

$$TFP_{ms,nt} = \frac{\tan a}{\tan z} = \left(\frac{\tan v}{\tan l}\frac{\tan s}{\tan t}\right)^{1/2}\left(\frac{\tan a}{\tan v}\frac{\tan t}{\tan z}\right)\left(\frac{\tan v}{\tan s}\frac{\tan l}{\tan t}\right)^{1/2}. \tag{4.8}$$

The first term in parentheses on the right-hand side represents technical change, the second term is the combined change in the measures of output-oriented technical and mix efficiency defined by (3.2) and (3.4), while the last term is the change in a measure of scale and mix efficiency that is yet to be defined in the efficiency literature. Observe that the decomposition given by (4.8) does not involve points of maximum productivity (points E and R in Figure 6) and is therefore feasible if such points do not exist.

5. Hicks–Moorsteen and Malmquist TFP indexes

With a view to eventually computing and decomposing Hicks–Moorsteen TFP indexes, it is convenient to represent the production technology using the Shephard (1953) output and input distance functions:

$$D_O^t(x, q) = \min_{\delta}\{\delta > 0 : (x, q/\delta) \in T^t\} \text{ and} \tag{5.1}$$

$$D_I^t(x, q) = \max_{\rho}\{\rho > 0 : (x/\rho, q) \in T^t\} \tag{5.2}$$

where T^t denotes the period-t production possibilities set. The output distance function measures the inverse of the largest radial expansion of the output vector that is possible while holding the input vector fixed, while the input distance function measures the largest radial contraction of the input vector that is feasible while holding the output vector fixed. The output distance is the Farrell (1957) output-oriented measure of technical efficiency, while the input distance is the inverse of the Farrell (1957) input-oriented measure. Irrespective of the returns to scale or scope properties of the technology, the output and input distance functions are linearly homogeneous and nondecreasing in output quantities and input quantities, respectively.

The homogeneity and monotonicity properties of Shephard distance functions make them natural candidates for use as output and input aggregator functions. Indeed, for the types of binary comparisons discussed in Section 2, it is common to implicitly aggregate output quantities using the period-t output distance function defined over the inputs of firm n in period t: $Q(q) = D_O^t(x_{nt}, q)$. If this aggregator function is used, then the index that compares the outputs of firm n in period t with the outputs of firm m in period s is the period-t Malmquist output quantity index of Caves *et al.* (1982, p. 1400):

$$Q_{ms,nt} = \frac{Q(q_{nt})}{Q(q_{ms})} = \frac{D_O^t(x_{nt}, q_{nt})}{D_O^t(x_{nt}, q_{ms})}. \tag{5.3}$$

The monotonicity and homogeneity properties of the output distance function ensure that this index satisfies the basic axioms of index number theory, including monotonicity, homogeneity, identity and proportionality. The identity axiom, for example, means the index only departs from unity as q_{nt} departs from q_{ms}. Another index of output change that satisfies these axioms is obtained by aggregating outputs using the geometric average of the period-t and period-s distance functions defined over the input vectors of the comparison and reference firms:

$$Q(q) = \left[D_O^t(x_{nt}, q) D_O^s(x_{ms}, q)\right]^{1/2}. \quad (5.4)$$

The associated Malmquist output quantity index is

$$Q_{ms,nt}^M = \left(\frac{D_O^t(x_{nt}, q_{nt}) D_O^s(x_{ms}, q_{nt})}{D_O^t(x_{nt}, q_{ms}) D_O^s(x_{ms}, q_{ms})} \right)^{1/2}. \quad (5.5)$$

Similarly, if inputs are aggregated using the geometric average of the period-t and period-s input distance functions,

$$X(x) = \left[D_I^t(x, q_{nt}) D_I^s(x, q_{ms})\right]^{1/2}, \quad (5.6)$$

then the associated Malmquist input quantity index is

$$X_{ms,nt}^M = \left(\frac{D_I^t(x_{nt}, q_{nt}) D_I^s(x_{nt}, q_{ms})}{D_I^t(x_{ms}, q_{nt}) D_I^s(x_{ms}, q_{ms})} \right)^{1/2}. \quad (5.7)$$

Finally, the index formed as the ratio of the Malmquist output and input quantity indexes given by (5.5) and (5.7) is a TFP index that has been attributed by Diewert (1992, p. 240) to Hicks (1961) and Moorsteen (1961):

$$TFP_{ms,nt}^{HM} = \frac{Q_{ms,nt}^M}{X_{ms,nt}^M} = \left[\frac{D_O^t(x_{nt}, q_{nt}) D_O^s(x_{ms}, q_{nt})}{D_O^t(x_{nt}, q_{ms}) D_O^s(x_{ms}, q_{ms})} \frac{D_I^t(x_{ms}, q_{nt}) D_I^s(x_{ms}, q_{ms})}{D_I^t(x_{nt}, q_{nt}) D_I^s(x_{nt}, q_{ms})} \right]^{1/2}. \quad (5.8)$$

By the manner of its construction, this so-called Hicks–Moorsteen index is consistent with the fundamental definition of TFP as the ratio of an aggregate output to an aggregate input – in the terminology of O'Donnell (2008), it is multiplicatively complete. Two related indexes that are *not* multiplicatively complete are

$$TFP_{ms,nt}^{IM} = \left[\frac{D_I^t(x_{ms}, q_{ms}) D_I^s(x_{ms}, q_{ms})}{D_I^t(x_{nt}, q_{nt}) D_I^s(x_{nt}, q_{nt})} \right]^{1/2} \text{and} \quad (5.9)$$

$$TFP_{ms,nt}^{OM} = \left[\frac{D_O^t(x_{nt}, q_{nt}) D_O^s(x_{nt}, q_{nt})}{D_O^t(x_{ms}, q_{ms}) D_O^s(x_{ms}, q_{ms})} \right]^{1/2}. \quad (5.10)$$

These are input- and output-oriented Malmquist TFP indexes of the type proposed by Caves *et al.* (1982). This class of indexes cannot in general[6] be expressed as the ratio of an output quantity index to an input quantity index. Nevertheless, Malmquist TFP indexes such as these are widely used in practice, possibly because they can be easily decomposed. For example, Färe *et al.* (1994, p. 71) rewrite the input-oriented index as

[6] A sufficient condition is that the input and output distance functions exhibit constant returns to scale (CRS) and inverse homotheticity – see Färe, Grosskopf and Roos (1998, p. 136).

$$TFP^{IM}_{ms,nt} = \frac{D^s_I(x_{ms}, q_{ms})}{D^t_I(x_{nt}, q_{nt})} \times \left(\frac{D^t_I(x_{nt}, q_{nt}) D^t_I(x_{ms}, q_{ms})}{D^s_I(x_{nt}, q_{nt}) D^s_I(x_{ms}, q_{ms})}\right)^{1/2}. \quad (5.11)$$

The first term on the right-hand side is the change in the input-oriented technical efficiency measure defined by Equation (4.4) – it is unambiguously a measure of pure input-oriented technical efficiency change. The second term is interpreted by Färe *et al*. (1994, p. 71) as a measure of technical change. There are no measures of scale or mix efficiency change in this decomposition.

