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Stochastic efficiency analysis with risk aversion bounds: a comment

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A recent contribution by Meyer *et al.* (2009, p. 521) corrected an error of fact by Hardaker *et al.* (2004b, p. 253) about the comparison between stochastic dominance with respect to a function (SDRF) and stochastic efficiency with respect to a function (SERF). While both methods compare risky prospects for a bounded range of degrees of risk aversion, SERF, unlike SDRF, also demands an assumption that a chosen measure of risk aversion is constant over all levels of outcomes being evaluated. It is argued that it is generally reasonable to make such an assumption, especially when the form of the utility function and the bounds on the degree of risk aversion are carefully chosen. Then SERF has the advantage that it can lead to a smaller efficient set than that identified by SDRF. SERF also has advantages of ease and transparency in use.

Key words: risk aversion, stochastic dominance, stochastic efficiency.

This comment refers to the correction by Meyer *et al.* (2009) in this *Journal* about the paper by Hardaker *et al.* (2004b), also in this *Journal*. There were four authors of this latter paper to which the published correction referred, two being Richardson and Schumann, who also coauthored the correction, and two being ourselves. We were aware of a draft comment by Meyer drawing attention to an error of fact in the 2004 paper, for it was shared among the four original authors. However, we were unaware of the intention of our two coauthors to join with Meyer in publishing the correction. We would have wished to have contributed to the published correction if given the opportunity. As that was not possible and because we have a somewhat different perspective on some of the issues addressed in the published correction, we have written this comment.

First, we accept that there was an error in the original paper by Hardaker *et al.* (2004b) where it was wrongly claimed that stochastic efficiency with respect to a function (SERF) is based on the same assumptions as stochastic dominance with respect to a function (SDRF) (Meyer 1977). Meyer's method does indeed allow risk aversion to vary

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within defined bounds as the outcome measure varies, whereas that is not possible in SERF. Both SDRF and SERF compare risky prospects for a range of degrees of risk aversion between specified lower and upper bounds, but SERF imposes an additional restriction that the measure of risk aversion used is held constant as the level of payoffs changes, thereby potentially contracting the efficient set.

That said, we think it is often reasonable to make an assumption of a constant risk aversion coefficient (absolute or relative) within a range of outcome values, provided such an assumption is made with care. There are a number of reasons for this, which are outlined below.

We seldom if ever know what the exact utility function is – which is why a stochastic dominance method must perforce be used. So we usually have to make some informed guesses about risk aversion.

As explained by Hardaker *et al.* (2004a, p. 103), it is usually accepted that the absolute risk aversion function for wealth $r_a(w)$ will decrease with increases in wealth w, but there is less consensus about the properties of the relative risk aversion function $r_r(w)$. Eeckhoudt and Gollier (1996, p. 46) argued that $r_r(w)$ must be a non-decreasing function of w. On the other hand, Arrow (1965) suggested that the value of $r_r(w)$ is likely to 'hover about 1.0', while introspection and some empirical evidence suggests a somewhat larger range, with some suggestion that $r_r(w)$ may sometimes decrease with increase in w (e.g. Hamal and Anderson 1982). In these circumstances, appeal to Occam's Razor leads us to assume that $r_r(w)$ will be more or less constant for moderate variations in w. Taken together, the above assumptions lead to the view that it is not plausible (and not supported by utility function elicitations) that there will generally be large 'kinks' in utility functions for wealth, causing sudden, large changes in risk aversion as w varies.

Contrary to what Meyer, Richardson and Schumann at one point imply, SERF is not confined to utility functions with constant absolute risk aversion (CARA) properties. The procedure can be applied to any utility function for which risk aversion can be parameterised within given bounds.[1] However, because SERF uses a constant risk aversion coefficient over the assessed range of outcomes, and as we believe that $r_r(w)$ is more or less constant with changes in w, we generally recommend use of the constant relative risk aversion (CRRA) function for application of SERF to evaluate outcomes calculated in terms of terminal wealth w. The CRRA function has the additional advantage that it implies diminishing absolute risk aversion as w increases, as is generally held to be the case.