6. Using DEA to compute and decompose TFP indexes

In principle, any multiplicatively complete TFP index can be decomposed using the framework developed by O'Donnell (2008). This section develops the DEA problems needed to compute and decompose the Hicks–Moorsteen index. This index was selected from among the class of multiplicatively complete indexes primarily because it is a distance-based index and DEA methodology for estimating distances is relatively straightforward. A second reason is that it is closely related to the Malmquist index that has for some time been the index number of choice in the productivity decomposition literature. For purposes of comparison, this section also presents the DEA problems used by Färe *et al*. (1994, p. 71) to compute and decompose the Malmquist index.

6.1 The DEA approach

Both input- and output-oriented DEA models are underpinned by the assumption that the production frontier is locally linear. In the input-oriented case, local linearity means that for any input vectors in the neighbourhood of x_{nt}, the production frontier takes the linear form

$$\mu' q_{nt} = \alpha + \upsilon' x_{nt} \quad (6.1)$$

where μ and υ are non-negative and α is unsigned. The fact that α is unsigned means the technology potentially exhibits VRS: if $\alpha < 0$ the technology exhibits local increasing returns to scale (IRS); if $\alpha \geq 0$ it exhibits local NIRS; if $\alpha > 0$ it exhibits local decreasing returns to scale (DRS); and if $\alpha = 0$ it exhibits local constant returns to scale (CRS). In the output-oriented case, local linearity means that for output vectors in the neighbourhood of q_{nt}, the production frontier takes the form

$$\eta' q_{nt} = \beta + \phi' x_{nt} \quad (6.2)$$

where η and υ are non-negative and the intercept β is again unsigned to allow for VRS. Different notation is used for the parameters in (6.1) and (6.2) to make it clear that they are defined with reference to possibly different neighbourhoods.

Associated with the (local) frontiers (6.1) and (6.2) are the (local) input and output distance functions

$$D_I^t(x_{nt}, q_{nt}) = \frac{\upsilon' x_{nt}}{\mu' q_{nt} - \alpha} \geq 1 \text{ and} \tag{6.3}$$

$$D_O^t(x_{nt}, q_{nt}) = \frac{\eta' q_{nt}}{\beta + \phi' x_{nt}} \leq 1. \tag{6.4}$$

DEA involves selecting values of the unknown parameters to minimize the value of the input distance function (6.3) and/or maximize the value of the output distance function (6.4).

A word of caution is in order concerning notation. Strictly speaking, the unknown parameters in Equations (6.1) to (6.4) should also have firm and time subscripts to indicate that these relationships only hold for observations in the neighbourhoods of x_{nt} (in the input-oriented case) and q_{nt} (in the output-oriented case). Different functions (i.e., different parameters) may be relevant in the neighbourhoods of other input and output vectors x_{ms} and q_{ms}. In this paper, these subscripts are suppressed, partly for notational simplicity but mainly for consistency with the way DEA problems are presented in the efficiency literature. However, it needs to be remembered that these parameters may change from point to point, so the ratios on the right-hand sides of Equations (6.3) and (6.4) cannot be blindly substituted into Equations (5.4) and (5.6) to identify the aggregate inputs and outputs corresponding to different input and output vectors.

6.2 Primal problems

The primal input-oriented DEA problem involves selecting values of μ, υ and α to minimize $D_I^t(x_{nt}, q_{nt})$ (or, equivalently, choosing parameters to maximize its inverse). Aside from the non-negativity restrictions on μ and υ, the only constraints on the parameters are that they must satisfy $D_I^t(x_{ir}, q_{ir}) \geq 1$ for $i = 1, \ldots, N$ and $r = 1, \ldots, t$. Imposing these constraints at these particular data points (i.e., at the input–output choices of all firms in all periods up to and including period t) implicitly prohibits technical regress; if technical regress is to be permitted, then the constraints should only be imposed for $i = 1, \ldots, N$ and $r = t$ (all firms in period t only). Irrespective of the number of points at which the constraints are imposed, there are infinitely many solutions to the resulting minimization problem.[7] A common method of identifying a unique solution is to set $\upsilon' x_{nt} = 1$. With this normalizing constraint, the input-oriented DEA LP for firm n in period t is

[7] To see this, simply observe from the structure of the input distance function (6.3) that if (α^*, μ^*, υ^*) minimizes $D_I^t(x_{nt}, q_{nt})$, then ($\lambda\alpha^*$, $\lambda\mu^*$, $\lambda\upsilon^*$) also minimizes $D_I^t(x_{nt}, q_{nt})$ for all $\lambda > 0$.

$$D_I^t(x_{nt}, q_{nt})^{-1} = \max_{\alpha,\mu,\upsilon} \quad \mu' q_{nt} - \alpha \tag{6.5a}$$

$$\text{s.t.} \quad \mu' q_{ir} - \upsilon' x_{ir} - \alpha \leq 0 \quad \text{for } i = 1, \ldots, N \text{ and } r = 1, \ldots, t \tag{6.5b}$$

$$\upsilon' x_{nt} = 1 \tag{6.5c}$$

$$\mu, \upsilon \geq 0 \tag{6.5d}$$

Primal output-oriented DEA problems involve selecting values of η, ϕ and β to maximize $D_O^t(x_{nt}, q_{nt})$. In the case where technical regress is prohibited, the unknown parameters are constrained so that $D_O^t(x_{ir}, q_{ir}) \leq 1$ for $i = 1, \ldots, N$ and $r = 1, \ldots, t$. A local solution can be identified using the normalization $\eta' q_{nt} = 1$, in which case the output-oriented analogue of LP (6.5) is

$$D_O^t(x_{nt}, q_{nt})^{-1} = \min_{\beta,\phi,\eta} \quad \beta + \phi' x_{nt} \tag{6.6a}$$

$$\text{s.t.} \quad -\eta' q_{ir} + \phi' x_{ir} + \beta \geq 0 \quad \text{for } i = 1, \ldots, N \text{ and } r = 1, \ldots, t \tag{6.6b}$$

$$\eta' q_{nt} = 1 \tag{6.6c}$$

$$\eta, \phi \geq 0 \tag{6.6d}$$

Problems (6.5) and (6.6) can be solved using standard LP software packages. However, sometimes it is more convenient and enlightening to obtain solutions after rewriting the problems in an alternative, dual, form.

6.3 Dual problems

Every normal primal LP has a dual form with the property that if both the primal and the dual LPs have feasible solutions, then the optimized values of the two objective functions are equal. The dual form of the normal maximization LP (6.5), for example, is

$$D_I^t(x_{nt}, q_{nt})^{-1} = \min_{\rho,\theta} \quad \rho \tag{6.7a}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \sum_{r=1}^{t} \theta_{ir} q_{ir} \geq q_{nt} \tag{6.7b}$$

$$\rho x_{nt} - \sum_{i=1}^{N} \sum_{r=1}^{t} \theta_{ir} x_{ir} \geq 0 \tag{6.7c}$$

$$\sum_{i=1}^{N} \sum_{r=1}^{t} \theta_{ir} = 1 \tag{6.7d}$$

$$\rho, \theta_{ir} \geq 0 \quad \text{for } i = 1, \ldots, N \quad \text{and} \quad r = 1, \ldots, t. \tag{6.7e}$$

If the production possibilities set is convex, then this dual input-oriented problem has a very simple interpretation. Convexity of the production possibilities set means that any convex combinations[8] of observed data points, such as the double-summations in (6.7b) and (6.7c), are technically feasible output and input levels. This particular LP seeks to scale down the input vector while holding the output vector fixed. Here, the role of the constraints (6.7b) and (6.7c) is to ensure that the observed output and scaled-down input vectors are technically feasible. The constraint (6.7d) holds with strict equality because α in the primal problem (6.5) was unsigned to allow for VRS. If $\alpha \geq 0$ (NIRS), then the constraint (6.7d) becomes

$$\sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir} \leq 1. \tag{6.8}$$

If $\alpha = 0$ (CRS), then the constraint (6.7d) is absent from the dual problem altogether. Irrespective of the form of the returns to scale constraint, a basic feasible solution (BFS) to this problem is $\rho = 1$ and $\theta_{ir} = I(i = n, r = t)$, where $I(.)$ is an indicator function that takes the value 1 if the argument is true and 0 otherwise.

The dual form of the output-oriented problem (6.6) has a similar structure:

$$D_O^t(x_{nt}, q_{nt})^{-1} = \max_{\lambda,\theta} \quad \lambda \tag{6.9a}$$

$$\text{s.t.} \quad \lambda q_{nt} - \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}q_{ir} \leq 0 \tag{6.9b}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}x_{ir} \leq x_{nt} \tag{6.9c}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir} = 1 \tag{6.9d}$$

$$\lambda, \theta_{ir} \geq 0 \quad \text{for } i = 1, \ldots, N \quad \text{and} \quad r = 1, \ldots, t. \tag{6.9e}$$

This particular LP seeks to scale up the output vector while holding the input vector fixed. The constraints (6.9b) and (6.9c) ensure the observed input and scaled-up output vectors are technically feasible, while the constraint (6.9d) allows for VRS. Again, to allow for NIRS we simply replace (6.9d) with (6.8), and to allow for CRS we omit (6.9d) altogether. A BFS is $\lambda = 1$ and $\theta_{ir} = I(i = n, r = t)$.

Several other LP problems are needed to compute and decompose the Malmquist and Hicks–Moorsteen TFP indexes given by Equations (5.8) to

[8] A *convex combination* is a linear combination of points where all coefficients are non-negative and sum to one. A convex combination of two points lies on the straight line segment connecting those two points.

(5.10). To avoid repetition, the remainder of this section only considers input-oriented problems. Corresponding output-oriented LPs are presented in the Appendix.

6.4 Additional LPs for computing and decomposing the input-oriented Malmquist index

Either of the dual LPs given by (6.7) or (6.9) can be used to identify the production technology (production frontier), but this alone does not allow us to compute and decompose Malmquist TFP indexes. In the input-oriented case that involves solving the following additional LPs:[9]

$$D_I^t(x_{ms}, q_{ms})^{-1} = \min_{\rho,\theta} \quad \rho \tag{6.10a}$$

$$\text{s.t.} \quad \sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} q_{ir} \geq q_{ms} \tag{6.10b}$$

$$\rho x_{ms} - \sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} x_{ir} \geq 0 \tag{6.10c}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} = 1 \tag{6.10d}$$

$$\rho, \theta_{ir} \geq 0 \quad \text{for } i = 1, \ldots, N \quad \text{and} \quad r = 1, \ldots, t. \tag{6.10e}$$

and

$$D_I^s(x_{nt}, q_{nt})^{-1} = \min_{\rho,\theta} \quad \rho \tag{6.11a}$$

$$\text{s.t.} \quad \sum_{i=1}^{N}\sum_{r=1}^{s} \theta_{ir} q_{ir} \geq q_{nt} \tag{6.11b}$$

$$\rho x_{nt} - \sum_{i=1}^{N}\sum_{r=1}^{s} \theta_{ir} x_{ir} \geq 0 \tag{6.11c}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{s} \theta_{ir} = 1 \tag{6.11d}$$

$$\rho, \theta_{ir} \geq 0 \quad \text{for } i = 1, \ldots, N \quad \text{and} \quad r = 1, \ldots, s. \tag{6.11e}$$

These problems do not always have a solution, even under the assumption of CRS. For example, problem (6.10) is always infeasible if technical regress is

[9] Computing the input-oriented Malmquist index also involves computing $D_I^s(x_{ms}, q_{ms})$. This distance measure is simply the value of $D_I^t(x_{nt}, q_{nt})$ for the base firm in the base period. Thus, it can be computed using LP (6.7).

permitted and no firms in period t produce a positive amount of an output that is produced by firm m in period s. In this case, irrespective of the returns to scale assumption, there are no values of θ_{ir} that can satisfy constraint (6.10b).

6.5 Additional LPs for computing and decomposing the Hicks–Moorsteen index

Computing (but not decomposing) the Hicks–Moorsteen TFP index involves solving two slightly different, and also possibly infeasible, input-oriented problems:

$$D_I^t(x_{ms}, q_{nt})^{-1} = \min_{\rho,\theta} \quad \rho \tag{6.12a}$$

$$\text{s.t.} \quad \sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} q_{ir} \geq q_{nt} \tag{6.12b}$$

$$\rho x_{ms} - \sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} x_{ir} \geq 0 \tag{6.12c}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} = 1 \tag{6.12d}$$

$$\rho, \theta_{ir} \geq 0 \quad \text{for } i = 1, \ldots, N \quad \text{and} \quad r = 1, \ldots, t. \tag{6.12e}$$

and

$$D_I^s(x_{nt}, q_{ms})^{-1} = \min_{\rho,\theta} \quad \rho \tag{6.13a}$$

$$\text{s.t.} \quad \sum_{i=1}^{N}\sum_{r=1}^{s} \theta_{ir} q_{ir} \geq q_{ms} \tag{6.13b}$$

$$\rho x_{nt} - \sum_{i=1}^{N}\sum_{r=1}^{s} \theta_{ir} x_{ir} \geq 0 \tag{6.13c}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{s} \theta_{ir} = 1 \tag{6.13d}$$

$$\rho, \theta_{ir} \geq 0 \quad \text{for } i = 1, \ldots, N \quad \text{and} \quad r = 1, \ldots, s. \tag{6.13e}$$