On the other hand, when the range of outcomes is small relative to w, as say in planning the cropping program for a farm for next year, or the

[1] For example, it can be applied using the sumex utility function (Patten *et al.* 1988). These authors also illustrate how SERF can be incorporated into a mathematical programming model for farm planning.

application of fertiliser to a single crop, it will often be convenient to work with partial outcomes, x, measuring 1-year income or losses/gains. Only then can it plausibly be assumed that the partial absolute risk aversion function $r_a(x)$ will be more or less constant.

We are encouraged to think that the somewhat stronger assumption about attitude to risk for SERF compared with Meyer's SDRF will be reasonable by our experience that, for real problems, the optimal set is seldom sensitive to the form of the utility function for consistent levels of risk aversion. Hence, results are not likely to be sensitive to assumptions about plausible changes in risk aversion over the range of outcome values. Moreover, when there is a change in the efficient set with change in the form of the utility function, we have found that the cost of being wrong, measured by the difference in certainty equivalents, is almost always trivially small. This finding flows from the fact that $CE = E - RP$, where CE is certainty equivalent, E is expected outcome and RP is risk premium, with RP typically small relative to E for plausible degrees of risk aversion (at least for most partial and short-term risky decisions by capitalist farmers) (Hardaker *et al.* 2004a, pp. 113–118). It then follows that small errors in estimating RPs seldom affect the ranking of risky prospects.

If it is accepted that it is reasonable to assume constant risk aversion over a range of levels of outcomes, the fact that SERF can yield a smaller efficient set than SDRF can be seen as an advantage. Indeed, the aim in stochastic dominance analysis is to keep the efficient set as small as is justifiably possible. To this end, it is important to give careful consideration to the bounds set on the risk aversion coefficient used. Yet, there is often confusion about picking appropriate bounds, for a couple of reasons. First, coefficients of absolute risk aversion are defined in the inverse of the currency units used. There are therefore obvious dangers in transporting such coefficients from one study to another, as has sometimes been done. Second, the magnitudes of risk aversion coefficients generally depend on the way the outcome variable is defined, i.e. whether as wealth, transitory or permanent income, losses and gains, or some other measure (Hardaker *et al.* 2004a, pp. 110–112). As argued there, if an assumption of asset integration is made, implying that partial outcomes such as losses and gains are viewed as changes in wealth (which is what they really are), it is possible to set bounds on risk aversion starting from a plausible range for $r_r(w)$ of, say, 0.5 (hardly risk averse at all) to 4 (extremely risk averse) (Anderson and Dillon 1992). As values of $r_r(w)$ are pure numbers, such values are legitimately comparable between applications. Then, using information about the level or likely range of wealth, it is simple to derive bounds for $r_a(w)$, which can be assumed to apply to $r_a(x)$, if this partial risk aversion measure is needed and appropriate. Recall, however, our preference for working with outcomes in terms of terminal wealth evaluated using bounds on $r_r(w)$, when possible.

Narrowing the bounds on the degree of risk aversion will tend to reduce the chances that SDRF and SERF (and some other procedures) will produce different efficient sets. Indeed, in the limit, the same single optimum (or tied optima) will be identified by both methods. In other words, it may be more important for analysts to focus on the appropriate range in risk aversion for a given client or group of clients than to worry about which method of stochastic efficiency analysis to use.

Unfortunately, we have noticed a number of empirical studies based on unrealistically wide bounds on risk aversion (e.g. Ribera *et al*. 2004). Including in recommendations to farmers choices based on near-paranoid attitudes to risk is arguably unrealistic and potentially misleading.

To conclude, while we accept that SDRF is based on a less stringent assumption about risk attitudes than SERF, there are advantages of SERF over SDRF. Properly applied, SERF may well lead to a smaller efficient set among which client decision makers would find a most preferred option. Moreover, SERF offers some important advantages in terms of transparency and ease of use. No specialist software is needed to apply SERF beyond a spreadsheet program, so there is flexibility in the choice of the form of utility function to use, and the graphical presentation of the results, at least for relatively few alternatives, provides a readily understood representation of the efficient set, switching points and differences between alternatives.

Finally, we return to an issue that we have raised recently in another paper, namely that our profession seems overly concerned about the finer points relating to risk aversion while seriously neglecting the careful establishment of distributions of outcomes for alternative risky choices (Hardaker and Lien 2010). Getting 'good' probabilities about the risks to be faced is almost always much more important than slight differences in methods of stochastic efficiency analysis.

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