Decomposing the index into the components identified by O'Donnell (2008) then involves solving a further two LPs. To motivate these additional LPs, it is convenient to rewrite the dual problem (6.7) in the alternative form

$$\bar{X}_{nt}/X_{nt} = \min_{\rho,\theta,v} \quad (\iota'_K x_{nt})^{-1}(\iota'_K v) \tag{6.14a}$$

$$\text{s.t.} \quad \sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} q_{ir} \geq q_{nt} \tag{6.14b}$$

$$v - \sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} x_{ir} \geq 0 \tag{6.14c}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} = 1 \tag{6.14d}$$

$$v - \rho x_{nt} = 0 \tag{6.14e}$$

$$\rho, v, \theta_{ir} \geq 0 \quad \text{for } i = 1, \ldots, N \quad \text{and} \quad r = 1, \ldots, t. \tag{6.14f}$$

where ι_K denotes a $K \times 1$ vector of ones, and $\bar{X}_{nt}$ and X_{nt} are the aggregate inputs used in Section 4 to define input-oriented technical efficiency: $ITE_{nt} = \bar{X}_{nt}/X_{nt} = D_I^t(x_{nt}, q_{nt})^{-1}$. To see that the two problems (6.7) and (6.14) are equivalent, substitute the equality constraint (6.14e) into both the inequality constraint (6.14c) and the objective function (6.14a). The fact that $\iota'_K x_{nt}$ is a known scalar means the objective function is still linear in the decision variables, meaning the problem (6.14) is still a LP. This formulation is useful because the constraint (6.14e) makes it explicit that input-oriented technical efficiency involves holding the input mix fixed. Input-oriented mix efficiency measures the improvement in TFP when this constraint is relaxed. When (6.14e) is relaxed, the dual input-oriented DEA problem becomes

$$\hat{X}_{nt}/X_{nt} = \min_{\theta,v} \quad (\iota'_K x_{nt})^{-1}(\iota'_K v) \tag{6.15a}$$

$$\text{s.t.} \quad \sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} q_{ir} \geq q_{nt} \tag{6.15b}$$

$$v - \sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} x_{ir} \geq 0 \tag{6.15c}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} = 1 \tag{6.15d}$$

$$v, \theta_{ir} \geq 0 \quad \text{for } i = 1, \ldots, N \quad \text{and} \quad r = 1, \ldots, t \tag{6.15e}$$

where $\hat{X}_{nt}$ is the maximum aggregate output that is possible holding the output vector fixed, and $\hat{X}_{nt}/X_{nt} = IME_{nt} \times ITE_{nt}$ is the product of the input-oriented measures of mix and technical efficiency introduced in Section 4. A BFS to this problem is $v = x_{nt}$ and $\theta_{ir} = I(i = n, r = t)$. At this BFS, the value of the objective function is one, implying that the minimized value of the objective function (a measure of the product of technical and mix efficiency) lies in the unit interval.

Observe that problem (6.15) allows the input vector to be chosen freely while holding the output vector fixed. A closely related LP that relaxes all constraints on both the output and input vectors is

$$TFP_t^* = \max_{\theta,z,v} \quad (\iota_J' z) \tag{6.16a}$$

$$\text{s.t.} \quad z - \sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} q_{ir} \leq 0 \tag{6.16b}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} x_{ir} - v \leq 0 \tag{6.16c}$$

$$\iota_K' v = 1 \tag{6.16d}$$

$$z, v, \theta_{ir} \geq 0 \quad \text{for } i = 1, \ldots, N \quad \text{and} \quad r = 1, \ldots, t \tag{6.16e}$$

where $TFP_t^* = Q_{nt}^*/X_{nt}^*$ denotes the maximum TFP that is possible using the technology available in period t (see Section 3). The constraints (6.16b) and (6.16c) ensure that the input and output vectors are technically feasible, while the normalizing constraint (6.16d) identifies a unique solution to the problem in much the same way that constraint (6.5c) identified a unique solution to problem (6.5). A BFS is $z = 0_J$, $v = \iota_K \times K^{-1}$ and $\theta_{ir} = 0$ for all $i = 1, \ldots, N$ and $r = 1, \ldots, t$, where 0_J denotes a $J \times 1$ vector of zeros. At this BFS, the value of the objective function is zero, implying $TFP_t^* \geq 0$.

Solutions to the LPs given by (6.5), (6.12) to (6.16) and their output-oriented counterparts are all that are required to decompose the Hicks–Moorsteen index into the measures of efficiency change defined by O'Donnell (2008). They are also sufficient to separately identify *levels* of technical efficiency, scale efficiency and mix efficiency for all observed input–output combinations:

$$ITE_{nt} = D_I^t(x_{nt}, q_{nt})^{-1}, \tag{6.17}$$

$$ISE_{nt} = D_I^t(x_{nt}, q_{nt})/H_I^t(x_{nt}, q_{nt}) \text{ and} \tag{6.18}$$

$$IME_{nt} = D_I^t(x_{nt}, q_{nt}) \times (\hat{X}_{nt}/X_{nt}) \tag{6.19}$$

where $H_I^t(x_{nt}, q_{nt})$ is the input distance under the assumption of CRS.

7. Empirical example

Coelli and Rao (2005) use DEA to compute and decompose output-oriented Malmquist TFP indexes of agricultural productivity change for 93 countries from 1980 to 2000. They make an assumption that is necessary (but not sufficient) for the Malmquist TFP index to be a reliable measure of productivity change, namely that the technology exhibits CRS. This paper computes and decomposes Hicks–Moorsteen TFP indexes using the methodology described in Sections 3–6. The Hicks–Moorsteen index is multiplicatively complete under any returns to scale assumption, so in this paper the technology is permitted to exhibit VRS.

Like Coelli and Rao (2005), this paper allows for technical regress. Technical regress can be narrowly conceptualized as the contraction in the production possibilities set that occurs when we forget the things we know. Conversely, technical progress can be thought of as the expansion in the production possibilities set that comes about through scientific discovery. In this paper, technical change is viewed as something involving more than just changes in technical know-how – it is a measure of the change in the production possibilities set caused by *any* changes in the external environment in which production takes place. In agriculture, for example, weather and climate typically have an influence on the amount of output that can be produced using a given set of inputs. This paper accounts for omitted environmental variables by allowing for technical regress – the production possibilities set is permitted to expand or contract with (unmeasured) changes in the environment as well as changes in the stock of knowledge.

The TFP and efficiency measures reported in this section were computed using the DPIN software written by O'Donnell (2010).

7.1 Data

Data on two outputs (crops and animals) and five inputs (land, labour livestock, tractors and fertilizer) were sourced from the FAO.[10] Details concerning the construction of the variables are available in Coelli and Rao (2005, pp. 121–122). The data were quantity data for 88 countries over the period 1970–2001. Some descriptive statistics are reported in Table 1. The FAO database contains no reliable data on farm incomes or costs, so profitability indexes were constructed as needed using data on agricultural incomes and costs reported by statistical agencies in selected countries.

7.2 Results

Indexes that measure changes in Australian agricultural profitability (PROF), productivity (TFP) and the TT are presented in Figure 7 and in the first few columns of Table 2. The profitability index was computed using data on the gross value of farm production and farm costs published by ABARE (2007); the TFP index is a Hicks–Moorsteen index computed using the DEA methodology described in Section 6; and the terms of trade index was computed

[10] I am grateful to Tim Coelli and Prasada Rao for providing the data.

Table 1 Descriptive statistics

	Mean	SD	Min	Max
Crop output	1.000	2.695	0.003852	32.24
Animal output	1.000	2.604	0.009128	31.37
Area	1.000	2.063	0.01215	12.84
Labour	1.000	4.420	0.005800	42.97
Livestock	1.000	2.401	0.01012	20.77
Tractors	1.000	2.830	1.406e−5	24.69
Fertilizer	1.000	3.142	8.273e−5	33.44
Crop output/area	2.068	2.671	0.001237	20.25
Crop output/labour	4.132	6.932	0.1424	52.22
Crop output/livestock	1.446	1.859	0.008842	13.29
Crop output/tractors	29.29	201.5	0.05071	5382.0
Crop output/fertilizer	17.90	82.66	0.09166	2098.0
Animal output/area	2.764	4.900	0.009106	39.07
Animal output/labour	10.92	22.69	0.04081	142.4
Animal output/livestock	1.282	1.248	0.1186	7.868
Animal output/tractors	12.68	38.53	0.1931	718.6
Animal output/fertilizer	8.555	27.37	0.1420	529.8

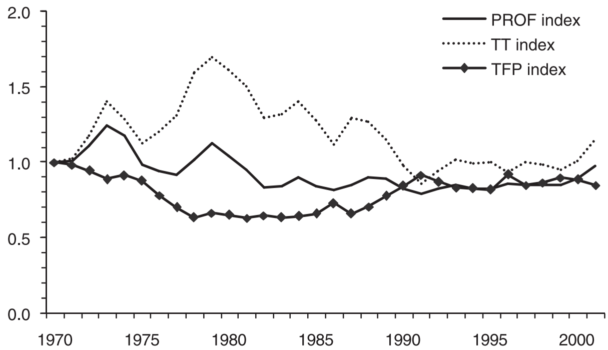

Figure 7 Components of profitability change: Australia, 1970–2001.

residually using Equation (2.4).[11] These indexes indicate that changes in the TT have been an important driver of changes in Australian agricultural profitability and that the TT effect on profitability has been moderated by compensating changes in TFP. This is consistent with the theoretical argument, developed in Section 2, that improvements in the TT encourage technically efficient optimizing firms to expand their operations (further) into the region of decreasing returns to scale (and scope), with the result that increases in profitability are associated with falls in productivity. Conversely, adverse

[11] ABARE (2007) also reports a terms of trade index computed using Equation (2.4), but the TFP index used in those calculations is a Fisher index of rates of growth in a much more disaggregated set of inputs and outputs. Both terms of trade indexes reveal a similar pattern of relative price movements over much of the study period (for the period 1977–2000, the correlation coefficient is 0.92).

Table 2 Indexes of changes in agricultural profitability, total factor productivity (TFP) and terms of trade (TT): Australia (base 1970 = 1)

Year	PROF index	TT index	TFP index	Tech change	TFPE index	OTE index	OSE index	OME index	ROSE index	RME index
1970	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1971	1.008	1.027	0.982	1.105	0.888	1.000	1.000	1.000	0.888	0.888
1972	1.113	1.179	0.944	1.172	0.805	1.000	1.000	1.000	0.805	0.805
1973	1.246	1.403	0.888	1.132	0.785	1.000	1.000	1.000	0.785	0.785
1974	1.173	1.285	0.913	1.115	0.819	1.000	1.000	1.000	0.819	0.819
1975	0.986	1.125	0.877	1.110	0.790	1.000	1.000	1.000	0.790	0.790
1976	0.942	1.206	0.781	1.104	0.708	1.000	1.000	0.995	0.711	0.708
1977	0.920	1.310	0.702	1.084	0.648	1.000	1.000	0.980	0.661	0.648
1978	1.015	1.599	0.635	1.063	0.598	0.997	0.994	0.851	0.704	0.603
1979	1.125	1.693	0.665	1.035	0.642	1.000	1.000	0.900	0.714	0.642
1980	1.047	1.609	0.650	1.066	0.610	1.000	1.000	0.887	0.688	0.610
1981	0.947	1.502	0.630	1.052	0.599	0.944	0.993	0.902	0.704	0.639
1982	0.837	1.293	0.647	1.117	0.580	0.947	0.987	0.918	0.667	0.620
1983	0.841	1.322	0.636	1.115	0.571	0.960	0.995	0.894	0.665	0.597
1984	0.902	1.403	0.643	1.108	0.581	1.000	1.000	0.876	0.663	0.581
1985	0.843	1.280	0.658	1.116	0.590	0.940	0.995	0.958	0.655	0.631
1986	0.817	1.121	0.729	1.048	0.696	1.000	1.000	1.000	0.696	0.696
1987	0.853	1.292	0.661	1.079	0.613	0.973	0.997	0.999	0.630	0.631
1988	0.899	1.273	0.706	1.062	0.665	1.000	1.000	1.000	0.665	0.665
1989	0.894	1.148	0.778	1.051	0.740	1.000	1.000	1.000	0.740	0.740
1990	0.822	0.975	0.843	1.109	0.760	1.000	1.000	1.000	0.760	0.760
1991	0.789	0.863	0.915	1.078	0.849	1.000	1.000	1.000	0.849	0.849
1992	0.821	0.941	0.872	1.134	0.769	1.000	1.000	1.000	0.769	0.769
1993	0.850	1.021	0.833	1.125	0.741	1.000	1.000	1.000	0.741	0.741
1994	0.823	0.994	0.827	1.120	0.739	1.000	1.000	1.000	0.739	0.739
1995	0.821	1.001	0.820	1.188	0.690	1.000	1.000	1.000	0.690	0.690
1996	0.856	0.930	0.921	1.195	0.771	1.000	1.000	1.000	0.771	0.771
1997	0.853	1.004	0.850	1.170	0.727	1.000	1.000	1.000	0.727	0.727
1998	0.847	0.980	0.864	1.216	0.711	1.000	1.000	1.000	0.711	0.711
1999	0.851	0.947	0.899	1.306	0.688	1.000	1.000	1.000	0.688	0.688
2000	0.892	1.007	0.885	1.306	0.678	1.000	1.000	1.000	0.678	0.678
2001	0.980	1.153	0.850	1.310	0.649	1.000	1.000	1.000	0.649	0.649

OME, output-oriented mix efficiency; OTE, output-oriented technical efficiency; PROF, profitability; ROSE, residual output-oriented scale efficiency.

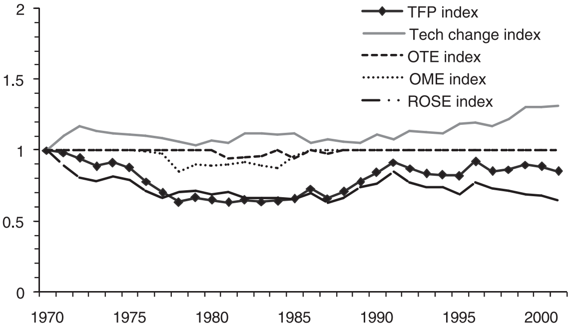

Figure 8 Components of total factor productivity change: Australia, 1970–2001.

movements in the TT can lead to simultaneous reductions in profitability and improvements in productivity.

Further evidence of this type of optimizing response can be seen in the indexes of technical change and output-oriented efficiency change presented in Figure 8 and the remaining columns of Table 2. These indexes indicate that important components of Australian agricultural TFP change have been changes in OME and ROSE – precisely the measures of efficiency we would expect to vary when technically efficient profit-maximizing firms respond optimally to changes in the TT. Observe that there were two periods in the 1970s when Australian farmers experienced significant improvements in the TT: 1970–1973 when the TT improved by 40 per cent, and 1975–1979 when it improved by 50 per cent. Each of these periods was associated with a fall in a combined measure of scale and mix efficiency – the product of the OME and ROSE efficiency measures plausibly fell by 22 per cent and 19 per cent in each period, respectively. Conversely, there were several periods when significant declines in the TT were associated with increases in levels of scale and mix efficiency (e.g., 1987–1991).

If (anticipated) changes in the TT have been driving changes in Australian agricultural TFP, and if Australian farmers are price-takers in output and input markets, then policy-makers should not be overly concerned that Australian agricultural TFP in 2001 was only 85 per cent of what it had been in 1970 – it is likely that Australian farmers have rationally changed the scale and mix of their operations in response to (anticipated) changes in relative output and input prices and that productivity declines have been associated with increases in (expected) net returns.

Figure 9 presents estimates of the efficiency components of New Zealand agricultural TFP change. It is evident from this figure that in the 1970s, the main source of variation in New Zealand agricultural TFP was variations in OTE. These variations coincided with decisions taken by the New Zealand government to increase levels of assistance to agriculture by, for example, expanding farm lending programs and providing floor price schemes for all major agricultural commodities. These types of measures can provide farmers with opportunities and incentives to adopt relatively risky production plans, and if New Zealand farmers adopted such plans then large variations in seasonal conditions would explain the large variations in OTE and TFP evident in Figure 9. The smoking gun is provided by Johnson (2000) who reports that by the early 1980s the amount of government assistance provided to the New Zealand pastoral sector by way of support prices had reached almost 10 per cent of farm GDP.

In the second half of the study period, the main component of New Zealand agricultural TFP change appears to have been changes in ROSE. These changes coincided with a program of major economic reform initiated by the newly elected Labour government in 1984. The reform program involved floating the exchange rate and phasing out all price support and input subsidies for agriculture. Johnson (2000) reports that these reforms led to large

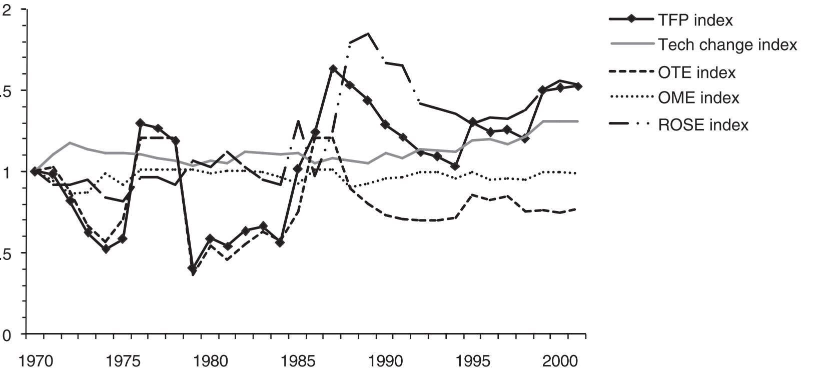

Figure 9 Components of total factor productivity change: New Zealand, 1970–2001.

and generally unfavourable variations in the agricultural TT – in the year to June 1984, for example, the TT improved by 8 per cent, but in the year to June 1986 it fell by 13 per cent. According to Johnson (2000), the 1980s was a period of significant adjustment in which 'some farmers had to exit the industry, others had to consolidate their resources, others had to change their enterprise mix' (p. 32). The adjustments Johnson (2000) describes are consistent with the estimated fluctuations in ROSE evident in Figure 9.

The US agricultural TFP experience is summarized in Figure 10. This figure reveals that US agricultural productivity has been steadily increasing over the study period and that the rate of productivity growth has outpaced the rate of technical progress, particularly in the latter half of the sample period when all output-oriented measures of efficiency improved. Since 1990, US agricultural productivity growth appears to have been primarily due to technical progress and changes in both mix and scale. The US Department of Agriculture estimates that between 1990 and 2001 US farmers experienced a 17 per cent fall in their TT. In the same period, this paper estimates that the product of OME and ROSE plausibly increased by 27 per cent and that TFP increased by 50 per cent.

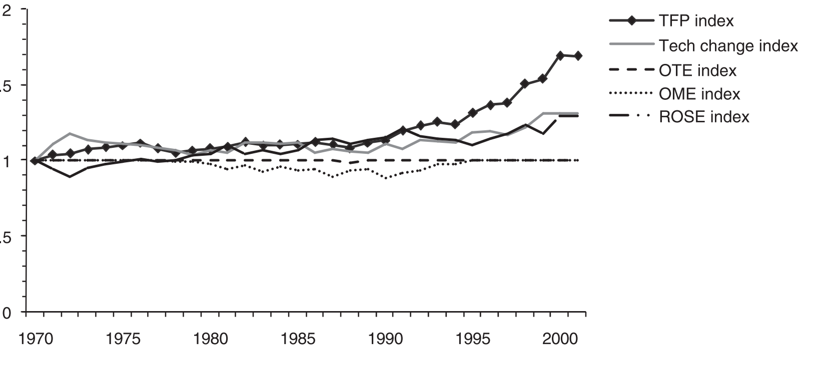

Figure 10 Components of total factor productivity change: United States, 1970–2001.

Observe from Table 2 and Figures 8–10 that global agriculture has periodically experienced periods of rapid technical change and that expansions in the production possibilities set have in some countries been associated with falls in TFP. For example, productivity in Australian agriculture fell by about 1.8 per cent in 1971, at a time when the production possibilities set was expanding at a rate of 10.5 per cent per annum (at least in the region of local CRS). The reasons for these large variations in the estimated measure of technical change are threefold. First, recall that technical change is a broad measure of the change in the production possibilities set caused by any changes in the environment in which production takes place. Thus, the measure will capture the effects of annual variations in seasonal conditions as well as the longer-term effects of new scientific discoveries and climate change. Second, DEA methodology has been used to estimate the production frontier, and DEA makes no allowance for measurement errors or other sources of statistical noise. Third, recall from Section 4 that technical change is measured as the difference in TFP at points R and E in Figure 6. When TFP at these points is estimated using DEA, the measure of technical change will be sensitive to the measured TFP of only a few efficient firms operating at the point of local CRS (i.e., relatively small efficient firms). Table 3 reports TFP-maximizing countries over the study period and reveals that TFP was maximized by Nepal during the 1970s, Nepal and Zimbabwe during the 1980s and Nepal and Thailand during the 1990s. The measures of technical change reported in Table 2 and Figures 8–10 are indexes of change in the maximum TFP values reported in Table 3. The average rate of technical change from 1970 to 2001 was 1.0 per cent per annum, marginally less than the 1.1 per cent reported by Coelli and Rao (2005) using a Malmquist approach.

Finally, the efficiency indexes reported in Table 2 and Figures 8–10 are estimates of efficiency *change*. Estimates of efficiency *levels* for a selection of countries in a small number of years are reported in Table 4, while estimates of efficiency levels for Australia for the entire study period are presented in Figure 11. Methods for estimating levels of residual mix and residual scale efficiency are not currently available, so Table 4 and Figure 11 only report pure technical, scale and mix efficiency scores. Note that the measures of technical and scale efficiency are the ones that have been reported in the efficiency and productivity literature for decades – it is only the measures of mix efficiency that are new. Also note that the output-oriented efficiency scores depicted in Figure 11 are the ones used to construct the corresponding efficiency indexes presented earlier in Table 2 and Figure 8 (the OTE and OME series depicted in Figure 8 are identical to those in Figure 11 because OTE = OME = 1 in the base period, 1970). Figure 11 reveals that Australian farmers were highly technically, scale and output mix-efficient throughout the study period, but less than 40 per cent input-mix inefficient. This low level of input-mix efficiency reflects the relatively high land-to-labour and land-to-capital ratios that are characteristic of Australian agriculture.

Table 3 Total factor productivity (TFP)-maximizing countries

Year	Country	TFP
1970	NEP	1.032
1971	NEP	1.140
1972	NEP	1.209
1973	NEP	1.168
1974	NEP	1.150
1975	NEP	1.145
1976	NEP	1.139
1977	NEP	1.118
1978	NEP	1.096
1979	NEP	1.068
1980	ZIM	1.100
1981	NEP	1.085
1982	NEP	1.152
1983	NEP	1.150
1984	NEP	1.143
1985	ZIM	1.151
1986	ZIM	1.081
1987	ZIM	1.113
1988	ZIM	1.095
1989	ZIM	1.084
1990	NEP	1.144
1991	NEP	1.112
1992	THA	1.170
1993	NEP	1.160
1994	NEP	1.156
1995	NEP	1.226
1996	THA	1.232
1997	NEP	1.207
1998	THA	1.254
1999	THA	1.347
2000	THA	1.347
2001	NEP	1.351

8. Conclusion

Several methods can be used to compute indexes of TFP change, but few methods are available for breaking these indexes into economically meaningful components. This paper develops DEA methodology for computing and decomposing Hicks–Moorsteen indexes of TFP change. The Hicks–Moorsteen index is a member of the class of multiplicatively complete TFP index numbers, which means it can be decomposed into a simple measure of technical change and several recognizable measures of efficiency change (O'Donnell 2008).

Being able to identify the components of TFP change is critically important for public policy-making. If welfare functions are increasing in net returns, then only some drivers of TFP growth are unambiguously desirable. Included among these are improvements in technical efficiency (movements towards the production frontier) and technical progress (upward movements in the production frontier). Public policies that promote these outcomes include the provision of agricultural extension services and investment in scientific

Table 4 Measures of technical, scale and mix efficiency

Year	Country	OTE	OSE	OME	ITE	ISE	IME
1970	ANG	0.5011	0.9777	0.9992	0.4901	0.9997	0.2210
1970	ARG	1.000	1.000	1.000	1.000	1.000	0.5178
1970	AUS	1.000	1.000	1.000	1.000	1.000	0.3464
1970	AUT	0.9046	0.9274	0.9849	0.9903	0.8472	0.09643
1971	ANG	0.4880	0.9392	0.9967	0.4656	0.9843	0.2244
1971	ARG	1.000	1.000	1.000	1.000	1.000	0.5288
1971	AUS	1.000	1.000	1.000	1.000	1.000	0.3337
1971	AUT	0.8417	0.9408	0.9040	0.9689	0.8173	0.09870
1975	NZL	0.5845	0.8753	0.9083	0.6278	0.8151	0.5283
1976	NZL	1.000	1.000	1.000	1.000	1.000	0.3654
1977	NZL	1.000	1.000	1.000	1.000	1.000	0.3924
1978	NZL	1.000	0.9743	1.000	1.000	0.9743	0.4081
1979	NZL	0.3000	0.8612	1.000	0.7081	0.3648	0.5967
1999	USA	1.000	1.000	1.000	1.000	1.000	0.5404
1999	URU	0.8574	0.7087	0.9448	0.7263	0.8368	0.8596
1999	VEN	1.000	1.000	1.000	1.000	1.000	0.4271
1999	VIE	1.000	1.000	1.000	1.000	1.000	0.9106
1999	ZIM	1.000	1.000	1.000	1.000	1.000	1.000
2000	USA	1.000	1.000	1.000	1.000	1.000	0.5559
2000	URU	0.8166	0.7091	0.9578	0.6781	0.8539	0.8944
2000	VEN	1.000	1.000	1.000	1.000	1.000	0.4467
2000	VIE	1.000	1.000	1.000	1.000	1.000	1.000
2000	ZIM	1.000	1.000	1.000	1.000	1.000	1.000
Mean		0.8468	0.8672	0.9565	0.8414	0.8728	0.5193
Minimum		0.2079	0.2981	0.4895	0.2328	0.2981	0.06205
Maximum		1.000	1.000	1.000	1.000	1.000	1.000

OME, output-oriented mix efficiency; OTE, output-oriented technical efficiency.

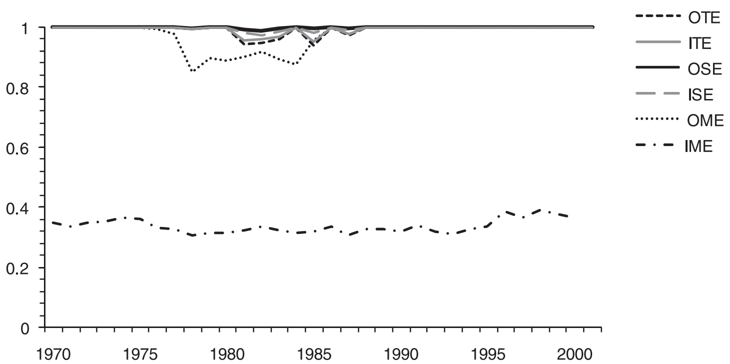

Figure 11 Output- and input-oriented measures of efficiency: Australia, 1970–2001.

research and development. However, even the provision of extension services can be wasteful if firms/sectors are already fully technically efficient (operating on the frontier).

Some drivers of TFP growth are not always associated with increases in net returns. Included among these are improvements in mix and scale efficiency (movements along the production frontier towards the point of maximum productivity). Policies that can lead to these outcomes include reductions in levels of output price support, removal of input subsidies, increases in tax rates and any other policies that cause deteriorations in the agricultural TT. These policies can have the effect of simultaneously increasing productivity and reducing net returns.

To illustrate some of these ideas, this paper used FAO country-level data to compute and decompose Hicks–Moorsteen indexes of agricultural TFP change for the period 1970–2001. The average rate of technical change was estimated to be 1.0 per cent per annum, slightly less than the 1.1 per cent reported by Coelli and Rao (2005) using a Malmquist index and a similar FAO data set. The results indicate that agricultural productivity in Australia, New Zealand and the United States has been responsive to changes in the agricultural TT. This was especially noticeable in New Zealand in the second-half of the 1980s when the newly-elected government implemented an extensive program of agricultural reform – the phasing out of output price support programs and the removal of input subsidies appears to have caused a rapid deterioration in the agricultural TT and simultaneous improvements in measures of mix efficiency, scale efficiency and TFP.

Any multiplicatively complete TFP index can be decomposed within the aggregate quantity-price framework developed by O'Donnell (2008). To illustrate, this paper used new DEA methodology and FAO data to compute and decompose the Hicks–Moorsteen index. A problem with DEA is that it makes no allowance for statistical noise, so any measurement errors in the data will be reflected in estimates of both efficiency and TFP. In this paper, the measurement error issue is complicated by the fact that the decomposition of TFP change is exhaustive, so if one component of TFP change is estimated poorly (e.g., technical change), then at least one other component must also be estimated poorly (e.g., technical efficiency). One solution is to estimate the technology using an econometric methodology that allows for statistical noise (e.g., stochastic frontier analysis); another possibility is to aggregate any estimated components of TFP change that are regarded as unreliable or poorly identified (e.g., mix efficiency change and residual scale efficiency change). The most appropriate way forward is likely to depend on the empirical context.

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Appendix

Output-oriented problems that correspond to the input-oriented problems (6.10) to (6.13) and (6.15) in the main text are:

$$D_O^t(x_{ms}, q_{ms})^{-1} = \max_{\lambda,\theta} \quad \lambda \tag{A.10a}$$

$$\text{s.t.} \quad \lambda q_{ms} - \sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} q_{ir} \leq 0 \tag{A.10b}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} x_{ir} \leq x_{ms} \tag{A.10c}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{t} \theta_{ir} = 1 \tag{A.10d}$$

$$\lambda, \theta_{ir} \geq 0 \quad \text{for } i = 1, \ldots, N \quad \text{and} \quad r = 1, \ldots, t. \tag{A.10e}$$

$$D_O^s(x_{nt}, q_{nt})^{-1} = \max_{\lambda,\theta} \quad \lambda \tag{A.11a}$$

$$\text{s.t.} \quad \lambda q_{nt} - \sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir}q_{ir} \leq 0 \tag{A.11b}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir}x_{ir} \leq x_{nt} \tag{A.11c}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir} = 1 \tag{A.11d}$$

$$\lambda, \theta_{ir} \geq 0 \quad \text{for } i = 1, \ldots, N \quad \text{and} \quad r = 1, \ldots, s. \tag{A.11e}$$

$$D_O^t(x_{nt}, q_{ms})^{-1} = \max_{\lambda,\theta} \quad \lambda \tag{A.12a}$$

$$\text{s.t.} \quad \lambda q_{ms} - \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}q_{ir} \leq 0 \tag{A.12b}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}x_{ir} \leq x_{nt} \tag{A.12c}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir} = 1 \tag{A.12d}$$

$$\lambda, \theta_{ir} \geq 0 \quad \text{for } i = 1, \ldots, N \quad \text{and} \quad r = 1, \ldots, t. \tag{A.12e}$$

$$D_O^s(x_{ms}, q_{nt})^{-1} = \max_{\lambda,\theta} \quad \lambda \tag{A.13a}$$

$$\text{s.t.} \quad \lambda q_{nt} - \sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir}q_{ir} \leq 0 \tag{A.13b}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir}x_{ir} \leq x_{ms} \tag{A.13c}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{s}\theta_{ir} = 1 \tag{A.13d}$$

$$\lambda, \theta_{ir} \geq 0 \quad \text{for } i = 1, \ldots, N \quad \text{and} \quad r = 1, \ldots, s. \tag{A.13e}$$

$$\hat{Q}_{nt}/Q_{nt} = \max_{\theta,z} \quad (\iota_J' q_{nt})^{-1}(\iota_J' z) \tag{A.15a}$$

$$\text{s.t.} \quad z - \sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}q_{ir} \leq 0 \tag{A.15b}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir}x_{ir} \leq x_{nt} \tag{A.15c}$$

$$\sum_{i=1}^{N}\sum_{r=1}^{t}\theta_{ir} = 1 \tag{A.15d}$$

$$z, \theta_{ir} \geq 0 \quad \text{for } i = 1, \ldots, N \quad \text{and} \quad r = 1, \ldots, t. \tag{A.15e}$$